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MONOPOLY BUNDLING OF WARRANTY AND QUALITY WHEN QUALITY IS UNOBSERVABLE

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by

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This paper examines the problem of a monopolist using qualities and warranties to screen buyers who have different valuations for the product. Since quality is assumed unobservable, warranties act as a signal of quality. A local technique cannot be used to characterize the solution to the monopolist's problem because valuation of quality and warranty do not always increase in buyer type. Instead a technique is used which shows the solution to a relaxed problem imposing the downward incentive constraints is feasible when all the incentitve constraints are imposed. An example is used to show that warranty legislation can have ambiguous welfare effects.

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1. Introduction

This paper examines the problem of a monopolist using qualities and warranties to screen buyers who have different valuations for the product. An important feature of the model is that buyers cannot observe quality. Warranties signal quality to the buyers since a higher warranty provides an incentive for the seller to produce higher quality.

The model captures some of the features of the market for new and used cars. Some dealers sell both products. New cars are covered by a warranty for at least 12 months or 12,000 miles. On the other hand, it is sometimes difficult to get any warranty on a used car. Mufflers are another example in which the top of the line products have a long warranty period (usually the lifetime of the car) and the economy versions have a short warranty period (usually one year). A third example comes from the airline passenger market. When passengers with regular tickets are bumped, they receive a ticket for a later flight plus additional compensation. When standby passengers are denied boarding, they receive no compensation. In all three markets, the seller must be wary of lower priced products "cannibalizing" sales of higher quality, higher margin products. All three examples are also instances in which it is difficult for the consumer to directly assess the probability of sucessful product performance.

The model predicts that a monopolist in situations such as these will provide less than the socially efficient quality and warranty to consumers with a low valuation for the product. The seller does this to discourage high demand buyers from switching to low margin products. This is analgous to the result obtained by Mussa and Rosen (1978) in which quality provision alone was studied. An analysis including warranties is of interest to economists because in many cases it is easier to determine warranty provision than to measure quality levels. In a similar vein, the model is useful to policy makers because in some cases it is simpler to require a warranty than to design and enforce quality specifications for each attribute of a product. It is shown that legislation requiring that the warranty be as large as the price will force the seller to increase the quality of the low tier product, if he continues to sell it. However, such a law actually leads to a decrease in social welfare if the seller opts to stop distributing this product. This occurs when the improved low quality product becomes too attractive to buyers of the high quality products.

Previous studies of warranties as a signal of quality examine either the case of monopoly with one consumer type or the case of perfect competition. These studies show that when the consumer can make correct inferences about quality based on the warranty, the market provides efficient quality and warranty. Legislation requiring money back guarantees is redundant since this is the outcome of a laissez faire policy. The first paper to make this point is Spence (1977). He shows that in a competitive equilibrium, consumers with differing preferences for quality are each allocated the efficient quality for their type and that each type of consumer is fully insured.[1] Grossman (1981) studies the problem of a monopolist selling a good with exogenously given quality. Any attempt by the seller to offer a low warranty causes the buyer to infer that the good is a low quality product. The result is that the product is sold with full warranty coverage and there is no social inefficiency. Courville and Hausman (1979) also show that either competition or monopoly result in efficient quality and warranty allocations. This literature contrasts with an earlier school of thought (described in Priest (1981)) which held that sellers who face no competitive pressure offer low warranties to save on costs. The later work recognizes that the monopolist has an incentive to offer high warranties because consumers are willing to pay more to get them.

The contract design problem with quality and warranty has been studied by Matthews and Moore (1984) for the case of observable quality. Though this is a difficult three dimensional screening problem, by making reasonable assumptions on preferences towards risk (namely, that they satisfy nonincreasing absolute risk aversion) and by use of an intricate argument, they are able to characterize the optimum. The present study uses their formulation to examine the unobservable quality case. This reduces the screening problem to two dimensions since warranty signals quality. However, the technique commonly used to solve such problems, namely, analysis of a relaxed problem with only local incentive constraints, cannot be used because an important assumption of this approach does not hold here. This assumption is often referred to as the Spence signalling condition (see Spence (1974)) and in this model would be that the marginal rate of substitition between quality and money increases in type. We use instead a new techique developed by Matthews and Moore to analyze the solution. This technique examines a relaxed problem which imposes global incentive constraints rather than only local ones.

The paper is organized as follows. Section 2 presents the model. Section 3 examines the cases of competition and of a perfectly discriminating monopolist. Section 4 characterizes the optimum for the imperfectly discriminating monopolist. Section 5 analyzes the effect on social welfare of legislation requiring warranties. Section 6 concludes the paper. All proofs are relegated to the appendix.

2. The Model

As in Matthews and Moore, each consumer chooses a contract $x = (p,q,w)$ where p is the price, q is the quality (interpreted as the probability that the product works), and w is the warranty. Consumers vary by the dollar amount, θ, that they value a working product. Their preferences towards risk are described by a concave utility function u( ). The utility of a contract x to a consumer of type θ is:

$$U(x,\theta) = qu(\theta-p) + (1-q)u(w-p).$$

There are n different types of consumers, $0 < \theta_1 < \theta_2 < \ldots < \theta_n$. The fraction of consumers of type θ_i is $f_i > 0$.

The risk neutral firm can produce a unit of quality q at the constant marginal cost of C(q), with $C'(q) > 0$ and $C''(q) > 0$. We also assume that $C'(0) < \theta_1$ and $\theta_n < C'(1)$, which implies that it is optimal for each type to have a product which works with positive probability but not with certainty. The expected unit cost, denoted D(q,w), of offering quality q and warranty w is:

$$D(q,w) = C(q) + (1-q)w.$$

If the buyer cannot observe quality and if certification is not possible, the seller cannot commit himself to a quality level. If the product has warranty w, the seller will select the quality that minimizes D(q,w). The first order conditions require that q solve:

(1) $\quad C'(q) = w.$

Buyers base their expectations of product quality on observations of warranty and price, using an inference function $q^e = \phi(w,p)$. From (1), with correct expectations, in equilibrium $\phi(w,p) = C'^{-1}(w)$. Since $C''(q) > 0$, higher warranty signals higher quality.

For convenience I will now let $x = (p,q)$ denote the contract offered by the seller. By (1) there is a unique warranty $w(q) = C'(q)$ associated with any quality q. The total unit cost of a contract with quality q is then:

(2) $\quad D(q,w(q)) = C(q) + (1-q)C'(q)$

For $q < 1$, unit cost is strictly increasing in quality since:

(3) $$\frac{dD}{dq} = C'(q) + (1-q)C''(q) - C'(q) = (1-q)C''(q) > 0, \text{ for } q < 1.$$

3. Efficient Allocations

Suppose quality is observable. As noted in Matthews and Moore, in a Pareto optimal allocation, the risk averse buyers are fully insured by the risk neutral firm. Therefore, the efficient warranty for type θ_i, denoted w_i^*, is equal to θ_i. It follows that the efficient quality q_i^* maximizes the total expected surplus of $q\theta_i - C(q)$, so q_i^* solves the first order condition $\theta_i = C'(q_i)$. Since $C'(0) < \theta_1$ and $\theta_n < C'(1)$, $0 < q_i^* < 1$ for all i. The competitive allocation is $(x_1^c, x_2^c, \ldots, x_n^c)$ where $q_i^c = q_i^*$, $w_i^c = w_i^*$, and $p_i^c = D(q_i^*, w_i^*)$, the price at which the firm breaks even on the contract. Each type selects the contract meant for his type since this contract maximizes his utility subject to the zero profit constraint. In the monopoly allocation x_1^m, with only one type of consumer, $q_1^m = q_1^*$, $w_1^m = w_1^*$, and $p_1^m = \theta_1$, the price that extracts all surplus. If there is more than one type, if the monopolist can distinguish each type, and if he is legally allowed to discriminate, the efficient qualities and warranties are allocated to each type at prices which extract all surplus. However, if the monopolist cannot distinguish types, this set of contracts is not feasible since higher types attain positive surplus from contracts meant for lower types.

An interesting result, which was first observed by Spence (1977) for the competitive case, is that the competitive and perfectly discriminatory allocations do not change when quality is unobservable. Since $w_i^* = \theta_i = C'(q_i^*)$, q_i^* is the quality the firm produces when the waranty is w_i^* and it is the quality that the consumers infer from such a warranty. Unobservable quality poses no incentive problem since, with fully insured consumers, the seller's behavior affects only his own costs. The policy implication is that there is no justification for legal intervention when quality is unobservable. Laws requiring warranty greater than the price are superfluous since products offered already fulfill this requirement.

4. The Imperfectly Discriminating Monopolist

The monopoly allocation above is not feasible if the monopolist cannot perfectly discriminate. The seller in this case offers the set of contracts which maximizes profit subject to the following two sets of contraints: (1) the incentive constraints (IC), which require that each type select the contract meant for his type, and (2) the voluntary participation constraints

(VP), which require that no type is assigned a contract which makes him worse off than not buying any contract at all. (The seller may not wish to service a particular type in which case the type is assigned a contract with zero price, warranty, and quality). There is also a constraint that the seller offer non-negative quality. The monopolist's problem (M) is formally expressed as follows:

$$\text{(M)} \qquad \underset{x_1,\ldots x_n}{\text{Maximize}} \sum_{i=1}^{n} f_i \, \pi(x_i)$$

subject to

$$\text{(IC)} \qquad U(x_i,\theta_j) \leq U(x_j,\theta_j) \quad \text{for all } j \neq i.$$

$$\text{(VP)} \qquad U(x_1,\theta_1) \geq u(0)$$

$$q_i \geq 0 \text{ for all } i$$

where

$$x_i = (p_i,q_i)$$

$$\pi(x_i) = p_i - C(q_i) - (1-q_i)C'(q_i) \quad \text{and}$$

$$U(x_i,\theta) = q_i u(\theta-p_i) + (1-q_i)u(C'(q_i)-p_i).$$

This is identical to the problem stated in Matthews and Moore, except there is an additional constraint $w_i = C'(q_i)$ substituted directly into the problem. The monopolist is worse off in this model than in their study since the choice set is smaller. Note that (VP) is explicitly imposed only for type θ_1, since (IC) implies that all higher types obtain their reservation level of utility if type θ_1 does.

Before examining the more difficult case of risk averse consumers, the instructive case of risk neutral consumers is considered. When quality is observable, risk neutrality reduces the problem to the one studied by Mussa and Rosen because in this case, warranties cannot be used to screen buyers. The seller offers the same set of contracts as he does when there are no warranties except that he raises the price of any contract by the expected cost of the warranty. Because buyers and the seller are indifferent to any warranty that is paid for by an increase in price, warranty provision is indeterminate.

When quality is unobservable, warranty provision is determinate. The seller offers the same qualities as above. Warranty coverage is limited to the levels which signal these qualities. The presence of unobservable quality changes neither quality provision nor the welfare of any party.

The result that unobservable quality does not effect qualities or welfare continues to hold in the more general case of preferences which satisfy constant absolute risk aversion (CARA). In this case, Matthews and Moore find that when quality is observable, $w_i = C'(q_i)$. This means that the contracts satisfy the signalling condition.

Risk Aversion

In the general case of preferences which do not satisfy CARA, the solution when quality is observable is not feasible when quality is unobservable. For instance, Matthews and Moore provide an example of the observable quality case in which, in the solution, one consumer receives a higher quality but lower warranty than another, which clearly violates the signalling condition.

A common method of approaching a two dimensional screening problem is to use a local approach. A problem is examined with only the local incentive constraints and it is shown that the solution to this problem satisfies the global constraints. In applications to discrete models the following three sets of constraints are imposed: (1) the adjacent downward incentive constraints which require that each type prefers his contract to the contract meant for the type directly below his, (2) voluntary participation of the lowest type, and (3) the monotonicity condition that higher type buyers are allocated higher quality products. This last constraint is implied by the global constraints due to the presence of the important assumption that the marginal rate of substitution between money and quality is increasing in type. Because any solution feasible for the global constraints is feasible for this relaxed problem, when it is shown that the solution to the relaxed problem is feasible for the global constraints, then it solves the general problem.

In the present model, this important condition on the marginal rate of substitution does not hold. The marginal rate of substitution of type θ at x between money and quality (with the effect of a change in quality on warranty directly imposed) is:

$$(4) \qquad MRS(x,\theta) = -\frac{\frac{\partial U(x,\theta)}{\partial q}}{\frac{\partial U(x,\theta)}{\partial p}} = \frac{u(\theta-p) - u(C'(q)-p) + (1-q)u'(C'(q)-p)C''(q)}{qu'(\theta-p) + (1-q)u'(C'(q)-p)}$$

When utility is increasing in quality, $MRS(x,\theta)$ is increasing in θ. The intuition is that higher types not only prefer quality more than a lower type but they mind an increase in price less because of the concavity of u( ). Suppose utility is decreasing in quality, however. This occurs at $q = 1$ and by continuity for some $q < 1$. In this case, the buyer is overinsured and higher quality is a bad because it lowers the chance of collection of the warranty.

At such a point, the effect of an increase in θ is not clear because it makes the numerator less negative and the denominator less positive. It can be shown, however, that for $C'(1)$ large enough, $MRS(x,\theta)$ is decreasing in θ at $q = 1$ and therefore for some $q < 1$. This case is illustrated in Figure 1. In this case, it is feasible for the monopolist to offer a lower quality to type θ_{i+1} than he offers to type θ_i. Therefore the local approach described above is invalid because it eliminates from consideration feasible solutions.

The above is an explanation of why we use a new technique developed by Matthews and Moore which does not impose monotonicity of the allocation. Instead, it imposes all the downward constraints. These are defined as:

(DIC) $\quad U(x_i,\theta_j) \leq U(x_j,\theta_j)$ for all $i < j$.

This constraint requires that each type prefers his own contract to that of any lower type but places no restriction on his preference for contracts meant for higher types. Define problem (M') as (M) with (DIC) substituted for (IC). It will be shown that the two problems have the same set of solutions.

The results are as follows. Lemma 1 shows that in solutions to (M') the utility of each type at his contract is nondecreasing in quality. Proposition 1 shows that in these solutions no type is allocated a quality that is higher than the efficient level for that type. Proposition 2 derives monotonicity of the allocation. Propostion 3 shows that in solutions to (M') quality allocations to all but the highest type are <u>strictly</u> less then the efficient level. Finally, Proposition 4 shows that the problems (M) and (M') have the same set of solutions. Proofs of the results are in the appendix.

Lemma 1: Every contract in a solution to (M') satisfies:

$$\frac{\partial U(x_i,\theta_i)}{\partial q} \geqslant 0.$$

The intuition for this proposition is that contracts in the problem region assign a quality level that is excessive to be profit maximizing. In the proof, it is shown that if type θ_i is awarded a contract x_i at which he prefers an decrease in quality (as in figure 1) then there exists a contract x' with the same price but with lower quality to which type θ_i is indifferent. Otherwise, type θ_i's indifference curve through x_i looks something like that in figure 2. This would imply that the buyer is indifferent to x_i and a contract with positive price and zero quality, which contradicts voluntary participation. (Actually, to show (VP) is violated, it must be shown that the price is greater than C'(0), the limit of the warranty as quality goes to zero). Since type θ_i is indifferent between x_i and x' and since x_i has higher quality at the same price, higher types do not prefer x' to x_i, as shown in figure 1. Recalling (3), it yields a greater profit to allocate x' to type θ_i but does not violate the downward incentive constraints. This means that x_i can not be part of a solution to (M').

Proposition 1: A solution $(x_1,\ldots,x_n)$ to (M') satisfies:

$$\begin{array}{lll} q_i \leqslant q_1^* & w_i \leqslant w_i^* = \theta_i & \text{for } i \leqslant n-1 \\ q_n = q_n^* & w_n = w_n^* = \theta_n & \end{array}$$

If some type θ_i is assigned a contract x_i with quality less than the efficient level, a contract x' with a local price and quality reduction is possible which leaves type θ_i indifferent but provides greater profit to the firm, Important for showing that this is feasible for (DIC) is that the utility of type θ_i is increasing in quality at x_i This implies that $MRS(x_i,\theta_j) > MRS(x_i,\theta_i)$ for $\theta_j > \theta_i$ which means that higher types prefer x_i to x' and therefore their own contracts to x'. This argument breaks down if $MRS(x_i,\theta_j) < MRS(x_i,\theta_i)$ for some $\theta_j > \theta_i$. Another important part of the argument is that only downward constraints are imposed since any type $\theta_j < \theta_i$ prefers x' to x_i.

Lemma 1 and Proposition 1 are also true for a relaxed problem with only the adjacent downward constraints. However, the next proposition, which shows monotonicity of the allocations, is valid only when all the downward constraints are imposed. The proposition also shows that higher quality products yield higher profits. (Since quality is unobservable, the testable implication is that firms earn higher profits on higher warranty products).

Proposition 2: For $i < n$ a solution to (M') satisfies:

(i) $U(x_i,\theta_{i+1}) = U(x_{i+1},\theta_{i+1})$,

(ii) $\pi(x_i) \leqslant \pi(x_{i+1})$,

(iii) $p_i \leqslant p_{i+1}$ and $q_i \leqslant q_{i+1}$.

Propositions 1 and 2 can be used to show

Proposition 3: A solution $(x_1,\ldots,x_n)$ to (M') satisfies:

$$q_i < q_i^* \qquad w_i < w_i^* = \theta_i \text{ for } i < n.$$

The intuition for the strict inequality in this proposition is that if for some $i < n$, type θ_i had the efficient quality for his type, price and quality can be lowered at only a second order loss in profits. However, since a higher type is no longer interested in the lower quality product, the price of the high quality product can be raised for a first order gain in profits. The proof of this depends on the fact that the contract of type θ_i is a binding constraint in the contract choice of a higher type. This was shown in Proposition 2.

Parts (i) and (iii) of Proposition 2 along with the fact that if one type is indifferent between two contracts then all lower types prefer the one with a lower price and quality (shown as Lemma A.3 in the appendix) yield the following equivalence result:

Proposition 4: Problem (M') and (M) have the same set of solutions.

Proposition 4 of Matthews and Moore, which states that if there is more than one non-zero contract offered, the one offered to the lowest type necessarily has warranty less than the price, is also true here. This is because in the solution, the lowest type buying a good has expected utility equal to zero. Since his utility is positive in the state that the product works, his utility must be negative in the state of product failure, implying that warranty must be less than the price.

5. The Social Benefit of Warranty Legislation

The merit of legislation requiring warranties has been an important issue. For instance, several states are currently considering laws which would require that automobile dealers offer warranties for used cars. One reason for the appeal of such legislation to policy makers is that it may be a relatively simple way to police the quality of products offered. It is much easier to require a dealer to be responsible for all problems which arise in the first 30 days of use than to impose quality standards for every aspect of the product, particularily when quality is difficult to observe.

In the context of this paper, warranty legislation can be modeled by a law requiring that warranty be no less than the price. In the cases of perfect competition, monopoly with one consumer type, or the perfectly dicriminating monopolist, such a law has no effect. In perfect competition, sellers offer warranty greater than the price; in the monopoly cases, the seller offers warranty equal to the price. However, when the monopolist uses warranty for screening purposes, warranty for the low quality product is strictly less than the price. In this case, such legislation forces the seller to change his product line.

It turns out that such legislation has an ambiguous effect on social welfare. This can be shown with a simple example with two types of consumers. For simplicity, risk neutrality is assumed. Let $C(q) = q^2/2$, $\theta_1 = .3$ and $\theta_2 = .6$. If $.66 \leq f_1 \leq 1.0$ (recalling that f_1 is the fraction of low type buyers) then it can be shown that the seller changes his product line in the following two ways. First, the law forces him to increase the warranty and thus the quality of the product offered the low type. It can be shown that warranty and quality are increased to the socially efficient level. Second, the seller lowers the price of the high quality good since the low tier product is now more attractive to the high types. The legislation yields the socially efficient quality levels and all the additional surplus goes to the buyers of high quality, (who also receive a transfer of surplus from the monopolist).

If $.50 \leq f_1 \leq .66$, then the outcome is much different. Because the high type buyers make up a larger percentage of the population than in above case, the price cut needed to prevent them from buying the lower quality good leads to a much greater loss in profits. Instead of raising the quality of the low tier good, the seller opts to stop serving the low types. He then raises the price for the high quality good since there is no longer any lower quality good to constrain this. All parties are worse off because of this law! If $0.0 \leq f_1 \leq .5$, then the law has no effect since the seller does not serve the low types in either case[2].

The above analysis is related to Shapiro (1981) which analyzes the welfare effects of a minimum quality standard. Quality is unobservable but premiums for reputation evolve rather than warranties to solve the moral hazard problem of quality production. The market structure is perfect competition. Higher quality standards reduce the short term benefit to a seller of forfeiting his reputation and producing the minimum quality good. It follows that, with a higher quality standard, the premium required to prevent sellers from forfeiting their reputations is lower meaning that prices for high quality goods are lower. Therefore, in his model, higher minimum quality standards always benefit high types. They hurt low types if their prefered quality is less than the minimum standard. In the analysis of this paper, legislation (which in this case concerns warranties) may also benefit consumers of high quality products by lowering the price of these goods. On the other hand, the legislation can hurt the higher types by raising the price of the high quality goods in the cases when the lower quality products are dropped. If a minimum quality standard was imposed in this model instead of a warranty law, there would be an analgous result, (assuming that it is feasible for the government to observe quality).

6. Conclusion

This paper offers one reason for why an automobile dealer typically refuses to offer warranty protection for used cars. It is to discourage potential new car buyers from buying the used models. Because the dealer only does this when he has monopoly power, an implication of the paper is that isolated dealers will tend to provide less warranty coverage for used cars than dealers who face nearby competition. Legislation requiring better warranties for used cars may result in the seller expending greater effort in the preparation of the used car to avoid warranty claims. On the other hand, higher warranties for used cars may cut into new car sales so much that a dealer may discontinue selling used cars.

One area for future research is to generalize the type of product studied. In the present model, the product either works or fails and if it fails the consumer suffers a loss which is not recoverable. A reservation for an airline flight is an example of such a product. If the plane is overbooked there is a loss to the consumer which is not fully compensated for by a ticket for a latter flight. For such products, the warranty must be greater than the the price if the consumer is to be fully insured. For many products, however, the consumer is content if the product is repaired or replaced, and we would expect that the warranty only cover these costs. Also, durables, a type of product commonly associated with warranties, have a life span which may be variable and do not either work or fail as in the model. In practice, warranties for durables tend to cover only a small percentage of the expected useful life of the product. However, if defects tend to surface early on, a short warranty period may be sufficient to ensure that the firm produces a quality product.

Appendix

This appendix contains three preliminary results and the proofs of the results stated in the text.

Lemma A1: If $(x_1, x_2, \ldots, x_n)$ is a solution to (M'), then it satisfies:

(i) $U(x_1, \theta_1) = u(0)$; $U(x_i, \theta_j) = U(x_j, \theta_j)$ for some $i < j$;

(ii) $\pi(x_i) \geqslant 0$;

(iii) $p_i > C'(0)$ for all x_i such that $q_i > 0$.

Proof:

(i) and (ii), left to reader.

(iii) $p_i - C'(0) = p_i - D(0, C'(0)) > p_i - D(q_i, C'(q_i)) = \pi(x_i) \geqslant 0$, where the strict inequality holds because $D(q, C'(q))$ is strictly increasing in q at $q = 0$. Q.E.D.

Definition: The marginal rate of substitiution for the firm is defined as

$$MRS(x,f) = -\frac{\frac{\partial \pi(x)}{\partial q}}{\frac{\partial \pi(x)}{\partial p}} = (1-q)C''(q)$$

Lemma A2: $MRS(x,\theta) \gtreqless MRS(x,f)$ as $\theta \gtreqless C'(q)$.

Proof: As $\theta \gtreqless C'(q)$:

$$MRS(x,\theta) = \frac{u(\theta-p) - u(C'(q)-p) + (1-q)u'(C'(q)-p)C''(q)}{qu'(\theta-p) + (1-q)u'(C'(q)-p)}$$

$$\gtreqless \frac{(1-q)u'(C'(q)-p)C''(q)}{qu'(\theta-p) + (1-q)u'(C'(q)-p)}$$

$$\gtreqless (1-q)C''(q) \quad = \quad MRS(x,f)$$

where the last relation holds because

$$\frac{u'(C'(q)-p)}{qu'(\theta-p) + (1-q)u'(C'(q)-p)} \gtreqless 1 \text{ as } \theta \gtreqless C'(q),$$

by the concavity of u( ). Q.E.D.

Lemma A3: Let x and x' be two contracts. Suppose $p' \geqslant p$ and $q' \geqslant q$ with one of these inequalites strict. If $U(x',\theta^o) \geqslant U(x,\theta^o)$, then $U(x',\theta) > U(x,\theta)$ for $\theta > \theta^o$ and if $U(x',\theta^o) \leqslant U(x,\theta^o)$, then $U(x',\theta) < U(x,\theta)$ for all $\theta < \theta^o$.

Proof: If $q' = 0$ and $p' > p$ then $U(x',\theta) < U(x,\theta)$ for all θ. Suppose that $q' > 0$. Let:

$$\begin{aligned} \Delta(\theta) &\equiv U(x',\theta) - U(x,\theta) \\ &= q'u(\theta-p') - qu(\theta-p) + (1-q')u(C'(q')-p') - (1-q)u(C'(q)-p). \end{aligned}$$

Differentiating yields $\Delta'(\theta) = q'u'(\theta-p') - qu'(\theta-p)$. If $q' > 0$, $q' \geqslant q$, and $p' \geqslant p$ with one of the weak inequalities being strict, then $\Delta'(\theta) > 0$. Q.E.D.

Lemma 1: Every contract in a solution to (M') satisfies:

$$\frac{\partial U(x_i,\theta_i)}{\partial q} \geq 0.$$

Proof: Suppose there is some contract x_i for which $\frac{\partial U(x_i,\theta_i)}{\partial q} < 0$. For some $q' \in (0,q_i)$, $U(p_i,q',\theta_i) = U(x_i,\theta_i)$. Otherwise, by continuity of $U(x,\theta_i)$ in q, $U(p_i,q,\theta_i) > U(x_i,\theta_i)$ for all $q \in (0,q_i)$. This implies $u(C'(0)-p_i) = \lim_{q\to 0} U(p_i,q,\theta_i) \geq U(x_i,\theta_i)$. By Lemma A1(iii), $u(0) > u(C'(0)-p_i)$ so $u(0) > U(x_i,\theta_i)$, which contradicts (VP).

Define a new contract set $(x_1^o,x_2^o,\ldots,x_n^o)$ by

$$x_j^o = x_j, \text{ if } j \neq i,$$
$$q_i^o = q',$$
$$p_i^o = p_i.$$

By equation (3), $\pi(x_i^o) > \pi(x_i)$. This implies that the total profit for the new set of contracts is greater than for the original. By Lemma A3 and the fact that θ_i is indifferent to x_i^o and x_i, the new set of contracts is feasible. This contradicts the assertion that x_i was part of a solution to (M'). Q.E.D.

Proposition 1: A solution $(x_1,\ldots,x_n)$ to (M') satisfies

$$q_i \leq q_i^* \qquad w_i \leq w_i^* = \theta_i \quad \text{for } i < n \text{ and}$$
$$q_n = q_n^* \qquad w_n = w_n^* = \theta_n.$$

Proof: The downward incentive constraints (DIC) imply that if $q_i > 0$ for some i, then $q_j > 0$ for all $j > i$. Let k be the index of the lowest type receiving positive quality. If $k > 1$ then $q_i = 0 < q_i^*$ for $i < k$.

Since $(x_1,\ldots,x_n)$ solves (M'), since $q_i = 0$ for $k < n$, and since the constraint $q_i \geq 0$ is not binding for $i \geq k$, $(x_k,\ldots,x_n)$ must solve

$$\text{Maximize} \quad \sum_{i=k}^{n} f_i \, \pi(x_i) \text{ subject to:}$$

$$\text{(DIC')} \qquad U(x_i,\theta_j) \leq U(x_j,\theta_j) \quad j > i \geq k.$$

$$\text{(VP')} \qquad u(0) \leq U(x_k,\theta_k).$$

Let λ_{ij} be the multiplier on the constraint that type θ_j prefers x_j to x_i, let μ_k be the multiplier on the constraint (VP'), and define μ_j for $j > k$ as

$$\text{(A1)} \qquad \mu_j = \sum_{i=k}^{j-1} \lambda_{ij}$$

The first order conditions for the above problem are:

$$\text{(A2)} \qquad \frac{\partial \pi(x)}{\partial p} - \sum_{j>i} \lambda_{ij} \frac{\partial U(x_i,\theta_j)}{\partial p} + \mu_i \frac{\partial U(x_i,\theta_i)}{\partial p} = 0$$

$$\text{(A3)} \qquad \frac{\partial \pi(x)}{\partial q} - \sum_{j>i} \lambda_{ij} \frac{\partial U(x_i,\theta_j)}{\partial q} + \mu_i \frac{\partial U(x_i,\theta_i)}{\partial q} = 0$$

for $i < n$ and

$$\text{(A4)} \qquad \frac{\partial \pi(x_n)}{\partial p} + \mu_n \frac{\partial U(x_n,\theta_n)}{\partial p} = 0$$

$$\text{(A5)} \qquad \frac{\partial \pi(x_n)}{\partial q} + \mu_n \frac{\partial U(x_n,\theta_n)}{\partial q} = 0.$$

Since $\frac{\partial U(x,\theta)}{\partial p} < 0$ and $\frac{\partial \pi(x)}{\partial p} > 0$, $\mu_i > 0$ for all i. Noting that $\frac{\partial \pi(x)}{\partial p} = 1$, solving (A2) for μ_i , and substituting this into (A3) yields

$$\frac{\partial \pi(x)}{\partial q} - \sum_{j>i} \lambda_{ij} \frac{\partial U(x_i,\theta_j)}{\partial q} - \left[\frac{1 - \sum_{j>i} \lambda_{ij} \frac{\partial U(x_i,\theta_j)}{\partial p}}{\frac{\partial U(x_i,\theta_i)}{\partial p}} \right] \frac{\partial U(x_i,\theta_i)}{\partial q} = 0.$$

for $i < n$. This can be rewritten as:

$$\text{(A6)} \quad MRS(x_i,\theta_i) - MRS(x_i,f) = \sum_{j>i} \lambda_{ij} \left[\frac{\partial U(x_i,\theta_j)}{\partial q} - \frac{\partial U(x_i,\theta_j)}{\partial p} MRS(x_i,\theta_i) \right]$$

By Lemma 1 $MRS(x_i,\theta_i) \geqslant 0$, which means that $MRS(x_i,\theta_j) > MRS(x_i,\theta_i)$ for $\theta_j > \theta_i$. This implies that the bracketed coefficient of each λ_{ij} in (A6) is strictly positive. Since $\lambda_{ij} \geqslant 0$, this implies $MRS(x_i,\theta_i) \geqslant MRS(x_i,f)$, which by Lemma A2 implies that $q_i \leqslant q_i^*$. (A4) and (A5) imply that $MRS(x_n,f) = MRS(x_n,\theta_n)$, which by Lemma A3 implies that $q_n = q_n^*$. Q.E.D.

Proposition 2: For $i < n$ a solution to (M') satisfies:

(i) $U(x_i,\theta_{i+1}) = U(x_{i+1},\theta_{i+1})$;

(ii) $\pi(x_i) \leqslant \pi(x_{i+1})$;

(iii) $p_i \leqslant p_{i+1}$ and $q_i \leqslant q_{i+1}$.

Proof: Suppose the solution does not satisfy all of the above conditions. Let $\underline{i}$ be the smallest i such that one of the above conditions does not hold. It will be shown that all of the conditions must hold for $\underline{i}$ which is a contradiction.

Suppose condition (i) does not hold which means that $U(x_{\underline{i}},\theta_{\underline{i}+1}) < U(x_{\underline{i}+1},\theta_{\underline{i}+1})$. By (DIC), $U(x_j,\theta_{\underline{i}}) \leq U(x_{\underline{i}},\theta_{\underline{i}})$ for $j < \underline{i}$. Since for $j < \underline{i}$, $p_j \leq p_{j+1}$ and $q_j \leq q_{j+1}$, by Lemma A3, $U(x_j,\theta_{\underline{i}+1}) \leq U(x_{\underline{i}},\theta_{\underline{i}+1})$. This implies that $U(x_j,\theta_{\underline{i}+1}) < U(x_{\underline{i}+1},\theta_{\underline{i}+1})$ for $j < \underline{i}+1$ contradicting the fact that some lower contract is a binding constraint (Lemma A1 (i)).

Suppose condition (ii) does not hold which means that $\pi(x_{\underline{i}}) > \pi(x_{\underline{i}+1})$. Offering $x_{\underline{i}}$ to type $\theta_{\underline{i}+1}$ does not violate (DIC) and increases profits, a contradiction.

Suppose condition (iii) does not hold. Then one of the following holds:

(a) $p_{\underline{i}} > p_{\underline{i}+1}$ and $q_{\underline{i}} \leq q_{\underline{i}+1}$,

(b) $p_{\underline{i}} > p_{\underline{i}+1}$ and $q_{\underline{i}} > q_{\underline{i}+1}$,

(c) $p_{\underline{i}} \leq p_{\underline{i}+1}$ and $q_{\underline{i}} > q_{\underline{i}+1}$.

(a) cannot hold since it contradicts $\pi(x_{\underline{i}}) \leq \pi(x_{\underline{i}+1})$. Suppose (b) holds. Since $U(x_{\underline{i}},\theta_{\underline{i}+1}) = U(x_{\underline{i}+1},\theta_{\underline{i}+1})$, by Lemma A3, $U(x_{\underline{i}+1},\theta_{\underline{i}}) > U(x_{\underline{i}},\theta_{\underline{i}})$. Since $\pi(x_{\underline{i}}) \leq \pi(x_{\underline{i}+1})$, replacing type $\theta_{\underline{i}}$'s contract with $x_{\underline{i}+1}$ is also a solution. However, with this set of contracts there is no binding constraint on the utility of type $\theta_{\underline{i}}$, yielding a contradiction. Finally, (c) cannot hold since from Proposition 1, $C'(q_{\underline{i}}) < \theta_{\underline{i}} < \theta_{\underline{i}+1}$ implying

$U(x_{\underline{i}+1},\theta_{\underline{i}+1}) < U(x_{\underline{i}},\theta_{\underline{i}+1})$, which contradicts (DIC). Q.E.D.

Proposition 3: If $(x_1,\ldots,x_n)$ is a solution to (M') then

$$q_i < q_i^* \text{ and } w_i < w_i^* \text{ for } i < n.$$

Proof: As in the proof of Proposition 1 let k be the index of the lowest type with a nonzero contract. It $k > 1$, then the proposition holds for $i < k$ since $q_i^* > 0$ for all i.

From (A1) and the fact that $\mu_{k+1} > 0$, we have $\lambda_{k,k+1} > 0$. It then follows from (A6) that $MRS(x_k,\theta_k) > MRS(x_k,f)$, which shows that $q_k < q_k^*$.

If we show that if $q_i < q_i^*$ and $i+1 < n$ then $q_{i+1} < q_{i+1}^*$, by induction the proposition is proved.

Suppose that $q_i < q_i^*$ and $i+1 < n$. If $q_i = q_{i+1}$ then $q_{i+1} = q_i < q_i^* < q_{i+1}^*$. If $q_i \neq q_{i+1}$ then by Proposition 2(iii), $q_j < q_{i+1}$ for $j \leq i$. Since for $j \leq i$, $U(x_j,\theta_{i+1}) \leq U(x_{i+1},\theta_{i+1})$, by Lemma A3 $U(x_j,\theta_{i+2}) < U(x_{i+1},\theta_{i+2})$. It follows that $U(x_j,\theta_{i+2}) < U(x_{i+2},\theta_{i+2})$ for $j \leq i$, which implies

that $\sum_{j=k}^{i} \lambda_{j,i+2} = 0$. But since $\mu_{i+2} = \sum_{j=k}^{i+1} \lambda_{j,i+2} > 0$,

$\lambda_{i+1,i+2} > 0$. From (A6) this implies that $MRS(x_{i+1},\theta_{i+1}) > MRS(x_{i+1},f)$. Q.E.D.

Footnotes

[1] Spence is mostly interested in cases in which the consumer does not make correct inferences from the warranty. In these cases he finds that regulation may be welfare improving.

[2] If quality were observable in the example, the law would have no effect. Warranty would be raised and the price would be increased by the expected value of the warranty increase till warranty equaled price.

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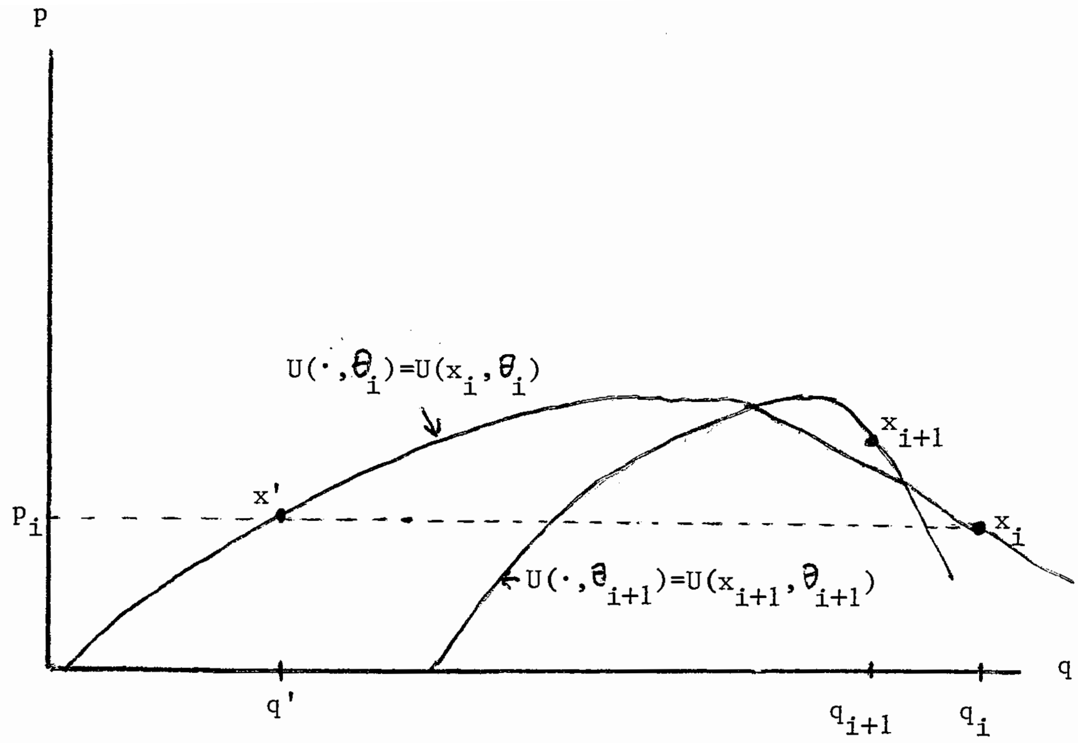

FIGURE 1

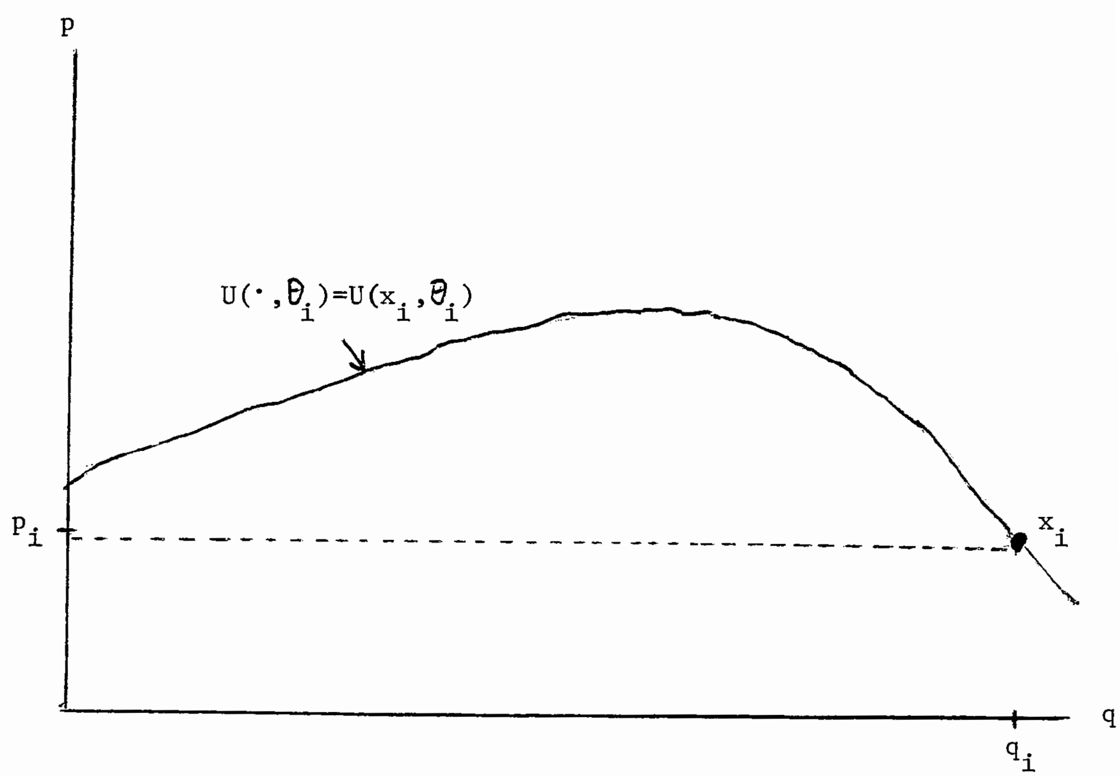

FIGURE 2