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MANAGERIAL TASK ASSIGNMENT AND PROMOTIONS

by

Joan E. Ricart i Costa

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J.L. Kellogg Graduate School of Management

Northwestern University

Evanston, Illinois 60201

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1 Introduction

A manager's ability is gradually revealed through time from his past and current perfomance in the job he has been assigned. The current manager's employer can observe the actual perfomance which is not completely verifiable by other prospective employers. The current employer (referred to as the firm) can use this information to place the manager in the job which best fits his specific ability and talent. Other employers (referred to as the market) can infer some information about the manager's talent by observing the assignments, promotions and demotions of the manager. Therefore, the firm must take into account the signaling effect of job assignments when it places the manager in a specific job.

In the model we present here, the manager can either be talented or not but he does not know which type he is. The prior probability of being talented is assumed to be common knowledge. This can be interpreted as information obtained from his schooling perfomance and therefore observable by all the parties.

We study a two period model. In the first period, the manager bargains with the firm a long term contract covering both periods. Because of reputation considerations in the labor markets, it is assumed that the firm commits itself to the contract but not to the promotion rule to be used. On the other hand, because of legal restrictions on placing oneself in involuntary servitude, managers are unable to irrevocably bind themselves to a firm. Furthermore, we assume that there are many firms competing for the manager's expertise and therefore he will have all the bargaining power.

After the output is realized at the end of the first period, the firm and the manager will be able to update their beliefs about the manager's ability. The firm will use this information to promote or demote the manager depending on the firm's best interest. The market will use the assignment as the only information in its bargaining process with the manager in the second period. The manager will effectively use his bargaining power to obtain the best possibilities from the market place. Our interest is to find the long term contract and promotion rules which, consistent with the bargaining assumptions, brings the market to an equilibrium.

Greenwald[1980] presents a model where the worker himself does not learn about his ability. The current employer can perfectly infer it after one period. Workers are paid a flat wage every period, which is determined in the spot labor market. Long term contracts are not considered in the model. Finally, there is a random turnover among workers of the firm who are paid their market value. In his model only pooling contracts can be used and therefore we have an adverse selection problem where the low types jeopardize the high ones. Very interesting implications for labor markets are obtained from this fact.

Waldman[1983] goes further ahead in a model closestly related to ours. He introduces the idea that the outside employers can only observe the task assignments and can use them as signals about the worker's ability. In his model the worker can be placed in either of two jobs. The current employer is able to perfectly infer the worker's ablity after the first period. Furthermore there is some firm specific human capital. This is essential in order to make the worker more valuable to the current employer. In his model, it is not relevant whether the worker himself knows his ability or not because he never exploits this knowledge.

In this paper, we try to go further. We believe that it is reasonable, in the managerial context, to assume that the manager will exploit any knowledge he has about his ability. The manager is able to exploit this knowledge by accepting more or less risk in his wage schedule. In this way, he can effectively use his bargaining power and his private information to separate himself from lower ability managers.

We first analyze, as a bench mark, the solution for the case where the market can verify the manager's perfomance and the symmetry in beliefs is mantained. The results we obtain are consistent with Harris and Holmstrom [1981] and Holmstrom and Ricart i Costa [1984].

Then we go on to study the asymmetric information case where we find several interesting results. First of all, wages for old managers are primarily associated with jobs and to a lesser degree with individual ability. For this reason, we find a wage differential for people assigned to the same job. Second, wages do not equal productivity. Managers who are promoted will receive wages below their productivity even with competing firms and without any firm specific human capital being included in the model. Hence, the firm can collect rents from promoted managers in the second period. On the other hand, demoted managers will be paid more than their productivities because of the insurance included in the contract. Third, consistent with observed stylized facts, the contract is downward rigid. Finally, from an effiency standpoint, there is a misassignement of old managers to jobs. In particular, managers will be promoted less than is optimal, while they will be demoted efficiently. In this context, a promotion is said to be efficient if the manager is assigned to the job he is best suited for.

Our results are very similar to those of Waldman [1983]. The main difference is that we find a wage differential associated with ability for people assigned to the same job. The difference arises because we allow the manager to exploit his private information, while Waldman does not. Furthermore, we do not need to use any firm specific human capital in our model. The only advantage of the current employer is to have superior information than the alternative ones.

The outline of the paper is as follows. Section 2 presents our model and characterizes the different jobs in the managerial structure. Section 3 analyzes the model under symmetric information. Section 4 studies the case of asymmetric information. Section 5 discusses the results obtained, the limiting assumptions and the relation with previous work. Finally section 6 concludes.

2 The model

The manager can either be talented (T) or not (N). Let η be the prior probability that the manager is talented, which is common knowledge. We assume $0 < \eta < 1$. We can think about this information as being obtained from his schooling and/or past experience.

The manager can be assigned to any of n available jobs. Let the subindex $i \in A = \{1,...,n\}$ represent the corresponding job. The manager's ability will affect the results obtained in each of the jobs. In particular, the output $y_i \in Y_i = [\underline{y}_i, \bar{y}_i]$, in monetary terms, obtained by a manager t=T,N assigned to job i, is determined by a density $\lambda_i^t(y_i)$ on Y_i. That is, a talented manager assigned to job i will obtain results in Y_i with probability density $\lambda_i^T(y_i)$.

We consider a two period model. In the first period the manager is assigned to some job i. As a result of his expertise and managerial ability, a monetary outcome will be realized. This outcome will allow the current employer, as well as the manager, to update their beliefs about the manager's talent. Depending of this update, the manager can be reassigned to another job k (promotion/demotion) for the second period, which is also the last.

The overall density on output for the manager with reputation η assigned to job i is

$$(1) \qquad p_i(\eta,y) = \eta\, \lambda_i^T(y) + (1-\eta)\, \lambda_i^N(y) \qquad y \in Y_i$$

and the posterior beliefs are calculated using Bayes' rule

(2) $$\eta_i(y) = \lambda_i^T(y)\ \eta\ /\ p_i(\eta,y).$$

To simplify the notation we define

(3) $$z_i(\eta) = \int_{Y_i} y\ p_i(\eta,y)\ dy \qquad i \in A, \quad \eta \in (0,1)$$

which is the expected return from a manager η assigned to job i. We will also use the following likelihood ratio:

(4) $$k_i(y) = \lambda_i^T(y)\ /\ \lambda_i^N(y) \qquad y \in Y_i, \quad i \in A.$$

Finally, the cumulative distribution function for each type of manager in job i is

(5) $$F_i^t(y) = \int_{\underline{Y}_i}^{y} \lambda_i^t(x)\ dx \qquad t = T,N, \quad y \in Y_i, \quad i \in A.$$

We will assume the maximum likelihood ratio property for each job i:

<u>AS1</u> (MLRP)

For each i, $k_i(y)$ is nondecreasing in y.

This assumption will assure that for each job i, higher output indicates more talent (See Milgrom [1981]). To fully understand the effect of assumption 1 in our model we need a lemma adapted from Rogerson[1983]:

<u>Lemma 1</u>

AS1 implies $\lambda_i^T >_1 \lambda_i^N$ for all i ε A where $>_1$ is the first order stochastic dominance ordering.

Proof:

Define $y_i^* = \max\{ y \in Y_i : k_i(y) \leqslant 1\}$

Clearly, if $y \leqslant y_i^*$ then $F_i^T(y) \leqslant F_i^N(y)$.

For $y > y_i^*$ suppose $F_i^N(y) < F_i^T(y)$ then:

$$1 = \int_{Y_i} \lambda_i^N(x)\ dx = \int_{\underline{y}_i}^{y} \lambda_i^N(x)\ dx + \int_{y}^{\bar{y}_i} (\lambda_i^T(x) \ /\ k_i(x))\ dx \leqslant$$

$$\leqslant F_i^N(y) + 1/k_i(y) \int_{y}^{\bar{y}_i} \lambda_i^T(x)\ dx \leqslant F_i^N(y) + (1-F_i^T(y)) < 1$$

where the first inequality uses AS1, the second uses $k_i(y) > 1$ and the strict inequality was our hypothesis. We obtain a contradiction and therefore

$F_i^N(y) \geqslant F_i^T(y)$ for all y in Y_i

which is the first order stochastic dominance equivalent condition. Q.E.D.

<u>Lemma 2</u>

$z_i(\eta)$ is linearly increasing in η for each i.

Proof:

We can write

(6) $\qquad z_i(\eta) = z_i(0) + \eta(z_i(1) - z_i(0))$

and using lemma 1

$$z_i(1) - z_i(1) = \int_{Y_i} y(\ \lambda_i^T(y) - \lambda_i^N(y))\ dy > 0. \qquad \text{Q.E.D.}$$

Therefore, each job i in A is characterized by a linearly increasing function of the manager's reputation and as a consequence, more able managers are better suited in any job (absolute advantage).

Since we want to be able to interprete increasing values of i as promotions we must give some ordering in A. In particular we want higher jobs to be more sensitive to the manager's talent. We can obtain this by assuming that the slope of $z_i(.)$ is increasing in i, i.e. higher jobs have higher slope. But note that if one job, say i, has higher slope than i-1 and also higher intersect at $\eta=0$ (i.e. $z_i(0) \geq z_{i-1}(0)$) , then job i clearly dominates job i-1. Therefore we can eliminate job i-1 from the managerial structure. Hence we can assume without loss of generality:

<u>AS2</u> (job ordering)

i. $z_i(0)$ is decreasing in i.

ii. $z_i(1)$ is increasing in i.

We can easily obtain AS2 from a more basic assumption in the case where the support of the distribution on output is independent of the job, i.e. $Y_i=Y$ for all i in A. In this case we can assume:

<u>AS2'</u>

i. $\lambda_i^N >_1 \lambda_{i+1}^N \qquad i = 1,\ldots,n-1.$

ii. $\lambda_{i+1}^T >_1 \lambda_i^T \qquad i = 1,\ldots,n-1.$

where $>_1$ is the first order stochastic dominance relation.

We can easily prove:

lemma 3

If $Y_i = Y$ for all i in A, then AS2' implies AS2.

Proof: (trivial).

Finally since we can always add a new job corresponding to "do nothing", characterized by $z_0(0) = z_0(1) = 0$, there is no loss in generality in assuming that $z_1(0) \geqslant 0$. That is, at least at the worst job the nontalented manager is not harmful for the firm.

With the stated assumptions, we can represent the managerial job structure as in figure 1 (picture for 3 jobs). Higher jobs are characterized by higher slopes (more sensitive to talent) and higher expected losses if the manager is not talented. As done in the figure we can define $\eta^0 = 0$, $\eta^n = 1$ and η^i, $i=1,\ldots,n-1$ be given by $z_i(\eta^i) = z_{i+1}(\eta^i)$.

Recall that for simplicity we assume that any job in the managerial structure which is strictly inferior for all reputations has been eliminated from the set A. For later use we also define the manager's marginal product

$$(7) \qquad v(\eta) = \max_i z_i(\eta) = z_k(\eta) \qquad \text{if } \eta^{k-1} \leqslant \eta < \eta^k.$$

Finally the firm as well as the other firms in the market are assumed to be risk neutral while the manager is assumed to be risk averse with atemporal utility function on money $u(.)$ strictly increasing and concave. We use a common discount factor $0 < \beta < 1$.

3 Symmetric information.

As a bench mark reference, let us assume that the output is public information. Then the market will be able to update probabilities and no asymmetry in information is generated. Since the market can completely distinguish any type of manager and because of the competition for the manager's expertise, the manager will be able to obtain his marginal product $v(\hat{\eta})$ from the market, where $\hat{\eta}$ is the updated probability of being talented.

Since the second period assignment does not reveal any information unknown to the market, the firm's optimal promotion rule will be

$$(8) \qquad a^*(\hat{\eta}) = k \qquad \text{if} \quad \eta^{k-1} \leq \hat{\eta} < \eta^k.$$

A long term contract for a manager with reputation η assigned to i in the first period is specified by

$$W_i = (\bar{w}_i, w_i(y))$$

corresponding to the first and second period wages respectively. The best contract is the solution of

P1

$$\max_{W_i} u(\bar{w}_i) + \beta \int_{Y_i} u(w_i(y)) \, p_i(\eta, y) \, dy$$

S.T.

$$(9) \qquad w_i(y) \geq v(\eta_i(y)) \qquad \text{for all } y \in Y_i$$

$$(10) \qquad z_i(\eta) - \bar{w}_i + \beta\int_{Y_i} (v(\eta_i(y)) - w_i(y))\, p_i(\eta,y)\, dy \geq 0.$$

Where (9) assures that the manager will not be bid away in the second period and (10) is the nonnegative profits condition for the firm. The solution of P1 is contained in the following proposition:

Proposition 1

The optimal long term contract W_i, for a manager with reputation η assigned to job i in the first period in the symmetric information case is $(\bar{w}_i^*, w_i^*(y))$ where

$$(11) \qquad w_i^*(y) = \max \{ \bar{w}_i^*, v(\eta_i(y))\} \quad \text{for } y \in Y_i$$

and $\bar{w}_i^*$ solves (10) with equality.

Proof: (See Holmstrom and Ricart i Costa [1984]).

Therefore the optimal contract is a rigid one where wages are nondecreasing in time and they only increase to match the market offers. That is, wage increases occur only when the manager's market value is higher than his current wage.

Now we should characterize the first period assignment. A manager with reputation η will be assigned to the job i^* that solves

$$\operatorname*{Max}_i\ u(\bar{w}_i^*) + \beta\int_{Y_i} u(w_i^*(y))\, p_i(\eta,y)\, dy.$$

As done in Holmstrom and Ricart i Costa, we can better interpret the factors affecting the assignment decision by proving

Proposition 2

The optimal first period assignment for a manager with reputation η is

$$(12)\quad i^*(\eta) = \underset{i\varepsilon A}{\operatorname{argmax}}\; u'(\bar{w}_i^*)[(z_i(\eta)+\beta v(\eta))+\beta(\int_{Y_i} v(\eta_i(y))p_i(\eta,y)dy - v(\eta))+H_i(\eta)]$$

where $H_i(\eta)$ is given by

$$(13)\quad H_i(\eta) = (\beta/u'(\bar{w}_i^*)) \int_{Y_i} [u(w_i^*(y)) - u(\bar{w}_i^*) - u'(\bar{w}_i^*)(w_i^*(y)-\bar{w}_i^*)]\; p_i(\eta,y)\; dy.$$

Proof:

Given proposition 1, the optimal assignment is the solution of

$$\underset{i,w}{\operatorname{Max}}\; u(w) + \beta\int_{Y_i} u(\max\{w,v(\eta_i(y))\})\; p_i(\eta,y)\; dy$$

S.T.

$$z_i(\eta) + c_i(\eta) - w - \beta\int_{Y_i} \max\{w,v(\eta_i(y))\}\; p_i(\eta,y)\; dy = 0.$$

For each w satisfying the constraint, the optimal i maximizes the Lagrangian with $u'(w)$ as multiplier. Since the optimal w for each i is $\bar{w}_i^*$ we need to compare only the n different Lagrangians, one for each i. The resulting optimization problem is exactly (12). Q.E.D.

Each term in (12) can be easily interpreted. The first bracketed term, $z_i(\eta) + \beta v(\eta)$ is the expected return from a manager η assigned to job i in the first period and optimally in the second, provided that there is no learning about the manager's talent. We call it the return in physical capital. The second bracketed term is the additional expected return due to the learning about the manager's reputation that takes place in the first period. This term can be rewritten as $\beta(E[v(\hat{\eta})] - v(E[\hat{\eta}]))$ where $\hat{\eta}$ is the second period reputation and E(.) is the expected value operator. Then, since v(.) is a convex function, by Jensen's inequality this term is positive. The first period learning about the manager's talent allows the firm to place him better in the second period and therefore obtain additional returns. We call it the human capital return.

The last term $H_i(.)$ is a risk premium penalizing for the variance in the human capital return. Since it is impossible to fully insure the manager (because of the market constraints), he has to bear some risk and therefore, one can easily check that $H_i < 0$. Furthermore, note that if the manager were risk neutral or were able to commit himself to the contract, this last term would be zero. To fully understand this term, we can use a quadratic expansion for $u(w_i^*(y))$ around $\bar{w}_i^*$ and H_i transforms to

$$H_i(\eta) \cong (\beta/2)(u''(\bar{w}_i^*)/u'(\bar{w}_i^*)) \int_{Y_i} (w_i^*(y) - \bar{w}_i^*)^2 \, p_i(\eta,y) \, dy$$

$$=(\beta/2)(u''(\bar{w}_i^*)/u'(\bar{w}_i^*))(\mathrm{var}[w_i^*] + (E[w_i^*] - \bar{w}_i^*)) \ ,$$

where var[.] is the variance operator. That is, the H_i term is proportional to the coefficient of absolute risk aversion at $\bar{w}_i^*$ and to the expected quadratic variation of second period wages with respect to the first period wage.

Finally, $u'(\bar{w}_i^*)$ is the price to get the value of each assignment in comparable terms.

4 Asymmetric Information.

We study now the case where the market is not able to verify the actual output obtained by the manager The only information available in the market is that obtained from observing the manager's assignment, promotions and demotions. While the manager and the firm can update the second period manager's reputation, the market will only be able to compute a probability distribution over all possible updates given the assignment pair (i,k). Furthermore promotions or demotions will be taken as signals about the manager's true value.

The firm commits itself to the long term contract negociated with the manager in the first period. Recall that this is a commitment on wages only. The firm will choose the promotion rule which maximizes its profits, consistent with its beliefs about the outside market offers. On the other hand the market will bargain with the manager,after he is reassigned in the second period, a contract which gives nonnegative profits to the alternative employer given his beliefs about the promotion rule followed by the firm. In equilibrium the expectations must be fulfilled. Finally, the long term contract maximizes the manager's expected utility given the promotion rule and the market opportunities in the second period. The market equilibrium is obtained when the manager can not obtain a better bargain in any of the two periods, each alternative employer makes zero expected profits given its offers and the long term employer chooses the promotion rule to maximize its second period profits but has zero expected profits from the long term contract.

We study the strategies of each one of the parties in different subsections. In this section the first period assignment i is kept constant. Therefore we will supress the subindex i to simplify the notation. And as we did before, we will use $\hat{\eta}$ to represent the manager's reputation in the second period as updated by him and the firm.

4.1 Bargaining Solution

At this point it is important to recall that the manager is not only the best informed part in the bargaining process, but (by assumption) he also has all the bargaining power. Furthermore several firms are competing for the expertise of the manager. Due to this characteristics peculiar of managerial labor markets, we assume that the market will offer contracts which separate out the different types of managers assigned to each job k. To do so, the market offers a menu of contracts designed in such a way that each type of manager will choose a different contract and the employer will earn nonnegative profits given this choice (i.e. the contract is "safe" in Myerson's[1982] terminology). Hence we follow the informational consistency approach introduced by Riley[1979].

It is not unrealistic to start by assuming that the firm's promotion rule has the following form

$$(14) \qquad a(\hat{\eta}) = k \qquad\qquad \text{if } \bar{\eta}^{k-1} \leq \hat{\eta} < \bar{\eta}^{k}$$

where $0 = \bar{\eta}^0 \leq \bar{\eta}^1 \leq \ldots \leq \bar{\eta}^n = 1$.

Let $U_k(\hat{\eta})$ be the expected utility that a manager can obtain in the market place if his reputation is $\hat{\eta}$ and he was assigned by the firm to job k in the second period. Define the equivalent market value $x_k(\hat{\eta})$ by

$$(15) \qquad u(x_k(\hat{\eta})) = U_k(\hat{\eta}).$$

Note that the market value function depends on both the assignment k and the reputation $\hat{\eta}$, because the market separates out different types of managers in each job, via self-selection.

Given our behavioral assumptions the market value of the manager that comes out of the bargaining process will be characterized by:

Proposition 3

The market value of a manager given the promotion rule a(.) satisfies:

c1. $\qquad x_k(\bar{\eta}^{k-1}) = v(\bar{\eta}^{k-1})$

c2. $\qquad x_k(\hat{\eta}) < v(\hat{\eta}) \quad \text{for all } \hat{\eta} \varepsilon (\bar{\eta}^{k-1}, \bar{\eta}^{k})$

c3. $\qquad x_k(\hat{\eta})$ is nondecreasing in $\hat{\eta}$ but not necessarily constant.

Proof:

First of all it is clear that a manager of type $\bar{\eta}_{k-1}$ can always declare his type and obtain $v(\bar{\eta}_{k-1})$. Hence

$$x_k(\bar{\eta}_{k-1}) \geqslant v(\bar{\eta}_{k-1}).$$

Since the manager has all the bargaining power, if he obtains an incentive compatible offer which pools him with other lower reputation types, he can always use his bargaining power to get another firm to offer him a contract which will be acceptable to him if and only if he is in the upper tail of the distribution. Hence the previous offer will only attract the lower tail of the distribution and the affected firm will have negative profits. Since this process can be continued, at least up to a point where types are too close to be effectivelly differentiated, we have proven c3.(Some technical details are discussed below).

It is also clear that this process will push the lowest type $\bar{\eta}_{k-1}$ to obtain just his marginal product. Hence c1 is satisfied.

In order to be able to differentiate himself from the lower types a manager has to accept some risk while the firm profits are zero, c2 must be satisfied.

Q.E.D.

Proposition 3 presents all the relevant properties of the bargaining solution we propose and is going to be used later on. It is not an easy task to characterize the separating solution we are defending even for a finite number of types. In the appendix we present a reasonable way to calculate the bargaining solution we use here for any finite number of types.

Provided that one can compute a solution for any finite case, we interpret our continuous case as the limit of the finite case in the following sense: Consider an equal size partition of $\bar{\Delta}_k$ with N classes. For each class choose a representative value (for instance the middle point) and solve the finite case. Consider as solution for the continuous case as the limit of the solutions to the finite case as N grows large. Therefore our continuity is only an abstraction representing the limit of finite output sets.

Finally it is important to note that a feasible solution satisfying c1-3 does exist. Just give the managers the choice between a constant wage $v(\bar{\eta}_{k-1})$ and a contingent wage giving him the output y. Furthermore, allow him to decide the job assignment. This solution is a nondecreasing lower bound for any separable solution.

As indicated before, this solution paralleles Riley's reactive equilibrium and therefore it is the Pareto dominant informationally consistent bargaining solution. Although this solution is somewhat controversial we believe it to be aceptable in the managerial context of this paper.

We can also defend the separable bargaining solution we are using here by transforming the bargaining process into a signaling game of the type exposited in Kreps[1984]. By using the stability concepts introduced in Kohlberg and Mertens[1982], Kreps is able to defend the intuitive "complete screening" solution which corresponds to the solution we present in the apendix for finite number of types.

The bargaining process in this section does not exactly fit Kreps' signaling game but can be transformed into it in the following way. First transform the problem to a finite one, by considering a finite number of types (or classes of types) as well as finite output sets and by considering wages in units of one cent. The manager will be the signaling player. The firm action is the wage schedule the manager is paid. The manager signals his type by the amount of risk he is willing to accept in the wage schedule. As in Kreps' game, the signal is costly since the manager is risk averse.

As an illustration, consider the case where the wage schedules are constrained to be linear, i.e. $w(y) = a + by$. Now the manager can signal by choosing the coefficient b in a discrete highly dense subset of [0,1]. Then the firm action, after observing the signal, determines the parameter a. The solution obtained by applying program P2 (See Appendix) to the case where the manager's types are $\eta_1 < \eta_2 < \ldots < \eta_N$, is a menu of linear schedules $w_j(y) = b_j y + (1-b_j)v(\eta_j)$ for $j=1,\ldots,N$, where $0=b_1 < b_2 < \ldots < b_N < 1$. And this solution coincides with the intuitive solution proposed by Kreps.

4.2 Long Term Contract

Given the market value functions $x_k(\eta)$ and the firm promotion rule a(.), the long term contract is the wage schedule $(\bar{w}, w(y))$ (recall i is fixed) which solves:

P3

$$\text{Max } u(\bar{w}) + \beta\int_Y u(w(y))\ p(\eta,y)\ dy$$

S.T.

$$(16) \qquad w(y) \geq x_{a_y}(\eta(y)) \qquad \text{for all } y \in Y$$

$$(17) \qquad z(\eta) - \bar{w} + \beta\int_Y (z_{a_y}(\eta(y)) - w(y))\ p(\eta,y)\ dy \geq 0$$

where a_y is a shorter notation for $a(\eta(y))$, the second period assignment given by the firm promotion rule a(.). Hence the contract maximizes the manager's expected utility subject to nonnegative expected profits for the firm and assuring that the manager won't be bid away in the second period.

Using weak duality in the same way it is done in Holmstrom and Ricart i Costa[1984], one can easily prove:

Proposition 4

The optimal solution for P3 is a rigid contract where

$$(18) \qquad w^*(y) = \max\{ \bar{w}^*, x_{a_y}(\eta(y)) \} \qquad y \in Y$$

and $\bar{w}^*$ solves (17) with equality.

4.3 Promotion Rule

Since the firm does not commit itself to a predetermined promotion rule, it will choose to promote a manager if and only if the benefit obtained by promotion is higher than otherwise. This decision is clearly dependent upon the long term contract agreed upon with the manager in the first period. Suppose that the manager's reputation at the start of the second period is $\hat{\eta}$. If he is assigned to job k, he will produce a return $z_k(\hat{\eta})$ to the firm. Furthermore, he must be paid $\max\{\bar{w}^*, x_k(\hat{\eta})\}$ as especified in the long term contract. Therefore, the firm will assign the manager to the job k^* which maximizes the difference $z_k(\hat{\eta}) - \max\{ \bar{w}^*, x_k(\hat{\eta}) \}$ over all jobs in A. That is

$$(19) \qquad a(\hat{\eta}) = \underset{k}{\operatorname{argmax}} \{ z_k(\hat{\eta}) - \max\{ \bar{w}^*, x_k(\hat{\eta}) \} \}$$

4.4 The Equilibrium

The market equilibrium is obtained when the manager can not obtain a better bargain in any of the two periods, each alternative employer makes zero expected profits given its offers and the long term employers chooses the promotion rule to maximize its second period profits and has zero expected profits from the long term contract.

Theorem 1

The market equilibrium is given by:

A. A long term contract characterized by

$$w^*(y) = \max\{\bar{w}^*, x_{a_y}(\eta(y))\} \qquad y \in Y$$

where $\bar{w}^*$ solves

$$z(\eta) - \bar{w}^* + \beta \int_Y (z_{a_y}(\eta(y)) - \max\{\bar{w}^*, x_{a_y}(\eta(y))\} p(\eta,y)\, dy = 0.$$

B. A firm's promotion rule characterized by

$$a(\hat{\eta}) = k \qquad \text{if } \bar{\eta}^{k-1} \leq \hat{\eta} < \bar{\eta}^k$$

where

$$0 = \bar{\eta}^0 \leq \bar{\eta}^1 \leq \ldots \leq \bar{\eta}^n = 1$$

and for $k = 1,\ldots,n-1$, $\bar{\eta}^k$ is given by

i. If $z_k(\eta^k) \leq \bar{w}^*$ then $\bar{\eta}^k = \eta^k$, i.e. the promotion rule is efficient.

ii. Otherwise $\bar{\eta}^k > \eta^k$ i.e. the promotion rule is inefficient and the cutoff value is given by

(20) $\qquad z_k(\bar{\eta}^k) = x_k(\bar{\eta}^k)$.

C. A valuation function $x_k(.)$ given in section 4.1.

Proof:

To simplify the proof we assume that no job in the managerial structure is eliminated by the equilibrium structure.

Point A is a restatement of proposition 4.

The market value $x_k(.)$ derived in section 4.1 is optimal given our behavioral assumptions for a promotion rule defined in B. We need the market value function for reputations outside the equilibrium intervals. In fact it is enough to note that if a manager with reputation $\eta < \bar{\eta}^{k-1}$ is assigned to job k, he can obtain $x_k(\bar{\eta}^{k-1})$. On the other hand, if a manager with reputation $\eta > \bar{\eta}^k$ is assigned to job k, then he can obtain $x_k(\hat{\eta})$, the extension of the market value function. These two observations will be enough to support the equilibrium.

Because of (19), the promotion rule in b will determine an equilibrium if we can show that for every k in A, if $\bar{\eta}^{k-1} \leqslant \hat{\eta} < \bar{\eta}^k$, then

$$z_k(\hat{\eta}) - \max\{\bar{w}^*, x_k(\hat{\eta})\} \geqslant z_j(\hat{\eta}) - \max\{\bar{w}^*, x_j(\hat{\eta})\} \quad \text{for all } j \in A$$

If $v(\hat{\eta}) < \bar{w}^*$ let k be the efficient assignment for $\hat{\eta}$, then

$$z_k(\hat{\eta}) - \max\{\bar{w}^*, x_k(\hat{\eta})\} = v(\hat{\eta}) - \bar{w}^* \geqslant z_j(\hat{\eta}) - \max\{\bar{w}^*, x_j(\hat{\eta})\} \text{ for all } j \in A$$

and therefore part i in b is the best response to the contract and the market value function.

If $v(\hat{\eta}) \geqslant \bar{w}^*$, let k be such that $\bar{\eta}^{k-1} \leqslant \hat{\eta} < \bar{\eta}^k$, where $\bar{\eta}^k$ is given by (20).

For any job $j > k$ we have

$$\max\{\bar{w}^*, x_j(\hat{\eta})\} = x_j(\bar{\eta}^{j-1}) = v(\bar{\eta}^{j-1}) > z_j(\hat{\eta})$$

and therefore

$$z_j(\hat{\eta}) - \max\{\bar{w}^*, x_j(\hat{\eta})\} < 0.$$

For any job $j < k$ we have

$$\max\{\bar{w}^*, x_j(\hat{\eta})\} \geqslant z_j(\hat{\eta})$$

and therefore

$$z_j(\hat{\eta}) - \max\{\bar{w}^*, x_j(\hat{\eta})\} \leqslant 0.$$

On the other hand we have

$$z_k(\hat{\eta}) - \max\{\bar{w}^*, x_k(\hat{\eta})\} \geqslant 0$$

which implies

$$z_k(\hat{\eta}) - \max\{\bar{w}^*, x_k(\hat{\eta})\} \geqslant z_j(\hat{\eta}) - \max\{\bar{w}^*, x_j(\hat{\eta})\} \text{ for all } j \in A$$

thus, part ii in B is proven. Q.E.D.

Figure 2 helps in understanding the equilibrium obtained as well as the proof. It represents the same managerial structure as figure 1. In addition we have drawn the market value functions (broken lines) and the optimal contract from proposition 4 (thick line). To check that we really have an equilibrium it is enough to prove that the firm will choose the promotion rule assumed by the market as well as the manager.

If $\hat{\eta} \leq \eta^*$, the manager will be paid $\bar{w}^*$ and promotions are efficient. Now suppose that $\eta^* < \hat{\eta} < \bar{\eta}^2$. Then, if the manager is promoted to a job different from job 2 we have the following net returns:

$$z_3(\hat{\eta}) - x_3(\bar{\eta}^2) < 0$$

or

$$z_1(\hat{\eta}) - \max\{\bar{w}^*, x_1(\hat{\eta})\} < 0.$$

while returns are positive if assigned to job 2.

Similarily, if $\hat{\eta} \geq \bar{\eta}^2$, and the manager is assigned to job 3, the returns are nonnegative, while if assigned to any other job we have:

$$z_1(\hat{\eta}) - \max\{\bar{w}^*, x_1(\hat{\eta})\} < 0$$

or

$$z_2(\hat{\eta}) - \max\{\bar{w}^*, x_2(\hat{\eta})\} \leq z_2(\hat{\eta}) - x_2(\bar{\eta}^2) = z_2(\hat{\eta}) - z_2(\bar{\eta}^2) < 0.$$

Therefore figure 2 represents an equilibrium for the asymmetric case.

Finally, note that the equilibrium is not difficult to calculate. For a fixed $\bar{w}^*$, we know that the low jobs are assigned efficiently. Also, to calculate the market value function we need only know the lowest reputation value in a specific job. Hence, by starting at zero reputation with $\bar{w}^*$ fixed we can calculate the market value functions and promotion rules for each job k. Finally proposition 4 allows us to calculate the contract. If the optimal value of $\bar{w}^*$ intersects v(.) at the same job we supposed at the begining we are done. If not we must increase of decrease the initial fix value of $\bar{w}^*$ and repeat.

In the next section we discuss some of the properties of our solution as well as some modifications of our assumptions.

5 Discussion of the solution.

Several properties of the equilibrium solution are easily derived from our results in section 4:

1. The long term contract is downward rigid.

This result is not surprising given our assumptions and previous knowledge about wage dynamics. But it is relevant that the result is robust to the asymmetry in information.

2. Wages for old managers are highly dependent on the job assignment but there are small wage differential reflecting individual ability as well.

This result is consistent with the observations described in Doeringer and Piore[1971]. This work is more related to blue collar labor but their evidence support our result at least for lower level management. Note that managers with small ability diffentials may have big wage differentials if one is promoted and the other is not. This discontinuity in the wage schedule is what we interpret as wages assigned to jobs. The wage discontinuities arise because the promotions help to reduce the informational gap between the current and alternative employers.

3. Old managers with high reputation have a market value well below their actual productivity.

Hence the firm is cashing in from these managers. It is important here to note that this result is obtained only due to asymmetries in information since we have not included in the model any kind of firm specific human capital. In a more realistic setup we expect the firm to be able to cash in still more from old managers because of the investment in firm specific human capital.

The intuition behind this result is as follows. Since the manager is able to exploit his knowledge about his ability, the wages are dependent on ability. But because of the cost of comunicating this information, wages are kept below actual productivity.

Harris and Holstrom[1981] paper has been criticized because of their result that wages equal productivity (for high ability workers). Our model supports their statement once productivity has been modify to account for the mobility cost, which here is represented by the cost of comunicating information.

Note also that because of the downward rigidity of the contract, the old managers with low reputation will be paid more than their marginal product. Hence in general wages do not equal productivity and the internal labor market is replacing the external one.

These results are consistent with the findings of Johnson et al.[1984]. Specifically, they find a significant negative association between the executive's position in the firm's decision-making hierarchy and the excess returns in stock prices after the sudden death of the executive. Among other interpretations, as firm specific human capital or transaction costs, this result may indicate the the firm losses, due to the sudden death, the expected quasi-rents from a promoted manager, and gains from the sudden death of demoted executives. Note also that our model is consistent with no association between the excess returns and ability, since we predict that for managers in the same position, the quasi-rents first increase with ability but then decreses back to zero.

4. There is a misassignment of old managers to jobs.

In particular old managers with high reputation are promoted less than it is efficient because promotions are signals to rival firms and will force wage increases. This creates an efficiency loss in spite of the fact that the firm knows that the manager will be more productive if promoted. On the other hand, demotion of low reputation managers is done efficiently since they have a wage guarantee and the signal to the market is irrelevant. See also Waldman[1983].

5. The long term contract is easily implementable.

The manager is only assured a guaranteed wage while wage increases and promotions are decided expost by the firm/market interaction. But, obviously, the wage guarantee must reflect both the insurance for bad results as well as the benefits that the firm will collect from good managers.

We want to discuss some criticism of our solution. One may say that since the manager has all the bargaining power he can enforce the promotion rule by accepting only contracts which explicitly contain the promotion rules. That is, we may choose in the first period the promotion rule that maximizes the manager's exante expected utility. This change does not affect the results obtained in the paper and its introduction would have unnecessarily complicated the exposition. The only change will be in the determination of the cutoff values for promotions of high reputation managers. One can prove that the cutoff values will be higher that η^k. Hence there will still be some inefficiency in the reassignment. The reason for this is that the market constraints are binding and the manager is willing to pay a premium (in the form of efficiency losses) to get better insurance.

A new factor behind our results is the argument that the market will produce a separating bargaining solution. Specifically, the fact that we obtain an increasing market value function is essential in obtaining our equilibrium. We believe that this is realistic in the context of managerial labor markets.

However we don't think this argument can be a valid one in the context of blue collar labor markets where individual productivity is sometimes very difficult to measure. In this case, we usually observe the use of collective bargaining (instead of individual bargaining) and the effect of ability over the job based wage is usually much smaller. Also, the collective bargaining representative and the firm can agree to a pooling type contract with average zero profits where the high types compensate for the low types of workers. (See Doeringer and Piore[1971], Miyazaki[1977], Willianson, Wachter and Harris[1975], Waldman[1983]).

One may also wonder what will be the effect in our model if we allow the firm to make counteroffers to the offers the manager obtains in the market place (recall that the long term contract is fixed at period 1). It is easy to see that this point has no effect in our model. The firm is able to calculate the best offer that the market can make to a given manager considering the cost of comunication. The long term contract already matches this best offer. Therefore, no counteroffer process is necessary.

Another point we want to discuss is an assumption in the proof of the theorem. Note that even if the market value increases, it can increase slow enough that once it intercepts $z_k(.)$, the manager should be promoted to the job k+t for t>1. Thus we can lose some jobs in the managerial structure which are never filled in the second period. One can easily check that this is not essential for our results and we omit the details and the necessary modifications in the proofs to account for this possibility. Still it is interesting to observe that this situation will determine some jobs which are filled only with new managers. These jobs may be natural ports of entry in the managerial structure.

Let us compare the symmetric and asymmetric cases. First, we have pointed out that there is some efficiency (or welfare) loss due to the misassignment of old managers in the asymmetric case. Second, the asymmetric case presents discontinuities in the wage schedule not obtained in the symmetric case. Finally, the managers may obtain better insurance in the asymmetric case since the binding market constraints are lessened, giving less variation to the second period wages. Depending on the size of the efficiency loss the manager may obtain a higher entry wage and even higher exante expected utility in the asymmetric case, since the manager has to bear less risk than before (a welfare improvement). Which effect is greater, we do not know, but even if expost the manager would prefer the market to be able to observe the actual output (and obtain his full marginal product), exante he may prefer the asymmetric situation. In fact, if the contract specifies the promotion rule then the manager is always better off in the asymmetric case. Thus even in cases where output may be verifiable, the manager can exante agree on a contract which legally forces him to keep the actual output as confidential information. Thus the introduction of contracts allows the manager to benefit from the adverse selection among old managers.

Greenwald[1980] presents a simple model where the current employer can perfectly infer the ability of the worker after one period of employment (while the worker himself cannot!). We have seen that the introduction of long term contracts as well as signaling opportunities, through assignments and risk sharing, drastically reduces the "lemons effect" of the adverse selection problem. In fact, the adverse selection effect is exante positive for the manager since it allows him to obtain a more uniform consumption stream.

The same two effects discussed above appear in the choice of the first period assignment for the asymmetric case. First, the return on human capital will decrease due to the misassignment of the manager in the second period. Second the cost or risk premium factor H_i will get smaller (as a cost) since there is less variation in the second period wage, i.e. less variance in the human capital return. Besides these changes, proposition 2 is still true for the asymmetric case (when one introduces the obvious modifications). Hence the first period assignment rule is essentially unchanged.

To finish this section, we want to emphasize that the only point which is driving all the results is the current employer's ability to learn about the manager's value while the market can not do so. In our model there is no firm-specific human capital, learning by doing, on the job training or any other effect which can increase the value of the manager to the current employer but not to the other firms. The manager is as valuable in one firm as in any contract which legally forces him to keep the actual output as confidential information. Thus the introduction of contracts allows the manager to benefit from the adverse selection among old managers.

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Waldman[1983] uses the signaling properties of the task assigments in a two period model where workers can be placed in either of two jobs. Given his assumptions, the outside market can only offer a pooling contract for any worker assigned to the same job. Therefore he obtains that wages are assigned to jobs independently of individual ability. He also finds that long term contracts are rigid and that there is misassignment of old workers to jobs.

Our model is very close to waldman's. The main difference is that our model allows the manager to exploit his knowledge about his ability. This additional assumption introduces the dependence of wages on ability and allows us to eliminate from the model any firm specific human capital consideration. The validity of this assumption can be determined by empirical evidence. If we find the ability does affect wages for managers assigned to the same job, the the manager must be exploiting his knowledge in the bargaining process.

6 Conclusions

The firm currently employing a manager has some advantages over other employers who are trying to hire the manager away from the firm. In our model we have reduced such advantages only to having better information about the manager's talent. Still the market can infer some information from the task the manager has been assigned to since this can usually be considered public information. This transforms the assignment to a signal which is strategically useful for the current employer.

We have made strong use of the specific characteristics differential in managerial labor markets to defend the use of separable incentive contracts for managers assigned to the same job. This has allowed us to obtain some interesting results.

We have proven that wages are primarily assigned to jobs but with a remaining differential depending on ability. Contracts are shown to be rigid and therefore to have the properties studied in Harris and Holmstrom[1981]. Yet the market value of the promoted managers is below actual productivity due to the cost of comunicating information. There is misassignment of old managers to jobs and therefore managers are promoted less than what would be efficient. Finally the long term contract is easily implementable and reflects the situations we encounter in real life.

Besides all the results obtained, we have presented a model which we believe to be useful for further studies on internal labor markets and hierarchical structures. In particular, we believe that the model can be used to study dual market situations where internal and external markets coexist as well as the effect of this situation on wages structures and promotion policies.

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Appendix

If we have a finite output set we will have only finite number of manager's type for each assignment k. Number them in an increasing order such that $\{\eta_1,\ldots,\eta_N\}$ are the types assigned to job k. Then the bargaining solution described in proposition 3 can be calculated with the following recursion:

P2

$$U_k(\eta_1) = u(v(\eta_1))$$

and for $j = 2,\ldots,N$

$$U_k(\eta_j) = \max_{w(\cdot),\, \ell\varepsilon A} \sum_{y\varepsilon Y_\ell} u(w(y))\; p_\ell(\eta_j, y)$$

S.T.

(A1) $$U_k(\eta_t) \geq \sum_{y\varepsilon Y_\ell} u(w(y))\; p_\ell(\eta_t, y) \quad \text{for all } t < j$$

(A2) $$\sum_{y\varepsilon Y_\ell} (y - w(y))\; p_\ell(\eta_j, y) \geq 0.$$

Since P2 is a finite program we can use Kuhn Tucker conditions. For every η_j assigned to job ℓ we have

(A3) $$1/u'(w(y)) = 1/\alpha - \sum_{t<j} (\mu_t/\alpha)\; (\,p_\ell(\eta_t, y)/p_\ell(\eta_j, y)) \qquad \text{for } y\varepsilon Y$$

where $\alpha > 0$ and $\mu_t \geq 0$ for $t<j$ are the corresponding multipliers. Now we can easily prove:

Lemma 4

i. The program P2 has a solution for every j.

ii. The wage schedule optimal for each j is increasing in y.

iii. If the optimal wage schedule for j obtained from P2 is feasible for $t > j$ then $U_k(\eta)$ is strictly increasing in η. Therefore the market value function $x_k(.)$ is strictly increasing in η.

Proof:

Since $w(y) = y$ is always feasible, P2 always has a solution. Furthermore the solution is bounded below by

$$U_k(\eta_j) \geqslant \max\{ u(v(\eta_1)), \sum_{y \varepsilon Y} u(y)\ p(\eta_j, y)\}.$$

where Y and p refers to the job efficiently optimal for η_j, $a^*(\eta_j)$.

To prove ii, first note that the probability ratio in (A3) can be writen as

$$\text{(A4)} \qquad p_\ell(\eta', y)/p_\ell(\eta, y) = [(1-\eta')+\eta' k_\ell(y)]/[(1-\eta)+\eta k_\ell(y)]$$

which is decreasing in y by AS1 whenever $\eta' < \eta$. Hence from (A3) w(y) is increasing in y for all j.

To prove iii, suppose we choose as wage schedule for j, the same that was optimal for j-1. Since, given our assumptions, it is feasible, by the MLRP assumption and ii we have $U_k(\eta_j) > U_k(\eta_{j-1})$. Q.E.D.

Some remarks are necessary at this point. First of all, we need to comment on the condition introduced in iii. This condition states that any wage schedule chosen for a manager of type t using P2 must provide nonnegative profits to the employer if it were chosen by a higher type j. At a first glance the condition may look to be an intuitive fact but we have not been able to prove it.

The necessary condition here is the incentive constraint assuring that higher types will not jeopardize lower types. But this restriction is not easily manageable. Before sugesting some ways of partially solving the problem, let us expose why we think our condition is reasonable.

For each j, we choose the wage schedule that being feasible maximizes the η_j-manager's expected utility. Since w(y)=y is feasible, the solution must be an improvement over it. Therefore it must insure the manager against some of the lower outcomes while decreasing his wage for some higher outputs. Hence, it seems that such a solution will have $r(y) = y - w(y)$ being negative for low values of output, while positive for the high ones. Such a schedule will be feasible for higher type managers because of the MLRP assumption.

Note also that the condition in iii is stronger that necessary since it is enough to assure that j has a feasible solution preferred to any lower type solution, instead of requiring any optimal solution for t to be feasible for any higher type manager. But this last condition is more tractable than the necessary one. To see this we check the changes in P2 if we impose our condition. We must add the following restriction:

(A5) $\sum_{y \varepsilon Y_\ell} (y - w(y))\ p\ (\eta_t, y) \geq 0 \qquad t \geq j.$

But note that

$$\sum_{y \varepsilon Y_\ell} (y - w(y))\ p_\ell(\eta, y) = \sum_{y \varepsilon Y} r(y)\ p_\ell(\eta, y) = \sum_{y \varepsilon Y} r(y)\ \lambda_\ell^N(y) + \eta \sum_{y \varepsilon Y} r(y)\ (\lambda_\ell^T(y) - \lambda_\ell^N(y))$$

Hence (A5) is satisfied if and only if

(A6) $\sum_{y \varepsilon Y_\ell} r(y)\ (\lambda_\ell^T(y) - \lambda_\ell^N(y)) \geq 0.$

Thus we only need to add (A6) to P2. Note that $w(y) = y$ is still a feasible solution.

The problem now is that $w(y)$ may fail to be increasing if (A6) is binding. Therefore, even if now we are assuring that any solution is feasible for higher types, we can not assure that higher types are better off. Thus we can not prove that $x_k(.)$ is strictly increasing.

In summary, if after solving the recursion P2 we can check that the high types will not deviate, then the solution obtained is optimal. Otherwise we can still calculate a lower bound of the solution by solving P2 with the additional constraint (A6). This last solution will be feasible and incentive compatible, but since (A6) will be binding for some j, it is not necessarily optimal.

As an ilustration that we use in the text, consider the case where the wage schedule is restricted to be a linear one, i.e. $w(y) = a + by$. Solving P2 in this class of wage schedules we obtain:

$$w_i(y) = b_i y + (1-b_i)\ v(\eta_i) \qquad \text{for type } \eta_i,$$

where

$$0 = b_1 < b_2 < \ldots < b_N < 1.$$

In particular, b_j for $j > 1$ is obtained in the following way:
For $t < j$, let b_j^t be the solution of

$$\sum_{y \varepsilon Y_\ell} p_\ell(\eta_t, y)\ [u((1-b_t)\ v(\eta_t) + b_t y) - u((1-b_j^t)v(\eta_j) + b_j^t y)] = 0.$$

It is easy to check, by induction, that $b_t < b_j^t < 1$. Then

$$b_j = \max_{t<j} \{b_j^t\}.$$

Furthermore, the assignment in the market is the efficiently optimal.

Finally, we must check that (A5) is satisfied to assure global incentive compatibility:

$$\begin{aligned}\sum_{y \varepsilon Y_\ell} (y - w(y))\ p_\ell(\eta_t, y) &= z_\ell(\eta_t) - (1-b_j)\ v(\eta_j) - b_j z_\ell(\eta_t) \\ &= (1-b_j)\ (z_\ell(\eta_t) - z_\ell(\eta_j)) > 0 \quad \text{for } t > j.\end{aligned}$$

Hence the solution is optimal in the class of linear wage schedules.