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BANKING PANICS, INFORMATION AND RATIONAL
EXPECTATIONS EQUILIBRIUM

by

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1. Introduction

Cyclical contractions in economic activity in the United States prior to World War II were almost invariably accompanied by banking panics. The typical pattern extensively documented by Friedman and Schwartz (1983), was a reduction in the deposit to currency ratio (which is, of course, a sensitive indicator of the state of public confidence in the banks) followed by a contraction in economic activity. From 1870 to 1940 there were seventeen major periods during which the deposit to currency ratio fell. Each of these periods also witnessed a cyclical downturn in real output. Of the eight most severe contractions identified by Friedman and Schwartz, six were characterized by major crises in banking involving runs on banks and widespread suspension of convertibility of deposits to currency.¹ In sharp contrast, no suspensions have occurred since then. It is remarkable that so unstable an industry has not met with a single such experience in the last fifty years. This amazing turnaround has been widely attributed to the advent of deposit insurance and close regulation of banking. The recent moves toward deregulation of this industry raise some natural questions about the optimal

¹Reductions in the deposit-currency ratio occurred in 1872-73, 1876-78, 1883-84, 1887, 1890-91, 1893, 1896-97, 1899-1900, 1902-04, 1907, 1912-13, 1914, 1917-18, 1920, 1923, 1930-33, 1936-37. Each of these periods also witnessed a contraction in economic activity. We follow Friedman and Schwartz (1963) in identifying major contractions 1873-79, 1884, 1890-91, 1893, 1907, 1920-21, 1929-33, 1937-38. Only in 1920-21 and 1937-38 were there no banking crises. The experience, or lack thereof, of 1937-38 is an immediate implication of our analysis since deposit insurance was effective then. The lack of a bank run in 1920 is a puzzle that we attempt to solve in this paper.

structure of deposit insurance. Our intent is to develop a deeper understanding of the nature of bank runs and the role of deposit insurance in preventing crises in banking.

A somewhat cursory examination of the data for the United States yields what seems to be a dramatic change in regime since 1934 when deposit insurance was instituted. Table 1 shows the sample correlation between the growth rate in real per capita income and the growth rate in the ratio of money stock to high powered money.²

Table 1. Sample Correlation between the first differences in the logarithm of per capita income and the first differences in the logarithm of the ratio of the money stock to high powered money.

Country	Period	
	1872-1933	1934-1975
USA	+0.537	-0.200
UK	0.070	+0.003

Source: Friedman and Schwartz (1982).

There are two striking things about the correlations shown in Table 1. The correlation for the US has declined sharply while the UK, with similar institutional features, does not seem to have undergone a regime shift. One major institutional difference in the two economies was the existence of a powerful central bank in the UK and either no central bank or a relatively ineffective one in the US. Any model which seeks to explain bank runs must be

²The salient features do not change if the ratio of deposit to the currency held by the public is used instead. The correlation between the growth rate in real per capita income and the growth rate in deposit to currency ratio in the U.S.A. was 0.456 for the period 1892-1933, and -0.121 for the period 1934-1960.

consistent with the different performances of these economies.

Clearly, this regime shift requires some explanation above and beyond the trivial observation that deposit insurance affected the operating characteristics of the banking system.

These issues have received some attention in the literature recently. Diamond and Dybvig (1983) have pioneered the attempt to model bank crises. Their model has many equilibria, some of which can be interpreted as a bank run. In their model those who wish to withdraw early have a higher marginal utility of consumption than those who would rather wait. The nature of the optimal contract is that the "waiters" insure the "withdrawers." The technology does reward those who wait with a higher yield but if any person who would rather wait feared that everyone else would withdraw then it is optimal for that individual to take his money and run. Essential to this story is that the bank must honor a sequential service constraint. Payments to an individual must be independent of the length of the "line" at the bank. It seems very difficult to generate this kind of a contract which is sometimes considered the essence of a demand deposit endogenously in an equilibrium model and Diamond and Dybvig impose it as a constraint on the equilibrium. In fact, under some circumstances, it is true in their model that by threatening to suspend convertibility in the case of a run, the bank can ensure that it is not in anyone's best interest to withdraw just because the others might. This presents a major problem since an important feature of bank runs in the United States was the accompaniment of widespread suspensions of convertibility. Jacklin (1983) in a very similar framework to ours addresses the question of the choice between deposit and equity contracts given that individuals may get information about future returns. Again, a key characteristic is that banks are not allowed to make deposit contracts

contingent upon the number of people who desire to withdraw.

The traditional "story" is that bank runs are initiated by fears of insolvency of particular banks and that the public by attempting to withdraw deposits endangers other banks, thus causing all deposits to become riskier assets. The essence of this story is that the general public observing some people withdrawing their assets becomes concerned about the solvency of banks and by its actions cause many banks to become insolvent. The policy implication frequently drawn from this is that there is social value in preventing people from withdrawing their assets for other than "fundamental" reasons. If policies are implemented based on this scenario, it seems essential to model this phenomenon explicitly.

Our simple model, which is based in large part upon the models of Diamond and Dybvig and of Jacklin, tries to capture the heart of this story. It is based upon the notion that if some individuals obtain information that future returns are likely to be low, then they have an incentive to withdraw. Uninformed individuals observing this also have an incentive to liquidate their investments. Essential to this story is that some individuals "need" to withdraw for other than informationally based reasons. Thus, if the random realization of such a group of individuals is unusually large, then the uninformed individuals will be misled and will precipitate a run on the bank. Long lines do, of course, have an informational content which makes it socially desirable to reduce investments when they occur. The technology is such that a large volume of withdrawals involves liquidation costs. This implies that when there is a run on the bank, it may be optimal to suspend convertibility of demand deposits to currency. The threat of suspension of convertibility changes the essential character of the equilibrium. Those individuals who "need" to withdraw their assets because they care a lot about

current consumption may in fact get a smaller return than otherwise. A central feature of bank runs prior to 1929 was the restriction of payments on demand deposits which served to ensure that "the panic[s] had a reasonably small effect on the banking structure . . . and gave time for the immediate panic to wear off" (Friedman and Schwartz, pp. 166-167). However, "they were regarded as anything but a satisfactory solution by those who experienced them, which is why they produced such strong pressure for monetary and banking reform" (ibid., p. 329).

An important implication of our model is, indeed, that a policy of restricting cash payments may improve on the ex ante utility levels of agents. Such a policy, however, implies ex post regret. An alternative policy is the provision of deposit insurance. The model provides a role for this, although the mechanism is somewhat subtle. Since bank runs are an economy wide phenomenon, the investment risks we model are aggregate, uninsurable risks. The government cannot insure against these risks. The government can, however, in the event of a threatened run, tax those who withdraw their assets and pay off those who do not withdraw. In this case, the incentives for early withdrawal are reduced and informed individuals may prefer not to liquidate their assets, thereby assuring that there are no bank runs. Such a taxation scheme is readily implemented through an inflation tax which can, of course, tax currency holders and depositors differently.

Section 2 develops the environment we consider. Section 3 contains the definition and characterization of equilibrium. The relationship between our equilibrium construct and a rational expectations equilibrium is also developed and is of independent interest. Rational expectations equilibria sometimes have the undesirable property that they reveal more information than a fully informed social planner could possess (Dubey, Geanakoplos, and Shubik

(1982)). In addition, nonexistence and multiplicity of equilibria are sometimes problems within the rational expectations construct (Grossman and Stiglitz (1980)). We circumvent these problems by using a game-theoretic equilibrium concept. Section 4 contains a discussion of extra-market constraints such as suspension of convertibility or deposit insurance which may improve on the equilibrium allocations. Section 5 concludes the paper.

2. The Model

This section considers an environment where people live for three periods: a planning period, time 1, and time 2. There is a single commodity. An investment decision is made during the planning period which yields a sure return at time 1. If resources are reinvested in period 1 they generate a random return at time 2. If resources are not reinvested, there is a liquidation cost which depends upon the level of consumption. there are a large number of individuals (technically, a continuum on the interval $[0,1]$ on which the Lebesgue measure is induced) each of whom has access to the blueprint technology.

Technology

An investment plan for an individual is a pair of numbers (k_0, k_1) representing investment in periods 0 and 1, respectively. Realized output is a pair of numbers (y_1, y_2) in periods 1 and 2, respectively. Investment decisions are costly to liquidate in period 1. In particular, the cost of liquidation depends upon the aggregate investment decisions made in the economy. Let K_1 represent the aggregate volume of investment. Then, output for any individual's technology is

$$(1) \quad y_1 = k_0 - k_1 \quad \text{if } K_1 \geq \bar{K}$$

$$y_1 = (1 - a)(k_0 - k_1) \text{ if } K_1 < \bar{K}$$

where $0 < a < 1$ and \bar{K} is given.

Output in period 2 is random and is given by

$$(2) \quad y_2 = \tilde{R}k_1$$

$$\tilde{R} \in \{H, L\}, \quad H > L$$

with probabilities p and $(1 - p)$, respectively.

For simplicity, assume that $L = 0$. Output in this model is to be interpreted as resources available for consumption.

Preferences

All agents in the economy are risk neutral and maximize expected utility of consumption. There are two types of individuals in the economy. Type I agents care only about consumption in period 1. It will be assumed that they die at the end of period 1. Type II agents derive utility from consumption in both periods 1 and 2. The utility functions of the respective types are given by

$$(3) \quad \begin{aligned} U^1(c_1, c_2) &= c_1 \\ U^2(c_1, c_2) &= c_1 + c_2 \end{aligned}$$

where the pair (c_1, c_2) represents consumption levels of the commodity in periods 1 and 2, respectively.

No individual knows his type at the planning period. A random fraction t of agents are type I. Assume that t can take on only finitely many values.

For ease of exposition also assume that t can take on one of three values, $t \in \{0, t_1, t_2\}$ with probabilities $r_0, r_1,$ and $r_2,$ respectively. The first element is set at zero without loss of generality.

Endowments

All agents are endowed with one unit of the good at the planning period. In addition, type 2 agents alone have an endowment of w units in period 2.

Information

At time 1, before k_1 is decided, agents can at very small cost acquire a signal about time 2 returns. Assume that only type II agents can be informed. The signals can take on three values, $s \in \{0, \underline{s}, \bar{s}\}$; $s = 0$ is an uninformative signal while $s = \underline{s}, s = \bar{s}$ are perfectly informative signals revealing the exact returns, low and high, respectively, in period 2. The signal is not perfectly correlated across individuals. A random fraction $\alpha \in \{0, \bar{\alpha}\}$; with $\bar{\alpha} > 0$, of those who invest resources to acquire information receive the informative signal while the rest receive the uninformative signal. Let $\Pr\{s \neq 0 \text{ and } \alpha = \bar{\alpha}\} = q$. The cost of acquiring this signal will be denoted by $\delta > 0$ and will be assumed to be arbitrarily small. It will also be assumed that this cost is purely psychic and does not affect the aggregate resource flows. This assumption is not crucial but makes the analysis easier. If no information is acquired then the value of the signal is set at 0. It will be assumed that (t, R, α) are drawn independently.

No individual at the planning periods knows whether he will be informed. Furthermore, the realization of $t, s,$ or α are not observable by other individuals in the economy. The only information public is the aggregate decision. To put it differently, what is observed is the fraction

of the population which chooses to continue investing rather than the reasons for doing so. This number will be referred to as the "bank's assets." Informally, one can think of individuals investing through a "bank" during the planning period and queuing up to withdraw in period 1. Then, what is observed is the resources left in the bank.

Parameter Restrictions

In order to ensure that individuals have a nontrivial signal extraction problem upon observing the bank's balance sheet we clearly need "confounding." Assume that

$$(4) \quad t_1 = \bar{\alpha}$$

$$(5) \quad t_2 = t_1 + \bar{\alpha}(1 - t_1)$$

Further, assume that, absent any information, it is desirable to continue the investment. Thus:

$$(6) \quad pH + (1 - p)L > 1 \quad \text{(PR1)}$$

In some of what follows, results do depend upon the magnitude of \bar{K} . In general, any concave transformation of investment into consumption will suffice for the results. It will be assumed, for reasons that will become apparent, that

$$(7) \quad \bar{K} = 1 - t_2 \quad \text{(PR2)}$$

and that

$$(8) \quad t_2 + \bar{\alpha} \neq 1 - \bar{\alpha} \quad (\text{PR3})$$

3. Equilibrium

The decision problem in the planning period is trivial since no individual cares about period 0 consumption. So is the decision problem of type I agents (those who die). The sequencing of the decisions of other agents is as follows. At time 1, first t is realized and every individual knows his own type. The information acquisition decision is then made and a random fraction $\alpha \in \{0, \bar{\alpha}\}$ receive the informative signal if agents decide to acquire the information. In order to avoid the problems caused by the fact that "everybody moves last" in traditional rational expectations formulations it will be assumed that the consumption decisions occur in two stages. In stage A individuals make their consumption decisions after having observed the signal. In stage B, they observe in addition the aggregate investment level in the economy after stage A. Agents then make their consumption decisions for stage B. The sum of the consumption at the two stages is the level of consumption in period 1.

Notation:

K_i	aggregate investment in stage i , period 1, $i = A, B$.
c_i	individual's consumption in stage i , period 1, $i = A, B$.
I	the fraction of the population of type II agents who acquire information.
$\theta \equiv (t, R, \alpha I, s)$	state of the world.
$F(K_A, K_B, \theta)$	joint distribution of aggregate investment and states of the world.

The technology is such that there is a potential loss to "late" withdrawers, i.e., those who withdraw in stage B. Returns in stage A depend upon aggregate withdrawals in stage A whereas returns on stage B depend upon aggregate withdrawals in both stages A and B. The problem faced by a representative individual is

$$(9) \quad \begin{aligned} & \text{Max}_{k_A, k_B} \quad c_A + \int c_B dF(K_A, K_B, \theta | K_A, s) + \int Rk_B dF(K_A, K_B, \theta | K_A, s) \\ & \text{s.t. } c_A = 1 - k_A \quad \text{if } K_A > \bar{K} \\ & \quad \quad \quad c_A = (1 - a)(1 - k_A) \quad \text{if } K_A < \bar{K} \\ & \quad \quad \quad c_B = k_A - k_B \quad \text{if } K_A + K_B > \bar{K} \\ & \quad \quad \quad c_B = (1 - a)(k_A - k_B) \quad \text{if } K_A + K_B < \bar{K} \end{aligned}$$

Let the solutions to this problem be denoted by $k_A(s)$ and $k_B(K_A, s)$.

Given the piecewise linearity of this problem, it is possible that there are multiple solutions. It will be assumed that if there are multiple solutions, the largest value of k is chosen. Since this is not a generic issue, this assumption is not crucial. A much less innocuous assumption is that we also rule out mixed strategies in the information acquisition decision. Let the value of the utility function at an optimum be denoted by $U(s)$. Then the information acquisition decision is made by examining the following inequality.

$$(10) \quad \int U(s) dF(K_A, K_B, \theta) - \delta > U(0)$$

If this inequality holds, resources are spent to acquire information. If not, then no information is acquired.

The aggregate level of consumption in the economy may now be defined in the two stages

$$(11) \quad K_A = \alpha(1 - t)k_A(s) + (1 - t)(1 - \alpha)k_A(0)$$

$$(12) \quad K_B = \alpha(1 - t)k_B(K_A, s) + (1 - t)(1 - \alpha)k_B(K_A, 0)$$

We have assumed that all those who die in period 1 consume in stage A. In a sense, this is an equilibrium outcome. This assumption is not at all important and avoids carrying around unnecessary notation.

A Nash equilibrium can now be defined for this game.

Definition of Equilibrium

A Bayesian Nash equilibrium for the two-stage game described above is a number $I \in \{0, 1 - t\}$ and a set of five functions

$$k_A(s): \quad S \rightarrow [0, 1]$$

$$k_B(K_A, s): \quad [0, 1] \times S \rightarrow [0, 1]$$

$$K_A(\theta): \quad \theta \rightarrow [0, 1]$$

$$K_B(\theta): \quad \theta \rightarrow [0, 1]$$

$$F(K_A, K_B, \theta): \quad [0, 1]^2 \times \theta \rightarrow [0, 1]$$

such that k_A, k_B solve the problem defined in equation (9), K_A, K_B are given by equations (11) and (12), respectively, $F(\cdot)$ is defined by equations (11) and (12) and the well-specified joint distribution of θ , and $I = 1 - t$ if inequality (10) holds, $I = 0$ otherwise.

The question of existence naturally arises in this context. Since we have restricted ourselves to pure strategy Nash equilibria, existence of equilibrium is not obvious. The following procedure will characterize the equilibria as existence is proved. With some abuse of notation, the dependence of c on K_A will be suppressed.

Proposition 1. Given restriction (PR1) in equation (6), $c_A(0) = 0$.

Proof. From equation (6) we have that:

$$pH + (1 - p)L > 1 - a$$

Since the random variables t , R and α are independent, it follows that if $s = 0$, then the posterior probabilities are the same as the prior probabilities. Thus,

$$\int R dF(K_A, K_B, \theta | s = 0) = pH + (1 - p)L > 1$$

and from the problem defined in equation (6)

$$c_A = 0.$$

Q.E.D.

The problem faced by an informed individual is slightly more

complicated. If $s = \bar{s}$, i.e., $R = H$, the decision problem is straightforward and $c_A(\bar{s})$ and $c_B(\bar{s}) = 0$. Necessarily, of course, if there are informed individuals in the economy, $I = 1 - t$.

It is useful to define a new set of variables. Let

$$(13) \quad \lambda_A = 1 - K_A$$

$$\lambda_B = 1 - (K_A + K_B)$$

The variable λ_A has the natural interpretation of being the "length of the line" at the bank if in equilibrium, as is to be expected, the agents show up either at stage A or at stage B, but not at both. λ_B has the interpretation of being the sum of the people who show up at the two stages.

It is also instructive to examine the states of the world as laid out in table 2. Let π_i , $i = 1, \dots, 12$, denote the states of the world with the index i as given in table 2. The rows denote states of the world and the columns will be seen to be equilibrium outcomes.

Proposition 2. In any equilibrium with $I \neq 0$, and $a > 0$, $k_A(\underline{s}) = k_B(\underline{s}) = 0$ (i.e., when the state of the world is known to be low, informed agents consume their resources.)

Proof. Suppose that the proposition is not true. Then $0 < k_A < 1$. First consider the case $0 < k_A(\underline{s}) < 1$. We will prove that in equilibrium the observation $s = \underline{s}$ will be revealed by the aggregate capital stock at the end of stage A, K_A , to the uninformed agents. Hence, $k_A(\underline{s}) = 0$ will dominate $0 < k_A(\underline{s}) < 1$. To prove that $s = \underline{s}$ is revealed it suffices to show that states 3 ($t = 0$) and 7 ($t = t_1$) are revealed to be low states.

Number	States			Line Length		Rational Expectations Equilibrium
	t	R	α	Nash Equilibrium		
				λ_A	λ_B	
1	0	L	0	0	0	0
2	0	H	0	0	0	0
3	0	L	$\bar{\alpha}$	$\bar{\alpha}$	$\bar{\alpha}$	$\bar{\alpha}$
4	0	H	$\bar{\alpha}$	0	0	0
5	t_1	L	0	t_1	t_1	t_1
6	t_1	H	0	t_1	t_1	t_1
7	t_1	L	$\bar{\alpha}$	$t_1 + \bar{\alpha}(1 - t_1)$	1	1
8	t_1	H	$\bar{\alpha}$	t_1	t_1	t_1
9	t_2	L	0	t_2	1	1
10	t_2	H	0	t_2	1	1
11	t_2	L	$\bar{\alpha}$	$t_2 + \bar{\alpha}(1 - t_2)$	1	1
12	t_2	H	$\bar{\alpha}$	t_2	$1 - \bar{\alpha}(1 - t_2)$	t_2

NOTE: $\bar{\alpha} = t_1$, $t_1 + \bar{\alpha}(1 - t_1) = t_2$

Table 2

From equation (8) and Proposition 1, we have

$$(14) \quad K_A(t, L, \bar{\alpha}) = (1 - t)(1 - \bar{\alpha})(1 - k_A(\underline{s}))$$

Note that

$$K_A(t, R, 0) = 1 - t$$

$$K_A(t, H, \bar{\alpha}) = 1 - t$$

Recalling equation (5)

$$t_2 = t_1 + \bar{\alpha}(1 - t_1)$$

we have from equation (14) that

$$(1 - t_2) < (1 - t_1)(1 - \bar{\alpha}(1 - k_A(\underline{s}))) < 1 - t_1$$

Thus, states 3 and 7 are revealed as such and necessarily in this case $K_B = 0$. Consequently, expected utility in this case is given by

$$\begin{aligned} EU &= \pi_3(1 - k_A) + \pi_7(1 - k_A) + \pi_{11}(1 - a)(1 - k_A) \\ &+ \pi_3 k_A(1 - a) + \pi_7 k_A(1 - a) + \pi_{11} k_A(1 - a). \end{aligned}$$

This is clearly dominated by the strategy setting $k_A(\underline{s}) = 0$. Then, we have

$$EU = \pi_3 + \pi_7 + \pi_{11}(1 - a).$$

In the case where $k_A(\underline{s}) = 1$, $K_A = 1 - t$ and no information is revealed. Again we can utilize the fact that $\bar{K} = 1 - t_2$ to show that expected utility can be increased by setting $k_A(\underline{s}) = 0$. Q.E.D.

The preceding propositions permit the characterization of λ_A completely. The inference problems faced by informed agents at stage B will now be examined. It will be assumed for the moment that $I \neq 0$. The decisions of uninformed agents clearly depend delicately upon the probabilities of the various states. A bank run equilibrium may now be defined as an equilibrium where individuals choose not to reinvest even though there has been no adverse information.

Definition.

A Nash equilibrium is a bank run equilibrium if $\lambda_B(t, R, 0) = 1$ for some t .

The following restrictions are enough to guarantee that there exists a bank run equilibrium.

$$(15) \quad \bar{\alpha}\pi_3 - \delta > \pi_2L \tag{PR4}$$

$$(15) \quad \frac{(\pi_7 + \pi_9)L + (\pi_{10} + \pi_{12})H}{\pi_7 + \pi_9 + \pi_{10} + \pi_{12}} < 1 - a \tag{PR5}$$

$$(16) \quad \frac{(\pi_3 + \pi_5)L + (\pi_6 + \pi_8)H}{\pi_3 + \pi_5 + \pi_6 + \pi_8} > 1 - a \tag{PR6}$$

Theorem 1. Given restrictions (PR1-PR6) there exists a unique bank run equilibrium.

Proof. Suppose $I = 0$. Then, the minimum expected utility gain to an

individual choosing to invest in acquiring information is the product of the probability that $s = \underline{s}$, that he is informed, i.e., $\alpha = \bar{\alpha}$, and that $t = t_1$. In this case, the individual gets to consume 1 unit in period 1.

Note that the product of these probabilities is $\bar{\alpha}\pi_3$. Thus, we get the inequality below using (15)

$$\bar{\alpha}\pi_3 - \delta > \pi_3 L.$$

The left hand side of this inequality is the expected utility gain. Thus, $I = 1 - t$.

In this case, λ_A is given by Propositions 1 and 2. The information partitions of uninformed agents are now given by

$$\lambda_A = \begin{cases} 0 & \text{if states 1, 2 or 4 can occur,} \\ t_1 & \text{if states 2, 5, 6 or 8 can occur,} \\ t_2 + \bar{\alpha}(1 - t_2) & \text{if state 11 occurs,} \\ t_2 & \text{otherwise.} \end{cases}$$

Restrictions (15) and (16) now imply that the allocations given in Table 3 are the unique solutions to the individual's decision problem. It also follows immediately from (15) that it is optimal to invest in acquiring information even though everyone else is doing the same. There is a potential gain if state 3 is realized. Q.E.D.

Remarks.

(a). There is a relationship between the equilibrium defined here and the more traditional rational expectations equilibrium concept. Some of the problems with rational expectataions equilibria become apparent in this context. Suppose the two-stage process is collapsed into one stage, and

assume that all individuals can observe the line length before making a decision. Then the maximization problem faced by a representative individual is:

$$(9a) \quad \text{Max}_{c,k} \int \{c(s,K) + Rk(s,K)\} dH(K, \theta | K, s)$$

subject to

$$c \leq 1 - k \quad \text{if } K > \bar{K}$$

$$c \leq (1 - a)(1 - k) \quad \text{if } K < \bar{K}$$

where $H(\cdot)$ is the associated distribution function. A rational expectations equilibrium can be defined exactly as earlier. Now, for example, an allocation rule that specifies a line length of 1 whenever the returns are low, regardless of whether any resources are invested to acquire information or the realization of α is an equilibrium. If everyone thinks that a long line implies poor returns and, in fact, when returns are bad the line is long, we have a rational expectations equilibrium. The problem, of course, lies in the fact that there is no clear mechanism to translate individual's decisions into equilibrium "prices."

However, it is fairly straightforward to show that under precisely the restrictions we impose on the model, there exists a rational expectations equilibrium with costly acquisition of information which is very similar to our Nash equilibrium. This is displayed in table 2. It is easy to verify that this is indeed a rational expectations equilibrium. The incentive to acquire costly information is again in state 3. The only difference in the

allocations arises in state 12 (when $t = t_2$, $\alpha = \bar{\alpha}$ and $R = H$). This is, of course, because if individuals observe a line length of $1 - \bar{\alpha}$, they can infer that state 12 has occurred and decide not to join the line.

(b). The robustness of the bank run equilibrium is of some interest. Even if the information acquisition cost $\delta = 0$, but $a > 0$, the bank run equilibrium is unique. Of course, even if $\delta = 0$ and $a = 0$, the bank run equilibrium continues to be a Nash equilibrium. All that is lost is uniqueness. The liquidation cost can also be incorporated by allowing individuals at stage A to discount future consumption in stage B and precisely the same implications follow.

While it is true that generically the equilibria in this model are fully revealing (change $\bar{\alpha}$ by a small amount to see this) and do not involve bank runs we do not consider that a serious problem with this model. In such a case, existence of a fully revealing equilibrium depends crucially upon the liquidation cost being strictly positive and upon there being no cost to acquiring information. It should be clear from our model that the bank run equilibrium when $\delta = 0$ and $a = 0$ is the only equilibrium which is a limit of equilibria with $\delta > 0$ and $a > 0$. As such, by standard continuity of arguments, it merits serious consideration even if $\delta = 0$ and $a = 0$.

(c). Our equilibrium concept is a modification of that suggested by Dubey, Geanakoplos and Shubik (1983). They argue for equilibria which mimic, to some extent, fully revealing equilibria. As discussed above, the case for confounding equilibria, when they exist, seems appealing.

4. Optimality

The expected utility in the planning period of a representative individual who follows the equilibrium strategies described in the preceding

section is easily calculated. This will be denoted by U_{eq} .

$$\begin{aligned}
 U_{eq} = & \pi_2 H + \pi_3 \bar{\alpha} + \pi_4 H + \pi_5 t_1 + \pi_6 t_1 + \pi_6 (1 - t_1) H + \pi_7 (1 - a) \\
 & + \pi_8 t_1 + \pi_8 (1 - t_1) H + \pi_9 (1 - a) + \pi_{10} (1 - a) + \pi_{11} (1 - a) \\
 & + \pi_{12} (1 - a(1 - \bar{\alpha})(1 - t_2) + \pi_{12} H[\bar{\alpha}(1 - t_2)]].
 \end{aligned}$$

In the above, we have made use of the fact that $L = 0$.

This equilibrium is not necessarily ex ante Pareto optimal. A mechanism is described below which can, under appropriate circumstances, dominate the equilibrium allocations in an ex ante sense. This mechanism is closely linked to "suspension of convertibility."

The idea is that individuals form a queue at the bank but not all individuals are able to withdraw their assets at stage A. Again, the observable magnitude is the actual amount of withdrawals. Clearly, if the number of individuals permitted to withdraw exceeds t_1 , the equilibrium allocations are unaffected since we permit unlimited withdrawals at stage B. Thus, suspension of convertibility is meaningful only if the first t_1 individuals are permitted to withdraw at will but anyone else who desires to withdraw must wait until stage B. Intuitively, it is best to think of individuals queueing up at the bank but only the first t_1 individuals are paid off. Then,

$$(18) \quad \lambda_A \in \{0, t_1\}$$

The information sets of uninformed agents is quite different now, since they do not observe realizations of $\lambda_A > t_1$. It is useful to introduce

additional notation. Let

$$(19) \quad \hat{K}_A \equiv \text{Max}\{K_A, (1 - t_1)\}$$

$w_A \equiv$ the amount the individual desires to withdraw.

Almost by definition, with suspension of convertibility, there are no liquidation costs in stage A. Hence, they are ignored in setting up the individual's problem. Note that an individual's actual consumption in stage A is now random and the expected value is given by

$$E(c_A) = w_A \int \frac{(1 - \hat{K}_A)}{(1 - K_A)} dG(\hat{K}_A, K_A, K_B, \theta | s)$$

where G is the associated distribution function.

Consumption in stage B is now given by

$$(20) \quad c_B \leq k_A - k_B + w \left[1 - \frac{(1 - \hat{K}_A)}{1 - K_A} \right]$$

The analogue to the problem posed in equation (9) is

$$(21) \quad \begin{aligned} \text{Max}_{w_A, c_B, k_A, k_B} & \quad w_A \int \frac{(1 - \hat{K}_A)}{(1 - K_A)} dG(\hat{K}_A, K_A, K_B, \theta | s) \\ & \quad + \int c_B dG(\hat{K}_A, K_A, K_B, \theta | s, \hat{K}_A) \\ & \quad + \int Rk_B dG(\hat{K}_A, K_A, K_B, \theta | s, \hat{K}_A) \end{aligned}$$

s.t.

$$0 \leq w_A \leq 1 - k_A$$

$$0 \leq c_B \leq k_A - k_B + w \left(1 - \frac{(1 - \hat{K}_A)}{1 - K_A}\right), \text{ if } \hat{K}_A + K_B > \bar{K},$$

$$0 \leq c_B \leq (1 - a) \left(k_A - k_B + w \left(1 - \frac{(1 - \hat{K}_A)}{1 - K_A}\right)\right), \text{ if } \hat{K}_A + K_B < \bar{K}$$

Given this, we can define an equilibrium exactly as in section 3. An uninformed individual now has the following information partition:

$$\hat{\lambda}_A = 0: \{1, 2, 4\}$$

$$\hat{\lambda}_A = t_1: \{3, 5, 6, 7, 8, 9, 10, 11, 12\}.$$

Thus, if the following restriction holds

$$(22) \quad \frac{\pi_3^L + \sum_{i=5}^{12} \pi_i^R}{\pi_3 + \sum_{i=5}^{12} \pi_i} > 1 \quad (\text{PR7})$$

it is optimal for an uninformed person not to withdraw his assets at stage

B. This proves:

Theorem 2. Given (PR1-PR7), there exists an equilibrium with suspension of convertibility which has line lengths

$$\hat{\lambda}_A = \text{Min}\{\lambda_A, t_1\}$$

$$\hat{\lambda}_B = t + \alpha. \quad []$$

It is, of course, of some interest to compare the allocations that result in the two equilibria. The criterion used here is ex ante expected utility. Clearly, the two equilibria differ only when $\lambda_A > t_1$. This constitutes the set of states: $D \equiv \{7,9,10,11,12\}$. Thus, the expected utility of the allocations when restricted to D can be examined.

$$(23) \quad U_{eq}(D) = (1 - a) \sum_{i \in D} \pi_i + \pi_{12} \bar{\alpha} (1 - t_2) (H - (1 - a)).$$

Let U_s denote expected utility with suspension of convertibility

$$(24) \quad U_s(D) = [t_1 + (t_2 - t_1)(1 - a)] \sum_{i \in D} \pi_i \\ + \pi_{10} (1 - t_2) H + \pi_{11} \bar{\alpha} (1 - t_2) (1 - a) + \pi_{12} (1 - t_2) H$$

Inspection of equations (23) and (24) reveals that it is distinctly possible that suspension of convertibility may yield ex ante superior allocations to those achieved in an equilibrium without such suspension.³

A sufficient condition, for example, is that liquidation costs be sufficiently high, i.e.,

$$(25) \quad t_1 + (t_2 - t_1)(1 - a) > (1 - a).$$

³Some experimentation with numerical examples has convinced us that the desired inequality holds for a range of parameter values. We provide an example. Let $H = 2.005$, $p = q = .05$, $r_0 = r_2 = 0.125$, $r_1 = 0.75$, $\bar{\alpha} = 0.25$, $a = 5/9$. It is of interest to note that a simpler suspension scheme for this example also dominates the bank run equilibrium although it is strictly dominated by the suspension scheme outlined above. This scheme has no payments at stage B and at stage A only the first t_1 individuals are paid off. The returns, if any, are equally distributed among survivors.

Alternatively, if states 10, 11 and 12 have high enough probabilities the allocations with suspensions are superior in an ex ante sense.

Of course, such suspension of convertibility accompanied by random assignment of individuals to positions in the line leaves some individuals worse off than others who are identically situated but have a higher position in the queue. In a sense, therefore, the fact that suspension of convertibility was consistently practiced in every bank run but that there were many ex post complaints about the state of affairs is explained by this model.

The role of deposit insurance in our model will now be considered. The particular mechanism of deposit insurance examined does not lead to allocations which maximize ex ante expected utility. Instead, it is meant to capture what is frequently alleged to be the main operating characteristic of the system. In the framework of this model, the government (or other centralized agency) is assumed to announce ex ante the following scheme: In the event of a bank run, all those alive in period 2 who have withdrawn their assets will be taxed and the proceeds paid to those who choose not to withdraw. Note that this policy rule is an out-of-equilibrium rule. By definition, in a bank run there are no depositors left in the bank. This leaves open questions of the perfectness of the equilibrium. Since it is sometimes thought that "Deposit insurance is . . . a form of insurance that tends to reduce the contingency insured against" (Friedman and Schwartz (1963), pp. 440), the lack of perfectness of equilibrium seems an essential to deposit insurance.

It will be shown that deposit insurance of this sort can lead to allocations which dominate the Nash equilibrium. Essentially, the allocation which we attempt to duplicate is the suspension of convertibility

equilibrium. Consider a tax on period 2 endowments which depends upon the realization of λ , the line length, in stage A. If $\lambda < t_1$, the tax is zero. If $\lambda > t_1$, a tax is imposed on the endowments of all individuals who withdraw their assets at stage A and the proceeds are used to subsidize all those who do not withdraw at stage A. In this case, provided the tax is large enough, none of the informed individuals withdraw in stage A (since they lose in states 7 and 11). The line length conveys no information. Note that ex ante expected utility in this case is the same as in the case with suspension of convertibility.⁴ However, there is no ex post rationing by fiat; the rationing instead arises as a consequence of the tax rule which induces decisions by private agents which mimic the allocations achieved through the direct rationing implicit in suspension of convertibility.

The scheme suggested above requires much less information than might be supposed. To the extent that a government can levy an inflation tax and share the seignorage proceeds with private banks (and, presumably, their depositors), a differential tax on currency and bank deposits can be levied without the necessity of identifying individual depositors. This simple scheme allows the more complicated tax and subsidy procedure outlined above to be carried out effectively.⁵ These remarks are merely meant to be suggestive. An explicit monetary model would need to be constructed to examine these issues. A related issue is the inability of private market arrangements to implement the deposit insurance scheme as part of an optimal

⁴Note that the deposit insurance scheme outlined here strictly dominates the simple suspension scheme in footnote 3.

⁵More precisely, such an inflation tax leaves informed individuals indifferent between withdrawing at stage A and stage B. The inflation tax scheme is effective only if all informed individuals choose to wait until stage B.

contract. We conjecture that an important reason is that in our model, investment, k , is to be interpreted as the sum total of an individual's asset holdings in financial intermediaries rather than holdings in one particular bank. Monitoring this investment figure is apt to be difficult.

Mechanisms which implement full information allocations will now be considered. Such mechanisms are easy to construct within this present framework. Consider a scheme where individuals are asked to report if they are informed and, if so, the state of nature. In period 2 it is known if they have told the truth, and if so they receive a subsidy from a lump sum tax levied on the population. If they are found to have lied, their entire endowment is taxed away. An appropriate setting of taxes and subsidies will give the appropriate incentives to informed individuals to reveal the truth when the state of nature is low. Note that type 1 agents always report that the state is high to avoid the liquidation costs but those who report that the state of the nature is good are ignored. This scheme is informationally very burdensome since it requires observability of period 2 endowments. If some type 2 agents have zero endowment this scheme is not feasible.

5. Conclusion

In a sense this paper is an extended example. Expanding the number of states yields no major changes in the results so we have chosen to restrict the number of states in order to make the results more transparent. We have established that bank runs can be modeled as an equilibrium phenomenon in a model which has a unique equilibrium. Previous work (Diamond and Dybvig) generates bank runs as one of a series of possible multiple equilibria. Multiple equilibrium models, of course, suffer from the problem that they have limited predictive power. We have demonstrated that some aspects of the intuitive "story" that bank runs start with fears of insolvency of particular

banks and then spread to other sectors can be rigorously modeled. The essence of our model is that if individuals observe long "lines" at banks, they correctly infer that there is a possibility that the bank is about to fail and precipitate a bank run. Bank runs occur even if no one has any adverse information about future returns.

There is a close relationship between our equilibrium and a rational expectations equilibrium. The two result in allocations that, in general, differ in only one state. Our equilibrium does not suffer from the multiplicities endemic to rational expectations equilibria nor do they have the odd feature that the equilibrium reveals more information than anyone or everyone in the marketplace has. Reasoning based upon continuity arguments suggests to us that confounding equilibria such as the bank run equilibrium of this model are more "stable" than fully revealing equilibria.

We have also argued that extra-market constraints such as suspension of convertibility or "deposit insurance" can improve upon the equilibrium allocations. In that sense, this is a positive theory of these institutional arrangements. This occurs because in the model considered here there are two sources of social costs in bank runs. One is the cost involved in liquidating fixed investments, the other is the fact that bank runs occur in some states even though returns are high and are known by some individuals to be high. Essentially, the fear induced by a large number of withdrawers even though these withdrawals are not informationally based causes a run on the bank.

A feature often thought to be central to demand deposits is "sequential service." This model provides no explanation of this phenomenon. It is of some interest, though, that the explanation of bank runs here does not depend upon a sequential service constraint. We conjecture that by replacing liquidation costs with different discount rates for individuals of different

types, this model could be extended to explain the sequential service constraint as a device for separating the different types. Our explanation of deposit insurance rests crucially upon the ability to tax depositors and withdrawers differently. An inflation tax seems an attractive vehicle to implement such a policy rule. This would require extending the model to allow for the use of money.

Quite deliberately, we have chosen to assume risk neutrality. For the kind of aggregate risks discussed here, this may seem absurd. However, nothing crucial depends upon the assumption of risk neutrality. An important direction in which this model needs to be extended is to incorporate the linkage between failures of particular banks and runs on the banking system as a whole. We have imposed a liquidation cost on the technology to capture the idea that failures of many banks are more costly than failures of a few. It is not clear to us why this might be so. In addition, the aggregate risks in this model are exogenously imposed while the facts documented in the introduction suggest the need for an integrated business cycle model in which the financial intermediation industry plays a central role.

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