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COMPARISON OF ALTERNATIVE FUNCTIONAL FORMS IN PRODUCTION*

by

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Introduction

In his seminal article on testing non-nested hypotheses, D. R. Cox (1961) proposes three approaches to the problem of choice between alternative families of data densities: (1) the Bayesian approach which Cox regards as self-evident, (2) a generalized likelihood ratio testing approach, and (3) an exponential weighting method of nesting alternative distributions. Over the last decade, many workers in econometrics have been quick to take up Cox's second and third suggestions and apply these methods to a wide variety of testing problems (see, for example, the special issue of the Annals of Applied Econometrics [1983] edited by Halbert White on non-nested models). Few econometricians, however, have utilized Cox's first suggestion, the Bayesian approach.

In recent years, econometricians have proposed a number of flexible functional forms for use in the analysis of subsitution in production. The Translog form, which has enjoyed the most frequent application, is a local quadratic approximation to the underlying cost or production function. A recent arrival, the Fourier Flexible Form proposed by Gallant (1982), employs a uniform global approximation method. As shown in Gallant (1982) and reviewed below, there are strong theoretical reasons for favoring the FFF over the Translog. However, the increased flexibility of the FFF is achieved through a significant increase in the number of parameters and it is not clear in the small samples encountered in most production work that the FFF will dominate. In this paper, we restrict discussion to a comparison of the Translog and FFF. Other flexible functional forms such as the Laurent form proposed by Barnett (1983) could easily be compared to the Translog and FFF using the procedures developed here. Bayesian hypothesis-testing methods are developed for the choice between alternative functional forms. These Bayesian procedures do not appeal to large sample results and contain an explicit trade-off between the number of parameters and the goodness of fit.

We will focus on the specification of cost functions and systems of cost share equations. Once a particular functional form has been postulated for the cost function, the associated system of cost share equations can be derived by applying Shephard's Lemma (see, for example, Varian [1978] for details). In the case of the Translog and Fourier flexible forms, the associated system of cost shares is linear in the parameters. Choice between the Translog and Fourier flexible forms is thus reduced to the choice between two non-nested multivariate regressions.[1]

The Translog Cost Share System

A number of authors have used a Translog cost function to analyze the properties of the aggregate U.S. production technology. In most studies (see Rossi [1984] for discussion and references), a three input, non-homothetic cost function is postulated for U. S. aggregate manufacturing data. We use U. S. aggregate manufacturing data for the period 1947-1971 first collected and used by Berndt and Wood (1975). Since we are interested only in the flexibility of alternative functional forms, homogeneity will not be imposed on the cost function and the associated system of cost shares.

$$(1) \quad \ell nC = \alpha_0 + \delta_0 \ell ny + \alpha_L \ell nP_L + \alpha_K \ell nP_K + \alpha_M \ell nP_M$$

$$+ 1/2\ \gamma_{LL}(\ell nP_L)^2 + 1/2\ \gamma_{KK}(\ell nP_K)^2 + 1/2\ \gamma_{MM}(\ell nP_M^2)$$

$$+ \gamma_{ML}\ell nP_M \ell nP_L + \gamma_{KL}\ell nP_K \ell nP_L + \gamma_{MK}\ell nP_M \ell nP_K$$

$$+ \delta_L \ell ny \ell nP_L + \delta_K \ell ny \ell nP_K + \delta_M \ell ny \ell nP_M$$

where P_L = price of labor, P_K = price of capital, P_M = price of materials, and y = output. Differentiating (1) and applying Shephard's lemma, a system of Translog cost share equations is obtained.

$$(2a)\quad S_L = \alpha_L + \delta_L \ell ny + \gamma_{LL} \ell nP_L + \gamma_{KL} \ell nP_K + \gamma_{ML} \ell nP_M$$

$$(2b)\quad S_K = \alpha_K + \delta_K \ell ny + \gamma_{KL} \ell nP_L + \gamma_{KK} \ell nP_K + \gamma_{MK} \ell nP_M$$

$$(2c)\quad S_M = \alpha_M + \delta_M \ell ny + \gamma_{ML} \ell nP_L + \gamma_{MK} \ell nP_K + \gamma_{MM} \ell nP_M$$

where S_L = cost share of labor = $P_L L/C$, S_K = cost share of capital = $P_K K/C$, S_M = cost share of materials = $P_M M/C$.

In order to complete the stochastic specification of the system of cost shares, a multivariate normal disturbance is added to (2a-c). Since cost shares are bounded between zero and one, the multivariate normal distribution can only serve as an approximation to the true distribution of cost shares. In Rossi (1984), an alternative Logistic-normal stochastic specification for share equation systems is developed. In this study, we employ the additive normal specification. By definition, $S_L + S_K + S_M = 1$. This implies that the trivariate normal distribution of (S_L, S_K, S_M) is singular. For estimation purposes, one of the equations is dropped--S_M is arbitrarily chosen--to obtain a two-equation system.

Without imposing symmetry conditions,[2] (2a) and (2b) can be rewritten as a system of regression equations.

$$(3a)\qquad \underline{y}_1 = X\underline{\beta}_1 + \underline{\varepsilon}_1$$

(3b) $\underline{y}_2 = X\underline{\beta}_2 + \underline{\varepsilon}_2$

$$\underline{\varepsilon}' = (\underline{\varepsilon}_1', \underline{\varepsilon}_2') \sim MVN(\underline{0}, \Sigma \otimes I_T)$$

Here we have used the standard notation for multivariate regression in order to facilitate reference to the statistical literature. $\underline{y}_1$ and $\underline{y}_2$ are vectors of observations on the labor and capital shares, respectively. The X matrix contains observations on log factor prices. Writing (3a,b) in matrix form,

(4) $Y = XB + U$

$$U = [\underline{\varepsilon}_1 \vdots \underline{\varepsilon}_2], \; B = [\underline{\beta}_1 \vdots \underline{\beta}_2], \; Y = [\underline{y}_1 \vdots \underline{y}_2].$$

The likelihood function for the multivariate regression model given in (4) can be written as

(5) $\ell(B, \Sigma^{-1} | Y, X) = (2\pi)^{-T} |\Sigma^{-1}|^{T/2} \exp\{-1/2 \; tr(Y - XB)'(Y - XB)\Sigma^{-1}\}$

or, in stacked form

(6) $\ell(\underline{\beta}, \Sigma | \underline{y}, Z) = (2\pi)^{-T} |\Sigma^{-1}|^{T/2} \; \exp\{-1/2(\underline{y} - Z\underline{\beta})'(\Sigma^{-1} \otimes I_T)(\underline{y} - Z\underline{\beta})\}$

with $\underline{y} = vec(Y)$, $\underline{\beta} = vec(B)$, $Z = [I_2 \otimes X]$.

Fourier Flexible Form Share Equation System

The Fourier Flexible form cost function was first introduced by Gallant (1982). Gallant proves that the Fourier Flexible form approximation method can uniformly approximate the true cost function in the sense of Sobolov norm. By increasing the length of the Fourier series expansions and the number of directions in which the expansions are taken, the FFF can be made as close as possible to the true cost function as measured by Sobolov norm. The Sobolov norm includes measures of how close the derivatives are approximated as well as the function itself. This is especially important for the cost function since the factor demands are derivatives of the cost function. The local series expansion methods which are used to derive the Translog and other quasi-quadratic forms do not have these uniform approximation properties. However, the FFF cost function is parameterized by a large number of parameters even for low dimensional systems. The sine-cosine representation of the FFF[3] is

$$(7)\quad g(\underline{x}_t|\theta) = u_0 + \underline{b}'x_t + 1/2\ \underline{x}_t'C\underline{x}_t$$

$$+ \sum_{\alpha=1}^{A}\{u_{0\alpha} + 2[u_{1\alpha}\cos(\underline{k}_\alpha'\ \underline{x}_t) - v_{1\alpha}\sin(\underline{k}_\alpha'\ \underline{x}_t)]\}$$

with

$$\underline{b}' = (\underline{c}', b_{M+1}),\ \underline{k}_\alpha' = (\underline{r}_\alpha', k_{0\alpha}),\ \underline{S}_t' = (S_{L,t}, S_{K,t}, S_{M,t}).$$

$\underline{x}_t$ is a scaled vector consisting of both the log input price vector and output at time t. $\underline{x}_t$ is scaled so that each element lies between 0 and 2π, $\underline{x}_t' = [(\ell n\underline{p}_t - \underline{\delta})'/\lambda,\ (\ell ny - \delta_0)/\lambda]$. The matrix of the quadratic form, C, can be written

$$C = \sum_{\alpha=1}^{A} u_{0\alpha} \underline{k}_\alpha \underline{k}_\alpha'$$

The FFF can be viewed as a Translog with an appended set of univariate Fourier expansions along the directions, $\underline{k}_\alpha$. The matrix of the quadratic form is parameterized by A parameters instead of the usual M(M+1)/2 parameters. The $\{\underline{k}_\alpha ; \alpha = 1,2,\dots,A\}$ are the set of multi-indices which determine the directions along which the Fourier expansions are taken. Table 1 presents the set of multi-indices used in this paper. Homogeneity restrictions are not imposed on the FFF cost function by ruling out multi-indices with $\underline{k}_\alpha' \underline{\iota}^* \neq 0$ $(\underline{\iota}^{*'} = (\underline{\iota}', 0))$.

Differentiation of g in (7) yields the system of FFF cost share equations

$$(8) \quad \underline{S}_t = \underline{c} - \sum_{\alpha=1}^{A} \{u_{0\alpha} \underline{k}_\alpha' \underline{x}_t + 2[u_{1\alpha} \sin(\underline{k}_\alpha' \underline{x}_t) + v_{1\alpha} \cos(\underline{k}_\alpha' \underline{x}_t)]\} \underline{r}_\alpha$$

Equation (8) can be written as

$$\underline{S}_t = X_t \underline{\theta}$$

where $\underline{\theta}' = (\underline{c}', u_{01},\dots,u_{0A}, u_{11},\dots,u_{1A}, v_{21},\dots,v_{2A})$ is a $M + 3A = 3 + 21 = 24$ tuple (M is the number of inputs in the system).

Appending an additive multivariate normal error term to (8), a system of regression equations is obtained. Again, the standard statistical notation is employed where $\underline{y}_t$ is the vector of cost shares for each factor at time t and X_t is the matrix of transformed factor prices at time t.

TABLE 1

MULTI-INDICES FOR THE FOURIER FLEXIBLE

COST FUNCTION

L	1	0	0	0	1	1	0
K	0	1	0	0	-1	0	1
M	0	0	1	0	0	-1	-1
y	0	0	0	1	0	0	0
$\alpha =$	1	2	3	4	5	6	7

$A = 7$

$K = \sum_i |k_{\alpha,i}| \leqslant 2$

(9) $\underline{y}_t = X_t\underline{\theta} + \underline{\varepsilon}_t \quad \underline{\varepsilon}_t \sim MVN(0,\Sigma).$

Rearranging (9) and imposing the adding up constraint, a system of seemingly unrelated regression equations is obtained. In our three input case, the labor and capital cost share equations form a two equation seemingly unrelated regression model.

(10a) $$\underline{y}_1 = X_1\underline{\theta} + \varepsilon_1$$

(10b) $$\underline{y}_2 = X_2\underline{\theta} + \underline{\varepsilon}_2$$

Note that the coefficient vector is constrained to be the same in both share equations. It is more convenient to write the system (10a,b) in the stacked form

(11) $$\underline{y} = Z\underline{\theta} + \underline{\varepsilon} \qquad \underline{\varepsilon} \sim MVN(\underline{0},\Sigma \otimes I_T)$$

$$\underline{y}' = (\underline{y}_1', \underline{y}_2'),\ Z' = [X_1' \vdots X_2'], \text{ and } \underline{\varepsilon}' = (\underline{\varepsilon}_1', \underline{\varepsilon}_2')$$

The likelihood function for (11) is

(12) $$\ell(\underline{\theta},\Sigma^{-1}|\underline{y}, Z) = (2\pi)^{-T}|\Sigma^{-1}|^{T/2} \exp\{-1/2(\underline{y} - Z\underline{\theta})'(\Sigma^{-1} \otimes I_T)(\underline{y} - Z\underline{\theta})\}$$

Posterior Odds Ratios

Two hypotheses are entertained in this section:

H_1: The cost share system is Translog;

H_2: The cost share system is FFF.

In the Bayesian approach to hypothesis evaluation, the posterior probabilities of the hypotheses are compared. The posterior probabilities represent the weight of prior belief and sample evidence for a particular hypothesis. The posterior probabilities of hypotheses can be conveniently summarized in the posterior odds ratio.

$$K_{12} = \frac{p(H_1|D)}{p(H_2|D)} = \frac{p(D|H_1)}{p(D|H_2)} \cdot \frac{p(H_1)}{p(H_2)} \qquad \text{[D stands for the data.]}$$

It should be noted that it is not necessary for the set of hypotheses to exhaust the possibilities. We do not require that $p(H_1) + p(H_2) = 1$. The posterior odds ratio is designed to evaluate the relative merits of two models without assuming that these models are necessarily the only appropriate models. The quantity $p(H_1)/p(H_2)$ is called the prior odds ratio and is taken to be 1:1 in this study. The posterior odds ratio can be applied to non-nested hypotheses without modification, unlike the likelihood testing ratio procedure. In addition, the posterior odds ratio is completely symmetric and the decision outcomes are not dependent on which hypothesis is considered the "maintained hypothesis."

In a parametric setting:

$$K_{12} = \frac{\int p_1(D,\underline{\phi}_1)d\underline{\phi}_1}{\int p_2(D,\underline{\phi}_2)d\underline{\phi}_2} \cdot \Pi_{12}$$

$$= \frac{\int p_1(D|\underline{\phi}_1)p_1(\underline{\phi}_1)d\underline{\phi}_1}{\int p_2(D|\underline{\phi}_2)p_2(\underline{\phi}_2)d\underline{\phi}_2} \cdot \Pi_{12}$$

$p_1(D|\underline{\phi}_1)$ is the data density under H_1 and $p_2(D|\underline{\phi}_2)$ is the data density under H_2.

In the case of choice between share equation systems, the two parametric models for the Translog (4) and FFF (12) can be compared using the following posterior odds ratio:

$$(13)\quad K_{12} = \frac{\int p_1(B,\Sigma^{-1},Y,X)d\Sigma^{-1}dB}{\int p_2(\underline{\theta},\Sigma^{-1},\underline{y},Z)d\Sigma^{-1}d\underline{\theta}} = \frac{\int \ell_1(B,\Sigma^{-1}|Y,X)p_1(B,\Sigma^{-1})d\Sigma^{-1}dB}{\int \ell_2(\underline{\theta},\Sigma^{-1}|\underline{y},Z)p_2(\underline{\theta},\Sigma^{-1})d\Sigma^{-1}d\underline{\theta}}$$

$p_1(B,\Sigma^{-1})$, $p_2(\underline{\theta},\Sigma^{-1})$ are the prior distributions under each of the hypotheses. In order to evaluate (13), prior distributions for each set of parameters must first be specified.

<u>Prior on (B,Σ^{-1})</u>

We write

$$p_1(B,\Sigma^{-1}) = p(B|\Sigma^{-1})p(\Sigma^{-1})$$

and employ a diffuse prior on Σ^{-1} coupled with a conditional normal prior on B. Σ is the variance-covariance matrix of the cost shares. The flexible functional forms are introduced to give different specifications for the mean of cost shares. We assume that the variance-covariance matrix of cost shares has the same prior distribution under both cost share equation specifications. Since little prior information is available on Σ, the same diffuse prior is use in both the numerator and denominator of the odds ratios. Our odds ratio calculations are immune to the scaling problems sometimes encountered with the use of improper priors because any changes in the prior density of Σ under Translog must also be made to the prior density of Σ under the FFF specification.

(14) $p(\Sigma^{-1}) = |\Sigma^{-1}|^{-(2+1)/2} = |\Sigma^{-1}|^{-3/2}$

(15) $p(B|\Sigma^{-1}) = (2\pi)^{-k}|\Sigma^{-1}|^{k/2}|A|^{1/2} \exp\{-1/2 \operatorname{tr}(B - \bar{B})'A(B - \bar{B})\Sigma^{-1}\}$

where B is a $k \times 2$ matrix, Σ^{-1} is a 2×2 matrix.

The conditional normal prior on B given in (15) can also be expressed in stacked form as

$$p(\underline{\beta}|\Sigma^{-1}) = (2\pi)^{-k}|\Sigma^{-1} \otimes A|^{1/2} \exp\{-1/2 (\underline{\beta} - \bar{\underline{\beta}})'(\Sigma^{-1} \otimes A)(\cdot)\}$$

$$\underline{\beta} = \operatorname{vec}(B) \quad \text{and} \quad \bar{\underline{\beta}} = \operatorname{vec}(\bar{B}).$$

The conditional prior mean of $\underline{\beta}$ and the elements of the A matrix must be specified.

$$\underline{\beta}' = (\alpha_L, \delta_L, \gamma_{LL}, \gamma_{KL}, \gamma_{ML}, \alpha_K, \delta_K, \gamma_{KL}, \gamma_{KK}, \gamma_{MK}).$$

We take the prior means of α_L, α_K to be the mean labor share and mean capital share, respectively. The prior mean of all other parameters are taken to be zero.

$$\bar{\underline{\beta}}' = (\bar{S}_L, \underline{0}', \bar{S}_K, \underline{0}')$$

This choice of prior mean centers the prior specification of the cost technology over the Cobb-Douglas specification with constant returns to scale.

The prior covariance of $\underline{\beta}$ is

$$\Sigma \otimes A^{-1} = \begin{matrix} \sigma_{11}A^{-1} & \sigma_{12}A^{-1} \\ \sigma_{21}A^{-1} & \sigma_{22}A^{-1} \end{matrix}$$

The g-prior notion of Zellner (1980) can be generalized to the multivariate case by taking the A matrix to be of the form $gX'X$. We choose g small enough so that the prior standard deviation of $\underline{\beta}$ conditional on $\Sigma = I$ is 2.0. This results in a relatively non-informative or "locally-uniform" prior.

Prior on $(\underline{\theta}, \Sigma^{-1})$

For the FFF cost share system, it is specified that $(\underline{\theta}, \Sigma^{-1})$ are a priori independent. A diffuse prior on Σ^{-1} is coupled with a normal prior on $\underline{\theta}$.

(16) $p(\Sigma^{-1}) = |\Sigma^{-1}|^{-3/2}$

(17) $p(\underline{\theta}) = (2\pi)^{-\ell/2}|C|^{1/2} \exp\{-1/2\ (\underline{\theta} - \bar{\underline{\theta}})'C(\underline{\theta} - \bar{\underline{\theta}})\}$

where ℓ is the length of the theta vector.

As in the Translog prior specification, the prior mean of share equation intercepts is taken to be the mean cost shares and the prior mean of all other elements of $\underline{\theta}$ is zero. The prior precision matrix is specified in the g-prior form

$$C = fZ'Z$$

f is chosen to keep all prior standard deviations at least 2.0. Again, a locally-uniform prior is specified.

Calculation of the Numerator of Posterior Odds Ratio

The numerator of the posterior odds ratio in (13) can be written as

$$\int \ell_1(B,\Sigma^{-1}|Y,X)p_1(B,\Sigma^{-1})d\Sigma^{-1}dB$$

$$= \int (2\pi)^{-2T/2}|\Sigma^{-1}|^{T/2} \exp\{-1/2 \text{ tr } (Y - XB)'(Y - XB)\Sigma^{-1}\}$$

$$\cdot (2\pi)^{-k}|\Sigma^{-1}|^{(k-3)/2}|A|^{1/2} \exp\{-1/2 \text{ tr } (B - \bar{B})'A(B - \bar{B})\Sigma^{-1}\}d\Sigma^{-1}dB.$$

Combining common terms and completing the square in the exponent,

$$= \int (2\pi)^{-(T+k)}|\Sigma^{-1}|^{(T+k-3)/2} |A|^{1/2}$$
$$\cdot \exp\{-1/2 \text{ tr}[S_1 + (B - \tilde{B})(X'X + A)(\cdot)]\Sigma^{-1}\}d\Sigma^{-1}dB$$

where

$$S_1 = (Y - X\tilde{B})'(Y - X\tilde{B}) + (\bar{B} - \tilde{B})'A(\bar{B} - \tilde{B})$$

and $\tilde{B} = (X'X + A)^{-1}(X'X\hat{B} + A\bar{B})$, $\hat{B} = (X'X)^{-1}X'Y$.

Properties of the Wishart distribution are needed to integrate out Σ^{-1}.

$$\int \ell_1(B,\Sigma^{-1}|Y,X)p_1(B,\Sigma^{-1})d\Sigma^{-1}dB$$

$$= c_0 \int |S_1 + (B - \tilde{B})'(X'X + A)(B - \tilde{B})|^{-(T+k)/2}dB$$

Using properties of the matrix-t distribution to integrate out over B,

(18) Numerator $= C_0^{'} |S_1|^{-T/2} \, |X^{'}X + A|^{-m/2}$

where $C_0^{'}$ is the appropriate constant of integration.

$$C_0 = \pi^{(m(m-1)/4 \, - \, mT/2)} \prod_{i=1}^{m} \Gamma(T+i+1/2) \, |A|^{1/2}$$

Calculation of the Denominator of Posterior Odds Ratio

The denominator of the posterior odds ratio can be written as

$$\int \ell_2(\underline{\theta},\Sigma^{-1}|\underline{y},Z)p(\underline{\theta})p(\Sigma^{-1})d\Sigma^{-1}d\underline{\theta}$$

$$= \int (2\pi)^{-T}|\Sigma^{-1}|^{T/2} \exp\{-1/2 \; (\underline{y} - Z\underline{\theta})^{'}(\Sigma^{-1} \otimes I_T)(\underline{y} - Z\underline{\theta})\}$$

$$\cdot \; (2\pi)^{-\ell/2}|\Sigma^{-1}|^{-3/2}|C|^{1/2} \exp\{-1/2 \; (\underline{\theta} - \bar{\underline{\theta}})^{'}C(\underline{\theta} - \bar{\underline{\theta}})\}d\Sigma^{-1}d\underline{\theta}$$

Completing the square and collecting terms,

$$= \int K_0 \exp\{-1/2 \; (s_1 + (\underline{\theta} - \tilde{\underline{\theta}})^{'}(Z^{'}(\Sigma^{-1}\otimes I)Z + C)(\underline{\theta} - \tilde{\underline{\theta}})\}d\Sigma^{-1}d\underline{\theta}$$

$$\tilde{\underline{\theta}} = \left(Z^{'}(\Sigma^{-1} \otimes I)Z + C\right)^{-1}\left(Z^{'}(\Sigma^{-1} \otimes I)Z\hat{\underline{\theta}} + C\bar{\underline{\theta}}\right),$$

$$\hat{\underline{\theta}} = (Z^{'}Z)^{-1}Z^{'}\underline{y}$$

$$s_1 = (\underline{y} - Z\tilde{\underline{\theta}})^{'}(\Sigma^{-1} \otimes I)(\underline{y} - Z\tilde{\underline{\theta}}) + (\bar{\underline{\theta}} - \tilde{\underline{\theta}})^{'}C(\bar{\underline{\theta}} - \tilde{\underline{\theta}})$$

The integral with respect to Σ^{-1} above does not have an analytical solution. The integral can be approximated by conditioning on $\Sigma^{-1} = \hat{\Sigma}^{-1}$. Another alternative, which will be explored below, would be to integrate first over θ and then compute the integral over the three unique elements of Σ using numerical integration techniques.

$$\text{Denominator} \doteq \int K_0 \exp\{-1/2[\hat{s}_1 + (\underline{\theta} - \tilde{\tilde{\underline{\theta}}})'(Z'(\hat{\Sigma}^{-1} \otimes I)Z + C)(\cdot)]\}d\underline{\theta}$$

$$\hat{s}_1 = (\underline{y} - Z\tilde{\tilde{\underline{\theta}}})'(\hat{\Sigma}^{-1} \otimes I)(\underline{y} - Z\tilde{\tilde{\underline{\theta}}})' + (\bar{\underline{\theta}} - \tilde{\tilde{\underline{\theta}}})'C(\bar{\underline{\theta}} - \tilde{\tilde{\underline{\theta}}})$$

$$\tilde{\tilde{\underline{\theta}}} = \left(Z'(\hat{\Sigma}^{-1} \otimes I)Z + C\right)^{-1}\left(Z'(\hat{\Sigma}^{-1} \otimes I)Z\hat{\underline{\theta}} + C\bar{\underline{\theta}}\right)$$

$$(19) \qquad \text{Denominator} \doteq K_0'\left|Z'(\hat{\Sigma}^{-1} \otimes I)Z + C\right|^{1/2}$$

where $K_0 = (2\pi)^{-T}\left|\hat{\Sigma}^{-1}\right| \; |C|^{1/2}\exp\{-1/2\ \hat{s}_1\}$.

Posterior Odds Ratio

By combining the formula for numerator (18) and the approximate result for the denominator in (19), odds ratios to compare the Translog and FFF for aggregate U.S. manufacturing can be computed. These approximate expressions for the numerator and denominator contain two important terms-- the determinant of the prior precision matrices and the determinant of the residual sum of squares matrices for each of the cost share specifications. The posterior odds ratio depends chiefly on three characteristics about each model: 1. the complexity of the model as measured by the number of parameters, 2. the goodness of fit as measured by the generalized variance of residuals, and 3. the amount of prior information available for each model.

In order to evaluate the adequacy of the approximate integral results for the denominator, numerical techniques are used to compute the three dimensional integral over Σ. The integrand involved cannot be approximated well by an elliptically symmetric surface. Standard Monte Carlo integration techniques use an "importance" function to sample points in the region of integration. Usually, the importance function is taken to be an elliptically symmetric density based on the asymptotic form of the integrand. For example, Zellner and Rossi (1984) use a multivariate student t importance function. To compute the integral over the elements of Σ non-standard Monte Carlo integration techniuques due to Friedman and Wright (1980) are employed. The Friedman-Wright procedure breaks up the region of integration into hyper-rectangles within which the variation in the integrand is minimized. Points are sampled uniformly within each of the regions and then averaged (see Zellner and Rossi (1984) for details). It should be emphasized that this problem has been made tractable by the use of analytical integration wherever possible. The dimension of the integral which must be computed numerically is very small relative to the dimension of the model parameter space.

Table 2 shows the odds ratios for H_1: Translog vs. H_2: FFF as calculated for aggregate U.S. manufacturing data. The odds ratio calculations in parentheses are the result of using the approximate results in the text. The approximate and exact results agree quite closely. The FFF system is considered with a "Translog-like" quadratic term, $(\underline{x}_t' C \underline{x}_t)$, in the cost function and without the quadratic form. The FFF share system was found to be favored by slightly better than 3:2 odds with quadratic terms and by roughly 1:1 odds without quadratic terms. The conclusion is that the periodic terms resulting from the Fourier series expansion are important in providing a good approximation to the underlying cost structure.

TABLE 2

POSTERIOR ODDS RATOS:

TRANSLOG VS. FOURIER FLEXIBLE FORM*

H_1: Translog

H_2: Fourier Flexible Form

H_1 vs.	H_2 With Quadratic Terms	H_2 Without Quadratic Terms
K_{12}	.69	.98
	(.61)	(1.05)

*Estimated using U.S. aggregate manufacturing cost data, 1947-1971.
Numbers in parentheses are odds ratios calculated using approximate integrals.

Summary

In this paper, a Bayesian approach to choosing between two non-nested multivariate regression systems was developed. The Bayesian approach involves the calculation of the posterior probabilities of alternative hypotheses and formation of a posterior odds ratio. Specific results are obtained for the comparison of a multivariate regression and seemingly unrelated regression model. These odds ratio results are applied to the choice between Translog and Fourier flexible form cost share equation systems. When calculated for aggregate U.S. manufacturing cost data, the odds ratios favor the Fourier flexible form over the Translog form. Even in small samples, the Fourier series expansion appears to be an effective approximation technique.

Backnotes

[1]The cost function will not be appended to the system of cost share equations. This would involve numerous cross-equation restrictions which would make the evaluation of the integrals difficult.

[2]This paper is concerned with the relative flexibility of the Translog and FFF. Symmetry is not imposed on either form.

[3]$J = 1$ is the analysis. J is the length of the Fourier expansion in direction $\underset{\sim}{k}_{\alpha}$.

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