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THE CHARACTERISTICS MODEL, HEDONIC PRICES AND THE CLIENTELE EFFECT[1]

by

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I. Introduction

The characteristics model of differentiated products first developed by Lancaster (1971) and (1975) has found many uses in both theoretical and applied economics. One of the most useful aspects of the model is the hedonic decomposition of prices that the approach affords. That is, the price of a typical good is written as the sum of characteristics prices times the levels of the characteristics embodied in that good. This has proven quite useful for adjusting price indices for changes in the qualities of the goods in the market basket (see Griliches (1971)).

The purpose of this paper is to present a reexamination of Lancaster's characteristics model in light of some recent research on models of commodity differentiation (Mas-Colell (1975), Hart (1979), and Jones (1984a)). First, it is shown that the characteristics model can be viewed as arising from special restrictions on the allowed preferences within these more general models of commodity differentiation. Second, it is shown by example that, even in quite reasonable (and robust) circumstances, the decomposition of prices mentioned above can fail to hold in equilibrium.

Given this fact, the possibility of positive results concerning the form of equilibrium prices as a function of characteristics is explored. It is shown that even though price linearity may fail, prices are a convex function of characteristics in equilibrium (Proposition 1). Further, if all individuals have the same homethetic utility function over characteristics (but possibly different incomes), it is shown that equilibrium prices can be linearly decomposed through the hedonic technique (Proposition 3). Finally, it is shown (Propositions 2 and 4) that these two results concerning the form of the equilibrium price function hold independent of the underlying market structure (e.g., perfect versus monopolistic competition) as long as consumers act as price takers.

The fact that equilibrium prices cannot generally be linearly decomposed is a result of boundary problems arising in the consumer's maximization problem within the Lacasterian approach. This observation in and of itself is not new (see Muellbauer (1974) and Deaton and Muellbauer (1980), for example). These earlier authors were concerned primarily with consumers' decisions, however, and did not explore the impact of these considerations on equilibrium prices. In addition, the relevance of the underlying market structure was not discussed. It is a benefit of the power of the approach adopted here that the effect of the boundary considerations on consumer decisions can be translated directly to restrictions on the form of equilibrium prices. Further, the fact that these restrictions are independent of market structure can be easily seen within our framework.

In Section II, notation is introduced and a simple example of the model in which prices are not linear in characteristics is prsented. The positive results concerning the form of equilibrium prices are presented in Section III. Section IV contains analysis of price data on multiple vitamins as an empirical test of the predictions of Section III. Finally, a few concluding remarks are offered as Section V.

II. Some Notation and an Example

After first setting out some notation, an example will be presented in which prices cannot be hedonically decomposed.

In the economy we will consider goods are completely described by the levels of the various characteristics they possess. There are J characteristics of interest to consumers indexed by j.

Thus, a good can be described as a point, t, in $T = I_1 \times I_2 \times \ldots \times I_J$ where $I_j \subset \mathbb{R}_+$ describes the possible levels of the j-th characteristic a good can have.

Notice that under this formulation goods with the same characteristics are identified. Since our consumers will act as price takers and care only about the total amount of characteristics they receive, goods with the same characteristics must fetch the same price (if they are sold at all).[2]

Individuals choose consumption bundles which we will model as non-negative destributions (measures) on T.

This is just the familiar motion of probability modified to allow for total mass unequal to one. The collection of non-negative measures on T will be denoted by M and a typical consumption bundle will be written as m. Thus, we will follow approach to consumer choice introduced in Mas-Colell (1975) and used in Hart (1979) and Jones (1984a). The advantage of this approach is that it allows for consumers to specialize by choosing distributions concentrated on a few goods or generalize by choosing a distribution with a density.

Notice that modelling consumption in this way assumes that goods are perfectly divisible; see the related comments in Section V.

If a consumer purchases the commodity bundle m, the total amount of the j-th characteristic he consumes is given by

$$c_j(m) = \int_T t_j \, dm(\underset{\sim}{t}).$$

(This is the Lebesgue-Stieltjes integral as in probability theory.) Equivalently, if m_j is the marginal distribution of m on the j-th component of t,

$$c_j(m) = \int_{I_j} s \, dm_j(s).$$

The essence of the characteristics approach is that individuals rank the various consumption bundles through the total amount of the J characteristics. That is, the preferences of individual h are given by

$$\text{(1)} \qquad V^h(m) = U^h\ (c_1(m),\ \ldots,\ c_J(m))$$

where $U^h(\cdot)$ is a standard utility function over $\mathbb{R}^J_+$.

Thus, the preferences considered will be restricted to be of the linearly combinable variety.[3] That is, as can be seen form the definition of c_j, in the preferences we will allow, the characteristics of the individual goods are linearly combined into an aggregate characteristics bundle $(c_1(m),\ldots,c_j(m))$ which determines the utility of the consumption bundle.

We will assume that U^h is strictly increasing, strictly concave and twice continuously differentiable.

We can now define the marginal value to h of increased consumption of a good with characteristics t when h is consuming m. We will call this h's marginal utility of t at m, $MU^h(m;t)$:

$$\text{(2)} \qquad MU^h(m;t) = \sum_{j=i}^{J} U^h_j\ (c_1(m),\ldots,c_J(m)) \cdot t_j = \text{grad } U^h \cdot t$$

where U^h_j is the j-th partial derivative of U^h. This is the directional derivative of the utility function V^h in the direction of commodity t. (See Jones (1984a) for a more formal and more general definition).

Note that MU^h is linear in t. This provides a theoretical basis for the hedonic decomposition of prices. That is, if, in equilibrium, the U_j^h's are proportional across households (2) describes equilibrium prices as well. Thus, prices are linear and the U_j^h's give the "characteristics prices" in this case.

The problem of guaranteeing that prices are linear in characteristics is thus reduced to finding conditions under which the U_j^h's are proportional in equilibrium.

One would like to argue that if this were not the case, mutually advantageous trades in characteristics can be made as long as all indivduals are allotted positive amounts of all characteristics. This argument would lead us to believe that, in equilibrium, the U_j^h's should be proportional other than in the exceptional cases where some agent has a zero allottment of some characteristic.

Unfortunately, this argument is incorrect. The problem with it is that agents cannot trade characteristics directly, rather they must trade characteristics bundled as goods. Thus, it may not be possible to find mutually advantageous trades even though marginal utilities of characteristics are not proportional and all agents are consuming positive amounts of all characteristics.

This occurs because of what is known as (in the context of differential tax treatment) the clientele effect in the finance literature. That is, consumers divide themselves by groups, each group buying a different collection of commodities. The following example illustrates this phenomenon.

Example

Consider a two consumer world with no production in which goods have different levels of two characteristics. It is useful to think of the various goods as different foods described by their contents of protein and vitamin A. Here, $J = 2$ and steak is represented by a t with a large first component, carrots one with a large second component.

Let $I_1 = I_2 = [0,1]$ and suppose the aggregate social endowment is given by the uniform distribution on $T = [0,1] \times [0,1]$--i.e., the distribution with density equal to one everywhere in T.

Let B_1 be those commodities in T with larger first components than second, B_2 those with larger second components.

Consider the allocation which gives the first household the social endowment on B_1 and gives the second household the social endowment on B_2. Denote these by m_1 and m_2 respectively.

Suppose the utility functions for the two individuals are such that

$$\text{grad } U^1 \; (c_1(m_1),\; c_2(m_1)) = (2,1)$$

and

$$\text{grad } U^2 \; (c_1(m_2),\; c_2(m_2)) = (1,2).$$

In this case, (m_1, m_2) is a Pareto optimal distribution of the social endowment, even though both agents are assigned positive quantities of both characteristics and the marginal rates of substitutions between characteristics of the two disagree.

To see this, consider a trade in which ν_1 is taken from the first agent and given to the second and ν_2 is taken from the second agent and given to the first. By design, the first agent gives up more of the first characteristic than the second and the second agent gives up more of the second than the first. Thus, after completion of this trade (which leaves the first agent with $m_1 - \nu_1 + \nu_2$ and the second with $m_2 - \nu_2 + \nu_1$) the first agent has more of the second characteristic than he began with and less of the first. Let Δ_1^1 and Δ_2^1 represent the changes in the levels of consumption of the two characteristics by the first agent, and let Δ_1^2, Δ_2^2 be the corresponding quantities for the second agent. Then, for the first agent's welfare to be improved by the trade, it must be true that $\Delta_2^1 > 2\,\Delta_1^1$. Similarly, if the second agents' welfare is to be improved, $\Delta_1^2 > 2\,\Delta_2^2$. Both of these inequalities cannot hold however since $\Delta_1^2 = \Delta_1^1$ and $\Delta_2^2 = \Delta_2^1$.

Thus, (m_1, m_2) is indeed Pareto optimal. Hence, it can supported as the equilibrium of an exchange economy. The supporting prices are given by $p^*(t) = \max(2t_1+t_2,\ t_1+2t_2) = \max\ (MU^1(m_1;t),\ MU^2(m_2;t))$. For this to be an equilibrium any assignment of endowments such that $e_1 + e_2 = m_1 + m_2$ and $e_h \cdot p^* = m_h \cdot p^*$, $h = 1,2$, will do (e.g., $e_h = m_h$ or $e_1 = e_2 = \frac{1}{2}(m_1+m_2)$).

As stated above, the agents have divided themselves by types each consuming from different regions of the various types of commodities. To see more clearly what has occured, consider two goods t_1 and t_2 with $t_1 \in B_1$, $t_2 \in B_2$. Figure 1 illustrates the trading possibilities between these two goods when the agents start out at (m_1, m_2).[4]

The agents are currently at e and as can be seen from the figure, there is no possibility for mutually advantageous trade. Note that the agents are at the boundary of their consumption sets even though they both "own" positive quantities of both of the available characteristics.

One might think that by putting sufficiently strong restrictions on the form of the U^h's we could guarantee that all goods are consumed by all agents and thus obtain the desired linearity of prices in terms of characteristies. One is tempted to do this by making all goods "essential" as in Cobb-Douglas type utility functions. This is not possible with utility functions of the type exhibited in (1), however. Indeed, it cannot be done when preferences depend only on the total levels of the characteristics as we have assumed. The only way to obtain this effect is to go to preferences of the additively separable variety as seen in the continuous time growth literature. As can be readily seen from that literature, linear prices should not be expected in economies with this type of preference structure.

Before proceeding, three comments are in order.

First, many models of product differentiation feature consumption sets having indivisibilities as an essential feature (e.g., Rosen (1974), Mas-Collel (1975)). In fact, one of Rosen's principal criticisms of the characteristics model is that it assumes perfectly divisible goods. Introducing indivisibilities would not obviate the problem mentioned above in any way, however.[5] In fact, it would nullify some of the results to be presented below.

The problem is that, due to the bundling of characteristics in terms of goods, there are not enough trading possibilities. Adding indivisibilities only exacerbates this problem.

Note that contrary to Rosen's statement that perfect divisibility is equivalent to an assumption that commodities can be unbundled into characteristics, our example shows that this is not the case.

Second, a natural question to ask is whether or not the inclusion of production will restore the desired price linearity. Certainly there are assumptions on technology which will give rise to the desired resulted, but these are of the most ad hoc nature (CRS with $t = (t_1, \ldots t_j)$ requiring t_1 units of input 1, t_2 units of input 2, etc.). Beyond this, one would have to expect any such efforts to fail. After all is said and done, the cause of the problem is that different agents consume different goods causing much of the usual marginal analysis to break down. There is no reason to believe that the inclusion of production would serve to keep this from occurring.

Finally, within the finance literature there is a natural assumption which will restore the linearity of prices. This is allowing unlimited short-selling. As can be seen in Figure 1, this will serve to equate the appropriate marginal rates of substitution so that (2) is indeed valid (see Ross (1983)).

We turn now to a discussion of what can be said about the form of price functions when preferences are of the form given in (1) above.

III. Results

We begin our discussion of the equilibrium properties of prices with an examination of a perfectly competitive economy with no production.

Suppose there are H households indexed by h, with utility functions as described in (1) above. Each household is endowed with a distribution, e^h in M, over T. then, $e^* = \sum_h e^h$ is the aggregate social endowment. The collection of available commodities will be denoted by S. This is the support of e^*, $S = \text{supp } e^*$ and is the smallest closed subset of T having full e^*-mass.

Assume that S is bounded and that 0 is not in S. (This is for technical reasons concerning monotonicity of preferences.)

In this case, it has been shown (see Jones (1984a)) that there is a competitive equilibrium with prices which are continuous on S. Let p be such a continuous equilibrium price function.

Proposition 1: Under the assumptions outlined above,

(i) p is convex on S, i.e., if t_1, t_2 and $\alpha t_1 + (1-2)t_2$, $0 \leq \alpha \leq 1$, are in S, $p(\alpha t_1 + (1-\alpha)t_2) \leq \alpha p(t_1) + (1-\alpha)p(t_2)$.

(ii) p is linear on the rays of S, i.e., if t and βt are in S for some $\beta > 0$, $p(\beta t) = \beta p(t)$.[6]

Proof. Let S^h = supp m^h be the collection of commodities purchased by household h. Then a straightforward argument along those given in Jones (1984) shows that for each h there is a constant $\lambda_h > 0$ such that

$$\frac{MU^h(m^h;t)}{p(t)} \leq \lambda_h$$

with equality if t is in S^h. From this it follows that $p(t) \geq \max_h [\frac{1}{\lambda_h} MU^h(m^h;t)]$ for all t in S. Since the S^h's exhaust S, equality holds for some h and

$$(3) \qquad p(t) = \max_h [\frac{1}{\lambda_h} MU^h(m^h;t)].$$

Since max is a convex function and MU^h is linear in t, (i) follows.

The argument that (ii) holds is equally straightforward.

As can be seen from the proof of Proposition 1, a stronger statement can actually be made. Under the conditions of the Proposition, there are $a_1,\ldots,a_H \in \mathbb{R}^J_+$ with

$$p(t) = \max_h(a_h \cdot t) \text{ for } t \in S.$$

Even this is too weak, however. To see this, consider Figure 2. Evidently, if $S = \{t_1,t_2,t_3\}$, the conclusion of Proposition 1 places no restrictions on $p(t_3)$. However, as can be seen from the proof, it follows that $\gamma p(t_3) \leq \alpha p(t_1) + (1 - \alpha)p(t_2)$ where γ and α are defined by:

$$\gamma t_3 = t_4 \quad \text{and} \quad \alpha \cdot t_1 + (1 - \alpha)t_2 = t_4.$$

Convexity of the price function implies that prices are higher near the boundary of T than would be expected if the price function was linear. Intuitively, this is because there is more competition for goods near the boundary. That is, not only are these goods desirable to individuals with eccentric tastes, but also an individual with moderate tastes can satisfy his desires by buying commodities from each extreme.

Part (ii) of the proposition is interesting. Originally, one's intuition is that, given the structure of preferences, there are really only J goods, namely the various characteristics. The example presented in Section II shows that this is incorrect. In fact, there are infinitely many commodities in that example. However, (ii) shows that there is really only a one dimensional family of commodites, not two. In general, the model can be reduced to one of a J-1 dimensional set of commodities. These are conveniently summarized by the relative proportions of the various characteristics. Thus, this example differs only slightly from Example 3 of Jones (1984a). Note that if there are indivisibilities in consumption sets, neither (i) nor (ii) need hold.

Conditions (i) and (ii) above are virtually the only restrictions one can place on prices in these models. In fact, by allowing a continuum of consumers one can construct examples in which any function satisfying (i) and (ii) can be generated as equilibrium prices for an exchange economy.

The problem with Proposition 1 as stated is that it is tied too heavily to competitive equilibria. Really, these properties arise solely due to the price-taking nature of the households we consider. Because of this, the results are more general than they first appear. In fact, any model which features price-taking consumers maximizing utility functions of the type in (1) will give rise to prices satisfying (i) and (ii) of Proposition 1.

To see this, suppose that S is a finite set (this would be expected if there are any scale economies in production) and let $p(\cdot)$ be a price function on S. Define $p(t) = \infty$ if t is not in S.

Proposition 2. Suppose m^h maximizes V^h on h's budget set $\beta^h = \{m | m \cdot p \leq w^h\}$ and $\text{supp}(\sum_h m^h) = S$. Then

(i) p is convex on S, and

(ii) p is linear on the rays of S.

The proof is the same as that of Proposition 1.

Thus, we see that these two qualitative characterisics of prices follow simply due to the assumption of price-taking consumers with preferences of the form in (1). Hence, in any model of firm competition with these features will give rise to prices (for traded goods) with the properties listed.

Note that from the Propositions, a qualitative prediction about the success of linear regression applied to these models is possible. We would expect that the model would systematically underpredict the prices of goods in the extremes of T and overpredict the prices of those goods with significant levels of all characteristics.

We turn now to a discussion of a case in which the linear decomposition of prices. Although the assumptions are strong, they may serve as a useful approximation in some cases. In addition, the result gives us further insight into how the linearity of prices can fail. We return to the situation considered in Proposition 1.

Proposition 3. Suppose that there is some $U\colon \mathbb{R}^J_+ \to \mathbb{R}$ such that

(i) $U^h = U$ for all h (hence U is twice continuously differentiable, strictly concave, etc.)

(ii) U represents homothetic preferences over characteristics--i.e.,

$$U(x_1) = U(x_2) \text{ implies } U(\alpha x_1) = U(\alpha x_2) \text{ for all } \alpha > 0.$$

Then, p is linear in t.

Proof: As in Proposition 1, let m^h be h's equilibrium allocation and define $w^h = p \cdot m^h$. Let

$$A(w) = \{(c_1(m), \ldots, c_J(m)) \mid p \cdot m \leq w\}.$$

This is the collection of characteristics bundles that an agent with income w can afford at prices p.

It is straightforward to show that A(w) is compact and convex. Further, $A(tw) = tA(w)$.

Let $c(m^h) = (c_1(m^h), \ldots, c_J(m^h))$.

Then, it is easy to see that $c(m^h)$ is the unique (by strict-concavity of U) maximizer of U on $A(w^h)$.

Further, since $A(tw) = tA(w)$ and U is homothetic, it follows that $c(m^{h'}) = \frac{w^{h'}}{w^h} c(m^h)$ for all h, h'.

Using the homotheticity of U, it follows that

$$\text{grad } U^h(c(m^h)) = \alpha \text{ grad } U^{h'}(c(m^{h'}))$$

for some $\alpha > 0$.

The result now follows from (3) and the fact that $w^{h'} > 0$ for all h.

One might think that by mimicing this proof, the homotheticity of U could be dropped. That is, if all agents have the same utility function over characteristics, the $c(m^h)$ all lie on the U income expansion path in characteristics space and so the same argument should work. In fact, this argument is not valid since $A(w)$ is not necessarily of the form of usual budget sets.

That is, if $\pi = \text{grad } U(c(m^h))$ and if c^* is on the π - income expansion path for U at $w = \alpha w^h$, it need not be true that $c^* \in A(w)$. Since $A(\alpha w^h) = \alpha A(W^h)$, c^* is in $A(w)$ if preferences over characteristics are homothetic because $c^* = \alpha c(m^h)$ and $c(m^h) \in A(w^h)$.

The assumption that the U^h's are the same is easily seen to be essential. In fact, the example is quite consistent with the two agents having homothetic, but different, utility functions. Figure 3 shows the situation in characteristics space for the example. Note that $U^1 \neq U^2$ necessarily.

Note that both agents have income $w = \frac{5}{6}$ and $c(m^1) = (\frac{1}{3}, \frac{1}{6})$, $c(m^2) = (\frac{1}{6}, \frac{1}{3})$. The kink in $A(w)$ arises due to the non-linearity of prices on T.

Of course, Proposition 3 can be extended to cover other market structures in much the same way that Proposition 1 was extended. We have:

<u>Proposition 4</u>. Suppose S is finite and the h's satisfy the assumptions of Proposition 3. If m_h maximizes V^h on $\beta^h = \{m | m \cdot p \leq w^h\}$ with $w^h > 0$, p is the restriction to S of some linear function on T.

IV. <u>A Simple Empirical Example</u>

The considerations raised in the previous two sections concerning the possibility of nonlinearity in the price function would be of only limited interest if prices are in fact linear in all examples where the model is applicable. In this section, in an attempt to address this issue, we will examine an example for which the characteristics model seems a very good approximation. There are many examples in the literature of empirical estimates of hedonic price functions. Rather than trying to present a comprehensive list here we direct the reader to Griliches (1971) and the refrences in Deaton and Muellbauer (1980).

The example we will consider is that of nonprescription multiple vitamins. A data set consisting of 277 observations was constructed. Manufacturers' specifications listed as ingredients 24 separate vitamins and minerals. Of the 277 observations, 163 of the vitamins contained only one of the ingredients while the remainer were combinations of two or more of the ingredients.

Let t_i be the vector (in $\mathbb{R}^{24}$) consisting of the quantities of the vitamins and minerals in a single tablet of the i-th observation and let p_i denote its price. The two models we will consider are:

<u>Model I</u>: $p_1 = a't_i + \varepsilon_i$ where the ε_i are i.i.d. with $E(\varepsilon_i) = 0$ and a is a vector of parameters.

<u>Model II</u>: $p_i = f(t_i) + \gamma_i$ where the γ_i are i.i.d. with $E(\gamma_i) = 0$ and, as suggested by section 3, f is convex and linear on rays.

A preliminary regression was run to estimate Model I. The results of this estimation are summarized in Table 1. As can be seen from the table, the model fits quite well. The $\mathbb{R}^2$ of .89 implies that the regression is significant at even the .0001 level.[7] Most of the coefficients are of the correct sign and many are highly significant.

All of the vitamins and minerals were represented in the sample as pure, single-ingredient tablets. This suggests a simple way to test whether or not Model II is statistically indistinguishable from Model I for this data set. This is to estimate Model I using only the data on pure vitamins and minerals and then use the remainder of the data as a validation check.

Table 1

Ingredient (Units)	Coefficient Estimate (¢/unit)	t
Vitamin A (IU)	.00009	2.539
Vitamin C (mg)	.00759	16.531
Vitamin B1 (mg)	.03336	2.871
Vitamin B2 (mg)	.04777	3.935
Niacin (mg)	.00880	1.925
Choline (mg)	.00596	1.370
Vitamin B6 (mg)	.02850	10.972
Vitamin B12 (mcg)	.000009	2.958
Vitamin D (IU)	.00064	0.766
Vitamin E (IU)	.02380	22.355
Folic Acid (mcg)	.00323	2.243
Biotin (mcg)	.00652	0.941
Pantothenic Acid (mg)	.00049	0.820
Iron (mg)	.04743	3.550
Calcium (mg)	-.00375	-2.130
Magnesium (mg)	.01178	3.049
Copper (mg)	-.00468	-0.418
Zinc (mg)	.05536	2.920
Chromium (mcg)	-.05022	-3.064
Selenium (mcg)	.04613	3.294
Molybdenum (mg)	.02721	1.368
Manganese (mg)	.05851	1.125
Potassium (mg)	.02124	1.597
Iodine (mcg)	-.00168	-0.514

Let X_1 be the matrix of characteristics of the pure vitamins and let X_2 be the matrix of characteristics of those vitamins which are combinations. Similarly, we partition the observations on the prices $p' = (p_1', p_2')$.

If we define $\hat{a} = (X_1', X_1)^{-1} X_1' p_1$, we see that under the assumptions of Model I, $\hat{a}$ is an unbiased estimate of a. Define $\hat{p} = X_2 \hat{a}$ and $\delta = p - \hat{p}$. Finally, let

$$K = \frac{1}{n_2} \sum_{i=1}^{n_2} \delta_i = \frac{1}{n_2} \sum_{i=1}^{n_2} (p_i - \hat{p}_i)$$

where n_2 is the number of combination vitamins.

Then, assuming that Model I is correct, it is straightforward to verify that $E(K) = 0$ and $\mathrm{var}(K) = \frac{\sigma^2}{n_2} + \bar{t}' \mathrm{cov}(\hat{a})\bar{t}$ where $\sigma^2 = \mathrm{var}(\varepsilon_i)$ and $\bar{t}_j$ is the average level of the j-th characteristic within the sample of combined vitamins. (Note: by construction, X_1 is an orthogonal design matrix so that $\mathrm{cov}(\hat{a}_i, \hat{a}_j) = 0$ if $i \neq j$.)

If Model II is correct and Model I is not, it still follows (due to the linearity on rays of f) that $E(\hat{a}_i) = f(e_i)$ where e_i is the i-th unit vector. Further, since f is convex it follows that $E(\hat{p}_i) \geq E(p_i)$ for all i. Hence, $E(K) \leq 0$ if Model II holds. Thus, if Model II holds and Model I does not, we would expect K to be negative and significant.

Indeed, this is exactly what we find as

$$K = -.0937 \text{ and } \widehat{\mathrm{var}(K)} = .0014$$

whence

$$K/\hat{\sigma}_K = -2.492$$

which is highly significant. (Note: if the ε_i are normally distributed, $K/\hat{\sigma}_K$ has a t-distribution with 90 degrees of freedom.)

Finally, as further evidence of the nonlinearity of p(t), we note that the procedure outlined above over-predicted the true price (i.e., $\hat{p}_i > p_i$) for 101 of the 114 multiple vitamins in the data set.

V. Concluding Remarks

We close with a few brief remarks.

(1) Adding other goods to the model is a straightforward extension. For example, suppose there are K other goods to be considered. Let $x \in \mathbb{R}_+^K$ denote consumption levels of these goods. Then, consumption sets of households are of the form $Z = \mathbb{R}_+^K \times M$ where M is as before. If individuals have preferences

on Z of the form

$$V^h(x,m) = U^h(x,\ c_1(m),\dots,c_j(m))$$

where U^h is a standard utility function on $\mathbb{R}_+^{K+J}$, the conclusions of Propositions 1 and 2 concerning the form of the restriction of p to T are still valid.

In addition, if $U^h(x,c)$ can be written as $U^h(x,c) = U_1^h(x) + U_2(c)$ where U_2 is homothetic and does not depend on h, the conclusions of Propositions 3 and 4 remain valid. To see this, simply apply the argument of Proposition 3 using the equilibrium expenditure on goods in T in place of income.

Note that preferences of this form need not be homothetic on Z. In particular, it is possible that prices are linear on T even though expenditure on goods in T is not a constant percent of income. Thus, it is quite possible that a linear regression of prices on characteristics within an industry would perform quite well even though the income elasticity of demand for products from that industry is not 1 (cf. Muellbauer (1974)).

(2) Propositions 1 through 4 have obvious extensions to the case where

$$V^h(m) = U^h(\textstyle\int_T g_1(t)dm,\dots,\int_T g_K(t)dm)$$

where the g_i's are nonnegative and continuous functions of t. For example, p is convex in g and linear on "g-rays" in this case. In particular, if the g_i are convex functions of t (i.e., increasing marginal value of characteristics), we can still conclude that p must be convex in t.[8]

(3) The results reported here depend very heavily on the implicit

assumption of perfect divisibility. This can be readily seen in the proof of Proposition 1. If goods are not perfectly divisible, marginal analysis (over quantitites) is not in general valid and the argument breaks down. If the collection of produced goods (S) has a nonempty interior and we assume that households only buy one unit of the good, marginal analysis on the levels of the characteristics can be performed. This is the approach adopted by Rosen (1974).

We can relax Rosen's assumption that only one unit of one good in T can be bought by allowing for the possibility of purchase of more than one item while still retaining indivisibility. In this case, the consumption set $M' \subset M$ is given

$$M' = \{m \mid t \in \text{supp } m \text{ implies } m(t) \text{ is an integer}\}.$$

This is the basis of the model analyzed in Mas-Colell (1975) and has the appealing property that although consumption must be in integer quantities, this interger (e.g., number of television sets or automobiles consumed) is endogenously determined.

In this case, the linearly combinable perferences given in (1) are still a possible specification and analogs of Propositions 1 and 2 still hold. It is easy to see that in this case, $p(t_1 + t_2) \leq p(t_1) + p(t_2)$ if t_1, t_2 and $t_1 + t_2$ are in S. This does not imply that p is convex. For example, a p which is convex across rays in S and concave along rays in S will satisfy this restriction. In particular, the semilog price function which is often used in hedonic regression (see the articles by Griliches and Dhrymes in Griliches (1971), for example):

$$\ln p(t) = a_1 t_1 + \ldots + a_j t_j$$

is of the required form.

(4) It has been pointed out (e.g., Pepall (1983), Dreze and Hagen (1978)) that the state preference approach to modelling asset markets is a special case of the Lancasterian model. This can be readily seen by identifying each possible state with a characteristic in the discussion presented above.

Hence, the results reported here apply to that model as well. Realisitically, our results probably do not have much bite in this case, however. That is, if there is any independent variation in the profits of differnt firms, the number of states is of an order of magnitude equal to the number of firms. In this case, neither convexity nor linearity of p implies anything about the relative prices of securities written on the different firms.

(5) Due to the problem of indivisibilities discussed in (3), it is difficult to give examples of industries in which the results of Section III are directly applicable.

The results are interesting for another reason, however. This is that they show how interrelated the demands for the outputs of "monopolists" can be. Of course, the location model provides a "local" example of this phenomenon. What Propositions 2 and 4 show is how these relationships can place qualitative "global" restrictions on demands when the assumptions of the characteristics model are met.

NOTES

[1]This note was begun as an example in connection with Jones (1984a). It is largely due to the extremely useful comments and encouragement of Philip Dybvig and Paul Milgrom that it appears at all. They convinced me that the example and the results presented here were significant enough to appear separately. Their assistance is gratefully acknowledged. Finally, I would like to thank Sherwin Rosen for his comments on an earlier draft of the paper, Kim Maselli for her help in preparing the data for section IV, and Peter Rossi for his many helpful suggestions. Of course, remaining errors are the author's responsibility.

[2]This is not simply a theoretical possibility in these models as they are often used to generate theories of the equilibrium level of product diversity (cf. Lancaster (1975)).

[3]I am indebted to Sherwin Rosen for suggesting this terminology.

[4]Note that in this example, any trade offered can be considered as being between two goods. For example, for both agents, ν_1 (defined above), is equivalent to a trade of ν_1 (T) units of the good with characteristics

$$t_j^* = \frac{1}{\nu_1(T)} \int_T t_j d\nu_1(\underset{\sim}{t}).$$

$t^* = (t_1^*, t_2^*)$ is in B_1 since B_1 is convex and t^* is the mean of the probability $(\frac{1}{\nu_1(T)} \nu_1)$ on B_1. Similar reasoning holds for ν_2.

[5]Rosen's work shows that this is true. Rosen's second main objection to the characteristics model is one of realism. He points out that owning two

six foot cars is not the same as owning one twelve foot car. Although this is clearly true, it really has nothing to do with the divisible vs. indivisible issue. The above example is one in which utility does not depend only on total quantities of characteristics. If Rosen's framework were extended to allow for the possibility of the purchase of more than one car, more general preferences of the variety used in Mas-Colell would have to be allowed in order to overcome this second (and quite valid) objection.

[6]To my knowledge, the first statement of a result of this sort appeared in Jones (1981). That result rested on the presence of "money" or some other essential good. The elegant argument presented here was suggested by Paul Milgrom who also convinced me that money need not be included in the model for the result to hold.

An argument to this effect is also included in Rosen (1974). However, he concludes that p must be linear. Of course, this is not the case as the example of Section II shows. Rosen also assumed that p is twice continuously differentiable. This will not be true unless p is linear when there are finitely many agents. It can be true if there are infinitely many individuals without p being linear, but it need not be true even there (consider the example as presented but with infinitely many agents of each of the two types).

[7]Note that since no constant is being used in the regression, the definition and interpretation of R^2 are altered slightly; see Aigner (1971).

[8]Note that the convexity of g as a function of t is quite consistent with the concavity of V^h as a function of m. This will be satisfied as long as U^h is concave in its arguments. Note also that the concavity of V^h does not (by itself) imply any restrictions on the form of the equilibrium price function.

Figures

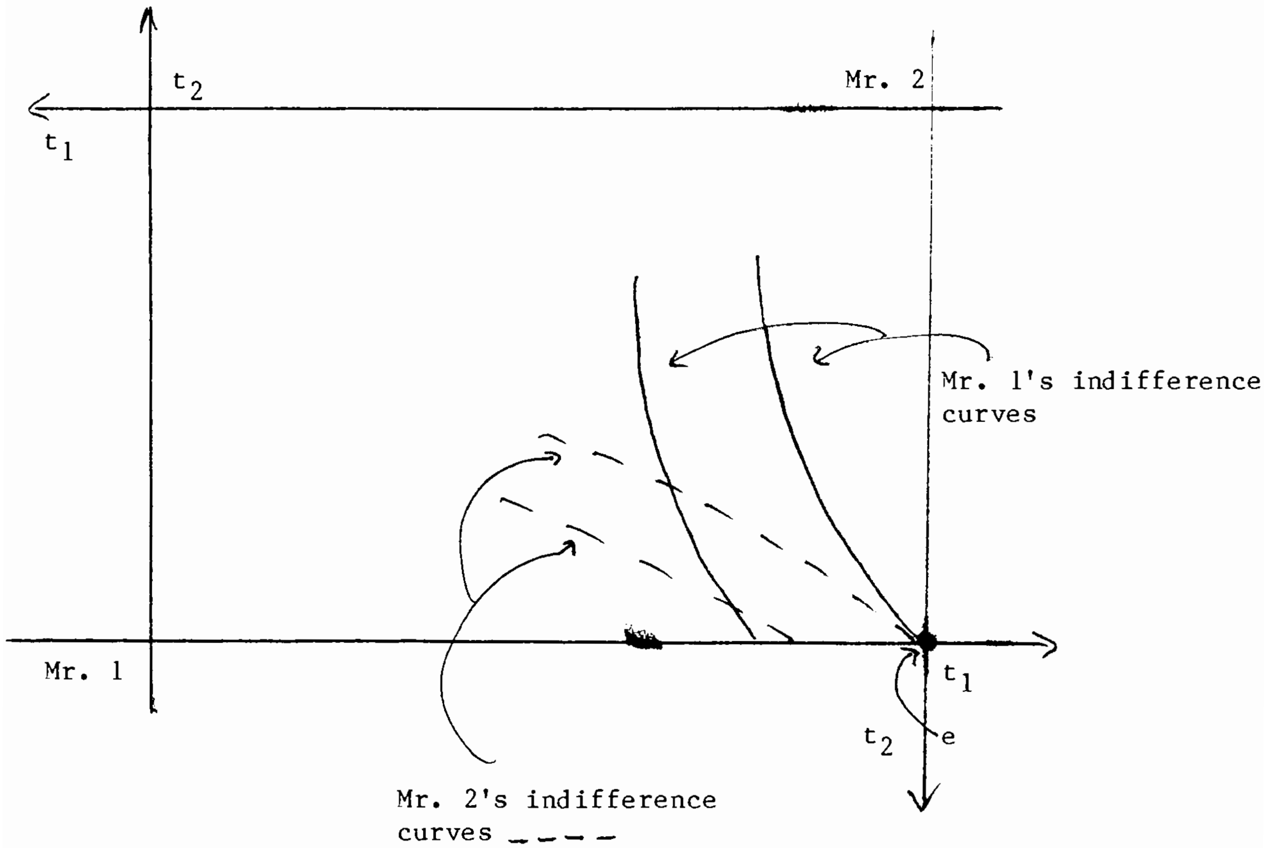

Figure 1

Note: The relative slopes of the two types of indifference curves depend on exactly what t_1 and t_2 are. However, they always have the relationship depicted.

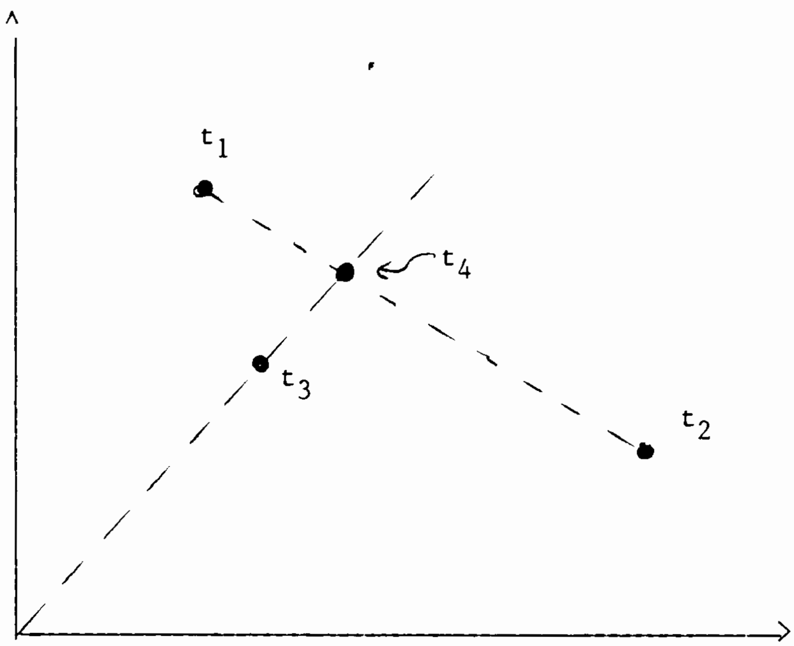

Figure 2

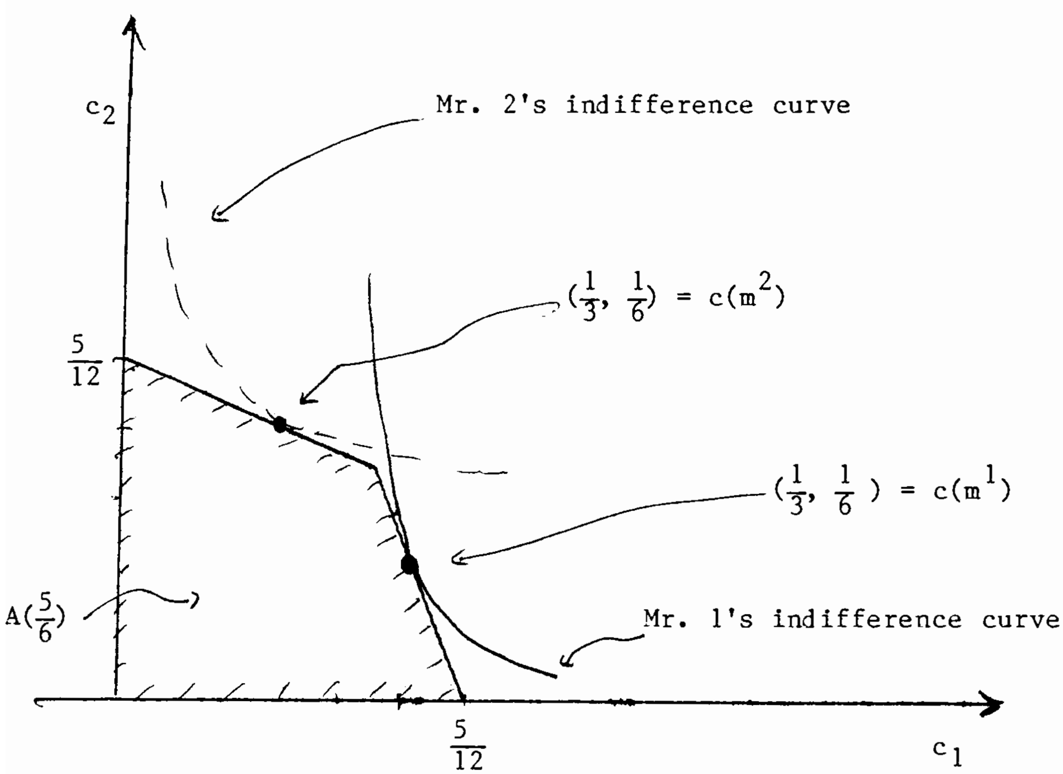

Figure 3

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