This paper reviews the development of public and private pensions in response to missing markets for providing insurance for consumption given uncertain lifetimes. A simple life-cycle model is used to demonstrate that even an actuarially fair, fully funded social security system can decrease national saving. Constrained access to publicly provided pension annuities provides an impetus to the growth of private pension annuities. When initial endowments are considered, the large partial equilibrium saving effects are mitigated for subsequent generations. Consideration of the welfare gains from introducing pension annuities requires an analysis of the tradeoff between benefits to early participants from access to the annuities and the costs to generations that follow a lower capital stock. Finally, empirical issues with respect to interpretation of econometric estimates of reductions in individual saving attributed to pensions are addressed.

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I. INTRODUCTION

Research on saving for old age and attempts to measure the impacts of pensions on household saving have occupied much of the literature in empirical public finance over the past decade. Identifying the channels through which pensions affect the intertemporal consumption decision can help to distinguish among motives for saving (e.g., for retirement consumption or for bequests) and help to explain empirical findings of the relationship between wealth and lifetime earnings. Proper quantification of the effects of pensions on saving is important for analyses of intergenerational equity, bequests and income distribution, and tax policy and saving.

Much of the discussion of the impact of pensions on non-pension saving has dealt with the social security system, beginning with the studies of U.S. time series by Feldstein (1974). The theoretical arguments of Feldstein (see also Barro, 1974, 1978) have concentrated on the funding status of social security, that is, the degree to which an unfunded social security system decreases national saving.2

Given this theoretical focus, it is not surprising that empirical tests of the effects of pensions on wealth accumulation have typically been conducted in the perfect certainty version of the life-cycle model (Modigliani and Ando, 1957; Modigliani and Brumberg, 1954).3 In that approach, pensions alter individual saving through their impact on individual intertemporal budget constraints. Disposable income falls by the amount of the contributions. With respect to social security, to the extent that the present value of benefits exceeds the present value of the taxes paid, an increase in lifetime resources is generated, raising consumption in all periods.
This paper focuses on the distinction of precautionary saving against uncertainty over length of life in the life-cycle framework and on the insurance aspects of social security and private pension annuities. The development of public and private pensions is examined in response to missing markets for providing insurance for consumption given uncertain lifetimes. The model of section II demonstrates that even an actuarially fair, fully funded social security system can reduce individual saving by more than the payroll contributions. Hence, previous partial equilibrium estimates of the impact of social security on saving measured only with respect to the intergenerational wealth transfer at the introduction of the system are, if anything, too small.\textsuperscript{4}

A related finding stems from the fact that under current U.S. law, social security taxes and benefits are calculated only up to an earnings ceiling. High-income individuals have incomplete access to the social security annuity system. Hence, even in the absence of an explicit bequest motive, the ratio of wealth to lifetime earnings could rise with the level of lifetime earnings. Constrained access to publicly provided pension annuities may provide an impetus to the growth of private pension annuities. This potential "annuity rationing" provides theoretical support for integration of private pension and social security benefit formulas.

Section III presents individual wealth-age profiles given uncertain lifetimes and public and private pension annuities. The large partial equilibrium saving effects of section II are mitigated for succeeding generations when initial endowments are considered. Unplanned bequests, which arise in the model because of lifetime uncertainty, provide an intergenerational link for saving decisions. For example, to the extent
that the introduction of social security reduces the size of accidental bequests, the net effect of social security on the consumption of subsequent generations is diminished. The tradeoff between benefits to early participants from access to the annuities and costs to generations that follow of a lower capital stock must be examined to consider the potential welfare gains from the introduction of pension annuities.

The fourth section addresses empirical issues arising from the wealth accumulation models derived in the paper with respect to the interpretation of econometric estimates of reductions in individual saving attributed to pensions. Using a model specification from the empirical literature, offsets are interpreted according to the presence or absence of a bequest motive and according to the ability of individuals to adjust participation in private pensions to counteract involuntary changes in social security.

Some conclusions and directions for future research are given in section V.

II. THE IMPACT OF PENSIONS ON LIFE-CYCLE SAVING

A. Consumer Saving Decisions

The solution to an economic agent's intertemporal consumption problem subject to a lifetime resource constraint involves the equalization of expected marginal utilities of consumption across time. Otherwise, an increase in consumption at one point in his life at the expense of consumption at another time would raise lifetime utility, indicating that the initial allocation was suboptimal. The introduction of uncertainty generates a demand for insurance to diversify risks.
Where insurance markets are incomplete or missing, the first-best optimum may be unattainable.

The type of uncertainty considered here is that over longevity; agents do not know when they will die. Yaari's (1965) seminal paper showed that with an uncertain lifetime, intertemporal utility maximisation can dictate saving for the possibility of living longer than the expected lifetime to avoid deprivation in old age (excessively high marginal utility of future consumption). That excess saving can be large. Kotlikoff and Spivak (1981, p. 379) found that for plausible underlying parameter values, the present expected value of unintended bequests represented almost 25 percent of initial wealth for a single male aged 35.

We begin with a simple life-cycle model of consumption. Agents are assumed to be selfish, in the sense that no bequests are desired. The retirement age $0$ is taken as exogenous, and individuals live $0$ periods for certain. The probability of having died in the interval $(0, t)$, is $p_t$ for each $t$; by assumption, $p_t$ is equal to zero in the interval $(0, 0)$. Individuals have an expected lifetime of $D$ years, with $D' > D$ being the maximum age to which one can survive. Individuals supply labor inelastically, and receive a gross wage $w_t$ in each period $t$ during their working life; wages are assumed to grow over the working period at a constant rate $g$, and are taxed at rate $f$.

Following Yaari (1965) and Barro and Friedman (1977), let utility be additively separable, and let $U(C_t)$ be evaluated contingent on being alive at time $t$. That is, the consumer's intertemporal choice model is given by
\[
\max_{t=0}^{\infty} \sum_{t=0}^{\infty} \left( 1 - p_t \right) U(C_t) (1+r)^{-t}
\]

subject to \(\sum_{t=0}^{\infty} C_t (1+r)^{-t} = K_0 + (1-\delta) \sum_{t=0}^{\infty} \frac{C_{t+1}}{1+\delta} \),

where \(C\), \(\delta\), and \(r\) represent consumption and the (constant) subjective discount rate and real interest rate, respectively. \(K_0\) represents initial resources from unplanned bequests from the previous generation.

Carrying out the optimization in (1) assuming \(U(c) = \frac{1}{\gamma} c^\gamma\) yields an optimal consumption stream of

\[
C_t = C_0 \left( \frac{1 + r}{1 + \delta} \right)^{\gamma/(1-\gamma)} \left( 1 - p_t \right)^{1/(1-\gamma)},
\]

where

\[
C_0 = \frac{K_0 + (1-\delta) \sum_{t=0}^{\infty} \left( \frac{1 + r}{1 + \delta} \right)^t}{\left( \frac{1 + \delta}{1 + \delta} \right)^{\gamma/(1-\gamma)}},
\]

and

\[
EPV_t = (1+r)^{-\gamma/(1-\gamma)} (1+r)^{1-\gamma} (1-p_t)^{1-\gamma}.
\]

The extent to which uncertainty over length of life affects the stream of consumption depends on agents' degree of relative risk aversion, \(\gamma\), the transformation of \(\gamma\), the elasticity of the marginal utility function. The higher is an individual's degree of relative risk.
aversion (or, equivalently, the lower is his intertemporal elasticity of substitution in consumption), the slower will his consumption grow over time.

B. The Introduction of Social Security

Access to a fair annuity market could remove the influence of lifetime uncertainty on consumption. Individuals could exchange a portion of their labor income when young to smooth consumption in old age. If all individuals were identical in terms of their probabilities of survival, then a competitive equilibrium in the provision of fair annuities would be possible. Since the individual deaths are presumably independent, there would be no social cost to the risk associated with any single individual's annuity, and annuities would be actuarially fair in the competitive equilibrium.

The existence of a competitive equilibrium may, however, be precluded by asymmetries of information between individuals and insurers, since individuals have better information concerning their life expectancy. This is, of course, the familiar "adverse selection" phenomenon discussed by Rothschild and Stiglitz (1976) and Wilson (1971). They found that the competitive outcome may be inefficient, in that the imposition of a common contract in addition to the competitively supplied contracts may be Pareto-improving. Eckstein, Eichenbaum, and Peled (1987) have interpreted this compulsory additional contract as social security. There may be additional "moral hazard" or "free-rider" barriers to the existence of an annuities market. If
individuals conjecture that the state will support them in deprivation, the need to purchase annuities is diminished.

Public provision of the annuities through public pensions is one possibility. Moral hazard problems still make voluntary participation difficult. Consider a compulsory social security program of the following form. Individuals pay a payroll tax at rate \( t_b \) on gross wages, from which the social security system is funded (i.e., in which contributions are invested and earn the market interest rate \( r \) in each period). During retirement they receive annuity benefits \( S_t \) in each period \( t \) until death. The budget constraint in (1) becomes

\[
\sum_{t=0}^{D'} C_t (1+r)^{-t} = K_0 + (1-g) t_b \sum_{t=0}^{Q} \frac{1}{(1+r)^t} + \sum_{t=Q+1}^{D'} S_t (1+r)^{-t},
\]

If benefits are set according to a replacement rate of the terminal wage, then the economy-wide actuarially fair benefit \( S \) satisfies the condition that

\[
\sum_{t=Q+1}^{D'} (1-p_t) (1+r)^{-t} = \sum_{t=0}^{Q} \omega_t \frac{1+r}{1+r}^t.
\]

Substituting the actuarially fair social security benefit into the budget constraint in (4) yields

\[
\sum_{t=0}^{D'} C_t (1+r)^{-t} = K_0 + (1-g) t_b (w-1) \sum_{t=0}^{Q} \frac{1+r}{1+r}^t,
\]

where \( w \) arises because of the difference in discount rates under certainty and uncertainty and is equal to
\[ \frac{D'}{\sum_{t=1}^{Q+1} (1+r)^{-t}} / \frac{D'}{\sum_{t=Q+1}^{n} (1-p_r)(1+r)^{-t}} \]

Since \( \omega \) is greater than unity, the system generates an increase in lifetime resources. Not that this increase in resources occurs even in a system which is actuarially fair and fully funded. In reality, the initial cohorts participating in social security received a rate of return greater than the actuarially fair return (see Hurd and Shoven, 1983). The point here is that because of the insurance feature of retirement annuities, the impact of social security on individual saving does not depend exclusively on such initial transfers. 

Depending on assumptions about the real interest rate and the social security payroll tax rate, the percentage increase in lifetime resources generated by an actuarially fair social security system can be large. Using actual data for survival probabilities for the U.S., when \( r=0.06 \) and \( t_e = 0.10 \), a 16 percent increase in lifetime resources is afforded by an actuarially fair social security system. Because the system generates an increase in lifetime resources, individual saving is reduced by more than the amount of the tax paid.

The existence of a social security annuity system does not guarantee that there will not be excess demand for old-age annuities. Suppose for example that not everyone has equal access to the retirement annuities provided by social security, and that effective participation is higher for low-income individuals than for high-income individuals. Let \( \bar{w} \) represent the ceiling on taxable income; the growth rate of the taxable wage base and the determination of the replacement rate are as before. The budget constraint in (6) then becomes
\[ \sum_{t=0}^{D^i} c_t (1+r)^{t-t} = k_0 + \sum_{t=0}^{Q} (1+g)^{t_{0}(\omega-1)}w^t_{0}(1+\omega)^{t-t} \]

where \( t_0 \) is equal to \( t_0 \left( \frac{\bar{w}}{\omega} \right) \). The impact of social security on an individual's lifetime resources depends on his income. As an annuity, social security administered in this way generates a smaller reduction in saving for high-income people than for low-income people.

C. Social Security as Incomplete Insurance—Development of Private Pensions

To the extent that high-income individuals (those for whom \( w_0 > \bar{w} \)) are constrained to less than their desired participation in social security, there is excess demand for annuities. Adverse selection and the possibility of multiple insurance\(^{16}\) still render unlikely the provision of such annuities by competitive insurance companies. Employer-sponsored private pension funds may act to fill this gap. Employers are likely to have better information on individual workers' life expectancies than would a disinterested insurance company. Second, by definition, such annuities can only be purchased at an individual's place of work; multiple insurance is not possible. Finally, the pension instrument may provide an added degree of freedom for the firm in influencing worker behavior.\(^{15}\)

The tax treatment of pension plans is an important consideration. Social security taxes are levied on gross earnings, and prior to the 1983 amendments to the Social Security Act, benefits were not considered taxable income. For private pension plans, employer contributions are a deductible business expense and are not regarded as taxable income to employees until benefits are paid. Pension fund earnings accumulate
tax-free until disbursement. Upon distribution, taxes paid on benefits are presumably less than corresponding wage tax payments, since earnings (and hence tax rates) are lower in retirement. Moreover, special retirement income credits further diminish effective tax rates on pension benefits.

At this point, it is assumed that covered workers take their participation in plans as given; the implications of relaxing that assumption will be discussed later. For simplicity, let $P$ be the actuarially fair pension benefit in retirement (determined by the product of a replacement rate and the terminal wage) corresponding to an implicit reduction in wages at rate $t_p$.\footnote{16}

In the context of this model, the worker bears only $(1-\theta)t_p$ of the wage reduction, where $\theta$ is the marginal income tax rate. Benefits are taxed at rate $\hat{\theta}$, where $\hat{\theta} > \theta$. We introduce a parameter $\beta$ to measure the extent to which benefits received are actuarially fair. That is, an actuarially fair pension benefit $P$ can be constructed just as in the case of social security annuity benefits in equation (5). Benefits received are equal to $\beta P$, where $P$ solves

$$
(8) \quad P \sum_{t=Q+1}^{D'} (1 - p_t)(1 + r)^{-t} = t_p \sum_{t=0}^{Q} w_t \left( \frac{1 + r}{1 + \hat{\theta}_t} \right)^t.
$$

For received annuity payments to be actuarially fair, it must be the case that $\beta = 1$; less-than-fair benefits are associated with $\beta < 1$.\footnote{16}
Given participation in social security, the budget constraint in (7) can be rewritten as

\[
(9) \quad \sum_{t=0}^{\infty} c_t (1+r)^{-t} = \frac{K_0^+}{(1-\delta)^{1-\delta}} \sum_{t=0}^{\infty} \psi_0 (1+p)^{-t} \left( \frac{t}{(1-p)} \right) + \frac{D^0}{(1+p)^{-t}} \sum_{t=0}^{\infty} \psi_0 (1+p)^{-t} \left( \frac{t}{(1-p)} \right) \]

\[
= K_0^+ \frac{(1-\delta)^{1-\delta}}{\delta} \sum_{t=0}^{\infty} \psi_0 (1+p)^{-t} \left( \frac{t}{(1-p)} \right) + \frac{D^0}{(1+p)^{-t}} \sum_{t=0}^{\infty} \psi_0 (1+p)^{-t} \left( \frac{t}{(1-p)} \right) \]

As shown before, \( w > 1 \). As long as \( \delta \) is close to unity, for any reasonable assessment of the relationship between \( \delta \) and \( \delta^2 \), \( (1-\delta)^{(1-\delta^2)} > 1-\delta^2 \).

This is certainly true for the estimated tax rates used by the Treasury in calculating the tax expenditure associated with pension tax subsidies, namely \( \delta = 0.23 \) and \( \delta^2 = 0.115 \) (see Vannelli, 1982, p. 44 for details). Because of the tax deductibility of pension contributions, even in a world of certainty over longevity (\( w=1 \)), a funded private pension can still generate an increase in lifetime resources for the individuals.

The tax treatment of pension contributions reinforces the role of private pension annuities in alleviating the rationing of public annuities. The effective contribution rates (participation rates) in the public and private pension systems both depend on the income of the
individual. Recall that \( \hat{t} = \tau \frac{\bar{w}}{w_0} \), where \( \bar{w} \) is the ceiling on taxable earnings. Under a progressive tax system, the marginal tax rate also depends on income (i.e., \( \tau (w_0) > 0 \)). Hence for given (assigned) nominal participation rates in social security and private pensions, high-income individuals receive a greater effective increase in lifetime resources from private pensions of the sort described here. This effect may be desirable if one reason for the private pension system is to supplement the rationed access to social security annuities for high-income workers.

IV. PENSIONS AND THE WEALTH-AGE PROFILE

A. Individual Saving Behavior

An important finding from the above analysis is that uncertainty over length of life alters the expected impact of compulsory social security annuities and private pension annuities on the level of non-pension wealth. We can use the earlier derivation of the impact of exogenous pensions on saving to consider wealth accumulation over the life cycle. For any time \( t \), the present value (at time 0) of an individual's accumulated stock of wealth, \( K_{0t} \) (i.e., the present value of the "accidental bequest" of an individual who died in period \( t \)), can be expressed as

\[
K_{0t} = K_0 + \sum_{t=0}^{\infty} (1+c)^{-t} \left( (1-\theta)_{[t]} \right) (1-\tau_p) w_t + S_t + P_t - C_t.
\]

Wages and retirement benefits (from social security and private plans) are the sources of income to the individual. \( w_t \) is zero in the
interval \([0^+, D')\), and \(S_e\) and \(P_L\) are zero in the interval \([0,0]\). Using the expressions derived before for \(w_L\), \(S_e\), \(P_L\), and \(C_L\) and denoting the present values of lifetime labor income, social security taxes, and implicit wage reductions to finance private pensions by \(V_L\), \(V_P\), and \(V_S\), respectively, we can construct wealth-age profiles relative to lifetime earnings. That is,

\[
(11a) \quad \frac{K_{Dt}}{V_L} = \frac{K_0 + (1 - \theta - \tau_s)w_O(V_L)}{V_L} \sum_{t=0}^{c} \frac{(1+r)^{t-1}}{D_{t}}, \quad t \in (0, D'),
\]

and

\[
(11b) \quad \frac{K_{DL}}{V_L} = \frac{(1 - \theta - \frac{V_S}{V_L})}{1 - \frac{V_P}{V_L}} + \frac{V_S}{V_L} \sum_{t=0}^{c} \frac{(1+r)^{t-1}}{D_{t}}, \quad t \in (0, D'),
\]

The ratio \(K_{DL}/V_L\) tracks an individual's accumulated stock of assets relative to lifetime earnings. With no uncertainty over longevity, \(K_{Dt}/V_L\) is simply a function of age, and the results of the basic life-
cycle model are reproduced, as the present values of pension contributions and benefits are equal. Considering first the case of social security alone, with lifetime uncertainty, wealth is still built up relative to earnings during the working period, but the rate at which consumption draws down accumulated wealth depends on survival probabilities and relative risk aversion. Because actuarially fair social security annuities generate an increase in lifetime resources, lifetime consumption rises. Much of this increase in consumption comes during an individual's working life, as the need to save for retirement is reduced.

The problem becomes more complicated when the insurance coverage provided by social security is not the same across individuals. Again, suppose that there is a ceiling on the level of earnings against which payroll tax rates and replacement rates are calculated. If that ceiling is $\overline{w}$ in the initial period and grows at the same rate as the wage base, then the effective tax rate is $\tau_s = t_s \frac{\overline{w}}{w_0}$. From equation (11), the ratio of wealth to lifetime earnings should rise with the level of lifetime earnings (at a decreasing rate). This finding has surfaced in some recent empirical studies of the impact of social security on saving (see for example Diamond and Hausman, 1982; and Hubbard, 1983). This relationship between saving rates and lifetime earnings occurs in the absence of any explicit bequest motive. As indicated earlier, the existence of private pensions amatiates this effect. The implications of these points for studies of the relationship between bequests and lifetime resources will be discussed in section IV.

The addition of private pension annuities complicates the evaluation of the effect of a change in compulsory social security
holdings on non-pension wealth. Suppose that individual participation in private pension annuities is not invariant to changes in social security annuities. Let \( \psi_{pS} \) represent the magnitude of that adjustment, i.e.,

\[
\psi_{pS} = \frac{dV_p}{dV_S}.
\]

Then from equation (11a), the impact of a change in social security wealth on the non-pension wealth of a non-retired individual is

\[
\frac{dK_{0t}}{dV_S} = \left[ w - 1 + \psi_{pS} ((1 - \delta)\bar{w} - (1 - \delta - \tau_S)) \right] \left[ \sum_{t=0}^{T} \frac{RPV_I}{P_I} \right] \left( \frac{1}{P_I} \right).
\]

If \( \psi_{pS} = 0 \), then the impact of a change in holdings of social security annuities has the same influence on lifetime resources as before. When \( \psi_{pS} < 0 \) (i.e., increases in involuntary social security annuitization can be at least partially undone through changes in private pension participation), the impact of social security on individual wealth accumulation will also depend on the extent to which private pension annuities are actuarially fair (i.e., on the value of \( \delta \) and on the tax advantages of pensions as compensation.

When coverage by social security is higher for low-wage earners than for high-wage earners, we can use equation (13) to examine the impact on non-pension wealth of change in the social security payroll tax rate (index of participation). First, since the effective tax rate

\[
\tau_S = \tau_S (\bar{w}/w_0),
\]

a given increase in the nominal tax rate translates into a smaller increase in \( V_S \) (and, ceteris paribus, a smaller displacement
of non-pension wealth) for high-income workers (for whom $w_0 > \bar{w}$) than for low-income workers (for whom $\bar{w} > w_0$). When private pension participation is responsive to changes in social security annuity holdings (i.e., when $\phi_{ps} < 0$), then for a given offset factor $\phi_{ps}$, high-income individuals receive a smaller total offset than low-income individuals.\(^7\)

**B. Long-Run Effects on Individual Saving**

Given uncertainty over length of life, actuarially fair pensions can reduce individual saving by more than the accumulated contributions. For plausible underlying assumptions about individual discount rates, survival probabilities, and the intertemporal elasticity of substitution in consumption, the magnitude of that reduction is substantial. The partial equilibrium conclusion is clear — estimates of the reduction in individual saving brought about by social security which focus only the extent to which the system delivers a present value of anticipated benefits greater the present value of taxes paid are, if anything, an underestimate. It is also important, however, to address the links among generations provided by unplanned bequests.

Consider the case of social security in the absence of private pensions. An initial bequest from an early death of a parent raises a beneficiary's consumption relative to lifetime earnings. In the model, the size of that bequest depends on the testator's coverage by social security and his age at death. By facilitating greater consumption out of lifetime earnings, social security reduces the accidental bequest. On that account, the initial resources available to the heir are
lower. Even within the partial equilibrium analysis, the impact of social security on the consumption and saving patterns of individuals in a given generation depends on the balance between the effective increase in lifetime resources made possible by access to a fair annuity and the reduction in inheritances because of that impact on the savings of the previous generation.

To see this more clearly, note that for an individual receiving an accidental bequest from a "parent" who died at age \( t \) in the interval \((0=1, D')\), the reduction in the bequest because of the parent's participation in social security is\(^{18}\)

\[
\frac{dK_t}{dS} = (1 + r)^t \left[ -1 + \frac{\sum_{i=0}^{t} (1+r)^{-i} \frac{\sum_{i=0}^{t} \text{EPV}_i}{\sum_{i=0}^{t} (1 - p_t)(1+r)^{-i}}}{\omega} \right].
\]

The role of family mortality history is important; individuals whose ancestors died early receive larger bequests relative to those whose "parent" lived a long time.

Members of the first generation to participate in social security benefit both from the bequests from the (uninsured) previous generation and from the gains from participation in the social security annuity system. The reduced value of accidental bequests permits smaller consumption gains for succeeding generations. While it is true that social security reduces individual saving to a lesser degree in the
generations after its introduction, there is still a reduction in the
long-run capital stock. Ultimately, to consider the potential welfare
gains from pension annuities, the tradeoff between the benefits to early
participants from access to the annuities and the costs to generations
that follow of a lower capital stock must be examined.

C. General Equilibrium Effects of Pensions on the Capital Stock

The partial equilibrium effects of pensions on individual saving
will be dampened in a general equilibrium analysis.\textsuperscript{19} The reduction in
individual wealth accumulation brought about by pension annuities will
induce changes in factor returns, exhibiting both income and
substitution effects on consumption. A higher real interest rate
decreases lifetime resources; in addition, a higher rate of interest
reduces the price of consumption in old age.\textsuperscript{20}

While detailed general equilibrium simulations are not performed
here, some simple calculations illustrate the basic points outlined
above. Suppose output is produced according to a Cobb-Douglas
production function in capital and effective labor, with a capital share
of one third. Factor markets are assumed to be competitive, so that
capital and labor are paid their marginal products. Again, labor is
inelastically supplied, and labor-augmenting technical change is assumed
to occur at a constant rate of 2 percent; let the population growth rate
be 1 percent. The individual optimization problem is assumed to be
parameterized by $r=0.06$, $\delta=0.03$, and $\gamma=1.00$. These assumptions
produce an average propensity to consume out of total income of about
0.82.\textsuperscript{21}
A fully funded, actuarially fair social security system with $t_s=0.10$ reduces the capital stock by about 60 percent, implying an increase in the interest rate of 40 percent and a reduction in output of about 20 percent. Those changes are, of course, upper bounds to the true steady-state changes, as both the saving rate and the effective increase in lifetime resources afforded by social security (indexed by $w$) are sensitive to the interest rate. The calculations do, however, point up the need to consider in welfare comparisons both the increase in the propensity to consume made possible by social security and the effects on consumption of the reduction in output accompanying a smaller capital stock. Access to the social security annuities facilitates an increase in the average propensity to consume (out of total income) of about 16 percent. Because of the fall in output, consumption per capita actually falls in the new steady state. If the output-reducing effect were large enough, lifetime welfare of a representative agent could actually decline in the new steady state following the introduction of social security.

These conclusions are obviously sensitive to the assumption of a lack of private provision of annuities. Private pension annuities, however, not only expand the effect of annuitization on the capital stock, but also affect the impact of social security on saving. That annuity markets are extremely imperfect in the real world is not evidence per se of a severe market failure, as individuals have some control over their participation in private pensions either explicitly (for participants in defined-contribution plans) or implicitly (through choice of employer). To the extent that individuals adjust their private pensions for variation in social security annuities, the
effective annuity market may be significant. The next section addresses these points in the context of interpreting econometric measures of the impact of pensions on individual wealth accumulation.

IV. EMPIRICAL ISSUES

Gathering econometric evidence of the impact of social security and private pension annuities on household saving in the context of lifetime uncertainty entails estimation of the wealth profiles consistent with equation (11). Two issues that have received attention in empirical work on individual saving behavior are the influence of pension wealth on the level of non-pension saving and the possibility of differing saving rates across earnings classes. The discussion of precautionary saving against lifetime uncertainty yields testable hypotheses for those issues that differ from those of the certainty model. Most previous empirical examinations of the impact of social security on non-pension wealth have employed specifications similar to:

\[
\left( \frac{W}{Y} \right)_t = g(A)_t + f(\text{SSU}) - s_p \left( \frac{\text{FFW}}{Y} \right)_t + \gamma T_t + \epsilon_t.
\]

Anticipated pension benefits are divided into two components, social security (SSU) and private pensions (FFW), to allow for different effects on saving. \(s_p\) and \(s_p\) are coefficients to be estimated. \(f\) is a function of age. Finally, the function \(g\) can be specified to test the nonlinearity in income of the ratio of wealth for permanent income.24

The appropriateness of forms (16) used in empirical studies depends on the structure of annuity markets and on whether or not a bequest
motive exists. The basic model presented earlier assumes complete market failure is the private provision of annuities and the absence of a bequest motive. Theoretical possibilities encompass assumptions along the dimensions of "perfectness" of private annuity markets and the presence or absence of a bequest motive.

For example, given the assumption of market failure in the provision of non-pension annuities, four potential cases can be considered along the two dimensions of (i) bequest motives and (ii) discretion in private pension participation. As a first case, suppose that there is no bequest motive and that private pension participation is exogenous to individual decisions. The reduction in non-pension wealth of a chance is compulsory social security annuities corresponds to the level described earlier; that is, the present value of anticipated (actuarially fair) social security benefits should displace non-pension wealth by more than dollar for dollar (in the absence of capital market restrictions). If effective replacement rates are nonlinear in earnings, high-income individuals are rationed in their access to social security annuities, and savings rates will rise with the level of permanent income.

Second, suppose that while there is no bequest motive, private pension participation is completely under individual control. In the limit, if private pension annuities are also actuarially fair (R=1 in equation (9)), there would be no restricted access to fair annuities, and individual saving rates would be independent of the level of lifetime earnings. Involuntary increases in compulsory annuities (social security) would be completely reflected in reduced holdings of private pension annuities and not in the level of non-pension wealth.
For intermediate versions of this second case, both a smaller offset to non-pension wealth from a change in social security benefits and a smaller effect of lifetime earnings on saving rates would be expected relative to the first case.

The existence of a bequest motive changes the predicted effect of changes in compulsory social security annuities on the level of non-pension wealth and complicates the distinction of "annuity rationing" effects from the data. The third and fourth cases embody the sort of "bequest motive" described above, evidenced by levels of non-pension wealth relative to permanent income that rise with permanent income.25

The third case is described by the existence of an operative bequest motive in conjunction with discretionary private pension participation. In this case, involuntary changes in social security participation will have no impact on non-pension wealth; the changes are counteracted by offsetting movements in private pension holdings. With discretion in pension participation, there is no restriction of annuity purchases, so that a nonlinear relationship between saving rates and lifetime earnings is traceable to the desire to leave bequests.

The fourth case combines a bequest motive with exogenous participation in private pensions. Again, the reduction in non-pension wealth attendant to an increase in holdings of social security annuities will be less than in the first case. An observation that saving rates out of permanent income increase with permanent income could reflect a combination of a bequest motive and rationed access to pension annuities.

The cases are summarized with respect to interpretations of the offset parameter α and nonlinearity of the ratio of non-pension wealth
to permanent income with respect to permanent income in Figures 1 and 2 below. Note that the predicted effects of changes in social security wealth and of changes in permanent income on individual wealth accumulation depend greatly on assumptions about bequest motives and on the size of the effective private annuity market afforded by access to private pensions. In reality, of course, the degree of discretion in private pension annuity holdings can vary anywhere between "none" and "complete." Estimation of the impact of changes in compulsory social security annuities on holdings of private pension annuities (e.g., equation (16) above) can help to allocate observed nonlinearities of saving rates with respect to the level of earnings between annuity rationing and bequest motives.
IV. CONCLUSIONS AND IMPLICATIONS

Assessing the impact of pensions on individual wealth accumulation is important for analyses of tax policy and saving, bequests and income distribution, and intergenerational equity. Previous research in the spirit of Feldstein (1974) has considered the funding status of social security and pensions. The emphasis here is on insurance features of
pension annuities with respect to the problem of uncertainty over length of life.

Section II of the paper considers the introduction of social security in an economy with market failure in the private provision of annuities. The principal findings are two. First, in such a world, even an actuarially fair, fully funded social security system can substantially reduce individual saving, though individual welfare is initially improved. Hence, partial equilibrium estimates of the impact of social security on saving which rely solely on the extent to which individuals earn a more than fair return on social security are underestimates of the true effect.

Second, constrained access to publicly provided pension annuities may provide an impetus to the growth of private pension annuities. A consideration of private pension annuities in an insurance framework is also presented in section II, along with implications for the tax treatment of private pensions and for the integration of social security and private pension benefits.

The third section demonstrates that the partial equilibrium impact of social security and private pension annuities on individual saving in a given generation is reduced when initial endowments are considered. For example, to the extent that the introduction of social security reduces the size of accidental bequests, the net effect of social security on the consumption of subsequent generations is mitigated. In addition, general equilibrium considerations can be expected to reverse part of the partial equilibrium impact. Because of these two considerations, the impact of social security on the steady-state capital stock is smaller than the partial equilibrium impact.
Ultimately, consideration of the welfare gains from compulsory social security requires an examination of the tradeoff between the benefits to early participants from access to the annuities and the costs to generations that follow of a lower capital stock.

To provide an interpretation of econometric measures of the impact of pensions on non-pension saving, two additional considerations are important. Theoretical possibilities encompass assumptions along the dimensions of "perfectness" of private annuity markets (in this case, the ability to adjust private pension participation in response to involuntary changes in social security annuities) and the presence or absence of a bequest motive.

Two immediate extensions to the models presented here are left as tasks for future research. First, additional research is needed on private annuity markets to determine the actual extent of market failure. Second, given the current political environment, introducing uncertainty over future social security benefits may be appropriate. That uncertainty would reduce the wealth impacts derived here.

Researching the relationships among social security, private pensions, annuity markets, and bequests facilitates close empirical scrutiny of models of individual and aggregate saving, permitting consideration of the welfare effects of compulsory pensions. In addition, while this paper has concentrated on annuity insurance, similar approaches could be used to study the impacts of other social insurance programs on national saving.
FOOTNOTES

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1Earlier studies for private pensions include those of Cagan (1965), Katona (1964), and Munnell (1974). Feldstein's results have by no means gone unchallenged; see for example LAIwer and Leanoy (1982) and the reply in Feldstein (1982). Microeconomic (cross-section) evidence has generally been supportive of the proposition that social security has reduced individual saving. See Feldstein and Pellechio (1979), Kotlikoff (1979b), Blinder, Gordon, and Wise (1981), Diamond and Hausman (1982), King and Dicks-Mireaux (1982), and Hubbard (1983).

2Empirical tests of the life-cycle model under certainty have tested the hypothesis of a hump-shaped wealth-age profile, but results have by no means unambiguously validated the model. See for example White (1978), Miller (1979), and Kurz (1981). Even after controlling for the effects of permanent income, Blinder, Gordon, and Wise (1981), Diamond and Hausman (1982), King and Dicks-Mireaux (1982), and Hubbard (1983) found results only mildly supportive of the basic theory. Other studies have addressed the possibility of their motives for saving. Kotlikoff and Summers (1981) retested the ability of the life-cycle model to explain wealth accumulation in the U.S., putting forth a major role for bequests.

3The underfunding hypothesis has also figured prominently in the examination of the impact of private pensions on national saving. To the extent that shareholders do not perceive the liability incurred from unfunded pension entitlements, national saving is reduced. See for example Feldstein and Seligman (1981), Feldstein and Mënich (1983), and Rugoff, Mënich, and Summers (1984).

4The intergenerational consequences of this point are taken up by Abel (1983), with the implication that the insurance feature of the social security system may reduce inequality in the distribution of wealth.

5The precise direction of the influence of this uncertainty for saving is unclear. Heightened uncertainty over the length of life may lead to more saving (because of a longer than expected lifetime) or to less saving (to maintain present consumption). In the argument of Yaari (1965), two individuals with identical tastes, income, and investment opportunities are compared. The difference between them is that one lives T periods for certain
while the other faces an uncertain lifetime of \( t \) periods, up to a maximum of \( T \) periods. Given a shorter expected life, uncertainty over length of life unambiguously leads to increased initial consumption. Campermane (1969) and Levhari and Mirman (1977), on the other hand, consider two agents with identical expected lives, but differing the distribution of length of life. In either case, the impact of uncertainty over the length of life on wealth accumulation of a risk-averse individual is ambiguous and depends on the parameters of the model.

6 Individuals will die on average prior to reaching age \( D' \), but the lifetime budget constraint reflects the possibility that the individual will live through \( D' \). In no case can the present value of consumption exceed lifetime resources. The problem is simplified here by making lifespan (and lifetime earnings) in the interval \([0,0]\) nonstochastic.

7 This role of annuities as a mechanism for sharing uncertainty about longevity is an integral part of Diamond's (1977) evaluation of the social security system, in which he focuses on the absence of complete markets for such contracts. Merton (1983) considers a Pareto-improving social security program in an intertemporal model in which human capital is not tradeable.

8 Rochschild and Stiglitz (1976) argue that there will be no "pooling equilibrium," where all buy the same contract, whereas Wilson (1977) shows that a pooling contract may result under slightly different assumptions concerning the nature of competition.

9 Previous work in this area in the context of pensions includes the contributions of Davies (1981) and Sheshinski and Weiss (1981). Davies used a life-cycle model under uncertain lifetime to address the phenomenon of slow dissaving if retirement. The presence of pensions in his simulation model (using Canadian data) reduced, but by no means eliminated, the effect of uncertainty on retirement consumption. In the model of Sheshinski and Weiss, the ultimate impact of social security on saving depends on the availability of a private annuity market. They found that, at the optimal, Yaari's (1965) result holds, namely that private savings are reserved for bequests, while social security benefits are used to finance retirement consumption.

10 The actuarially fair benefit is constructed with respect to economy-wide survival probabilities. It is true that individuals who believe they will die "young" will want to purchase less than the "average optimal" amount of social security annuities, while those who expect to live a long time will want more. Both groups are better off, however, with the mandatory social security than without it, since in its absence, adverse selection is assumed to foreclose the possibility of a market in private annuities. This assumption will be relaxed in the context of private pensions later in the paper.
While the imposition of the social security system increases lifetime resources, nothing has been said about the optimal tax rate. Current law prohibits the explicit leverage of anticipated social security benefits. The ability to implicitly borrow against future benefits will depend on differences in \(\alpha\) (differences in ability to procure "unsecured" loans). Under the assumption of complete (explicit and implicit) nonmarketability of benefits, we can easily demonstrate that there is an interior solution \((0 < t < 1)\) for the individual's optimal tax rate (a sufficient statistic of participation as long as benefits are actuarially fair). The intuition is that while the purchase of "social security retirement annuities" increases resources available in old age, it decreases the resources available for current consumption. The optimal tax rate \(t_s\) in such a world is just

\[
  t_s = \frac{1}{\omega + 1} \left( \frac{\sum_{i=0}^{D'} \frac{EY_i}{\omega + 1}}{\sum_{i=0}^{D'} \frac{EY_i}{\omega + 1}} \right),
\]

which is zero for individuals who "know" that they will die prior to retirement.

Uncertainty over future social security benefits would mitigate the effect shown here. Watson (1982) discusses the influence of uncertainty over benefits in assessing the impact of social security on savings. Herton, Robie, and Marcus (1984) show that many private pension integration arrangements remove much of this uncertainty.

A retirement age of 65 was assumed. Probabilities for survival were taken from Faber (1982).

The idea here is that an individual who thinks he will live a long time would buy several small annuities rather than one large one in order to misrepresent his assessment of his longevity. Companies know his participation in social security, but not the extent to which he has obtained insurance from other private sources. Pauly (1976) discusses certain situations in which market equilibria might occur after a compulsory insurance program is imposed.

Lazear (1983) has discussed this point, emphasizing the role of pensions in influencing turnover, retirement, and investment in human capital. Many arguments for the existence of private pensions have emphasized their favorable federal tax treatment. Tax treatment cannot be the complete explanation, since "defined contribution" plans would dominate. "Defined benefit" plans are instead prevalent. Mundell (1982) emphasizes both the tax benefits (to employers and to employees) and the inadequacy of social security in explaining the growth of private pension plans.
This ignores the possibility that firms may be willing to offer "more-than-fals" plans to achieve some other impact on worker behavior. See Lazear (1983).

This is just the characteristic of "integration" of the benefits of social security and private pension annuities. Since the passage of the Revenue Act of 1942, Congress has allowed public (social security) and private benefits to be considered together in determining whether a private plan discriminates in favor of low-income workers. For descriptions of typical integration provisions and discussions of their prevalence in the U.S. pension system, see Munnell (1982) and Kotlikoff and Smith (1983).

The implicit assumption, of course, is that the parent dies at the beginning of the child's (optimising) life, age twenty here. This assumption is made to highlight the point that the existence of social security for the previous generation mitigates the impact of the present generation's participation in social security on its own wealth accumulation. More general assumptions about the timing of a testator's death would complicate expressions like (14) in the text, but the qualitative point would remain.

The consumption of individuals of each age can be calculated from equations (17) and (18), given the initial wage. The growth rate of the population will determine the relative number of persons at each age. Aggregate consumption can be calculated by summing consumption over ages, weighted by the relative population size.

Kotlikoff (1979a), using a life-cycle model with no uncertainty over longevity and a Cobb-Douglas production technology, considered the general-equilibrium impact of a pay-as-you-go social security system. For plausible parameter values, he found that the positive lifetime wealth increment traceable to social security (because of the growth of the wage base) caused a 20-percent reduction in the steady-state capital stock. While this effect is certainly substantial, it is roughly half of his partial equilibrium effect, which is directly related to the extent to which the present value of benefits exceeds the present value of payroll taxes paid. Kotlikoff's analysis also incorporates the influence of social security on retirement age, which is taken as exogenous here. To the extent that social security lowers the desired retirement age, the partial equilibrium wealth replacement effect of social security on saving is dampened.

The calculation was performed as follows. Let \( Y, T, \) and \( n \) represent total income, labor income, and the population growth rate, respectively, then:

\[
\frac{\Delta}{Y} \left[ \frac{C}{Y} \right] \left[ \frac{C}{nY} \right] = \frac{1}{3} \sum_{d=0}^{\infty} \left( \frac{1}{1 + \gamma} \right)^d \left( \frac{1}{1 + \eta} \right)^d \left[ \frac{1}{1 - \gamma} \right] \left[ \frac{1}{1 - \eta} \right] \frac{1}{1 - \nu}
\]
Given the assumed values for \( a, r, n \) and \( \delta \) in the text, \( \frac{C}{Y} \approx 0.02 \).

22Another consideration not addressed here is the real-world limitation on borrowing against future income for current consumption. Hayashi (1982) found that approximately twenty percent of all consumption is accounted for by such liquidity constrained individuals.

23See also the discussion in Kotlikoff, Shoven, and Spivak (1983) and Hubbard (1984).

24Estimating a version of (16) in level form, Feldstein and Pellechio (1979) found that an extra dollar of social security wealth reduced non-pension wealth by approximately a dollar, using data from the Federal Reserve Board's 1962 Survey of Consumer Finances; they had no data on private pensions. Some of their specifications also found a positive relationship between the ratio of net worth to permanent income and the level of permanent income. Using data from the Retirement History Survey, Diamond and Hausman (1982) found a social security offset of 30 to 50 percent (with a smaller non-pension wealth reduction for changes in private pension wealth). They also found evidence of a positive relationship between \( Y/Y^* \) and \( Y^* \).

Employing a logarithmic form of (16) for Canadian data, King and Dicks-Mireaux (1982) estimated the offset to non-pension wealth from a one-dollar increase in social security wealth to be 2\(c\) cents (10 cents for private pensions), with offsets of approximately dollar-for-dollar for individuals in the top decile of the wealth distribution. Hubbard (1983) estimated a similar model for the U.S. (using data from the President's Commission on Pension Policy), finding a mean offset for social security wealth of 33 cents (16 cents for private pensions), with social security offsets in excess of dollar-for-dollar for those in the top decile of the wealth distribution.

25Such a bequest motive is usually grounded in work in the human capital literature (see for example Becker and Tomes, 1976, 1979). That is, if human capital investments initially yield a higher rate of return than that on financial assets, parents who "care" about their children invest first in human capital up to the level at which the returns to additional investment just equal the market return. Further transfers are exclusively financial. Hence observed (financial) bequests will be higher for children whose parents had large resources than for children with access to low parental resources. Despite serious data limitations, there have been some recent efforts to estimate the relationship between bequests and lifetime resources. The finding that the ratio of bequests to earnings rises with the level of earnings is corroborated in the careful empirical study of Meng and David (1983).
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