Discussion Paper No. 603

DYNAMIC LIMIT PRICING AND INTERNAL FINANCE*

by

Kenneth L. Judd**
and
Bruce C. Petersen***

June 1984

---

*This paper has greately benefitted from many suggestions and comments by Mark Satterthwaite, Ron Braeutigam, Steven Matthews, and John Panzar, and also from the research assistance of Scott McShan. We also gratefully acknowledge the financial support of the National Science Foundation and the J. L. Kellogg Graduate School of Management at Northwestern University (Ken Judd) and the Sloan Foundation.

**Department of Managerial Economics and Decision Sciences, J. L. Kellogg Graduate School of Management, Northwestern University, 2001 Sheridan Road, Evanston, Illinois 60201.

***Department of Economics, Northwestern University, Evanston, Illinois 60201.

I. INTRODUCTION

In this paper we examine the optimal pricing strategy of a dominant firm or a group of joint profit maximizing oligopolists facing expansion by a competitive fringe. The problem is of considerable interest because most concentrated industries consist of a large number of fringe firms alongside one or more dominant firms.[1] Furthermore, expansion by the competitive fringe appears to be a most important source of "entry," since full scale entry by new firms into significant oligopolistic markets appears to be a fairly rare event.[2]

The problem of the dominant firm facing expansion by a competitive fringe was first examined by Gaskins [1971]. He labeled the pricing strategy of the dominant firm "dynamic limit pricing." We believe a new formulation is in order because of two developments, on which we expand below: (1) Gaskins' model has received widespread application at the theoretical, empirical, and policy levels and (2) the strategic assumptions underlying his model have come under telling criticism in recent years. We believe that our formulation handles the basic criticisms of Gaskins' approach yet continues to yield a rich set of predictions about dominant firm pricing strategy.

Gaskins' model has become widely known and used by both economists and non-economists for further theoretical modeling, empirical research in industrial organization, and policy analysis. Some reasons for this wide range of application can be found in Scherer's [1980, pp. 236-243] excellent description of the model and its predictions. Scherer notes that the model "is compelling not only because it yields rich predictions, but also because these predictions appear to be consistent with a good deal of what we know about American industrial history." [p. 239] Scherer gives several examples, including the pricing strategies of U.S. Steel, American Viscose, American Can, Xerox, IBM, Alcoa, and General Motors.[3]

Along with the many applications have come some telling criticisms of Gaskins' formulation. These criticisms center around the ad hoc nature of the fringe expansion equation and the game theoretic foundation of the model. In particular, Gaskins' fringe expansion equation is not based on any maximization behavior on the part of the fringe. Some of the criticisms of the game theoretic foundations of the model apply with equal force to all but the most recent limit pricing models.[4] It has been pointed out by J. Friedman [1979] and Milgrom and Roberts [1982] that under complete information, if established firms' pre-entry actions do not influence post-entry costs or demand, these actions can not deter entry. The capital investment decision is one example of a pre-entry action which can affect post-entry conditions.[5] We are aware of no previous explanations, however, for how price could deter either entry or fringe expansion under complete information.

Our dynamic limit pricing formulation is based on the importance of internal finance (retained earnings) to fringe firms. In this respect our model is related to Spence [1979], in which internal finance plays the crucial role of the constraint on the expansion of later entrants into a new market. Spence, however, chose to examine capacity, not price, as the control variable of the first entrant.

We set up the dynamic limit pricing problem as a deterministic, non-cooperative, differential game between the dominant firm and the competitive fringe. The control variable of the dominant firm is price while the control variable of the fringe firms is their retention ratio. Fringe firms retain one-hundred percent of their income and invest full-out as long as it is in long-run interest to do so. The connection between current price and expansion is then obvious - current price determines fringe earnings which in turn determines the maximum possible rate of expansion of their capital stock. Todays pricing decision then does affect the future circumstances that dominant firms face.

We demonstrate several interesting features of the equilibrium outcome. First, in the case where the market's rate of growth is less than the dominant firm's discount rate, if the fringe is initially small (large) the price drops (rises) to fringe marginal cost at some finite time although the fringe market share continues to rise (fall) forever. This contrasts with Gaskins' analysis where the price only approaches the fringe marginal cost asymptotically, and then so only in the case of zero growth. We also examine the case where the market's rate of growth is greater than the dominant firm's discount rate and the game lasts for a finite time, a case not examined by Gaskins. We believe this to be an important case given the fact that demand for many goods grows very rapidly immediately following introduction. In the rapid growth case we find that equilibrium goes through as many as four possible stages. While our analysis is explicitly open-loop, we show that our equilibrium is also often a closed-loop equilibrium, and that the long-run steady state outcomes cannot differ.

The next section of the paper is a review and a critique of Gaskins' formulation. In section three we examine the key role of retained earnings as the source of finance for fringe expansion; we then derive the expansion equation in section four. In section five we solve our formulation of the dynamic limit pricing problem and compare the results with Gaskins'. Finally, in section six we discuss the alternative equilibrium concepts.

II. GASKINS' MODEL

For a number of reasons it is important to begin with a review and a critique of Gaskins' model. One reason is that we will retain some of its features in our formulation. A second reason is that Gaskins' paper is widely cited and has seen many applications. It is therefore desirable to point out the similarities and differences in the two formulations and in their results.

Before proceeding with a review of his model, we should mention some of its applications. To cite but a few of the theoretical extensions of Gaskins' model, Brock [1975] includes technological progress, Lee [1975] adds non-price policies and learning by doing, DeBondt [1977] includes scale effects and Encaoua and Jacquemin [1980] incorporate non-price policies. At the empirical level, Gaskins' model clearly demonstrates the possibility of a feedback relationship between price and market structure - the choice of a pricing policy affects market share over time, as well as market share determining pricing policy. While the vast majority of industrial organization studies continue to be cross-sectional, a few recent studies are dynamic, and more are likely to follow. Brock [1975], for example, estimates Gaskins' model econometrically for the computer industry; while Martin [1979] includes a concentration equation based on Gaskins' model in a system of simultaneous equations. Martin finds that a dynamic specification of concentration is critical to the specification of the profitability equation. Finally, Gaskins' model has seen application at the policy level, including frequent citations in law journals. It appears that a number of lawyers as well as economists interested in antitrust issues are familiar with the model, including Dunfee and Stern [1975], Easterbrock [1981], and Kaplow [1982].

Dynamic limit pricing differs from static limit pricing in that it allows more general strategies on the part of dominant firms. Firms following a static limit pricing strategy either charge the short-run profit-maximizing price and allow their market shares to decline, or they set price at the limit price and preclude all entry. Gaskins argues that there is no justification for this dichotomy; rather maximization of the present value of future profits entails a balancing between current profits and future market share.

In Gaskins' formulation the optimal pricing strategy maximizes:

$$V = \int_{0}^{\infty} (p(t)-c_d)q(p(t),t)e^{-rt}dt \tag{1}$$

where V = the present value of the dominant firms' profit stream, $p(t)$ = product price, c_d = average total cost of production (assumed to be constant over time), $q(p(t),t)$ = dominant firms' output, and r = dominant firms' discount rate.

Gaskins assumes that the level of dominant firms' current sales can be decomposed into additive univariate functions of price and time, such that:

$$q(p(t), t) = f(p(t))e^{\gamma t} - x(t) \tag{2}$$

where $f(p(t))$ is the market demand curve, γ is the market growth rate, and $x(t)$ is the output of the competitive fringe which is assumed to be fixed at any point in time. The net effect of fringe expansion, $\dot{x}$, is to shift the dominant firm's residual demand curve laterally.

Gaskins argues that if fringe firms view current product price as a proxy for future price then expansion will be a monotonically nondecreasing function of current price. He then assumes that expansion is a linear function of current price, given by:

$$\dot{x}(t) = k_o e^{\gamma t}(p(t)-\bar{p}) \qquad x(0) = x_o, \ \bar{p}>c \tag{3}$$

where $\bar{p}$ is the limit price, k_o is the response coefficient at time 0 $(k>0)$, and x_o is the initial output of the competitive fringe. Gaskins also assumes that the response coefficient $k(t) = k_o e^{\gamma t}$ is a growing exponential function of time. He argues that increasing disposable income should cause a proportional increase in the quantity of resources available to the fringe for investment in any particular market.

Equations (1), (2), and (3) allow the optimal pricing strategy of dominant firms to be solved analytically using the mathematics of optimal control. The objective is to maximize (1), using (2), subject to (3) where $x(t)$ is the state variable and $p(t)$ is the control variable. The Hamiltonian for the problem is given by

$$H = (p(t)-c_d)(f(p)e^{\gamma t}-x(t))e^{-rt}+z(t)k_o e^{\gamma t}(p(t)-\bar{p}) \tag{4}$$

where $z(t)$, the costate variable, is the shadow price of an additional unit of rival entry at any point in time. The first term in equation (4) is the change in present value accruing from current sales. The second term is the product of $z(t)$ and $\dot{x}(t)$, which is the effect of current entry on future profits. Thus, maximizing the Hamiltonian with respect to $p(t)$ can be thought of as balancing the present value of current and future sales.

The necessary conditions[6] for a p to maximize V can be used to obtain a system of differential equations describing the time path of prices and fringe market shares. If $w(t)=x(t)e^{-\gamma t}$ is the normalized size of the fringe, the resulting system of equations is:

$$\dot{w}(t) = k_o(p(t)-\bar{p}) - \gamma w(t), \qquad w(0)= x_o, \tag{5}$$

$$\dot{p}(t) = \frac{k_o(\bar{p}-c_d) - r(f(p)-w(t)+f'(p)(p(t)-c_d)) + \gamma w(t)}{-2f'(p) - f''(p)(p(t)-c_d)}. \tag{6}$$

Equations (5) and (6) define two possible optimal price trajectories, depending on the initial size of the fringe, its cost disadvantage vis-a-vis the dominant firm, and other factors. If the dominant firm is initially large, it will price initially above the steady-state level and lower it gradually over time, thereby causing the fringe to gain market share until the steady state is reached. This is the strategy which is consistent with a number of corporate histories described by Scherer. If the dominant firm is initially small, it initially sets price below the steady state level and raises it gradually over time, thereby causing the fringe to loose market share until the steady-state is reached. In both cases the present value of the profit stream is maximized by balancing the contributions of current price to profits with the loss or gain of future profits from the loss or gain of market share. For further details, we refer the reader to the original paper.

The weak point in Gaskins' formulation is that fringe firms (the entrants) are not treated as rational, maximizing economic agents. As Milgrom and Roberts [1982, p. 444] point out, this is common to most of the existing limit pricing literature. In addition, a number of issues can be raised about the exact specification of the fringe expansion equation, $\dot{x}(t) = k_o e^{\gamma t}(p(t) - \bar{p})$. One issue is the response coefficient, k. A priori nothing is known about this parameter which is unfortunate since $\dot{x}(t)$, $\dot{p}(t)$, and the steady state values of market share and price critically depend on its magnitude.[7] A second issue is the justification for the response coefficient growing at an exponential rate γ. Gaskins' justification, that increasing disposable income should cause a proportional increase in resources available to fringe firms in all industries, seems tenuous in an economy where new industries are emerging and competing for resources, some have matured, and others are declining. Another issue is why fringe expansion does not depend on the present size of the fringe, as well as price. One would expect that the larger the fringe, the greater $\dot{x}$, other things equal. Finally, is there any justification for a positive, much less a linear, relationship between fringe expansion and price? We return to these issues after formulating our own expansion equation in section four.

III. THE ROLE OF INTERNAL FINANCE

Similar to Spence [1979] the availability of internal finance is the constraint on the rate of expansion of the fringe in our formulation. Internal finance, particularly for small firms, has been the dominant source of finance historically[8] as well as during the post World War II era. While the importance of internal finance is widely known, we feel that it is worthwhile to review the relevant statistics pertaining to the different sources of corporate finance as well as the explanations for the dominance of retentions.

(A.) EXPLANATIONS FOR THE DOMINANCE OF INTERNAL FINANCE

Corporations may finance expansion with internal finance or with debt and new share issues, sources of external finance. In terms of comparative dollar values, retained earnings are largest, debt next, and new share issues are quite unimportant. There are a number of explanations for this pattern of finance, including corporate income taxation, flotation costs, costs of financial distress, agency costs, and limited capital markets. These explanations are discussed below.

One important reason for the small amount of new share issues on the part of unregulated firms is the design of the corporate tax system.[9] The United

States and a number of other countries employ what is known as a "classical" tax system. Among the provisions of this system is that capital gains are taxed at the personal level at a favorable rate compared to dividend and interest income.[10] A number of recent studies have examined the cost of equity finance (retentions and new share issues) under the classical tax system. A partial list includes King [1974, 1977], Auerbach [1979a, 1979b, 1982a, 1982b], Bradford [1981], Haley and Schall [1979] and Atkinson and Stiglitz [1981]. In each study, retained earnings is shown to dominate new share issues as a source of finance. The basic intuition is that no tax savings occur from the issue of new shares, while tax savings do occur when earnings are retained because a dividend tax is avoided for a lower tax on capital gains. Given the typical shareholders' marginal tax rate, the tax advantage of retentions over new share issues appears to be quite large.[11]

Flotation costs are a second reason why internal finance is a lower cost source than external finance. This is particularly true for small issues of debt or new shares - and therefore especially relevant to fringe firms - because the transaction costs tend to be largely fixed costs.[12]

The usual explanations given for the low debt-equity ratios employed by unregulated firms in the United States is the costs of financial distress and agency costs.[13] Financial distress refers to the set of problems that arise whenever a firm has difficulties in meeting its principal and interest obligations. Bankruptcy is the most extreme form of financial distress. There are several possible types of costs arising from financial distress short of bankruptcy including lawyers' and accountants fees, lost sales, higher costs of production, reduced output, foregone or delayed investment, higher financing costs, and general disruption of firm activities.[14] Another cost of debt finance is agency costs, which arise from the efforts of creditors of the firm to ensure that the firm honors its contractual obligations. These costs result from the attempts by creditors to modify or control firm decisions[15] and the failure to make some investments due to the pricing of debt contracts. Agency costs at the margin tend to rise with the debt-equity ratio because as this ratio increases, investments which maximize stockholder interests tend more and more to deviate from those that maximize the joint interests of stockholders and bondholders.

In summary, then, the favorable taxation of capital gains and flotations costs give advantages to retentions over new share issues. A further advantage, that of limited capital markets, arises if the common stock of a company is not freely traded in the capital markets, quite often the case for fringe firms. Finally, the existance of flotation costs, costs of financial distress, and agency costs explain the low debt-equity ratios of unregulated firms.

(B.) SOURCES OF FINANCE: SOME STATISTICS

We present below some statistics on firm financial patterns across size classes. Table I reports the average retention ratios for corporations broken down by asset categories for the last decade.[16] It is apparent that the percentage of income retained by small firms is very high - firms under 10 million dollars in assets retained on average approximately 80% of their income. We emphasize that these are average numbers. It is certainly true that many small firms in declining markets or in industries with limited investment opportunities retain little or no income. A sizable percentage of small firms, then, must be retaining virtually 100% of their income. A good example of a group of fringe firms retaining 100% of their income for a period of several years is the peripheral equipment and subsystems segment of the computer industry.[17] Getting slightly ahead of our line of analysis, it is interesting to note IBM'S response. According to Alan McAdams, an expert witness in U.S. vs. I.B.M., I.B.M. in the early 1970's adopted a strategy of setting low prices with the intention of lowering the earnings, and therefore the available finance, of the peripheral equipment supplies.[18]

TABLE I

RATES OF RETENTION BY ASSET CLASSES,

1970-1979 (assets classes in millions)

Under 1	1-5	5-10	10-25	25-50	50-100	100-250	over 250
82.6%	83.5%	78.9%	73.0%	67.6%	62.0%	50.4%	31.9%

Table I does not preclude the possibility that external finance is a large portion of total finance for some size categories. It turns out that this is not the case. For the last decade, IRS statistics show that corporations under 250 million in assets had debt/(debt + equity) ratios of approximately 25% and that very little variation existed across the size classes reported above.[19] Turning to new share issues, IRS statistics for the last decade show that less than 10% of all new equity for corporations under 250 million in assets came from this form of external finance.[20] Once again, there was very little variation across size classes.

IV. THE EXPANSION EQUATION

The importance of internal finance is clearly a reason why current expansion of the fringe (and future output) is a function of current price. The greater the p(t) established by the dominant firm, the greater the available current internal finance for the purchase of capital and the expansion of output.

The income of the fringe available for expansion is $[(1-T)(p(t) - c_f)x(t)]$, where T is the corporate income tax and c_f is the non-capital costs of production up to the capacity constraint x(t).[21] (We assume, as does Gaskins, constant returns to scale and a capacity constraint.) The fringe obviously will not retain 100% of its income in all time periods - eventually it must collectively pay some dividends. Let u(t) be the fraction of earnings retained by the fringe. (As we will see in the next section, u will be the control variable of the fringe.) Then the expansion equation of the fringe can be written as:

$$\dot{x}(t) = [(1-T)(p(t)-c_f)x(t)]\hat{J}u(t) \tag{7}$$

where $\hat{J}$ is the physical output-dollar value of capital ratio.[22] A useful way to think about the expansion equation is that if K(t) is the dollar value of the capital stock of the fringe at time t, then $x(t) = K(t)\hat{J}$, and thus $\dot{x}(t) = \dot{K}(t)\hat{J}$. $\dot{K}(t)$ is just the term in brackets in equation (7) when u = 1.

It should be noted that debt finance has not been included in the expansion equation. If debt can be increased by some fraction of a dollar for every additional dollar of new internal finance, then a multiplier equal to the ratio of (debt + equity)/equity could be included. Following Spence

[1979, p.4] we therefore do not include debt finance, but note that it could be easily incorporated in $\hat{J}$.

Before proceeding to the solution, it is appropriate to compare our fringe expansion equation with that of Gaskins', $\dot{x}(t) = k_o e^{\gamma t}(p(t) - \bar{p})$. Gaskins assumed a linear relationship between $\dot{x}$ and p. Interestingly enough, our expansion equation has a linear relationship between these variables - quite simply, fringe income varies proportionately with p for an $x \neq 0$. It is also true, of course, that fringe income varies proportionately with x for any p. Contrary to Gaskins' formulation, then, $\dot{x}$ is a function of x, as one might expect. Indeed, we can rewrite equation (8) as a rate of expansion: $\dot{x}(t)/x(t) = (1 - T)(p(t) - c_f)\hat{J}u$.

We noted that Gaskins' response coefficient, $k(t) = k_o e^{\gamma t}$, is completely unspecified. Something analogous to Gaskins' k can be found in our formulation by taking the partial derivative of equation (8) with respect to p(t):

$$\frac{\partial \dot{x}(t)}{\partial p(t)} = k(t) = (1-T)x(t)\hat{J}u.$$

Our "response coefficient" depends on the corporate tax rate, the current size of the fringe, the fringe retention rate, and the physical output-capital ratio. What is especially important is that the parameters $\hat{J}$ and T are knowable a priori - that is, for individual industries one could determine what the response coefficient is at any moment in time. It is apparent that our response coefficient will increase over time as long as $\dot{x}(t) > 0$. Our formulation does not, however, provide any economic justification for Gaskins' assumption that k(t) grows exponentially over time in every industry at some common rate γ.

V. Solution

We shall determine the nature of equilibrium in a dynamic game between the dominant firm and the competitive fringe. In this game the dominant firm chooses a price path, $p(t)$, and the fringe firms choose their reinvestment rate, $u(t)$. Equilibrium is that pair of $p(t)$ and $u(t)$ such that each is a best reply to the other. In examining the Nash open-loop equilibrium, we are implicitly assuming that at some initial time the players simultaneously make irreversible decisions concerning $p(t)$ and $u(t)$. We will see that our open-loop equilibrium is identical to the closed-loop subgame-perfect feedback equilibrium when the growth rate of demand is large relative to the rate of interest, and that when demand growth is slow steady-state outcomes for the both equilibrium concepts must be the same independent of the growth rate of demand. Closed-loop equilibrium is more realistic since it models continuous and sequential decision-making, but intractable. Since the two equilibrium concepts are so closely related in our problem, it is reasonable to examine the open-loop equilibrium.

Both players make their choices in order to maximize discounted profits, with the dominant firm taking into account its impact on fringe capacity. Following Gaskins' notation, we let $f(p)$ be demand at $t = 0$, and let x_0 be fringe capacity at $t = 0$. We assume that the before-tax interest rate is $r > 0$ and that market demand grows at the rate of $\gamma \geq 0$. Recall that c_f is the variable marginal cost for the fringe up to the capacity constraint x, where the absolute capacity constraint x can be increased by J units per dollar of gross profits where $J = \hat{J}(1 - T)$.

c_d is the marginal cost of production for the dominant firm. c_d may be interpreted in a number of ways. First, one could assume that the dominant firm is using a technology different from the fringe firms, one without marginal capital costs, i.e., there is a large initial set-up cost, but thereafter costs are proportional to output. Such an assumption would allow us to ignore the capacity problem of the dominant firm. While this is not an absurd assumption (since the dominant firm operates at a different scale of production, it is not unrealistic that it use a different technology of this type) the asymmetry is bothersome. If the dominant firm uses a technology similar to that of the fringe firms, it would also have a capacity choice, and we must make assumptions which are consistent with the imperfect capital market assumption that is crucial to our analysis. In our description of the constraints on the fringe firms, we are implicitly assuming that there is no possibility of leasing the equipment and that there is no resale value to the equipment, presumably due to *ex post* firm specificity of the equipment and high costs of monitoring care of leased equipment. We should also make these assumptions apply to the dominant firm. Nevertheless, we may often ignore the dominant firm's capacity choice in this case. If investment is irreversible and dominant firm's sales are always growing, then c_d must be interpreted as the total marginal cost. If the dominant firm's sales is declining, then c_d is the marginal variable cost, since the capital costs are sunk and unrecoverable. We will assume that c_d is constant through time. This means our analysis applies to two cases: either there is no marginal capacity cost for the dominant firm or, the dominant firm's sales are monotonic and capacity costs are admissible.

Recall that w is the fringe capacity expressed as a proportion of market size, that is, $w(t) = x(t)e^{-\gamma t}$. w is the state variable of interest to both players, the dominant firm wanting to keep it low and the fringe possibly wanting to increase it. For both, the state equation is

$$\dot{w} = (p - c_f)wuJ - \gamma w \tag{8}$$

which is derived from the fringe expansion equation, (7).

The dominant firm's problem is

$$\max_{p(t)} \int_0^F e^{\gamma t}(f(p) - w)(p - c_d)e^{-rt}dt$$

s.t.

$$\dot{w} = (p - c_f)\ wuJ - \gamma w$$

where $F \leqslant \infty$ is the time at which the game ends. It is appropriate to drop the tax rate, T, from the maximand since profits are pure rents. Since the tax rate is included in J, taxes will affect the equilibrium, but only to the extent that they deprive the fringe firms of investment funds. Let η be the dominant firm's shadow price for w. By the Pontryagin Maximum Principle

$$\dot{\eta} = r\eta + p - c_d - \eta u(p - c_f)J \tag{9}$$

and p(t) is chosen to maximize the current-value Hamiltonian (where $r - \gamma$ is the appropriate discount rate)

$$H(w,p,\eta) = \begin{cases} (p - c_d)(f(p) - w) + \eta((p - c_f)wuJ - \gamma w), & p > c_f \\ (c_f - c_d)\ f(p) - \eta\gamma w & ,\ p = c_f \end{cases} \tag{10}$$

implying that

$$p = c_f \quad \text{or} \quad 0 = (p - c_d)f' + f(p) - w + \eta uw. \tag{11}$$

The corner choice, $p = c_f$, cannot be ruled out since at that price the fringe shuts down, causing the Hamiltonian to look like the graph in either Figure 1a, where the corner choice of c_f is the solution, or Figure 1b, where the optimal p, p^o, is above c_f.

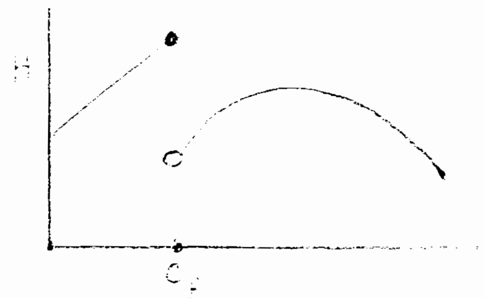

Figure 1a

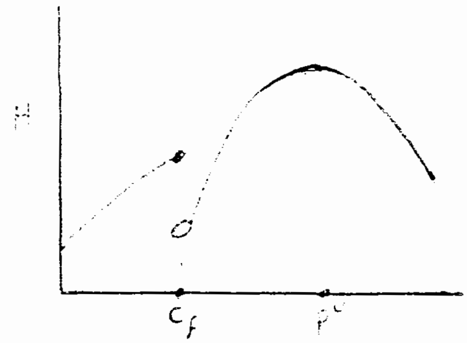

Figure 1b

Each fringe firm will maximize the present value of its net cash flow, taking prices as given. Since each firm is a price-taker, the fringe acts in the aggregate as a profit-maximizing price-taker. Again, since profits are pure rents, before-tax profits are maximized. Therefore, the competitive fringe solves the problem

$$\underset{u(t) \in [0,1]}{\text{Max}} \int_0^F e^{\gamma t}(p - c_f)w(1 - u)e^{-rt}dt$$

s.t.

$$\dot{w} = (p - c_f)wuJ - \gamma w$$

If λ is the shadow price for w from the point of view of a fringe firm, then its evolution is described by

$$\dot{\lambda} = r\lambda - (p - c_f)(1 - u) - \lambda(p - c_f)uJ \tag{12}$$

and the decision rule for a fringe firm is

$$u = \begin{cases} 1, & \lambda > J^{-1} \\ \in [0,1], & \lambda = J^{-1} \\ 0, & \lambda < J^{-1} \end{cases} \tag{13}$$

The fringe decision rule is bang-bang since both the payoff and equation of motion are linear in the control, u.

Case I: $\gamma < r$

We first examine the slow growth case where the rate of market growth is less than the interest rate. We consider the case of an infinite horizon, exactly the situation examined by Gaskins.

First, we determine the steady state outcome. It is not possible that in the steady state the dominant firm limit prices by setting $p = c_f$. Such a steady-state price would imply that the fringe firms would not invest since the quasi-rent would be zero, implying that the fringe would disappear, making it irrational for the dominant firm to set price equal to c_f. Hence, in any steady state, the firm must choose a price on the interior of its choice set. Under this assumption, we find that the steady states are:

(i) $w^{SS} = 0$ and u arbitrary; or

(ii) if $c_f + rJ^{-1} \geq c_d$, then

$$\eta = (c_f + rJ^{-1} - c_d)/(\gamma - r)$$
$$\lambda = J^{-1}$$
$$p = rJ^{-1} + c_f$$
$$u = \gamma/(p - c_f)J = \gamma/r$$
$$w^{SS} = ((p - c_d)f'(p) + f(p))/(1-\eta uJ) \text{ ; or}$$

(iii) if $c_f + rJ^{-1} < c_d$, the "dominant" firm is driven out and price equals $c_f + rJ^{-1}$, the fringe's long-run marginal cost.

In (i), the fringe does not exist so the dominant firm will set price at the monopoly level. In (ii), the fringe exists and has size w^{SS}. It is straightforward to check that the dominant firm will not set price equal to c_f, in this steady state. Hence the steady state is an equilibrium. In (iii) the fringe eliminates the dominant firm because of its superior cost structure.

We must next examine the evolution of the game out of steady state. We want to determine whether the game can converge to the steady state and the extent the dominant firm can affect this transition. Due to the specific nature of the manipulations involved in this transition analysis, we assume that demand is linear:

$$f(p) = a - bp.$$

A convenient reference price will be the dominant firm's monopoly price,

$$p^* \equiv \frac{a}{2b} + \frac{c_d}{2}$$

that is, if there were no fringe as in case (i) above, then the dominant firm would charge p^*. We will also concentrate on the more interesting case where $c_f + rJ^{-1} \geq c_d$. The alternative has a trivial steady state and the analysis of convergence to that condition requires only minor adjustments to the following analysis.

In general, the dominant firm's behavior is described by setting

$$p = c_f, \quad \text{or } p = p^* + \frac{w}{2b}(\eta uJ - 1), \tag{14}$$

whichever maximizes the Hamiltonian, H, which is given by substituting $a - bp$

for $f(p)$ in (10).

Next, we will show that for w close to w^{SS}, we can construct an equilibrium path which will converge monotonically to the steady state. We need to first establish that, given this convergence assumption, when w is close, but not necessarily equal, to w^{SS}, price and the fringe shadow price of capacity are equal to their steady-state values. To establish this we show that it is inconsistent for price, shadow price, and fringe capacity to all converge gradually to their steady-state values. Since w cannot jump, it must converge gradually to w^{SS}, if it converges. There would appear to be four combinations of p and λ converging asymptotically to their steady-state values from above or below. However, p and λ must move in the same direction. If λ is above and falling to its steady-state value, J^{-1}, then price must also be falling. This follows from the fact that $u=1$ when λ exceeds J^{-1}, implying via (12) that λ is falling if and only if p also exceeds its steady-state value, $c_f + rJ^{-1}$. Similarly, if λ is less than and rising to J^{-1}, then p also is less than and rising to its steady-state value.

Next suppose that p exceeds the steady-state price and λ is falling to its steady-state value. Then $u=1$ since λ exceeds J^{-1}. This together with (8) implies that the rate of growth in w would exceed $r - \gamma$, since p exceeds $c_f + rJ^{-1}$. Since we are examining the slow growth case, $r - \gamma$ is positive and w must hit w^{SS} at some finite time. To stay at w^{SS} at this point, u must fall to its steady-state value. However, (14) shows that price increases when u falls, implying that when w hits w^{SS}, price would have to rise and thereby stay above its steady-state value, a contradiction. Similarly, one can prove that if λ and p were to rise to there steady-state value from lower values, price would have to drop and stay below its steady-state value when w^{SS} were hit. Therefore, if w is to converge to its steady-state value monotonically,

λ and price must be at their steady-state values when w is close to w^{SS}.

Since p and λ are constant as w converges to its steady state, u must be changing. To determine the relationship between u and η as w converges to its steady state, we differentiate the price equation, (14), with respect to time. (Equations (14) and (8) are used to eliminate w and $\dot{w}$ from the resulting expressions.) The result shows that u and η must obey

$$\dot{u} = \frac{r - \gamma - u\eta J(r - \gamma) - Ju(c_f + rJ^{-1} - c_d)}{\eta J}$$

$$\dot{\eta} = r(1 - u)\eta + c_f + rJ^{-1} - c_d.$$

The phase diagram for this system is represented in Figure 2. Note the saddle point stability of the steady state. If w is close to w^{SS}, then there are unique η and u on the stable manifold such that $c_f + rJ^{-1} = p^* - \frac{w(\eta uJ - 1)}{2b}$ since any hyperbola of the form $\eta u = k$ has a unique intersection with the negatively sloped stable manifold. Therefore, for w near its steady-state value, w^{SS}, there is a unique η-u pair on the stable manifold of Figure 2 consistent with w and the pricing formula. As η and u converge to their steady-state values in Figure 2, the unique corresponding w also converges to w^{SS}. We have thereby demonstrated the existence of an equilibrium path near the steady state which converges monotonically to the steady state. We make no claim of uniqueness since we have not ruled out cycles. However, our arguments show that this equilibrium is the only one converging monotonically to steady state. In this equilibrium, u and η follow the stable manifold in Figure 2 to the steady state of that system, with w being determined by the price equation (14), since $p = c_f + rJ^{-1}$ along this path.

Finally, we show that there are equilibrium paths for arbitrary w. Suppose w is less than w^1, where $(1,\eta^1)$ is on the stable manifold of Figure 2 and w^1 is that value of w consistent with it. To analyze this we examine the phase diagram for η and w when u=1 displayed in Figure 3. This system is given by equations (8) and (9) with u set equal to one. Since the steady-state w in this phase diagram is associated with a price of $c_f + \gamma J^{-1}$ and u = 1, it exceeds the true steady state, w^{SS}. Since u = 1 and $p = c_f + rJ^{-1}$, w is increasing and our (η^1, w^1) pair is above the $\dot{w} = 0$ locus in Figure 3. Therefore, we can run time back from (η^1, w^1) and always remain above the $\dot{w} = 0$ locus. This implies from our pricing formula that price is above $c_f + rJ^{-1}$ and rising as we run time backwards. Hence, if λ is J^{-1} when we are at (η^1, w^1), then λ rises as we move back in time, proving that u = 1 is consistent with the evolution of λ during that time. Hence, we have constructed an equilibrium for small initial w.

If w is large, initially u is zero and $\lambda < J^{-1}$, but λ increases and hits J^{-1} exactly when η^0 and w^0 are hit, where these are the values of η and w consistent with u = 0 and η being on the stable manifold of Figure 3. This case is even more straightforward since it is just the solution to the piecewise linear ordinary differential equations:

$$\dot{\eta} = r\eta + \max(P - c_d, 0)$$

$$\dot{\lambda} = r\lambda + \max(P - c_f, 0)$$

$$\dot{w} = -\gamma w$$

where

$$P = \arg\max_p \pi(p)$$

$$\pi(p) = \begin{cases} (p - c_d)(a-bp-w), & p > c_f \\ (p - c_d)(a-bp) & p \leq c_f \end{cases}$$

and we impose the boundary conditions

$$\eta(T_0) = \eta^0$$

$$w(0) = w_0$$

$$w(T_0) = w^0$$

$$\lambda(T_0) = J^{-1}$$

where w_0 is the initial value of w. Note that T_0, the length of time that $u = 0$ is endogenous, being determined from the $\dot{w}$ equation.

The equilibrium that we have constructed has several interesting features. First, the dominant firm does use its price-setting power to restrain fringe firm expansion, since $\eta \neq 0$. However, its "limit pricing" behavior decays over time and does not have any long-run impact on performance since the steady-state price is fringe long-run marginal cost. The dominant firm prices high initially, but not as high as its static monopoly price. Its price is reduced as the fringe grows in size. At some finite time price equals fringe long-run marginal cost and remains there forever. This does not stop fringe expansion, but after this time, the fringe reinvestment rate drops monotonically from 1 to the steady-state rate and earnings are distributed to investors.

In summary, we can conclude:

Theorem 1: If $\gamma < r$ and $F = \infty$, there is an equilibrium where:

I. If w_0 is sufficiently small,

i) price is initially above $c_f + rJ^{-1}$;

ii) at some finite time, t_1, price is $c_f + rJ^{-1}$;

iii) price drops to $c_f + rJ^{-1}$ and $u = 1$ for $t < t_1$;

iv) price equals $c_f + rJ^{-1}$ for $t \geq t_1$;

v) u drops smoothly from 1 to its steady-state value for $t \geq t_1$;

II. If w_0 is sufficiently large, then

i) price is initially below $c_f + rJ^{-1}$;

ii) at some finite time t_1, price is $c_f + rJ^{-1}$;

iii) price rises to $c_f + rJ^{-1}$ and $u = 0$ for $t < t_1$;

iv) price equals $c_f + rJ^{-1}$ for $t \geq t_1$;

v) u rises smoothly from 0 to its steady-state value after t_1.

We can partially compare the optimal trajectories found by Gaskins with our results. Recall that the present-value Hamiltonian for Gaskins' formulation (when $w(t)$ is substituted for $x(t)e^{-\gamma t}$) is given by:

$$H(w,p,z) = (p(t)-c_d)(f(p) - w(t))e^{-rt} + z(t)[k_o(p(t) - \bar{p}) - \gamma w(t)]$$

and that the (present value) Hamiltonian for our formulation when $u = 1$ is:

$$H(w,p,\eta) = (p(t) - c_d)(f(p) - w(t))e^{-rt} + \eta(t)e^{-rt}[(p(t) - c_f)w(t)J - \gamma w(t)]$$

The first term in either Hamiltonian is the present value accruing from current sales while the second term reflects the effect of current entry on future profits. Differences between the two Hamiltonians occur only in the second terms and arise because of the different expansion equations. Note in particular that the value of the second, dynamic, term is proportional to the value of $w(t)$ in our Hamiltonian but not in Gaskins' because our rate of expansion, $\dot{w}(t)$, is proportional to $w(t)$. This imples that our game equilibrium analysis yields higher initial prices when fringe shares are small. This is expected since maximizing the Hamiltonian with respect to price involves a balancing of the first and second terms, and the value of our second term becomes small as fringe output becomes small. Since our terminal price is lower, and price attains this lower price at some finite time, the average rate of decline in price when the initial w is small must be greater in our game analysis than in Gaskins' model. This is intuitive since the reduced long-run effectiveness of limit pricing in the game analysis encourages the dominant firm to be more aggresive in acquiring profits through high prices in the initial stages when it has a greater market share.

The necessary conditions for a maximum value for the dominant firm's problem for either formulation can be written as a system of differential equations in $p(t)$ and $w(t)$. The system of equations for Gaskins' formultion was given in Section II, equations (5) and (6). The $\dot{w}$ and $\dot{p}$ equations for our model are determined by differentiating the price equations with u=1, and are given by[24]

$$\dot{w}(t) = (p(t) - c_f)w(t)J - \gamma w(t)$$

$$\dot{p}(t) = \frac{(c_f - c_d)w(t)J - (r - \gamma)[f(p) - w(t) + f'(p)(p(t) - c_d)] + \gamma w(t)}{-2f'(p) - f''(p)(p(t) - c_d)}$$

A comparison of Gaskins' $\dot{p}(t)$ equation with ours indicates that only the numerators differ. The differences arise only because our response coefficient is endogenous and depends linearly on $w(t)$. Therefore Gaskins analysis is very similar to our $u=1$ phase. However, u cannot always be one since this would be irrational for the fringe firms. The crucial difference is that rational firms will reduce their growth rate when their investment needs become smaller than available revenues.

When the price drops to the fringe long-run marginal cost, full reinvestment ceases and price remains constant in our game, whereas in Gaskins, the steady-state price is approached only asymptotically, and will always exceed long-run fringe marginal cost, except in the steady state of the no growth case. Gaskins finds this to be a "disturbing result" (p. 317) and provides numerical examples which show rather large deviations of price over marginal cost. We find instead that the steady-state price always equals fringe marginal cost and that it is reached in a finite period of time.

Case II: $\gamma > r$

Next we examine the case of rapid growth where $\gamma > r$. To keep payoffs bounded, we must assume that the game ends at some $F < \infty$. We have in mind two types of situations. First, one could think of the good as being faddish in nature with demand growing rapidly, but then dropping to zero at some time F. Second, and more realistically, this analysis will be directly useful in examining our third case where demand initially grows rapidly, but then slows down. In order to assure survival of the dominant firm, we assume $c_d < c_f + \gamma J^{-1}$; otherwise, even if the dominant firm charged only its breakeven price, the fringe would want to fully invest (since γ exceeds r) until nearly F, and if c_d exceeded $c_f + \gamma J^{-1}$, the fringe would grow more rapidly than the market, squeezing the dominant firm out. To avoid the trivial case of natural monopoly where the fringe shrinks relative to the market even if price were p^*, we assume that $c_f + \gamma J^{-1} < p^*$. Again, we assume demand is linear.

In this case, a crucial fact is that the price rises as long as $u = 1$ and price exceeds c_f. This follows from:

$$\dot{p} = \frac{\dot{w}}{2b}(\eta J - 1) + \frac{w}{2b}\dot{\eta}J$$

$$= \frac{w}{2b}((c_f - c_d)J + \gamma + (r - \gamma)\eta J)$$

which is positive since $r < \gamma$ and $c_d < c_f + \gamma J^{-1}$. If $u = 0$, then $p = p^* - w/2b$ and p rises since w falls with no reinvestment.

From this, we may further conclude that $\ddot{\lambda} < 0$ also when $u = 0$ or 1 and λ is falling since

$$\ddot{\lambda} = \begin{cases} r\dot{\lambda} - \dot{p}, & u = 0 \\ r\dot{\lambda} - \dot{p}\lambda, & u = 1 \end{cases}$$

is negative if λ is falling and p is rising.

Next we establish that in equilibrium, the fringe will go through possibly three phases, initially doing no expanding, then reinvesting at a 100% rate until some time $F_0 < F$, after which it does not reinvest, where $F - F_0$ is bounded above independent of F and the initial conditions.

First, we show that if λ approaches J^{-1} from above, then λ must pass through J^{-1} immediately. Since $\dot{\lambda} < 0$ during such an approach to J^{-1}, price must exceed $c_f + rJ^{-1}$ from (12). Since $u=1$, price is rising and cannot approach $c_f + rJ^{-1}$ from above as λ converges to J^{-1}. When λ hits J^{-1}, u cannot rise since it equals 1, implying that price cannot fall. Since λ is concave in time, when λ is J^{-1}, price must exceed $c_f + rJ^{-1}$ and $\dot{\lambda}$ must be negative. Once λ falls from J^{-1}, then $u = 0$ and λ continues to decline, implying that $u = 0$ thereafter. Also, $\dot{\lambda}$ is bounded above by $-r(p^* - c_f - rJ^{-1})$ when $u = 0$, implying that λ moves from J^{-1} to 0 in an amount of time bounded above independent of F and w. However, $\lambda(F) = 0$ is the fringe firm's transversality condition at the end of the game, proving that the length of the final stage is bounded above independent of F. In summary, we have shown that once λ hits J^{-1} from above it must continue to decline. Therefore, the fringe goes through at most the three stages described above since once it begins reinvesting all earnings, it never ceases that policy until it stops all reinvestment forever.

During the final stage of the game from $t=F_0$ to $t=F$, η follows

$$\dot{\eta} = r\eta + p^* - c_d - w/2b$$

Since $\dot{w} = -\gamma w$ during this stage and since $\eta(F_0) = 0$ also, it follows from solving these linear differential equations that $|\eta(F_0)|$ is also bounded above independent of F. Let N(w) be the value of η at F_0 if the fringe is w at F_0 and the fringe decides to stop expanding at F_0, i.e., $\lambda(F_0) = J^{-1}$. Since η, w, and λ are governed by the linear differential equations above, there is a unique such N(w). Let $W(\eta)$ be the inverse correspondence of N(w). $W(\eta)$ is then the possible sizes of the fringe at F_0 if the fringe ceases to expand when the dominant firms costate is η.

The next piece we need for our analysis is the phase diagram of the dominant firm's behavior in the intermediate stage when $u = 1$. This is described in Figure 4. We first describe the dominant firm's choice between setting price equal to the fringe marginal cost and making the interior choice. Since $\eta < 0$ (more w depresses the dominant firm's profits), it is clear from our price equation that when w and/or η are large in magnitude, p =

c_f will be chosen. This says that if the fringe is large or if the future lost profits from expansion of the fringe is large, then the dominant firm sets price equal to fringe firm costs, causing the fringe firm's expansion to cease. Let $M(w)$ be the η such that

$$\eta < M(w) \Rightarrow p = c_f$$

$$\eta > M(w) \Rightarrow p = p^* + \frac{w}{2b}(\eta J - 1)$$

$M(w)$ exists because H^{ℓ} evaluated at $p^* + \frac{w}{2b}(\eta J - 1)$ is strictly monotonic in η. Examination of the Hamiltonian also shows that $M(w)$ is increasing in w, that is, the larger the fringe is, the smaller is the critical η at which price is set at c_f by the dominant firm. $M(w)$ is therefore as displayed in Figure 4. Also, as c_d decreases and as c_f increases, $M(w)$ shifts up, increasing the likelihood that $p = c_f$ is chosen.

The final pieces needed for construction of our phase diagram are the stationary loci for w and η when $u=1$ and $\eta > M(w)$--that is, the fringe is not shut down. Straightforward calculations show that, if $\eta > M(w)$,

$$\dot{\eta} = 0 \Rightarrow w = \left(\frac{p^* - c_f}{1 - \eta J}\right)2b + \frac{r\eta 2b}{(1 - \eta J)^2} + \frac{(c_f - c_d)2b}{(1 - \eta J)^2}$$

$$\dot{w} = 0 \Rightarrow w = \left(\frac{p^* - c_f}{1 - \eta J}\right)2b - \frac{\gamma 2b}{(1 - \eta J)J}$$

At this point we use the assumption that $\gamma < (p^* - c_f)J$ to assure a positive w when $\dot{w} = 0$. This is reasonable since it just states that the growth rate γ is slow enough that the fringe can increase its market share if it fully reinvests and the price is the dominant firm's monopoly price.

It is straightforward to calculate that the $\dot{\eta} = 0$ and $\dot{w} = 0$ loci never intersect for $\eta \leq 0 \leq w$. Also, the w intercept of the $\dot{u} = 0$ locus exceeds the w intercept of the $\dot{w} = 0$ locus. We assume that at the w intercept of the $\dot{\eta}$ locus p exceeds c_f, that is, when there is no dynamic consideration and w is at the value where $\dot{\eta} = 0$, the incumbent does not choose to shut down the fringe. (This is true, for example, if $c_f \leq c_d$. In fact, for $c_f \leq c_d$, the $\dot{\eta} = 0$ locus lies to the left of the $\eta = M(w)$ locus.)

To piece together the analyses of u = 1 and $t > F_0$, note that just prior to $\lambda = J^{-1}$, u is 1. Since λ is falling, $p > c_f + rJ^{-1}$. Hence

$$\dot{\eta} = (c_f + rJ^{-1} - p)\eta J + p - c_d > 0,$$

proving that η is rising at the moment the fringe shuts down.

Putting these pieces together, we get the phase diagram presented in Figure 4. This phase diagram represents the possible paths of the game when the fringe is fully reinvesting.

We now can determine that the game goes through possibly four phases. First, the fringe may not want to expand and the dominant firm sets price equal to the static leadership price. This will be the case if w is initially large, making the static leadership price small and fringe investment unattractive. Eventually w will be sufficiently small and the static leadership price sufficiently large that the fringe will want to expand. At this point, the game begins to be described by Figure 4. The game may be below and to the right of the $\eta = M(w)$ locus where $p = c_f$ and the fringe capacity decreases relative to market size. In this case, the dominant firm decides to limit price and prevent any fringe growth. η also decreases, that is, the current value of the marginal cost to the dominant firm of fringe expansion increases. Eventually, the $\eta = M(w)$ locus is hit. For a while, the game may move along $\eta = M(w)$. This would be accomplished by the dominant firm using a "mixed" (using a generalized curve sense of mixing) strategy, alternating between limit pricing, $p = c_f$, and the alternative, $p = p^* - \frac{w}{2b}(\eta J - 1)$. Eventually, however, the game moves to a third phase (or is in this third phase when the fringe begins to want to expand) of price exceeding c_f and increasing but being kept low by the dominant firm to slow expansion of the fringe. During this phase the game proceeds through regions A, B, and C. In region A, the price is low enough that w and fringe market share are decreasing. However, η is also decreasing, meaning that marginal fringe capacity is increasingly more costly to the dominant firm. This decline in η is ended at some point where the $\dot{\eta}$ locus is crossed and the game moves from A to region B. In B, w is still falling, but η is rising. Since the marginal cost of fringe capacity is declining the dominant firm eases up on the price. This continues until the marginal cost of fringe capacity is so small that the dominant firm will allow it to grow. This happens when the stationary w locus is crossed and the game moves to region C. In C fringe market share is rising, continuing to rise until the $w = W(\eta)$ locus is reached, after which the fringe ceases to reinvest and the dominant firm engages in static leadership pricing. It is straightforward to check that p rises all through regions A, B, and C and jumps when the fringe ceases to expand.

In summary, we can conclude:

<u>Theorem 2</u>: In the equilibrium of our game with $\gamma > r$ and $\bar{F} < \infty$, there are up to four phases, which are, in order,

i. the fringe does not want to reinvest, and price is the static leadership price;

ii. the fringe wants to invest and price is set to c_f;

iii. the fringe sets $u = 1$ and price rises above c_f;

iv. the fringe sets $u = 0$, and price is the static leadership price and continues to rise.

Note that the fast and slow growth cases differ substantially. In the slow growth case, the fringe size moves in a monotonic fashion whereas in the fast growth case relative fringe size may go through phases of both expansion and contraction. Price movements also differ in the same fashion with prices being much more volatile in the fast growth case. The dominant firm is much more aggressive in the fast-growth case, or at least is more successful in limiting fringe firm growth. This is not surprising, since the value of limiting fringe firm growth is greater relative to the current sacrifices implicit in limit pricing as the future market is larger relative to the current market.

Case III: Fast Then Slow Growth

Casual empiricism suggests that for many goods demand first grows rapidly then slows as the industry matures. This can be modeled in our analysis by assuming that the rate of growth initially exceeds the rate of interest, as in Case II, then at some known time, R, the rate of growth drops to a level below the rate of interest, as in Case I. The analysis of this case is accomplished by a simple union of the two preceding cases. The time R denotes the end of the fast growth phase as did F in Case II, except that at R the shadow prices are not zero, but rather are given by the initial equilibrium relationship between the w and shadow prices in the equilibrium of a slow growth game. All that is altered is the terminal surface of the fast growth phase. Figure 4 shows the phase diagram for the fast growth game and continues to be the phase diagram for the fast growth phase with fringe firm reinvestment. The curve G in Figure 5 represents the equilibrium η-w relationship which is exists at the beginning of the slow growth infinite horizon game of Case I. In this case, it is the terminal surface of the fast growth equilibrium system. This terminal surface G represents the transition between the two growth phases.

In Figure 5 we combine the fast growth phase diagram when $u=1$ with the surface G. Unifying the analysis of the two phases in this fashion shows that the resulting equilibrium has some interesting features. Initially, in the fast growth phase, the dominant firm will be very aggressive, keeping price low to slow fringe expansion. As the fast growth phase nears its end, the dominant firm cashes in by letting price rise, reaping large profits because the fringe is small but the market has grown to a large size. When the slow growth phase sets in the fringe is better able to grow sufficiently rapidly to increase its market share and forces price down to its marginal cost, which is also the long-run price.

At this point it is clear that our strong assumptions about various methods of financing expansion are somewhat stronger than necessary. While the assumption that new equity financing is completely unavailable is quite strong, it is not necessary for our equilibria to be valid. A much more reasonable assumption is that equity financing is available but is more costly than retentions. As long as the marginal cost of equity financing exceeds the marginal value of capital, λ, a fringe firm will finance expansion solely through retentions and then only if λ exceeds J^{-1}. Therefore, as long as λ is below the cost of equity financing, our equilibrium remains an equilibrium if equity financing is also possible since fringe firms will not choose to use any equity financing. Since λ declines during the final stage of equilibrium in all three cases, the equilibrium of any terminal subgame after the time when λ drops below the cost of equity financing is unaffected by the introduction of equity financing. Similar comments can be made concerning debt financing. This shows that our analysis is more general than indicated by the initial assumptions.

VI . Alternative Equilibrium Concepts

We have characterized the open-loop equilibrium of the dynamic price leadership model. First note that continuous limit pricing is never an equilibrium of our open-loop game. If the dominant firm would choose a price path $\tilde{p}(t)$ with $\tilde{p}(t) < c_f + rJ^{-1}$ always, then the fringe will never expand. However, if the fringe firms are committed to no expansion, it is not rational for the dominant firm to react with such a price path. It may be that the dominant firm would make more money charging $\tilde{p}(t)$ with no fringe expansion than it does in our equilibrium, but that outcome is not an open-loop equilibrium unless the static leadership price is less than $c_f + rJ^{-1}$. It may be the outcome if the game were a Stackelberg game where the dominant firm could not only commit itself to a fixed price path, but could also communicate such a commitment to the fringe firms before they committed themselves to any investment policy.

The final issue we should discuss is the subgame perfection of our equilibria. While a complete closed-loop subgame perfect (also known as feedback) equilibrium analysis of this game is beyond our reach at this time, certain aspects are immediately apparent. The crucial difference between these equilibrium concepts is that the open-loop solution implicitly assumes no player will react to an unexpected deviation by the state variable, w, from its expected equilibrium path caused by unexpected behavior by another player. (See Intrilligator(1971) for a formal comparison between closed-loop subgame perfect equilibrium and open-loop equilibrium.) Since the fringe is composed of infinitesimal firms, neither the dominant firm nor any fringe firm will react to unexpected behavior by any fringe firm since the state variable can only be infinitesimally affected by a fringe firm. The only issue left is whether the fringe firms will react to deviations in w caused by the dominant firm. Their strategy is often a bang-bang policy alternating between no reinvestment and 100% reinvestment. The only condition in which the fringe would react to a marginal change in the state variable is when λ is equal to J^{-1}. Hence, outside the steady state of the slow growth case where λ sits on this value forever, the assumption of a zero reaction by the fringe to changes w is consistent with rational fringe behavior.

In the case of rapid growth, we found that λ would equal J^{-1} for only an instant. At all other times, the fringe was on a corner in its choice of reinvestment rate, and hence insensitive to marginal changes in w. Therefore, our open-loop equilibrium in the rapid growth case with a finite horizon is also a closed-loop feedback equilibrium, showing that there is no subgame perfection problems with our analysis in that case.

Similarly, when λ does not equal J^{-1} in the slow growth case, the fringe is on a corner and insensitive to unexpected changes in w. The only part of our open-loop equilibrium analysis in which the fringe may be sensitive to deviations in dominant firm behavior is the steady state and the final stage of convergence to w^{SS} in the slow growth case. However, a steady-state closed-loop equilibrium price in this game must also be $c_f + rJ^{-1}$ since a larger constant steady-state price would cause the fringe to become indefinitely large, implying that price would have to fall to the minimum of c_f and c_d, and a smaller steady-state price would imply that the fringe disappears, causing the dominant firm to charge p^* instead. Therefore, if a closed-loop equilibrium converges to a steady state, the long-run price must be the same as in the steady state of our open-loop solution. These same arguments also show that a cyclical closed-loop equilibrium must oscillate between being below and above $c_f + rJ^{-1}$. It therefore appears that the long-run behavior of a closed-loop equilibrium would be closer to our open-loop equilibrium than to that of Gaskins'. While we agree that a closed-loop analysis would be preferable, it appears that our open-loop solution is an acceptable substitute for the intractable closed-loop solution since it violates the crucial closed-loop condition only during this one phase in the slow-growth case.

VII. Conclusions

In this essay we have examined the optimal pricing strategy of a dominant firm facing expansion by a competitive fringe. This problem was first examined by Gaskins[1971], who labeled the pricing strategy of the dominant firm "dynamic limit pricing". While his analysis has received widespread application, its strategic assumptions have come under telling criticism in recent years. The principle differences between our formulation and Gaskins' is that we precisely specify the constraint on fringe expansion, restriction to internal finance, and we treat the fringe as a rational, maximizing economic agent. The capital market imperfection provides a rational basis to the dominant firm's choice to keep price low today in order to limit the rate of expansion of the fringe.

In solving the noncooperative differential game between the dominant firm and the competitive fringe, we first examined the case Gaskins considered, market growth less that the discount rate, but we also examined a case with temporary rapid growth. In the equilibrium of our "slow" growth case, we find that: i) if the fringe share of the market is sufficiently small, the dominant firm will set price above its cost of production; ii) at some finite time, price will drop to the fringe long-run marginal cost; and iii) the fringe firms will retain 100% of their earnings until price equals their long-run marginal cost. While our results resemble Gaskins' in that the dominant firm will use its power to slow fringe growth, equilibrium of our model converges to a lower and at a faster. Therefore, the long-run anticompetitive nature of dynamic limit pricing is much less in our game and performance is not as adversely affected.

When the market initially goes through a period of rapid growth, we cannot be as sanguine with regard to performance. In this case the dynamic incentives for reducing current fringe are sufficiently large relative to the current cost of limit pricing that the dominant firm will initially price to reduce and keep small the fringe market share. However, when this initial phase of rapid growth draws to a close, the dominant firm cashes in by raising prices, allowing fringe firms to accumulate the necessary earnings for growth. When the market growth rate slows, the fringe grows and prices drop, converging to the slow growth steady-state values.

In conclusion, we find that when fringe firms are faced with capital market imperfections limiting the availability of external finance, dynamic limit pricing will be an important feature of dominant firm decision-making. However, in equilibrium, the importance of this behavior will depend crucially on the rate of growth of demand. Also, the implications for long-run performance are more optimistic in the open-loop equilibrium than in the myopic framework analyzed by Gaskins.

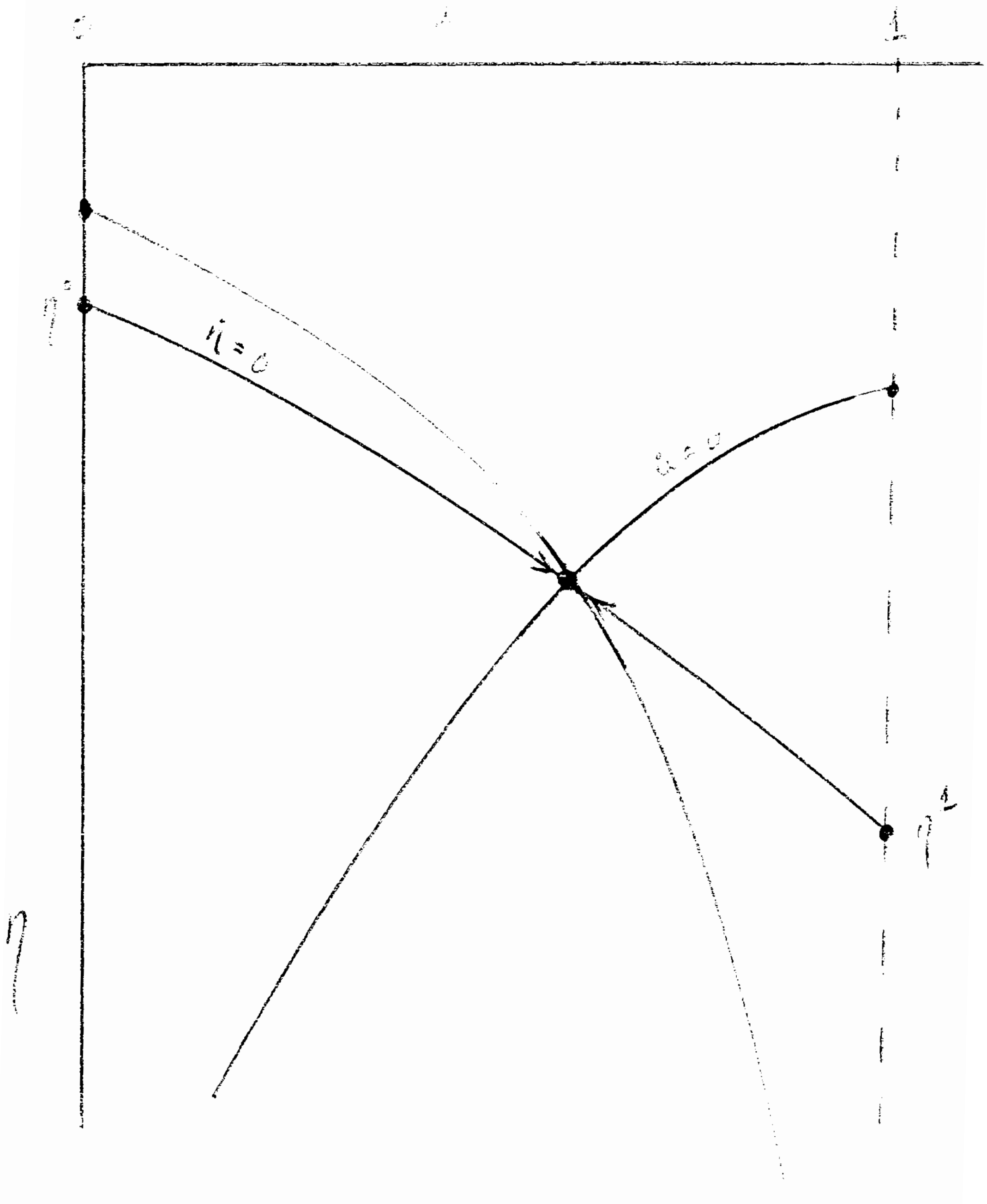

Fig. 2

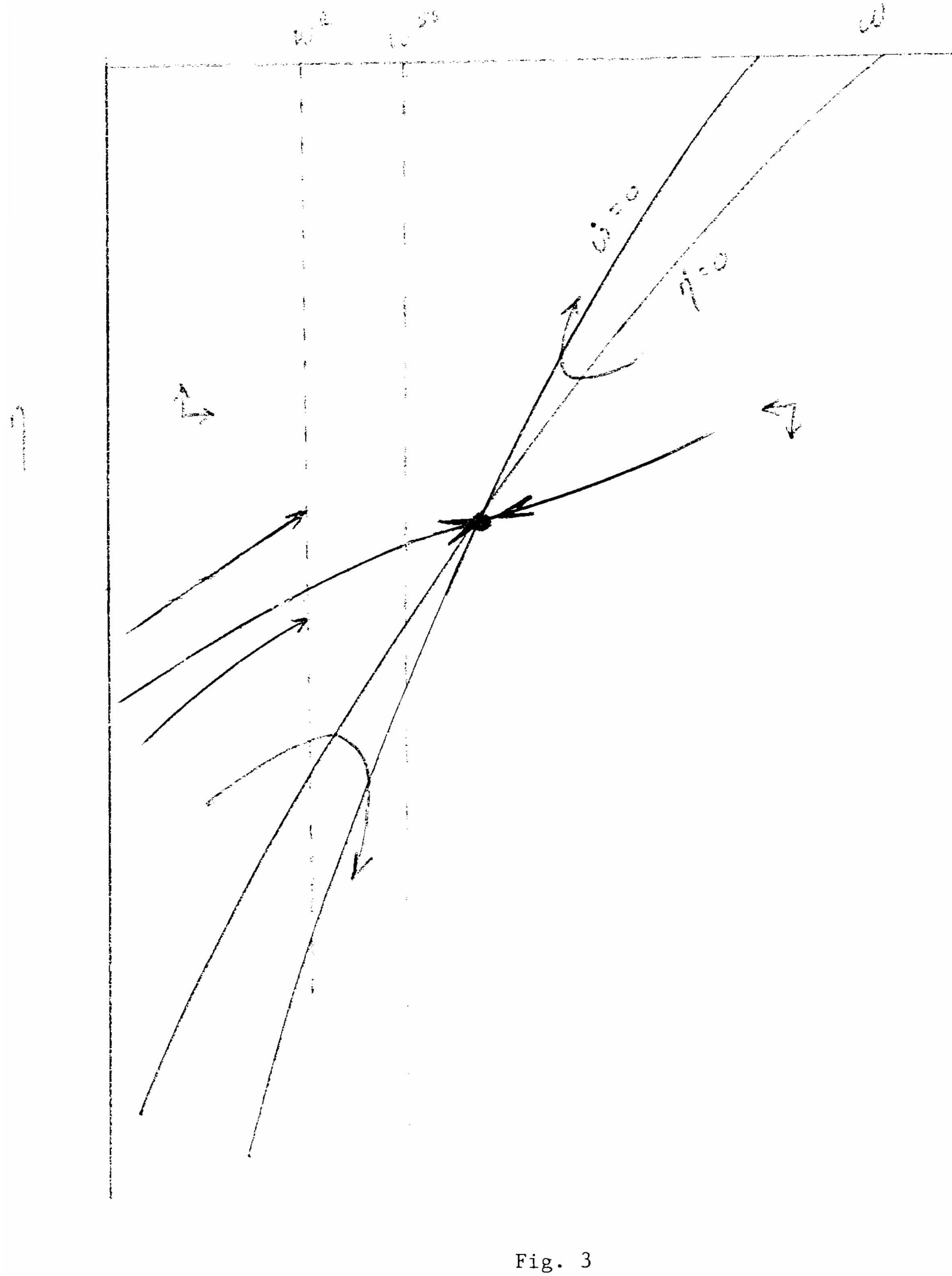

Fig. 3

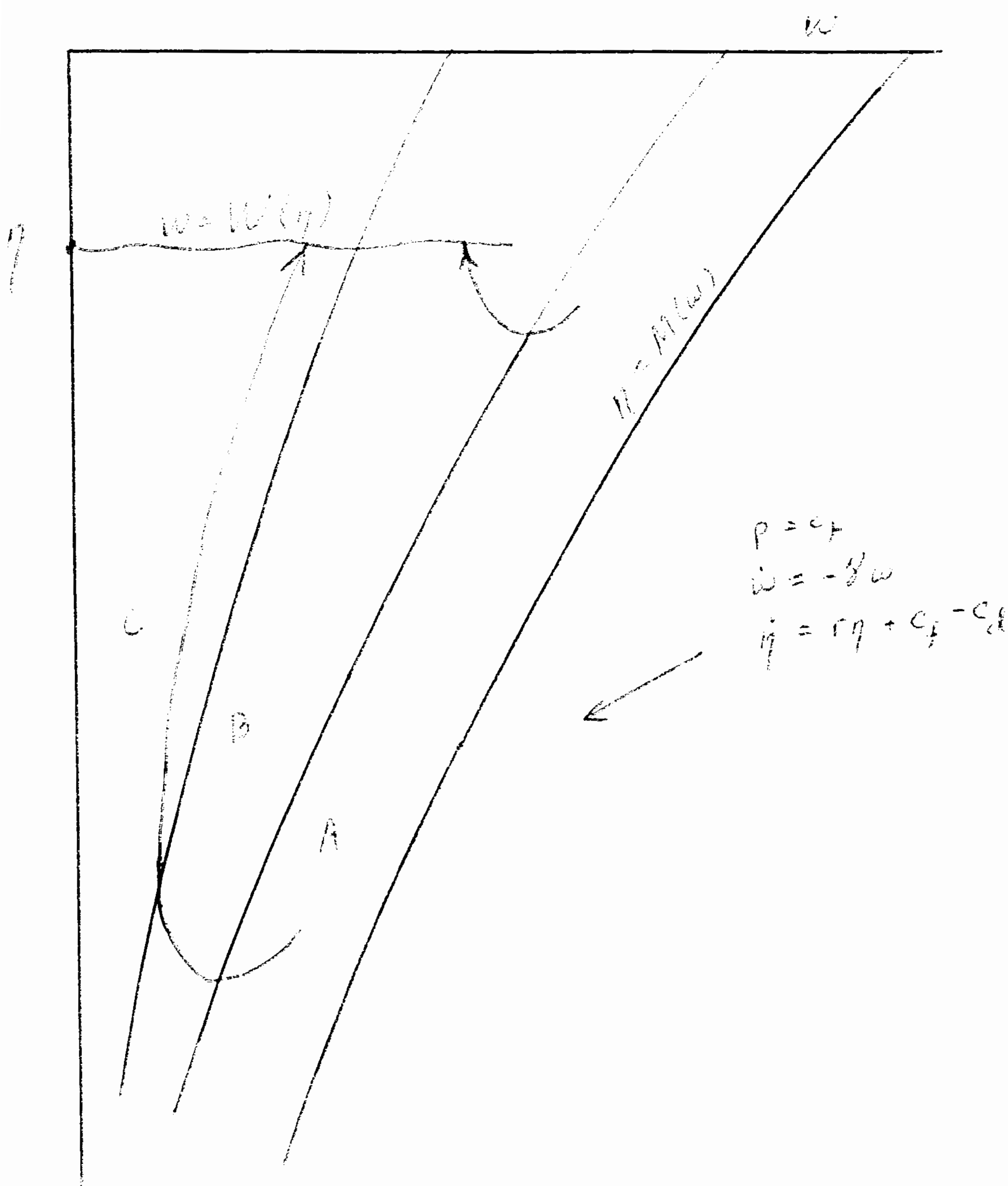

Fig. 4

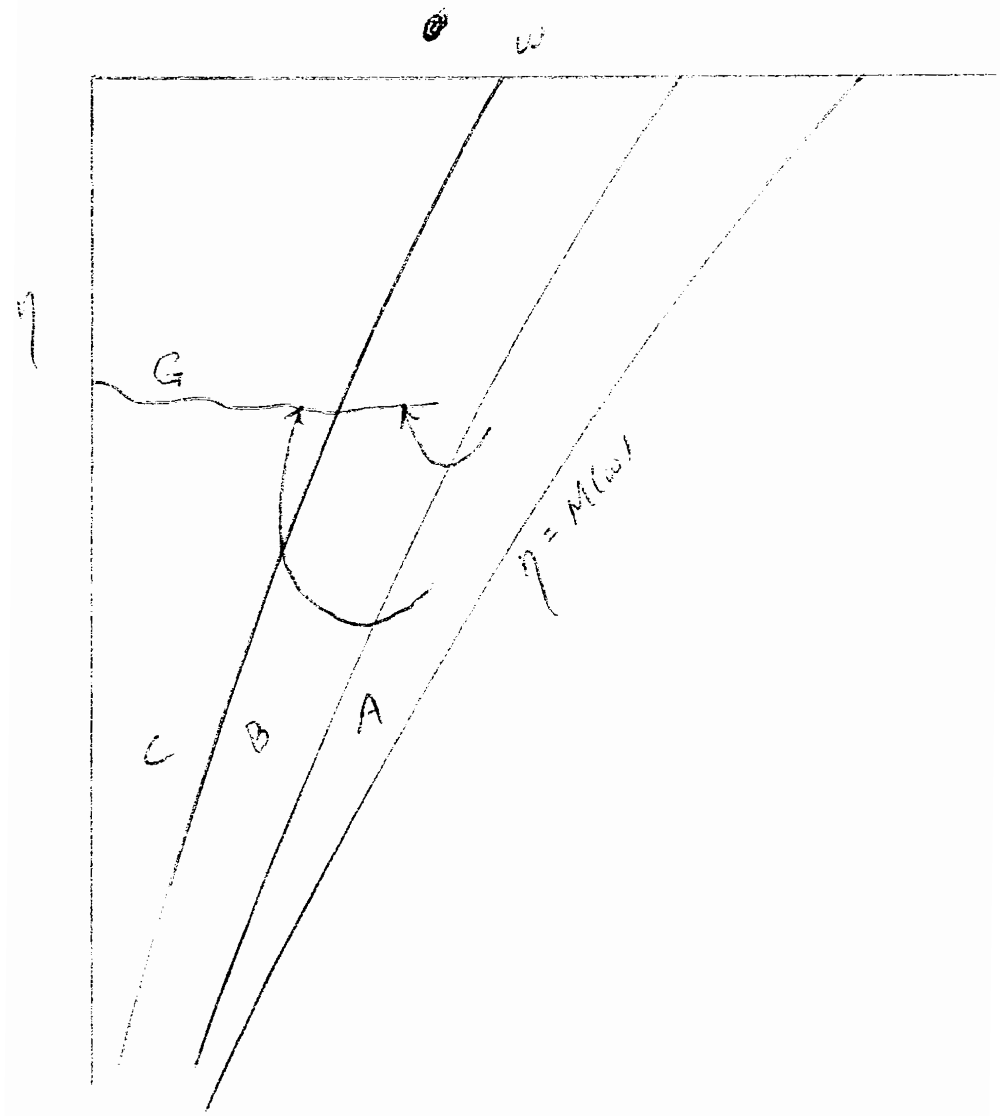

Fig. 5

Footnotes

1. For some examples of highly concentrated industries with a large number of fringe firms, see Scherer [1980, p. 62]. Some examples from Scherer of industries in 1972 with four firm concentration ratios of 90 or greater with a large number of fringe firms include flat glass (11), cereal breakfast foods (34), turbines and turbine generations (59), and electric lamps (103).

2. For a discussion of modes of entry and expansion, see Scherer [1980, p. 248].

3. Scherer argues that the first four companies are examples of firms with little or no cost advantage which sought to maximize the present value of profits by initially selecting high market prices and sacrificing market share, while the latter three companies are examples of firms with cost advantages who found it optimal to seek lower mark-ups (above the limit price) and higher long run market shares - all predictions of Gaskins' model.

4. A recent example is Matthews and Mirman [1983].

5. For an analysis of capital investment as a deterrent to entry, see Dixit [1980].

6. The necessary conditions in Gaskins' formulation are:

$$\text{(i)}\quad \dot{x}^*(t) = k_o e^{\gamma t}(p^*(t)-\bar{p}), \qquad x^*(0)=x_o;$$

$$\text{(ii)}\quad \dot{z}^*(t) = -\frac{\partial H}{\partial x}\,(x^*(t),z^*(t),p^*(t),t);$$

$$= (p^*(t)-c_d)e^{-rt}, \qquad \lim_{t\to\infty} z^*(t)=0;$$

$$\text{(iii)}\quad \frac{\partial H}{\partial p(t)} = ((f(p)e^{\gamma t}-x^*(t))+(p^*(t)-c)f')e^{-rt}+z^*(t)k_o^{\gamma t} = 0.$$

7. Gaskins' provides a numerical example at the end of his paper for a given demand curve and a given response coefficient. We recomputed the steady-state values of market share and price, along with the price trajectories for a range of response coefficients and demand parameters. We find that the results are very sensitive to the selection of the response coefficient and the demand parameters. Plausible results for any given demand curve can be obtained only by experimenting with the selection of k.

8. See Butters and Lintner [1945] for a review of the historical importance of retentions as a source of finance for expansion.

9. Regulated firms are subject to special tax treatment.

10. In the United States, the effective tax rate on capital gains is much lower than the tax rate on dividends for three reasons: (1) the 60% exclusion of long term capital gains (2) the taxation of such gains only upon realization, and (3) the tax is forgiven if the gain is not realized before death.

11. King [1974, p. 34; 1977, p. 238] and Auerbach [1979b, p. 442; 1982a, p. 19; 1982b, p. 26] calculate shadow prices for the cost of retentions $(\bar{r})$ and the cost of new share issues $(\bar{s})$. King and Auerbach find $\bar{r} = i/(1-T)(1-c)$ and $\bar{s} = i/(1-T)(1-\theta)$ where i is the required after-tax rate of return, T is the corporate income tax, c is the effective tax rate on capital gains, and θ is the tax rate on dividends.

12. Haley and Schall [1979, p. 375] report flotation costs for different sizes of new share and debt issues. As a percentage of the amount issued, these costs drop precipitously as the issue size increases. For example, for common stock issues in amounts between one-half and one million dollars, the flotation costs were 20.1%. For issues in amounts between five and ten million, the flotation costs declined to 8.7%. The issue costs for similar amounts of debt were somewhat lower.

13. See for example Haley and Schall, chapters fourteen and fifteen.

14. Ibid., p. 374.

15. Haley and Schall [1979, p. 377] state that the primary approach used by creditors to control firm decisions is the inclusion of restrictive covenants or requirements in the debt contract. Limits may be placed on new investment, disposal of assets, dividends, managerial salaries, etc.

16. The retention ratios are computed from the Internal Revenue Service, Statistics of Income, Corporate Income Tax Returns, 1970-1979, Table 5.

17. A list of peripheral equipment suppliers and their financial histories can be found in Standard and Poor's Corporation, Standard and Poor's Corporation Industry Surveys, 1970-1979.

18. Alan K. McAdams, "The Computer Industry," p. 268, in W. Adams, The Structure of American Industry.

19. See footnote 16 for full reference. We include only debt finance with a maturity of one year or greater in this percentage. Short-term debt (i.e. under one year in maturity) is usually used to finance short-term needs, such as accounts receivable, rather than capital expansion.

20. See footnote 16 for full reference. For each size class, we divided additions to equity stemming from new share issues by additions to equity from all sources (i.e. retentions plus new share issues).

21. This type of cost function is commonly used in theoretical work in industrial organization. See for example Spence [1977] and Dixit [1980].

22. It should be noted that $1/(Jp)$, not $1/J$ is the conventional capital-value of output ratio. Since $\dot{x}(t)$ is expressed in physical units of output, not in dollar value of output, J must also be expressed in physical units of output per dollar of capital per period of time. This presents no problem for applications as long as the distinction between $1/(Jp)$ and $1/J$ is kept in mind. As an example, suppose the after tax income of the fringe is \$15,000,000 and p = \$10,000 (e.g. output is automobiles) and $1/Jp = 3$ (the average value in the U.S.), then $J = 1/\$30{,}000$ and therefore $\dot{x} = 500$.

23. We have also not included new share issues in our expansion equation. We will assume throughout the remainder of the paper that prices set by the dominant firm are never high enough to warrant new share issues by the fringe.

24. The necessary conditions for a maximum value of the dominant firm's problem generates the simultaneous differential equations:

$$\text{(i)} \quad \dot{w}^*(t) = (p^*(t)-c_f)\, w^*(t)u^*(t)J - \gamma w^*(t)$$

$$\text{(ii)} \quad \eta^*(t) = (p^*(t)-c_d)e^{-rt} + \eta^*(t)(p(t)-c_f)Ju(t)$$

This system of differential equations can be converted into the autonomous system in the paper by eliminating $\eta(t)$.

BIBLIOGRAPHY

ATKINSON, A. B. and J. E. STIGLITZ, Lectures on Public Economics. New York: McGraw-Hill Book Company, 1980.

AUERBACH, A. J., "Share Valuation and Corporate Equity Policy," Journal of Public Economics 11 (1979a): 291-305.

____________, "Wealth Maximization and the Cost of Capital." The Quarterly Journal of Economics. (August 1979b): 433-446.

____________, "Issues in the Measurement and Determinants of Business Savings." Savings and Government Policy, Federal Reserve of Boston, 1982a, forthcoming.

____________, "Taxes, Firm Financial Policy and the Cost of Capital: An Empirical Analysis." N.B.E.R. Working Paper No. 955, August 1982b.

BRADFORD, D., "The Incidence and Allocation Effects of a Tax on Corporate Distributions." Journal of Public Econmics, 15 (April, 1981): 1-22.

BROCK, G., The U.S. Computer Industry, Cambridge, Mass.: Ballinger, 1975.

BUTTERS, J. K., and J. Lintner, Effects of Federal Taxes on Growing Enterprises Harvard University, 1945.

DE BONDT, R. "On the Effects of Retarded Entry," European Economic Review 8 (August 1977): 361-71.

DIXIT, A. "The Role of Investment in Entry-Deterrence," Economic Journal, 90 (March 1980): 95-106.

DUNFEE, T. W. and L. W. STERN, "Potential Competition Theory as an Anti-Merger Tool under Section 7 of the Clayton Act: A Decision Model," Northwestern University Law Review, 69, (1975): 821-871.

EASTERBROOK, F. H., "Maximum Price Fixing," The University of Chicago Law Review, 48 (1981): 886-910.

ENCAOUA, D. and A. P. JACQUEMIN, "Degree of Monopoly, Indices of Concentration and Threat of Entry," International Economic Review, 31 (1980) 87-105.

FRIEDMAN, J., "On Entry Preventing Behavior and Limit Price Models of Entry," in Applied Games Theory, eds. S. J. Brams, Warzburg, Vienna: Physical-Verlag: 236-53.

GASKINS, D. W. "Dynamic Limit Pricing: Optimal Pricing under Threat of Entry," Journal of Economic Theory 3 (September 1971): 306-22.

HALEY, C. W., and L. D. SCHALL, The Theory of Financial Decisions, 2nd ed. New York: McGraw-Hill Book Company, 1979.

INTERNAL REVENUE SERVICE, Statistics of Income, Corporation Income Tax Returns, U. S. Government Printing Office, Washington, D.,C., 1970-1979.

INTRILLIGATOR, M. D., Mathematical Optimization and Economic Theory, Prentice-Hall, Englewood Cliffs, N. J., 1971.

KAMIEN, M. I., and N. L. SCHWARTZ, "Limit Pricing and Uncertain Entry." Econometrica 39 (May 1971): 441-455.

KAPLOW, L. "The Accuracy of Traditional Market Power Analysis and a Direct Adjustment Alternative," Harvard Law Review, 95 (June 1982): 1817-1848.

KING, M. A., Public Policy and the Corporation. London: Chapman and Hall, 1977.

____________, "Taxation and the Cost of Capital." Review of Economic Studies 85 (1974): 21-35.

LEE, W., "Oligopoly and Entry," Journal of Economic Theory 13 (August 1975): 35-54.

MARTIN, S., "Advertising, Concentration, and Profitability: The Simultaneity Problem," Bell Journal of Economics, 10 (Autumn 1979): 639-47.

MATTHEWS, S. and L. J. MIRMAN, "Equilibrium Limit Pricing: The Effects of Private Information and Stochastic Demand, " Econometrica, 51 (July 1983): 981-996.

MC ADAMS, A. "The Computer Industry" in W. Adams, The Structure of American Industry, sixth edition, McMillan Publishing Co., 1982.

MILGROM, P. and J. ROBERTS, "Limit Pricing and Entry Under Incomplete Information: An Equilibrium Analysis," Econometrica, 50 (March, 1982): 443-459.

SCHERER, F. M., Industrial Market Structure and Economic Performance, Rand McNally Publishing Co., 1980.

SPENCE, M. "Entry, Investment and Oligopolistic Pricing," Bell Journal of Economics, 8 (Autumn 1977): 534-44.

SPENCE, A. M., "Investment Strategy and Growth in a New Market," Bell Journal of Economics, 10 (Spring 1979): 1-19.

STANDARD AND POOR'S CORPORATION, Standard and Poor's Corporation Industry Surveys Current-Basic Analysis, 1970-1979.