Discussion Paper #626S

SUSTAINABLE OUTLAY SCHEDULES*

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ABSTRACT

This paper studies the impact that idealized free entry may have on the structure of tariffs offered by firms operating under increasing returns to scale. We show: (1) without the feasibility of resale, some non-linearity in the equilibrium price schedule is necessary; (2) there exist optimal outlay schedules which are sustainable with constant marginal costs; and (3) there may be no optimal, sustainable outlay schedule if marginal costs are declining.

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1. Introduction

Over the last decade there has arisen a voluminous literature on the theoretical analysis of nonlinear pricing policies: i.e. models in which the total outlay of a customer cannot be expressed as price times quantity purchased. As pointed out by Leland and Meyer (1976), this is an area in which theory had lagged behind practice for decades (if not centuries). Firms producing products or services for which resale is impossible (or very expensive) have long engaged in pricing practices such as two-part or declining block tariffs. However, while the recent theoretical work has focused upon the efficacy of these (and more general) pricing innovations for the pursuit of profit or consumers' surplus, the impetus for their adoption in practice has come from the threat that large users might find it in their interest to "go it alone", producing (or contracting for) the service themselves. Thus, it might be argued that it is the credibility of this threat,

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resulting in price concessions to large users, which has imparted the nonlinearity into the tariff structures of public utilities.

In this paper, we theoretically examine the economic forces which lay behind this source of nonlinearity. The credibility of the implied threat of large users comes from the possibility that, at a uniform price, they are providing a subsidy in the sense that they pay the firm more in revenues than it would cost them to produce only their output. Thus, the natural approach to the problem leads to a generalization of the sustainability analysis of Panzar and Willig (1977) and others to the case of nonlinear outlay schedules. Section 2 provides the formal definition and characterization of sustainability in this context, as well as the required notation. In Section 3, we present our results, and Section 4 provides a brief summary.

2. Definitions and Notation

In order to make our arguments suitably precise, it is necessary to take some care in the specification of the problem. While the nonlinear tariffs actually offered tend to be relatively simple, capturing the richness of the full set of tariff options open to the firm ex ante is always difficult. We proceed following the line of Spence (1980) which posits a finite set of potential users rather than a continuum, as in Mirman and Sibley (1981) and the optimal taxation literature. Since our results do not require us to explicitly characterize the schedules in question, Spence's approach allows us to keep our formal apparatus to at least a local minimum. We begin by describing the agents in our model:

Definition 1: Consumers are indexed by $i \varepsilon \{1,...,n\} = N$ and are characterized by their gross benefit function $B_i(q,\overline{p},y) \geq 0$ reflecting the dollar amount they are willing to pay for q units of the good purveyed by the firm, for given income y and other prices $\overline{p}$. Suppressing these latter arguments, we assume $B_i^{'}(q) \equiv \partial B_i/\partial q \geq 0$.

For analytical convenience, we follow Spence and posit that the firm can assign quantities q_i and outlays r_i to consumers subject to the constraints that no consumer prefers the outlay, quantity pair assigned to any other consumer. Thus, we have:

Definition 2: An outlay schedule is a vector $(r,q) \varepsilon T \equiv \{(r,q) \varepsilon R_+^{2n} | B_i(q_i)-r_i \geq B_i(q_j)-r_j \;\forall\; i,j;\; q_i(B_i(q_i)-r_i) \geq 0 \;\forall\; i\}$.

While somewhat formidable, this definition allows us to treat all variables as decisions of the firm while capturing the essential feature that all the self-selection constraints dictated by free consumer choice are satisfied. The cost function of the firm, $TC(\cdot)$, is assumed to take the form

(1) $$TC(Q) = f+C(Q); \qquad f \geq 0,\ C(0) = 0$$

where $Q = \sum_{i \varepsilon N} q_i$. (For future reference, Q automatically inherits any supernotation placed upon q; e.g. $Q^0 = \sum_i q_i^0$, and $Q_V = \sum_{i \varepsilon V} q_i$.) The presence of a (sufficiently large) fixed cost f can ensure the presence of economies of scale, even if C(Q) is strictly convex; i.e. marginal costs are always rising.

An optimal schedule is characterized via statement of the standard program of maximizing the unweighted sum of consumers' plus producers' surplus subject to the financial viability of the firm:

(2) $$\max_{(r,q)\varepsilon T} W(q) = \sum_{i \varepsilon N} B_i(q_i) - TC(Q)$$

$$\text{s.t. } \pi(r,q) = \sum_{i \varepsilon N} r_i - TC(Q) \geq 0.$$

Definition 3: The set M of optimal outlay schedules is characterized by the solution set of (2): M = arg max W.

Program (2) yields a set of optimal solutions for obvious reasons. With an unweighted objective function, if it is possible for the firm to extract enough surplus from consumers to cover its costs via, say, a two-part tariff, it may be possible to achieve the same aggregate surplus for a wide range of values for the entry fee, since its level then merely serves to apportion the total between consumers and the producer. Similar situations can readily arise under more complicated tariff structures. We assume that M contains at least one point in addition to the origin. Note that this is a stronger requirement than that there exist a q such that net benefits are positive, i.e. that the "bridge be worth building". It also requires that costs can be covered by freely available tariffs which do not rely on price discrimination based upon the preferences (identities) of individual consumers.

To address the issue of the vulnerability of tariffs to entry, we first define subsidy in the natural way:

Definition 4: An outlay schedule $(r,q)\varepsilon T$ is free of subsidy if $\sum_{i \varepsilon V} r_i \leq TC(Q_V)$ for all $V \subseteq N$. Let $F \subseteq T$ denote the set of subsidy free outlay schedules.

Thus, to be subsidy free, an outlay schedule must have the property that no group of users pays more than the total cost of serving only themselves. Sustainability of a tariff implies a stronger notion of stability. Not only must users be unable to "go it alone", they must also be incapable of being wooed away by a more enticing output/outlay combination:

Definition 5: An outlay schedule $(r,q)\varepsilon T$ is sustainable if $\nexists\ (\hat{r},\hat{q})\varepsilon T$ and a $V \subseteq N$ such that $B_i(\hat{q}_i) - \hat{r}_i \geq B_i(q_i) - r_i$, $i\varepsilon V$ with $\pi(\hat{r},\hat{q}) = \sum_{i\varepsilon V} \hat{r}_i - TC(\hat{Q}_V) > 0$. Let $S \subseteq T$ denote the set of sustainable outlay schedules.

Finally, the concept of Pareto superiority is useful in comparing two outlay schedules:

Definition 6: An outlay schedule $(r,q)\varepsilon T$ is said to be strictly Pareto superior to the outlay schedule $(r^d,q^d)\varepsilon T$; $(r,q)P(r^d,q^d)$; iff $B_i(q_i) - r_i > B_i\left(q_i^d\right) - r_i^d\ \forall\ i$ and $\pi(r,q) > \pi(r^d,q^d)$.

3. Sustainability Results

We begin our discussion of sustainability by presenting (what should be) a rather obvious result which, surprisingly, leads directly to a rather startling conclusion.

Proposition 1: A Pareto dominated outlay schedule $(r^d,q^d)\varepsilon T$ is unsustainable.

Proof: By Definition 6, $\exists$ at tariff $(r,q)\varepsilon T$, s.t. $(r,q)P(r^d,q^d)$. But this contradicts the requirement for sustainability in Definition 5. $\square$

Clearly, a tariff which is not Pareto efficient cannot survive for long in a world of frictionless free entry. But consider the implication of that fact:

Proposition 2: Any uniform price not equal to marginal cost is unsustainable.

Proof: Willig (1978) proved that it is always possible to construct an outlay schedule Pareto superior to a uniform price unequal to marginal cost. Therefore, Proposition 1 applies. $\square$

This result reveals the potentially wide applicability of our analysis. When resale is impossible (or very costly), the forces of free entry will force a nonlinear outlay schedule upon all firms except those operating in the perfectly competitive arena. In such cases, Demsetzian competition *for* the market requires the introduction of, what amounts to, Pigouvian second degree price discrimination. While one would expect that few markets are, in reality, so readily contested that *only* sustainable outlay schedules could survive as equilibria, nevertheless, Proposition 2 reveals that there are powerful forces at work driving the tariff structures of increasing returns "monopolies" away from uniformity.

We continue our analysis by focusing on common properties of both sustainable and optimal outlay schedules. It follows immediately from Definitions 4 and 5 that all sustainable outlay schedules must be subsidy free: i.e. $F \subseteq S$. In general, the "lump sum" nature of the inframarginal charges of even optimal outlay schedules may preclude their being subsidy free. However, in the interesting special case of constant marginal costs ($TC(Q) = f+cQ$), it is possible, without loss of generality, to restrict our attention to the subset of optimal outlay schedules $M \cap F$ which are subsidy free:

Proposition 3: When marginal costs are constant, if a nontrivial optimal outlay schedule exists, then there exists one such schedule which is subsidy free. That is, $M \neq \emptyset \Rightarrow M \cap F \neq \emptyset$.

Proof: In the case of constant marginal costs, it is easy to see that $r \geq cq$ for all $(r,q) \varepsilon F$. Therefore, suppose that there is a $(r^0,q^0) \varepsilon M$ with $r_i < cq_i$ for $i \varepsilon V \subset N$. Consider the schedule (r^1,q^1), where $\left(r_i^1,q_i^1\right) = \left(r_i^0,q_i^0\right)$ for $i\varepsilon N-V$. To determine q_i^1, $i\varepsilon V$, define

$$(2) \qquad \hat{i} = \arg\max_j \left\{B_i\left(q_j^0\right) - r_j^0,\ j\varepsilon N-V;\ B_i\left(q_j^0\right) - cq_j^0,\ j\varepsilon V\right\}$$

Then let $q_i^1 = q_{\hat{i}}^0$, unless the above maximum net benefit is zero, in which case $q_i^1 = 0$. Now set $r_i^1 = r_{\hat{i}}^0$ if $\hat{i}\varepsilon N-V$ and $r_i^1 = cq_{\hat{i}}^0$ if $\hat{i}\varepsilon V$. Clearly $(r^1,q^1)\varepsilon F$ and $\pi(r^1,q^1) \geq \pi(r^0,q^0) \geq 0$ by construction. The change in total surplus is given by

$$(3) \qquad \Delta W = \sum_{i\varepsilon V} \left\{B_i\left(q_i^1\right) - B_i\left(q_i^0\right) - c\left(q_i^1-q_i^0\right)\right\} \geq 0$$

(From (2), we know that $B_i\left(q_i^1\right) - cq_i^1 \geq B_i\left(q_i^1\right) - cq_i^0\ \forall\ i\varepsilon V.$) $\square$

The intuition behind this result is straightforward. Simply raise the outlay for all consumers not covering their incremental cost cq_i to that level. Either the consumer continues to select the initial quantity, and there is no welfare effect, only a transfer, or the consumer alters his quantity (possibly by dropping off the system). Any such move raises welfare, the consumer's net benefits are greater by revealed preference and profits no lower by construction. Thus, we have established that if a nontrivial optimal outlay schedule exists, there exists an optimal outlay schedule which is subsidy free.

Next, we further restrict the set of subsidy free outlay schedules to those which yield only zero profit to the firm, for only these can be sustainable in view of Definition 5. Let $Z = \{(r,q) \varepsilon M \cap F \mid \pi(r,q) = 0\}$ denote this class of outlay schedules. (The further construction which shows that $M \cap F \neq \emptyset \Rightarrow Z \neq \emptyset$ is left to the reader.)

We are now in a position to state and prove our main result:

Theorem: When marginal costs are constant, any zero profit subsidy free optimal tariff is also sustainable. That is, $(r,q) \varepsilon Z \Rightarrow (r,q) \varepsilon S$.

Proof: Suppose the contrary, that there exists a $(r^*,q^*) \varepsilon Z$ that is unsustainable. Then there exists a $(\hat{r},\hat{q})$ and a $V \subseteq N$ such tha $B_i(\hat{q}_i) - \hat{r} \geq B_i(q_i^*) - r_i^*$, $i \varepsilon V$ and

$$(4) \qquad \pi(\hat{r},\hat{q}) = \sum_{i \varepsilon V} (\hat{r}_i - c\hat{q}_i) - F > 0.$$

Consider the outlay schedule $(\tilde{r},\tilde{q})$ defined by the "optimal mixing" of (r^*,q^*) and $(\hat{r},\hat{q})$:

$$(5) \qquad \tilde{r}_i,\tilde{q}_i = \arg\max[B_i(q_i^*) - r_i^*,\ B_i(\hat{q}_i) - \hat{r}_i]$$

Clearly, consumer benefits must be at least as large under $(\tilde{r},\tilde{q})$ as under (r^*,q^*), since each consumer, in effect, has the option to keep his initial allocation or improve upon it. To see how the firm would fare, note that

$$\pi(\tilde{r},\tilde{q}) = \sum_{i \varepsilon V} r_i - cq_i + \sum_{i \varepsilon N-V} r_i^* - cq_i^* - F$$

or

$$\pi(\tilde{r},\tilde{q}) = \pi(\hat{r},\hat{q}) + \sum_{i \varepsilon N-V} r_i^* - cq_i^* > 0,$$

by (4) and the assumption that $(r^*,q^*) \varepsilon Z$. Thus $W(\tilde{r},\tilde{q}) > W(r^*,q^*)$, contradicting the hypothesis that $(r^*,q^*) \varepsilon Z$. □

The intuition behind this result is also straightforward. If it is possible for a potential entrant to offer an outlay schedule which allows it to cover its fixed costs even though it attracts only a subset V of users, then it would certainly have been financially feasible for the monopolist to offer the "lower envelope" of the two schedules initially. This must be so because the users in N-V contribute revenues at least as large as the incremental cost of serving them. Such a "mixed schedule" would yield a strictly greater level of welfare than the original schedule. Therefore, the latter could not have been optimal.

The foregoing theorem suggests that there may be substantial scope for a utility to pursue efficiency through its tariff structure and yet remain invulnerable to competitive threats to its natural monopoly. (In this respect, our result is closely related to the Weak Invisible Hand Theorem of Baumol, Bailey, and Willig (1977), and probably could have been arrived at by "translating" their postulates to the present model.) However, this happy conclusion does not hold in general. And, somewhat surprisingly, if the economies of scale which may necessitate nonlinear outlay schedules are strengthened to include declining marginal costs, this sustainability property can easily disappear.

Next, we provide a simple example which precludes the extension of our theorem to the case of declining marginal costs. There are three consumers characterized by the following benefit functions:

$$B_1(q) = \begin{cases} 4q - \frac{3}{2}q^2, & 0 \le q \le \frac{4}{3} \\ \frac{8}{3}, & \frac{4}{3} \le q \end{cases}$$

$$B_2(q) = \begin{cases} 9q - 2q^2, & 0 \le q \le \frac{9}{4} \\ \frac{81}{8}, & \frac{9}{4} \le q \end{cases}$$

$$B_3(q) = \begin{cases} 29q - 2q^2, & 0 \le q \le \frac{29}{4} \\ \frac{841}{8}, & \frac{29}{4} \le q \end{cases}$$

Costs are given by:

$$C(q) = 11q - \tfrac{1}{2}q^2, \quad 0 \le q \le 11$$
$$MC(q) = 11 - q, \quad 0 \le q \le 11$$

The outlay schedule $(r_1^*, q_1^*) = (2\frac{1}{2}, 1)$, $(r_2^*, q_2^*) = (5\frac{1}{2}, 2)$, $(r_3^*, q_3^*) = (52, 7)$ is not only optimal subject to a profit constraint, but actually achieves a first best allocation. This is most easily seen in Figure 1, which depicts the individual inverse demand curves, $B_i'(q)$, and the marginal cost curve. At the quantities indicated each consumer's marginal willingness to pay is equal to marginal cost. Further, welfare decreases if either or both of the smaller consumers are excluded from the system; e.g. $B_1(1) = 2\frac{1}{2} > C(10) - C(9) = 1\frac{1}{2}$, $B_2(2) = 10 > C(10) - C(8) = 4$, and $B_1(1) + B_2(2) = 12\frac{1}{2} > C(10) - C(7) = 7\frac{1}{2}$. This allocation is also subsidy-free since each subset of users is charged less than its stand-alone cost. That is, $r_1 = 2\frac{1}{2} \le 10\frac{1}{2} = C(1)$, $r_2 = 5\frac{1}{2} \le 20 = C(2)$, $r_3 = 52 \le 52\frac{1}{2} = C(7)$, $r_1 + r_2 = 8$

$\leq 28\frac{1}{2} = C(3)$, $r_1 + r_3 = 54\frac{1}{2} \leq 56 = C(8)$, and $r_2 + r_3 = 57\frac{1}{2}$ $\leq 58\frac{1}{2} = C(9)$.

Unfortunately, it is equally easy to show that this optimal schedule is unsustainable because it offers an opportunity for the largest customer to gain by providing his own service. If customer 3 were to operate the facility on his own, he would choose an output level of 6; the point where his marginal benefit curve intersects the marginal cost curve. He would obtain a net benefit of $B_3(6) - C(6) = 102 - 48 = 54$. Therefore any schedule which does not yield a net benefit to customer 3 of at least 54 is not sustainable. Since the optimal allocation gives him 7 units and a gross benefit of 105, his payment cannot exceed 51.

Not only is _this_ outlay schedule unsustainable, any _first best_ allocation which covers costs must also be unsustainable. To see this, note that the strict concavity of $W(q_1,q_2,q_3) = \sum_{i=1}^{3} B_i(q_i) - C(q_1+q_2+q_3)$ over the relevant range guarantees that all first best allocations must involve the same quantities for each consumer. For 1 to participate, $r_1 \leq 2\frac{1}{2}$. And for 2 to select q_2^* rather than q_1^*, it is required that $r_2-r_1 \leq B_2(2) - B_2(1) = 3$. Taken together, this requires $r_1+r_2 \leq 8$, so that $r_3 \geq C(10) - 8 = 52$. But as we have just seen, any $r_3 > 51$ will give rise to a profitable entry opportunity. Hence, no optimal schedule can be sustainable.

4. Summary

This paper has called attention to the impact which idealized free entry may have on the structure of tariffs offered by firms operating under increasing returns to scale. We have obtained three major insights in our preliminary investigation: (1) without the feasibility of resale, some nonlinearity in the price schedule is *necessary* in equilibrium; (2) when marginal costs are constant, optimality and equilibrium are always compatible; and (3) declining marginal costs tend to make optimal outlay schedules *more* vulnerable to "entrants" which select out large users.

In general one can, speaking loosely, view the overall degree of scale economies as being determined by the level of "fixed" or start-up costs and the rate of decline of marginal costs. An increase in either will increase the degree of scale economies, *ceteris paribus*. However, while it is clear that the former effect favors the sustainability of an outlay schedule by raising the per unit "overhead" of small scale entry, our example has demonstrated that declining marginal costs may actually make schedules more vulnerable to entry.

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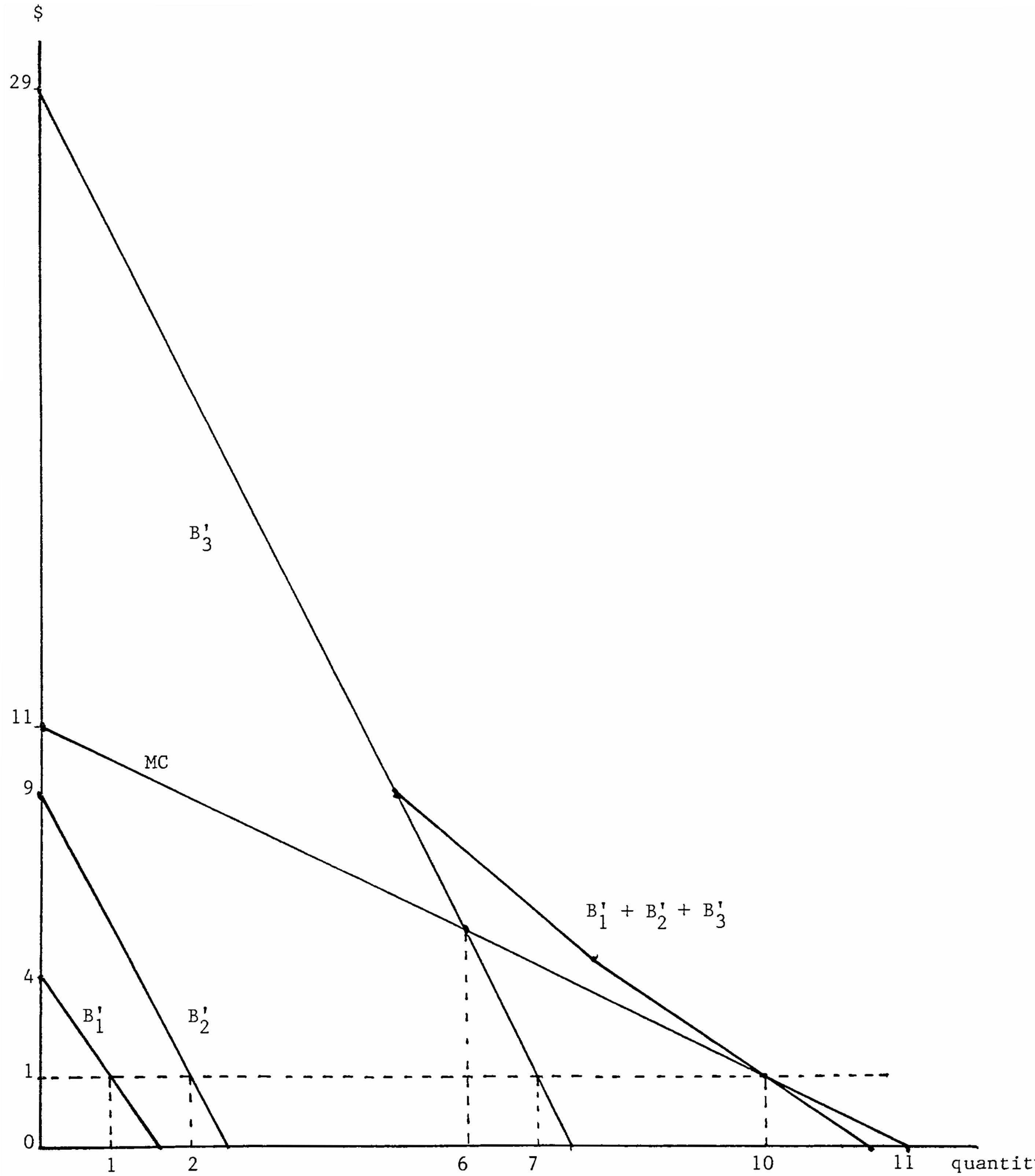

Figure 1