"Efficiency, Adverse Selection, and Production"

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EFFICIENCY, ADVERSE SELECTION, AND PRODUCTION

by

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I. Introduction

This paper investigates the efficiency of an industry where producers have private information concerning their state-contingent production function, resulting in a standard adverse selection problem. The model studied is a straightforward extension of the work of Rothschild and Stiglitz (1976), Wilson (1977), Miyazaki (1977), Jaynes (1978), and Spence (1979) on pure risk-sharing in the presence of adverse selection. This essay examines the implications of adverse selection for the nature and efficiency of the joint equilibrium in the risk-sharing and goods markets. We make the assumption that, ex ante, producers have varying opportunities outside of the industry being studied, giving rise to supply curves for producers of each risk class. This potential variation in total production makes it necessary to model demand: we assume that consumers have a deterministic nonincreasing demand curve for the produced good with consumers' surplus being a valid measure of utility.

We show that if the supply curves of the risk types and the demand curve have nonzero elasticities when both the good's market and the insurance market are in equilibrium, and if there is cross-subsidization in the insurance market equilibrium, then the resulting equilibrium may not be Pareto efficient among all feasible allocations, even when we take into account the informational constraints. The nature of the inefficiency depends of the equilibrium concept used. First we examine a reactive equilibrium when insurers take into account the effect their offers have on the entry of both potential competitors in the risk market and potential producers in the good's

market. In this case, we find that there may be too few low quality, high risk producers in that there is an incentive compatible change in the insurance contracts which leaves the utility of a low risk agent unchanged, improves a high risk agent's utility, induces entry of high risk agents and increases consumers' surplus by more than the loss on the new insurance contracts. On the other hand, if insurers are small, viewing the good's price and their competitors' behavior as fixed, we show that equilibrium may have too few low risk producers. In either case, some social intervention may be necessary to achieve efficiency when adverse selection problems arise in the insurance of production risks, even when such intervention is limited by the same informational constraints as the private insurance market.

One often hears appeals to transactions costs and imperfect information as reasons why competitive markets generally do not achieve efficiency. It is straightforward to give examples of this in the presence of transactions costs and demonstrate the possibility of Pareto-improving interventions which respect such transactions costs. For example, in Judd (1980), we find that the absence of insurance markets for producers may justify government intervention in the commodity market. However, when one examines equilibria in the presence of asymmetric information, the expected inefficiencies may not appear when we take into account the constraints put on a planner due to the informational asymmetries. For example, Shavell (1979) demonstrates constrained efficiency of equilibrium in a simple insurance model with moral hazard. This paper proves that equilibrium may be Pareto inefficient even when we take into account the informational constraints and that intervention may be Pareto-improving even when the government uses only public knowledge.

In Section II, the basic model is described. In Section III various interpretations of this model and its relationship to the literature will be

discussed. Section IV discusses the Wilson equilibrium as adapted by Miyazaki
and Spence for this model. Section V contains the main result of the paper
concerning efficiency of equilibrium. Since the Wilson equilibrium is a
controversial equilibrium concept, we also analyze the Jaynes equilibrium for
this model in Section VI. Section VII repeats the analysis for the case of
small, price-taking insurers. Section VIII summarizes the paper's key points.

II. The Model

The following story outlines our model. Suppose that there are many
potential wheat producers, each with alternative activities. If a potential
producer should decide to enter, his production will be either zero or one
unit of wheat. Some potential wheat producers, type L agents, are competent
farmers with a low probability, q^L, of crop failure. The other group, the
H's, have a higher probability, q^H, of crop failure. Output is independently
distributed across producers. The L's will be able to trade their risky
endowment for the state-contingent income x whereas H's will be able to trade
their risky endowment for the certain income y. Based on the attractiveness
of the available insurance trades compared to his alternative activities, each
potential agent will decide whether to enter, with the number of participants
of type i given by N_i as a function of the utility of participation.

The following notation describes the model:

q^L (q^H)   probability that a low-risk (high-risk) agent produces one
                  unit of the good (otherwise, production is zero); $q^L > q^H$;

q^M average probability of success;

u(I)   a continuously differentiable von Neumann-Morgenstern
                utility function of income, common to both types of agents;

$N_i(v_i)$ number of type i producers if a type i participant would receive expected utility equal to v_i, i=L,H; assumed to be continuously differentiable with finite derivative;

$x=(x_1,x_2)$ state-contingent income of type L agents after trading endowments for insurance with x_1 being his income if there is production, and x_2 otherwise;

$$\bar{x} \equiv q^L x_1 + (1-q^L)x_2$$

$y \in R^1$ the (riskless) income intended for type H agents after they have traded their endowment for insurance;[1]

$Q(x,y)$ total production, assuming that the production risks are independent across producers;

$P(Q)$ the continuously differentiable inverse demand curve for the good; assumed to have negative slope everywhere;

$CS(x,y)$ consumer surplus when contracts x and y are available;

$L(x,y;S)$ losses to an insurer on x and y when they are offered and compete with the other contracts, S;

$U^i(z)$ utility to type i agent of contract $z \in R^1 \cap R^2$.

III. Interpretations and Applications of the Model

The most straightfoward interpretation of the model is that there are insurance companies offering contracts x and y. Despite the fact that that particular institution does not play a significant role in spreading production risks, this model is an appropriate case to study. First of all, bank credit is modeled here: a banker does not know whether a prospective borrower has a high probability of failure, and may offer a menu of credit

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[1]Note that x and y are the state-contingent incomes, not the net trades. While the two approaches are identical, our choice is made to keep the exposition clear.

possibilities to him, usually increasing the cost of debt as it becomes more levered. This approach to the adverse selection problem in credit markets was discussed in Jaffee and Russell (1976). Second, some equilibria of our models may be considered to be equilibria of equity markets. Leland and Pyle (1976) examined the possibility of financial structure being a signal of the quality of an entrepreneur's venture--as the entrepreneur's personal stake in a project is greater as indicated by his observable portfolio, the outside investors view any equity he issues as being less risky. If x (y) has an expected value equal to the expected value of the endowment of type L (H) agent, then we can view $(p,0) - x ((p,0) - (y,y))$ as the state-contingent payoff to a security in a type L (H) firm. In such a case our analysis will apply to stock market equilibria with adverse selection.

Next note two features of the nature of production and entry. First, we are focusing on idiosyncratic risks since we assume independence across producers. It is doubtful that adding common risks will change the thrust of this paper since contracts can be contingent upon common knowledge. Second, this model is consistent with general equilibrium analysis. In fact, if we consider producers to be choosing between self-sufficiency and producing in order to trade for marketed goods, then we have a general equilibrium analysis under the maintained assumption that consumers' surplus is a valid measure of consumers' welfare. More generally, if producers receive their social marginal product in their alternative activity then welfare analysis can be confined to the industry considered here.

In summary, the basic model has a number of interesting interpretations and the analysis of efficiency will be consistent with general equilibrium.

IV. Reactive Equilibrium with Large Insurers

The equilibrium concept we first examine will be the Wilson reactive
equilibrium as modified by Miyazaki and Spence. The pair (x,y) will represent
the available risk-sharing opportunities, i.e., an agent may choose to trade
his endowment for either x or y. We will use the following axiomatic
definition of the equilibrium.

> Definition: (x,y) is an M-reactive equilibrium if and only if its total
> profits are nonnegative and there is no other pair of contracts, (x',y'),
> such that (x',y') makes money when (x,y) is offered, and continues to
> make money if (x,y) is withdrawn because it becomes unprofitable with the
> offer of (x',y') and after the producers have adjusted their entry
> decisions in response to the new risk-sharing opportunities. More
> formally, (x,y) is an M-reactive equilibrium if and only if
> (i) $L(x,y,\phi) \geqslant 0$; and
> (ii) $\neg \, \exists \, x',y' \left(L(x',y',\{x,y\} \geqslant 0 \text{ and } L(x,y,\{x',y'\}) \geqslant 0 \right.$
> $\left. \text{or } L(x',y',\phi) \geqslant 0 \text{ and } L(x,y,\{x',y'\}) < 0 \right)$

If our supply curves were perfectly inelastic, then we would have Miyazaki's
model. Hence, our model and equilibrium concept generalizes Miyazaki.

This equilibrium concept is the equilibrium to a simple extensive-form
game. First, the "incumbent" chooses an (x,y) pair. Second, the "entrant"
chooses an (x',y') pair. Third, the incumbent may withdraw the (x,y) pair,
but not just one of x or y. Fourth, the potential producers decide whether to
enter. It is straightforward to show that the equilibria of this game are
exactly the M-reactive equilibria. This extensive-form game is offered as a
highly stylized model of a competitive insurance industry, and shows exactly
the type of strategic interaction necessary to support an M-reactive

equilibrium as an outcome of a noncooperative game.

As in Miyazaki and Spence, we will use the following characterization of equilibrium:

<u>Theorem I</u>: The unique M-reactive equilibrium is the (x,y) pair which solves

(M) $\quad\quad$ $\underset{x,y}{\text{Max}}$ $\quad q^L u(x_1) + (1-q^L)\, u(x_2)$

$$\text{s.t.} \quad \text{(i)} \quad u(y) = q^H u(x_1) + (1-q^H)u(x_2)$$

$$\text{(ii)} \quad yN_H + \bar{x}N_L = P(Q(x,y))Q(x,y)$$

$$\text{(iii)} \quad y - q^H P(Q) \geqslant 0$$

and is least favorable to type H agents among such solutions. Therefore, the equilibrium maximizes the expected utility of the low risk agents subject to (i) the incentive compatibility constraint, (ii) the zero profit constraint, and (iii) the condition that high risk agents receive at least actuarially fair insurance.

<u>Proof</u>: This is a straightforward generalization of Spence (1979) and Miyazaki (1977). The crucial insight is that the logic of the corresponding results in Miyazaki and Spence is just as applicable here where the value of the endowment is endogenously determined. First, we see that (i)-(iii) are necessary conditions for equilibrium. (i) summarizes the condition that the pair (x,y) be incentive compatible, i.e., that a type H agent will choose y and a type L agent will choose x. In this model, it is the former constraint that will be binding with the other incentive compatibility constraint following as a consequence of $q^H < q^L$. If the value of y to an H were strictly greater than the value of x to an H, then a competing insurer could propose some less risky x' such that (x',y) were still incentive compatible, x' is preferred to x by L's and is profitable. The incumbent would only

attract H's and would withdraw if y lost money when sold to H's. If y broke
even on H's, the incumbent could stay, but effectively the market would be
offering the menu (x',y). In either case, (x,y) would not be an
equilibrium. Hence (i) holds.

(ii) states that profits must be zero; if this were not true then an
entrant could offer a pair (x',y') which both L's and H's strictly prefer,
would be profitable because the entry responses would be finite, and thereby
eliminate the incumbent. (iii) says that H's must receive at least an
actuarially fair contract; if not, an entrant could offer a single contract
which would be better for H's, be actuarially unfair for H's, and make money
since the firm would make money on sales to both groups.

That equilibrium is a solution to this maximization problem is also
straighforward. Suppose (x,y) satisfied (i)-(iii) but did not solve (M); we
shall show that (x,y) cannot be an equilibrium. There is another profitable
incentive compatible pair (x',y') with x' being better for L's. Since the L's
prefer x', no agent would purchase x. From (iii), we know that both y and y'
at best break even for the insurers. If $y > y'$, then y must lose money and
(x,y) would be withdrawn. If $y < y'$, then (x,y) has effectively been replaced
by (x',y'). In either case, (x,y) cannot be considered an equilibrium.

When there are many such solutions to the maximization problem,
equilibrium is that solution which is least favorable to the high risk
agents. Suppose (x,y) and (x',y') were two solutions with $y'>y$. If the
incumbent offered (x',y') then the entrant could offer (x,y^-) where y^- is
slightly less than y. The incumbent would have to exit since it would serve
all high risk agents, whereas low risk agents would be indifferent between x
and x' and presumably split their purchases. Even with (x,y) gone, the
entrant's contract will make positive profits because $y^- < y$. Therefore,

(x,y) cannot be equilibrium. This argument rules out all solutions to the
maximization problem M except that one which pays type H agents the least.
Hence, we have the desired characterization of equilibrium.

The characterization of equilibrium in Theorem I can be represented
graphically. In Figure 1, the horizontal axis is income conditional on
success and the vertical axis is income if failure occurs.[2] The endowment of
a producer will be $(P,0)$ if price is P.

Let ZZ' be the efficient locus of x contracts for type L agents such that
if y is the riskless contract of equal value to an H, the (x,y) pair breaks
even for the insurer. ZZ' represents the zero profit locus for an insurance
firm conditional on either it being the only active insurer or competing with
firms offering the same menu of contracts, and no other pair would break even
and be better for both types. The nature of ZZ' is relatively unrestricted.
Because of entry possibilities, it is not necessarily true that $L(x,y,\phi)$ is
monotonic in x when (x,y) are restricted to being incentive compatible. There
may be many segments to the locus of x's which belong to zero-profit incentive
compatible pairs, such as B. However, any x contract along B is dominated for
L's by some contract along ZZ'. ZZ' is the efficient locus of contracts for
L's which are part of some zero-profit incentive compatible pair.

Theorem I says that the equilibrium contract for the type L agents is E,
where a type L argument's utility is maximized subject to nonnegative profits
for the insurer; hence, E is the point of tangency between the ZZ' locus and a
type L indifference curve. The corresponding y contract for type H agents is

determined by drawing the type H indifference curve through x and finding its intersection with the 45° line.

While this equilibrium concept is controversial, we chose it because of its strong efficiency properties in the models of Miyazaki and Spence. Since we are interested in examining the likelihood of inefficiency of equilibria, we should study an equilibrium concept which is most likely to yield efficiency. Also, there will be cases where this equilibrium concept will be the Nash equilibrium examined in Rothschild-Stiglitz.

V. Efficiency of Reactive Equilibrium

To study the efficiency of a reactive equilibrium, we need a definition of equilibrium which takes into account the constraints imposed on any social planner by the asymmetry of information.

Definition: A pair of insurance contracts, (x,y), is R-S-constrained efficient if and only if

$$U^H(y) \geqslant U^H(x)$$

$$U^L(x) \geqslant U^L(y)$$

and there is no other pair, (x',y'), such that

$$U^H(y') \geqslant U^H(x')$$

$$U^L(x') \geqslant U^L(y')$$

$$U^H(y') > U^H(y)$$

$$U^L(x') > U^L(x)$$

$$CS(x',y') - L(x',y',\phi) > CS(x,y) - L(x,y,\phi)$$

We are calling a pair (x,y) efficient if it is incentive compatible and there is no other incentive compatible pair which makes all active agents better off. Implicit in this formulation is the assumption that each producer's total insurance is observable; hence, we call it R-S-constrained efficient.

Note that we are using the strong notion of Pareto efficiency; however, given the structure of our model, the weak and strong notions will be equivalent.

To examine the efficiency of equilibrium, we study the Lagrangian of the maximization problem, M, which characterizes equilibrium. Let μ, η, and λ to be the multipliers of the constraints (i), (ii) and (iii) in M. Then the equilibrium (x,y) satisfies (i), (ii), (iii), and

(1) $\quad \mu u'(y) + (\eta\{[P'Q + P]q^H - y\} - \lambda q^H P')N_H u'(y) - \eta N_H + \lambda = 0$

(2) $\quad (q^L - \mu q^H)u'(x_1) + (\eta[(P'Q + P)q^L - \bar{x}] - \lambda q^H P' q^L) q^L N_L u'(x_1) - \eta N_L q^L = 0$

(3) $\quad [(1-q^L) - \mu(1-q^H)]u'(x_2) + (\eta[(P'Q+P)q^L - \bar{x}] - \lambda q^H P' q^L)(1-q^L)N_L u'(x_2) - \eta N_L(1-q^L) = 0$

(4) $\quad \lambda(y - q^H P(Q)) = 0$

From the maximizing characterization of equilibrium, we make the following crucial observation: if changes in x_1, x_2 and y equal to dx_1, dx_2, and dy were imposed so that incentive compatibility were maintained and that type L utility were left unchanged, the change in profits would also be zero if $\lambda = 0$, in particular, if there is cross-subsidization in equilibrium (λ may also be zero if there is no cross-subsidization). If dx_1, dx_2 and dy were so chosen, then

(6) $\qquad q^L u'(x_1)dx_1 + (1 - q^L)u'(x_2)dx_2 = 0$

(7) $\qquad u'(y)dy - [q^H u'(x_1)dx_1 + (1 - q^H)u'(x_2)dx_2] = 0$

where (6) implies that incentive compatibility is maintained and (7) implies that type L utility is unchanged. When written in this linear form, it is

clear that dy can be positive if and only if $q^L \neq q^H$, which is the adverse

selection assumption, and if $u'(x_1) \neq u'(x_2)$, which is true since under

adverse selection, the low risk type will not be fully insured in

equilibrium. The first-order conditions imply that

$$(8) \qquad (1) \cdot dy + (2) \cdot dx_1 + (3) \cdot dx_2 = 0$$

It is straightforward to check that if $\lambda = 0$, (6)-(8) implies that the change

in profits due to dx_1, dx_2, and dy is also zero. The change in consumer

surplus, dW, is equal to the product of current production and the price drop

induced by the entry of type H producers:

$$(9) \qquad dW = - Q P' q^H N_H' u'(y) \, dy.$$

dW will be positive if dy is positive, if P' is negative, and N_H' is

positive. Therefore, we have proved Theorem II.

<u>Theorem II</u>: If $q^H \neq q^L$, $P' < 0 < N_H'$ in equilibrium, and there is cross-

subsidization in equilibrium, then the M-reactive equilibrium is not R-S-

constrained efficient. In fact, in such cases, any small change in the

contracts which preserves incentive compatibility, makes H agents better off,

and leaves the utility of type L agents unchanged will cause an increase in

consumer surplus sufficiently large to leave consumers better off after they

pay lump-sum taxes to finance the induced loss on the insurance contracts.

Since the assignment of lump-sum taxes may be impractical, it is of

interest that a commodity tax will also do the job.

<u>Corollary I</u>: In Theorem II, we may replace lump-sum taxes with an excise tax.

<u>Proof</u>: Since a small sales tax has no deadweight loss, it is initially

equivalent to a lump-sum tax.

The intuition behind Theorem II is clear. If there is cross-subsidization in equilibrium, we can ignore the last constraint in (M) when considering small changes in x and y. If we perturb (x,y) so as to keep type L utility unchanged and continue to satisfy the incentive compatibility constraint, then to a first order the zero profit constraint also continues to hold. Of course, the second order sufficiency condition in (M) would tell us that the zero profit constraint is violated. Because of the adverse selection problem we may choose a direction for the perturbation so that type H utility is increased, and increased by a first order magnitude. If the type H supply curve has positive elasticity, this change will induce a first order increase in production, which in turn will cause a first order decrease in price if demand is not perfectly elastic. The final result will then be a first-order increase in consumers' surplus, sufficient to cover the second-order violation of the zero profit constraint and leave consumers better off.

The inefficiency of equilibrium is displayed in Figure 1. The insurance market equilibrium is at E where the representative low-risk agent's indifference curve is tangent to the zero profit locus, ZZ'. If the insurance market is moved from (E,y) to (E',y'), where y' is the new contract which must be offered to type H agents to preserve incentive compatibility, then there is initially no first-order loss to type L utility nor is there a first-order violation of the zero-profit locus since the move from (E,y) to (E',y') is tangential to U^L and ZZ' at E. There is clearly a first-order gain to type H utility. This does not yet prove that equilibrium is inefficient since we cannot transfer some of the first-order gain of type H agents to type L agents without violating incentive compatibility. If this were the end of the story we would still have the R-S-constrained Pareto efficiency demonstrated by

Miyazaki. However, in our model, there is a spillover effect into the good's market. If the supply curve of type H agents is not perfectly inelastic, then the first-order gain to type H agents will cause a first-order increase in the level of output which will cause a first-order gain in consumer surplus if the demand curve is not perfectly elastic. This first-order gain in consumer surplus is sufficient to cover the second-order insurance losses at (E', p') and the second-order cost of improving (E', p') so as to put type L utlity back at the initial level. Because of this spillover into the product market, the M-equilibrium is inefficient, subject to the constraints on information implicit in the problem.

We have not been explicit about how this subsidization of insurance is carried out. One straightforward fashion would be for the government to offer insurance and prohibit any competition. On the other hand, a direct subsidization of the insurance industry may not work. To see this, recall that the insurance industry implicitly has a social welfare function putting all weight on type L agents. If the shadow price of the zero profits constraint is large, then a subsidy would raise type L utility, resulting in substantial type L entry, but this may be offset by a decline in type H utility and type H participation, leaving the impact on consumers' surplus ambiguous. The key fact demonstrated above is that there is a Pareto improvement which uses no more information than what already is common knowledge in the market equilibrium.

While it is not surprising that equilibrium may be inefficient, it is somewhat unintuitive that performance may be enhanced by encouraging entry of the less efficient producers. One usually views adverse selection as a mechanism by which the bad drive out the good. In fact, in our M-equilibrium, the inefficient producers often receive more than the actuarial value of their

production and the efficient receive less than they "should." However, when there is cross-subsidization in equilibrium, the marginal inefficient producer has two parts to his true social product: first, his production adds to social welfare directly, and second, his entry relieves the incentive compatibility constraint, allowing more of the efficient producers to enter, improving social welfare again since the efficient producers are paid less than their marginal product. To the extent that prices drop, these contributions cannot be appropriated by the insurance firms since they go to consumers. Since the insurance market equilibrium is concerned only with the utility of efficient producers, the contribution to consumers of marginal type L entry is ignored and equilibrium is inefficient. When there either is no entry response on the part of type H agents or no price response to entry, equilibrium is efficient. Finally, note that this inefficiency does not arise because of market power since we would have the competitive perfect information equilibrium if $q^H = q^L$.

It is an open question whether inducement of the marginal type L agents is Pareto-improving. However, any such change from the equilibrium (x,y) would not be tangent to the zero-profit locus and therefore would require a first-order increase in tax revenue. Since the perturbation examined in Theorem II requires only a second-order increase in tax revenue, we see that if the social welfare function places a positive value on consumers, the optimal direction of any intervention initially attracts only type H agents.

VI. Equilibrium and Efficiency with Incomplete Observation of Contracts

A crucial and arguably unrealistic assumption of the foregoing analysis is that an insurer may observe all insurance contracts purchased by a customer. In this section, we lift that assumption, following Jaynes (1978).

For the case of two types, Jaynes argued that in equilibrium there would

be one contract which would be purchased by all, would break even, and its
sellers would announce publicly who purchased it. This announcement would
prevent agents from purchasing too many of these contracts from different
firms since if one purchased more total insurance than desired by a low-risk
agent, each firm would infer that he were a high-risk agent. There would also
be another contract, sold by other insurers, which would break even and be
purchased in addition to the public contract only by high risk types, with
such a sale being confidential. This is formalized in:

> Definition: A contract x is a <u>J-equilibrium</u> if and only if
>
> > (i)   x breaks even when all active producers purchase it;
> >
> > (ii) there is no contract, x', which would make money in competition
> > with x, taking into account the altered level and composition
> > of active producers.
>
> where in (i) and (ii) we assume that type H producers end up receiving
> $y = q^H x_1 + (1 - q^H)x_2$.

A straightforward generalization of Jaynes' analysis shows that in
equilibrium the low risk types would end up with state-contingent income of
$x = (x_1, x_2)$ which solves

$$\text{(J)} \qquad \underset{x,y}{\text{Max}}\; q^L u(x_1) + (1 - q^L)u(x_2)$$

$$\text{s.t. } x_1(1-q^M) + x_2 q^M = q^M\, P(Q(x,y))$$

$$x_1 \leqslant P(Q((x,y))$$

That is, x is that contract on the market odds line above the endowment which
maximizes the utility of a low risk agent. Type H agents will end up with
$y = q^H x_1 + (1-q^H) x_2$, the expected value of x to a type H agent.

Equilibrium solves (J) where the effects of alternative x's on Q and P(Q) are taken into account. In the following, we will assume that only the zero-profits constraint is binding, i.e., low-risk agents actually insure against failure. The concept of constrained efficiency in this context is:

Definition: A contract x is <u>J-constrained efficient</u> if and only if there is no other contract x' such that

$$U^L(x') > U^L(x)$$

$$U^H(y') > U^H(y)$$

$$CS(x',y') - L(x',y',\phi) > CS(x,y) - L(x,y,\phi)$$

where

$$y = q^H x_1 + (1 - q^H)x_2$$

$$y' = q^H x_1' + (1 - q^H)x_2'$$

We are calling the public contract, x, efficient if there is no other public contract, x', which would make all active agents better off, when we take into account the ability of agents to make unobservable trades which are actuarially fair to H's.

If the number of risks of each type were fixed, as in Jaynes (1978), then the resulting allocation is Pareto efficient given the informational constraint since equilibrium solves (J). However, with the production risks and entry assumed here, the equilibrium allocation is again inefficient. This is best demonstrated graphically. (We forego the algebraic demonstration as it parallels that of Theorem II in the previous section.)

In Figure 2, suppose A is the state-contingent income of each agent after buying the publicly disclosed contract. Again, define ZZ' to be the locus of x contracts for L's which break even when sold to all and the H's end up with certain income y, the expected value of x for an H. From Theorem II, A is the

point of tangency of ZZ' with an indifference curve of a type L agent, U^L. B

will be the type H contract, with the vector $\overrightarrow{AB}$ being actuarially fair for

them. Note that the graph assumes that AB is flatter than U^L and ZZ' at A.

Suppose otherwise. Then the indifference curve for L through A would be

like $\overline{U}^L$, which intersects $\overrightarrow{AB}$ at D. However, D would make money for an insurer

since it would cause utility for both types to be the same as if A were the

public contract, but D would then lie below the market odds line through A

since AB is fair for type H agents. By continuity and concavity of utility

there are contracts to the northeast of D which would break even and be better

for low risk agents, contradicting the fact that A solves the above

maximization problem. Hence U^L is steeper than $\overrightarrow{AB}$ at A.

Since $\overrightarrow{AB}$ and U^L must intersect as demonstrated, suppose that the

government proposed a contract along $\overrightarrow{AE}$, where $\overrightarrow{AE}$ is tangent to ZZ' and U^L.

To a first-order, a low risk agent's utility is unchanged, and the insurance

contract still breaks even. However, $\overrightarrow{AE}$ is steeper than $\overrightarrow{AB}$ since $\overrightarrow{AE}$ is

tangent to U^L. Therefore, the expected and actual final income of type H

agents is increased to a first order, eliciting entry of type H agents,

causing a first order decrease in the price of the good and first order

increase in consumers' surplus. Hence, we have shown Theorem III.

Theorem III If $N_H' > 0 > P'$ and both types purchase insurance in the Jaynes

equilibrium, then equilibrium is not J-constrained efficient.

Again, we find that equilibrium may be inefficient and that efficiency

may be enhanced by attracting the relatively inefficient producers. Hence the

nature of the informational constraint is not crucial in determining

appropriate intervention.

VII. Price-taking Insurers

In both of the previous models we assumed that the insurers were aware of their impact on the equilibrium price of the produced good. This is realistic if there are few active insurers. However, if there were many insurers and no one insurer could efficiently serve a large fraction of the producers due to, say, diseconomies of scale, then in equilibrium insurers would presumably regard the good's price as fixed.

We first examine the equilibrium corresponding to M above except now firms take p, the good's price, as fixed. For a fixed p, we are in the same situation as examined originally in Rothschild-Stiglitz and Wilson. Again we shall use the solution concept of Miyazaki and Spence. That is, we use the following concept of equilibrium.

Definition: $\langle p,(x,y)\rangle$ is a price-taking M-equilibrium if and only if:
  (i) (x,y) is a Miyazaki equilibrium of insurance contracts when the endowment is (p,0) and the ratio of type H's to type L's is $N_L(U^L(x))/N_H(U^H(y))$;
  (ii) at p, demand equals supply, i.e.,
     $$p = P(q^L N_L(U^L(x)) + q^H N_H(U^H(y))$$

We may also use Myerson (1983) to justify our choice of equilibrium concept in this case. Myerson examines bargaining problems where the principal possesses information not available to the agent and the principal receives all the rent from the relationship. If we regard our potential producers as the principals, each of whom know their quality, and we view the insurers as the agents, with both parties taking p and the ratio of H's to L's as parametric, then we have a structure to which we can apply Myerson's analysis. While this description makes the producers the strategically active

players and insurers passive, we use it since Myerson shows that the result is the appropriate generalization of the concept of the core to these incomplete information environments and that the insurance market equilibrium appears competitive with zero profits. Furthermore, in our two-type case Myerson's analysis yields the same unique equilibrium as Miyazaki. More formally, following Myerson and his notation, define:

$$D_0 = \{(x,y) \; \varepsilon \; R^3 \mid 0 < x_1, \; x_2, \; y < p^{max} \} \; ^3$$
$$D_1 = \{L,H\}$$
$$D_2 = \{accept, \; reject\}$$
$$T_1 = \{L,H\}$$
$$T_2 = \{1\}$$

where p^{max} is the maximum price such that demand is nonnegative. In this game, the insured dictates the choice of $(x,y) \in D_0$, i.e., chooses the menu. Then the producer and the insurer play a Bayesian game, each simultaneously choosing from its set of alternatives. The producer may choose either L from D_1, indicating he is a low risk agent entitled to x, or H, thereby taking y. The insurer, choosing from D_2, may either accept or reject the proposal in D_0. If he rejects, then he neither loses nor gains money in any state. If he accepts, he pays either x or y, whichever is chosen by the producer and receives the value of the realized production.

Let Γ denote the collection of the decision sets, type sets, and the types' utility functions and probability distributions over types. Let n_i be the number of type i agents, i = L,H.

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[3]D_0 is finite in Myerson. Since D_0 is compact, we may think of D_0 as being a very fine mesh of grid points in R^3 without any change in the results.

Theorem IV. For a fixed price and number of both types, the Miyazaki equilibrium is also the unique principal's neutral optimum to the game Γ. Hence, for a given p, n_H, and n_L, the contracts x and y are chosen so as to

$$\underset{x,y}{\text{Max}} \; q^L u(x_1) + (1-q^L) \, u(x_2)$$

$$\text{(i)} \quad u(y) = q^H u(x_1) + (1-q^H) u(x_2)$$

$$\text{(ii)} \quad p \; Q - (y n_H + \bar{x} n_L) = 0$$

$$\text{(iii)} \quad y - q^H p \geqslant 0$$

Proof. We need to show that the mechanism where the producer chooses between x^M and y^M, the Miyazaki equilibrium pair of contracts, is an acceptable proposal to the insurer and is a neutral optimum. In Myerson's notation, this is represented as

$$\mu(z^m, \; L, \; \text{accept}|L) = 1, \; \mu(z^m, \; H, \; \text{accept}|H) = 1$$

where $z^m = (x,y)$. A necessary condition for $z^o = (x^o, y^o)$ to be a neutral optimum is that z^o solves, for some nonnegative weights, λ^L and λ^H,

$$\underset{z^o}{\text{Max}} \; \lambda^L U^L(z^o) + \lambda^H U^H(z^o)$$

subject to the incentive compatibility conditions and nonnegative profits for the insurer. Differentiation of the Lagrangian shows that $y_1^o = y_2^o$. Figure 1 can be used here since the insurers act as in Figure 1 except they believe that the supply curves are perfectly inelastic. Generally, z^o is represented by (x^o, y^o) where x^o is on ZZ' and an H is indifferent between y^o and x^o. In particular, x^M corresponds to E and y^M to y in Figure 1.

A neutral optimum x^o cannot lie below E on ZZ' since E would be better for both types. A neutral optimum cannot lie above E on ZZ'. Any such mechanism is blocked since an L could offer E, which is superior for L's and worse for H's, causing an insurer to infer L's true type. The insurer would then take the offer and reject all others. Hence E is the unique core allocation and also the unique neutral optimum of Γ.

Therefore, each p, n_H, and n_L triple will give rise to an insurance market equilibrium for (x,y). We next need to prove the existence of the joint equilibrium of the insurance and goods market. Theorem V follows directly from the continuity of our supply and demand functions.

Theorem V: If (i) sufficiently low utility of participation will lead to no entry by either type, and (ii) sufficiently high price will lead to no demand, then there exists a price-taking M-equilibrium.

We next examine the efficiency properties of our equilibrium. The appropriate efficiency concept is R-S-constrained efficiency. Suppose x were changed to $x + dx$, with the pair $(x + dx, y)$ remaining incentive compatible;

$$0 = q^H u'(x_1)\, dx_1 + (1-q^H)u'(x_2)dx_2$$

Consumer surplus would be affected by entry of type L agents and the change in the expected value of payments to current producers, that change being

$$dW = N_L'(q^L u'(x_1)dx_1 + (1-q^L)\, u'(x_2)dx_2)(q^L P - \bar{x}) - N_L(q^L dx_1 + (1-q^L)dx_2)$$

Expressing dx_2 in terms of dx_1, substituting the result into dW, we find that

$$dx_2 = \frac{q^H}{q^H-1}\ \frac{u'(x_1)}{u'(x_2)}\ dx_1$$

$$dW = q^L dx_1 \{ (N_L' \, u'(x_1) \, (q^L P - \bar{x}) - N_L) \left(1 - \frac{1-q^L}{q^L} \frac{q^H}{1-q^H}\right)$$

$$- N_L \left(1 - \frac{u'(x_1)}{u'(x_2)}\right) \frac{1-q^L}{q^L} \frac{q^H}{1-q^H} \}$$

Since $q^L > q^H$ and $x_1 > x_2$, entry of a type L agent occurs if $dx_1 > 0$.

If $q^L = q^H$, i.e., no adverse selection, then $x_1 = x_2$ in equilibrium and

$dW = 0$, corresponding to the usual efficiency of equilibrium under complete

information. However, with adverse selection, dW will generally not be zero

in equilibrium. In fact, consumers are indifferent with respect to entry of

the marginal type L agent only if $dW = 0$, clearly a very restrictive

condition. If the supply of producers were fixed, then $dW < 0$. However, if

N_L' were sufficiently large, $dW > 0$. Since an increase in x_1 raises type L

utility and y is left unchanged, equilibrium is inefficient if $dW > 0$.

Since the value of N_L' is independent of other parameters, if one had an

equilibrium where the elasticity of type L producer supply was not

sufficiently large to have inefficiency, then one could arbitrarily increase

the elasticity of the supply function at the equilibrium, without changing the

equilibrium. Therefore, we have proved Theorem VI.

Theorem VI: If the elasticity of supply of low risk producers is sufficiently

high, and we have cross-subsidization in equilibrium, then the price-taking M-

equilibrium is not R-S constrained efficient. Such inefficient equilibria

exist.

From our expression for dW, it appears that an equilibrium is more likely

to be inefficient as the probabilities of success are more unequal, as the

elasticity of supply of low risk producers is greater, as $q^L P - \bar{x}$, the social

rent from a marginal entrant, is greater, and as marginal utilities across

states differ more. Exact comparative statics are generally intractible since most of these indices of market imperfection are jointly determined.

To highlight the differences between the large insurer case and this price-taking insurer case, we next prove that marginal entry of type H agents cannot be Pareto-improving.

Theorem VII: In the price-taking M-equilibrium, if y is increased and x is altered to maintain incentive compatibility and type L utility, then consumers' welfare is decreased.

Proof: The change in consumer welfare is

$$dW = N_H' u'(y) dy (q^H P - y) + d\pi$$

where

$$d\pi = -N_H dy - N_L (q^L dx_1 + (1 - q^L) dx_2)$$

is the change in profits on the insurance contracts. Since $q^H P - y \leqslant 0$, the only possible source of gain is from the profits on insurance contracts, $d\pi$. However, if higher profits could be extracted from existing producers, they would be extracted in equilibrium by implementing (dy, dx) and using the extra profits to improve U^L in an incentive compatible fashion. Hence, dW and $d\pi$ are both negative.

We also find that equilibria with no cross-subsidization are efficient.

Theorem VIII. If the price-taking M-equilibrium has no cross-subsidy, then the outcome is R-S-constrained efficient.

Proof. To show constrained efficiency, it will be sufficient to compute nonnegative α and γ such that x^M, y^M solves

$$\text{Max } \alpha u(y) + U^L(x) + \gamma CS(x,y)$$
$$x,y$$

$$\text{s.t. } U^H(y) - U^H(x) \geqslant 0$$

Performing the necessary differentiations and assuming no cross-subsidy yields

the necessary conditions for α and γ in terms of the equilibrium (x,y):

$$q^L u'(x_1) - \gamma N_L q^L + \beta q^H u'(x_1) = 0$$

$$(1 - q^L)u'(x_1) - \gamma N_L(1 - q^L) + \beta q^H u'(x_2) = 0$$

$$\alpha u'(y) - \gamma N_H - \beta u'(y) = 0$$

where β is the social shadow price of the incentive compatibility

constraint. Solving the resulting linear equations for α, γ, and β yields

$$\Delta \begin{bmatrix} \alpha \\ \gamma \\ \beta \end{bmatrix} = \begin{bmatrix} q^L(1 - q^L)N_L(u'(x_2) - u'(x_1))u'(y) + N_H u'(x_1)u'(x_2)(q^H - q^L) \\ u'(x_1)u'(x_2)u'(y)(q^H - q^L) \\ N_L u'(y)q^L(1 - q^L)(u'(x_2) - u'(x_1)) \end{bmatrix}$$

where

$$\Delta = N_L^2 u'(x_2)[u'(y)]^2(q^H - q^L)$$

$$\times (q^H q^L(u'(x_2) - u'(x_1)) - q^L u'(x_2) + q^H u'(x_1))$$

Since the price-taking M-equilibrium solves (M) except for the assumption of

fixed prices, (x^M, y^M) is characterized by (1)-(3) with $P' = 0$. Combining (2)

and (3) shows that

$$u'(x_1) - u'(x_2) = \mu(\frac{q^H}{q^L} u'(x_1) - \frac{1 - q^H}{1 - q^L} u'(x_2))$$

implying that $\Delta < 0$ since $u'(x_1) < u'(x_2)$ and $\mu > 0$. Hence, $\gamma > 0$.

Last, we need to show that $\alpha \geqslant 0$. Straightforward calculation shows that $\alpha\Delta$ equals the rate of change of U^L as y is increased away from y^M, changing x so as to maintain incentive compatibility and zero profits. However, that change is equal to λ, the shadow price of $y \geqslant q^H p$ in (M). Since $\lambda, \Delta \geqslant 0$, α must be nonnegative. Hence the price-taking M-equilibrium is efficient.

In a completely analogous fashion, we may define price-taking J-equilibrium, and prove Theorem IX.

Theorem IX: A price-taking J-equilibirum exists if demand is zero at some price and if a sufficiently low price results in no supply. If the elasticity of the supply of the low-risk producers is sufficiently high, a price-taking J-equilibrium is not J-constrained efficient. Such inefficient equilibria exist.

We find that equilibrium may still be inefficient with a competitive atomistic insurance industry, however the nature of the inefficiency differs from the case when insurers were large. Here we see that the market is more likely to lead to too few efficient producers, corresponding to our standard intuition that the bad drive out the good. Therefore, any policy judgments are highly dependent on the market structure of the risk-sharing market.

IX.   Comments and Speculations

The focus of this note has been narrow and specific to illustrate the major point. Several unnecessary assumptions were made for the sake of ease of exposition. First, it is certainly not necessary to assume that the bad state is one of no production at all. The generalization to an arbitrary two point distribution of production is obvious. Second, Theorem II is clearly

true sometimes when there is no cross-subsidization in the competitive equilibrium since the shadow price of the last constraint in (M) may be zero when there is no cross-subsidization and the argument of Theorem II clearly holds when λ is small.

I would also conjecture that these arguments will continue to be valid in a dynamic context, if buyers can be forced to commit themselves ex ante to a contract. In that case, firms would generally offer a buyer a choice among experience-rated insurance contracts. Again, I would expect that equilibrium would be characterized by the analogous maximization problem and that with production we could still have inefficiency: i.e., there would be some incentive compatible perturbation leaving type L utility and profits unchanged, inducing desirable entry by type H agents. If in equilibrium all contracts were just single-period contracts, then this analysis would continue to hold since in each period the market would be segmented by the experience of the buyers, presumably an observable characteristic, and within each segment, the static analysis would hold. (The interesting case left would be where firms commit themselves to future contracts, but buyers are not bound to them. Then the problem is dynamic in an essential fashion, possibly invalidating the essentially static intuition of this note.)

In this essay, we have seen that market equilibrium risk-sharing allocations will often result in a Pareto inefficient allocation of resources among producers, calling into question the ability of competitive financial markets to bring about a correct allocation of productive resources across risky projects where there is asymmetry in information. In the context of internal labor markets as studied by Miyazaki, this analysis questions the efficiency of the allocation of laborers among their various opportunities. This line of analysis may help in our understanding of these issues.

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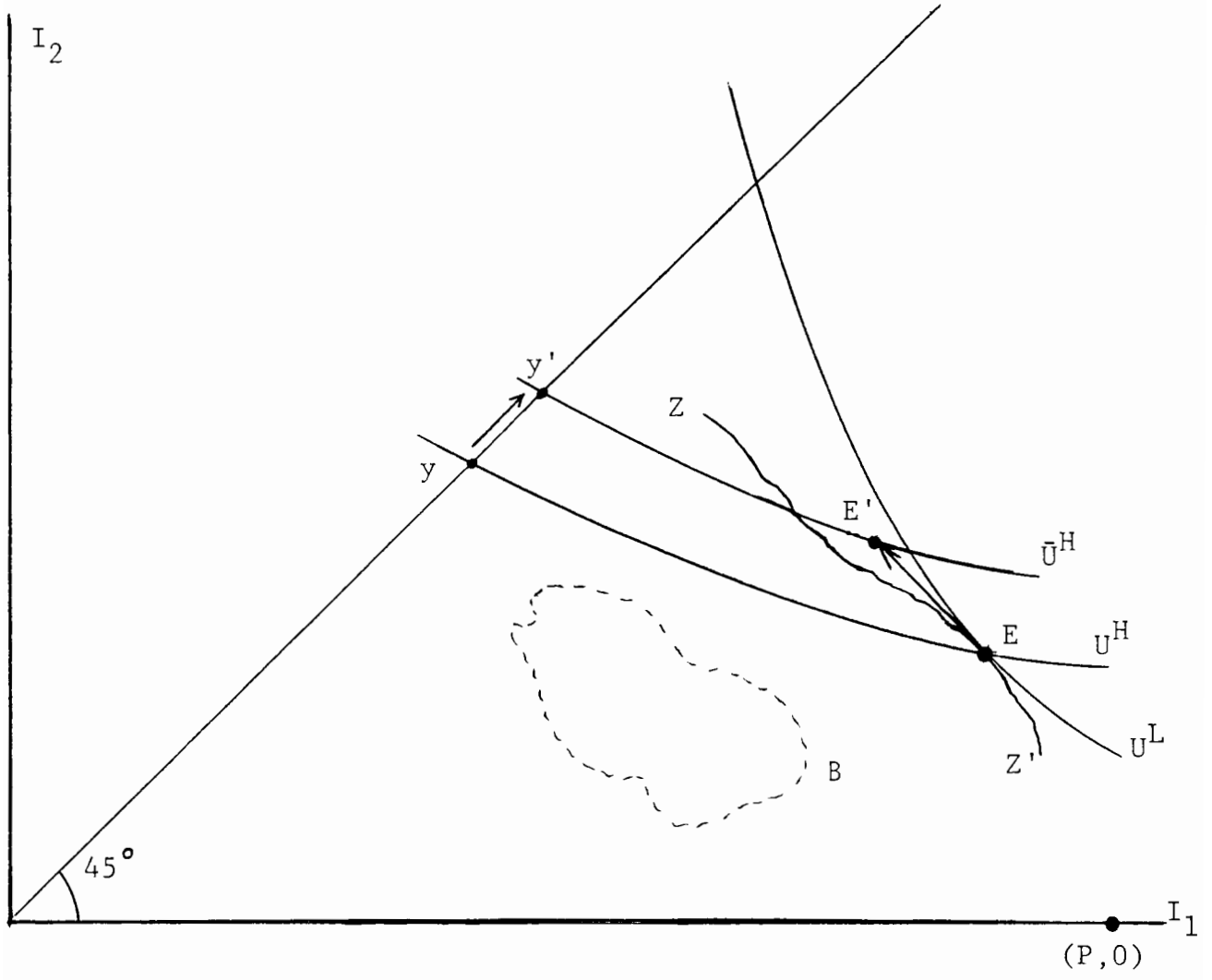
Figure 1

Figure 2