# UNCERTAIN LIFETIMES, SOCIAL SECURITY, AND INDIVIDUAL SAVING

by

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Uncertain lifetimes create a demand for annuities to provide for consumption in old age. Because such markets may be imperfect or missing, compulsory participation in social security system may fulfill some of this demand. A simple simple life-cycle model is used to show that a fair, fully funded social security system can reduce individual saving by more than the tax paid. Moreover, if access to the social security annuities is rationed by income because of tax and replacement rates which are nonlinear in earnings, saving rates will rise with earnings, even in the absence of a bequest motive. While the partial equilibrium impact of social security on individual saving is larger than that found in studies in which lifetime is certain, long-run effects may be smaller, because of the impact of social security on intergenerational transfers ("accidental bequests").

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## I. INTRODUCTION

The social security system in the United States acts as a major link in the provision of income security for the elderly. This income security is effected through a tax and transfer system which remains unfunded. That is, taxes are collected from "working" cohorts to pay benefits to "retired" cohorts. In addition to being intergenerational, the transfer is also implicitly intertemporal, since social security taxes are used to purchase an asset which is not marketable during the purchase period, namely the entitlement to social security benefits in the future. The impact of this transfer on household saving has been the subject of much debate since the pioneering paper by Feldstein (1974). In a recent review of the economic impacts of the social security system, Aaron (1982) concludes that, given the wide variation in empirical results since Feldstein's first study, it is difficult to discern any effect of social security on individual saving.

Most empirical tests of the effects of social security on saving have been conducted in the perfect certainty context of the life-cycle model of consumption and saving (cf. Modigliani and Ando, 1957; and Modigliani and Brumberg, 1954). In that approach, social security affects wealth accumulation through its impact on individual (or household) intertemporal budget constraints. Disposable income falls by the amount of the tax. To the extent that the present value of benefits exceeds the present value of tax paid, an increase in lifetime resources is generated, raising consumption in all periods.

This paper focuses on the role of uncertainty over length of life in explaining the impact of social security on saving. The theoretical model shows that even an actuarially fair social security system (in which the tax price paid for social security is equal to the expected present value of benefits) can reduce household saving by more than the tax paid. Moreover, rationing of "social security annuities," so that low-income individuals receive greater coverage than high-income individuals, can generate individual (or household) saving rates which rise substantially with income (a finding of many recent studies of wealth accumulation). Depending on the particular way in which participation in social security is determined and depending on the extent to which an individual is constrained in capital markets, a wide range of "offsets" to saving may be traceable to social security. Focusing only on the wealth effect of the extent to which anticipated benefits exceed taxes paid is not sufficient to explain the influence of social security on household saving.

As an asset, the entitlement to social security benefits is a contingent claim, paying insurance benefits for retirement consumption (via earnings replacement rates) in the event that an individual survives to retirement. In an actuarially fair social security system, the "returns" on the social security asset are positively correlated with the marginal utility of consumption in retirement. Hence the introduction of an actuarially fair social security system raises an individual's lifetime resources, increasing consumption in all periods. Saving is reduced by more than the tax.[1]

The intuition here dates back to the seminal paper of Yaari (1965). With uncertainty over longevity, intertemporal utility maximization will dictate saving for the possibility of living longer than the expected lifetime to avoid deprivation (high marginal utility of future consumption). In the absence of fair annuities markets, a nonaltruistic individual will always leave "accidental" bequests. In their analysis of the family as an incomplete annuities market, Kotlikoff and Spivak (1981, p.379) found that for plausible underlying parameter values, the present expected value of unintended bequests represented almost 25 percent of initial wealth for a single male aged 55.

This role of annuities as a mechanism for sharing uncertainty about longevity is an integral part of Diamond's (1977) evaluation of the social security system, in which he focuses on the absence of complete markets for such contracts. Merton (1981) considers Pareto-improving social security programs in an intertemporal model in which human capital is not tradeable. Eckstein, Eichenbaum, and Peled (1983) consider the Pareto-improving potential of mandatory social security in the context of market failure in competitive insurance markets in the present of adverse selection in the paradigm of Rothschild and Stiglitz (1976) or Wilson (1977).

Sheshinski and Weiss (1981) directly considered the annuity aspects of the social security system in the framework of an overlapping generations model. In their model, the ultimate impact of social security on savings depends on the availability of a private market for (indexed) annuities. They found that, at the optimum, Yaari's (1965) result holds, namely that private savings are reserved for bequests, while social security benefits are used to finance retirement consumption. Under certain conditions, their results contradicted both the findings of Barro (1978) and of Feldstein (1974).

Tests of the impact of social security on saving in a life-cycle model are implicitly joint tests of the hypothesized channels of influence and the life-cycle model itself. Empirical tests of the life-cycle model under certainty have tested the hypothesis of a "hump-shaped" pattern for the wealth-age profile. The conclusions reached by no means constitute an unambiguous validation of the model. Mirer (1979), using a cross section of data for the United States of people over sixty-five, found no significant tendency for wealth to decline in advancing age. Other studies, such as those of Kurz (1981) and White (1978) found similarly ambiguous results. Brittain's (1978) study based on estate tax data found wealth to be an increasing function of age. Even after controlling for the effects of permanent income, Blinder, Gordon, and Wise (1981), Diamond and Hausman (1982), and Hubbard (1983) found results which only mildly supported the basic theory.

Davies (1981) used a life-cycle model under uncertain lifetime to address the phenomenon of slow dissaving in retirement. He examined the difference between consumption by an individual (with given age, earnings, and wealth) under certainty and uncertainty over longevity, finding a depressing effect of uncertainty on consumption in retirement. The presence of pensions in his simulation model (using Canadian data) reduced this effect, but by no means eliminated it. In their test of the life-cycle model (also using Canadian data), King and Dicks-Mireaux (1982) found that wealth did decline in retirement, but at a much slower rate than would be predicted by the basic theory. They suggested the possibility of significant roles for bequest motives or uncertainty over date of death.[2]

Much of the controversy surrounding social security and saving has resulted from time-series studies for the U.S. first done by Feldstein (1974). Based on an analysis of the aggregate consumption function, Feldstein estimated an equation of consumption as a function of real disposable income, the rate of unemployment, non-pension wealth, corporate retained earnings, pension saving, and social security wealth.[3] He found that the wealth effect (from the excess of the present value of benefits over the present value of taxes) dominated the tax effect (the reduction is disposable income as a result of the collection of social security payroll taxes). Using a point estimate for 1971, Feldstein claimed that the implied effect of social security is to reduce personal saving by 50% and to reduce the capital stock in the ong run by 38%.[4]

Microeconomic (cross-section) evidence has generally been favorable to the proposition that social security has reduced individual saving. Feldstein and Pellechio (1979) found (using the 1962 Survey of Financial Characteristics of Consumers) a significant substitution effect of social security wealth for non-pension wealth. A more recent study by Blinder, Gordon, and Wise (1981) found a smaller substitution of social security wealth for private wealth, a displacement effect of 39 cents. Kotlikoff (1979b) found that the accumulated value of employer and employee payroll tax contributions exerted a strong negative effect on individual saving. His "net wealth increment" variable (the amount by which the present value of social security benefits exceeded the present value of a worker's payroll tax contributions) did not have a significant impact on pre-retirement household wealth accumulation. The evidence of Diamond and Hausman (1982), based on data from the National Longitudinal Survey of Mature Men, implied offsets from a one-dollar increase in social security wealth fungible wealth in the range of 30 cents to 50 cents.

This paper provides a new framework for thinking about the influence of social security on life-cycle saving, yielding new interpretations of estimated "offsets" to saving of social security. For example, that individuals do not (in the presence of uncertain lifetimes) save significant portions of their wealth through annuities could indicate: (i) that annuities provided by the market are not fairly priced, (ii) a strong bequest motive in wealth accumulation, or (iii) that the annuity components of social security (and some private pensions) are sufficient to meet the demand for annuities. With respect to the first point, adverse selection and transactions costs (in negotiating individual annuity contracts) can inhibit the development of a market in fair annuities. The model of the next section integrates the annuity features of social security within the life-cycle model and examines the impact of an actuarially fair social security system on saving. Analyzing the extent to which public and private pension annuities are sufficient to address the risks of uncertain longevity can help to distinguish motivations for saving (e.g., for retirement or for bequests, etc.).[5]

Section II of the paper examines the impact of social security on saving in the context of the life cycle model of consumption under both certainty and uncertainty over longevity. The results of the theoretical model under uncertain lifetimes demonstrate that the commencement of even a funded, actuarially fair social security system can substantially reduce individual saving. Hence, previous partial equilibrium estimates of the impact of social security on saving drawn solely from consideration of the intergenerational wealth transfer at the introduction of the system are, if anything, too small. Moreover, to the extent that participation in social security annuities is rationed by income, observed individual saving rates may rise with the level of earnings, rendering accidental bequests an increasing function of lifetime earnings (even in the absence of an explicit bequest motive).

Section III analyzes dynamic wealth accumulation of a life-cycle consumer under uncertain lifetimes given social security, and constructs individual wealth-age profiles. While fair social security annuities permit greater dissaving in old age, the wealth-age profile will in general differ from that individually selected participation in actuarially fair annuites. The partial equilibrium effect noted in Section II is mitigated by two factors. First, accidental bequests, which arise because of lifetime uncertainty, provide an intergenerational link for saving and consumption decisions. To the extent that the introduction of social security reduces the size of accidental bequests, the net effect of social security on the consumption of succeeding generations is reduced. Hence, intergenerational transfers reduce the ultimate impact of social security saving, but in quite a different manner than that suggested by Barro (1974). Second, general equilibrium considerations, primarily the endogeneity of factor returns, reverse part of the partial equilibrium impact.

Section IV summarizes the principal findings of the paper and discusses directions for future research.

## II. SOCIAL SECURITY AND SAVINGS IN A LIFE-CYCLE MODEL

### A. Certainty Case

We begin with a life-cycle model of consumption similar to those used by Kotlikoff (1979b) and by Summers (1981), in which there is no uncertainty over income, length of life or social security benefits.[6] Capital markets are perfect. Agents are assumed to be selfish, in the sense that no bequests are desired. They work until age Q, then live until age D in retirement. The age of retirement is taken as exogenous. Individuals receive wage $w_t$ at each point t in their careers. The basic problem is to choose a consumption-saving path so as to maximize an intertemporal utility function subject to a budget constraint. That is,

$$(1) \qquad \max \sum_{t=o}^{D} U(C_t)(1 + \delta)^{-t}$$

subject to the condition that

$$\sum_{t=o}^{D} C_t(1 + r)^{-t} = \sum_{t=o}^{Q} w_t(1 + r)^{-t},$$

where C, $\delta$, and r represent consumption and the (constant) subjective discount rate and real interest rate, respectively.

Employing the assumption of a constant wage growth of g and using an isoelastic form of the utility function (where $U(C_t) = \frac{1}{\gamma} C_t^{\gamma}$), we can rewrite the maximization problem in (1) as

$$(2) \qquad \max \sum_{t=o}^{D} \frac{1}{\gamma} C_t^{\gamma}(1 + \delta)^{-t}$$

subject to

$$\sum_{t=o}^{D} C_t(1 + r)^{-t} = \sum_{t=0}^{Q} w_o(1 + g)^t(1 + r)^{-t}.$$

Equality of marginal utilities of consumption across time requires that

$$(3) \qquad C_t = C_o\left(\frac{1 + r}{1 + \delta}\right)^{t/(1-\gamma)} ,$$

so that initial consumption is determined according to

$$(4) \qquad C_o = \frac{w_o \sum_{t=o}^{Q} \left(\frac{1 + g}{1 + r}\right)^t}{\sum_{i=o}^{D} (1 + r)^{i\gamma/(1-\gamma)}(1 + \delta)^{-i/(1-\gamma)}} \; .$$ [7]

In that solution, the age-consumption profile is sensitive to the interest rate, where the sensitivity depends on the coefficient of relative risk aversion (a transformation of $\gamma$, the elasticity of the marginal utility function).

The social security system is easily analyzed in this framework. Consider a system which levies a payroll tax at rate $t_S$ on wages during a working life and gives benefits $S_t$ during retirement. Let $S_t$ be constant and be determined by an effective replacement rate R of the worker's final wage. Then the lifetime budget constraint becomes

$$(5) \qquad \sum_{t=o}^{D} C_t(1 + r)^{-t} = (1 - t_s)w_o \sum_{t=o}^{Q} \left(\frac{1 + g}{1 + r}\right)^t + \sum_{t=Q+1}^{D} S_t(1 + r)^{-t}$$

$$= w_o \sum_{t=o}^{Q} \left(\frac{1 + g}{1 + r}\right)^t + [Rw_Q \sum_{t=Q+1}^{D} (1 + r)^{-t} - t_s w_o \sum_{t=o}^{Q} \left(\frac{1 + g}{1 + r}\right)^t].$$

When longevity is certain, the extent to which the social security system affects individual saving (i.e., whether it leads to an increase in perceived lifetime resources and, hence, consumption in each period) depends on the relationship between benefits and taxes paid. A fair social security system in this context requires a replacement rate and a payroll tax rate such that

$$(6) \qquad \frac{R}{t_s} = \frac{w_o \sum_{t=o}^{D} \left(\frac{1+g}{1+r}\right)^t}{w_Q \sum_{t=Q+1}^{D} (1+r)^{-t}} .$$

In the basic life-cycle model, under a funded system, private saving should be offset dollar for dollar for taxes paid. If the system is unfunded, private saving is also offset when the present value of expected benefits exceeds the present value of taxes paid. That is, savings should be reduced by

$$Rw_Q \sum_{t=Q+1}^{D} (1+r)^{-t} - t_s w_o \sum_{t=o}^{D} \left(\frac{1+g}{1+r}\right)^t .$$

The effect of the imbalance on accumulated wealth depends on the individual's age.

We can consider a social security system which taxes earnings only up to a certain level or which provides an earnings replacement rate in retirement which is scaled against an earnings ceiling. Let the original maximum taxable wage be $\overline{w}$, and assume that the tax base grows each period at rate g, the growth rate of real wages. Then, if the social security benefit to be paid in retirement is expressed as the product of an effective replacement rate and the terminal wage, the fair benefit can be written as

$$(7) \qquad S = Rw_Q = \frac{t_s \sum_{t=o}^{Q} \overline{w} \left(\frac{1+g}{1+r}\right)^t}{\sum_{t=Q+1}^{D} (1+r)^{-t}} \, .$$ [8]

B. Uncertain Lifetime Case

Relaxing the assumption of a certain lifetime makes the problem more complex. The precise direction of the influence of this uncertainty for saving is unclear. Heightened uncertainty over the length of life may lead to more saving (as a precaution against the possibility of a longer than expected life) or to less (because future income is more uncertain and hence discounted more heavily). In the argument of Yaari (1965), two individuals with identical tastes, income, and investment opportunities are compared. The difference between them is that one lives T periods for certain while the other faces an uncertain lifetime of t periods, up to a maximum of T periods. Given a shorter expected life, uncertainty over length of life unambiguously leads to increased initial consumption in Yaari's model. Champernowne (1969) and Levhari and Mirman (1977), on the other hand, consider two agents with identical expected lives, but differing in the distribution of length of life. In either case, the impact of uncertainty over the length of life on wealth accumulation of a risk-averse individual is ambiguous and depends on the parameters of the model.

To extend the simple life-cycle model of the previous section to incorporate uncertainty over the length of life, we assume the following life pattern. Individuals live Q periods for certain. The probability of having died in the interval (0, t] is $p_t$ for each t. Individuals have an expected lifetime of D years, with $D' > D$ being the maximum age to which one can survive. That is, D is just the weighted average of the years t in (0, D'], with the weights being $(1 - p_t)$ for each t.

Following Yaari (1965) and Barro and Friedman (1977), utility is assumed to be additively separable, and the utility index $U(C_t)$ is evaluated contingent on being alive at time t; the utility index is zero at t if the individual is not alive. The optimization problem becomes

$$(8) \qquad \max \sum_{t=o}^{D'} (1 - p_t)\, U(C_t)\, (1 + \delta)^{-t}$$

subject to

$$\sum_{t=o}^{D'} C_t(1 + r)^{-t} = \sum_{t=o}^{Q} w_o\left(\frac{1 + g}{1 + r}\right)^t .$$ 9

Carrying out the optimization in (8) assuming $U(C) = \frac{1}{\gamma} C^{\gamma}$ yields an optimal consumption stream of

$$(9) \qquad C_t = C_o\left(\frac{1 + r}{1 + \delta}\right)^{t/(1-\gamma)} (1 - p_t)^{1/(1-\gamma)} \quad ,$$

where

$$(10) \qquad C_o = \frac{w_o \sum_{t=o}^{Q} \left(\frac{1 + g}{1 + r}\right)^t}{\sum_{i=o}^{D'} (1 + r)^{i\gamma/(1-\gamma)} (1 + \delta)^{-i/(1-\gamma)} (1 - p_i)^{1/(1-\gamma)}} .$$

The extent to which uncertainty over length of life affects the stream of consumption depends on agents' degree of relative risk aversion. The higher is an individual's degree of relative risk aversion (or, equivalently, the lower is his intertemporal elasticity of substitution in consumption), the slower will his consumption grow over time. Intuitively, a social security system may provide insurance in the form of a guaranteed income in retirement in exchange for taxes paid during the working period.

Access to a perfect annuities market could remove the influence of lifetime uncertainty on consumption. Individuals could exchange a portion of their labor income now to smooth consumption in the future. The fraction of his earnings which an individual would be willing to use in order to purchase an annuity can be found by comparing the values of the indirect utility function in the absence of and presence of annuities. In the case of an uncertain lifetime with no annuities,

$$(11)\qquad V_{Ut} = \frac{1}{\gamma}\left[w_0 \sum_{i=0}^{Q} \left(\frac{1+g}{1+r}\right)^i \left(\frac{1+r}{1+\delta}\right)^{i/(1-\gamma)}\right]^{\gamma} \left[\sum_{i=0}^{D'} (1+r)^{i\gamma/(1-\gamma)}(1+\delta)^{-i/(1-\gamma)}(1-p_i)^{1/(1-\gamma)}\right]^{1-\gamma}.$$

It is straightforward to show that with a competitive market in actuarially fair annuites.

$$(12)\qquad V_{At} = \frac{1}{\gamma}\left[w_0 \sum_{i=0}^{Q} \left(\frac{1+g}{1+r}\right)^i \left(\frac{1+r}{1+\delta}\right)^{i/(1-\gamma)}\right]^{\gamma} \left[\sum_{i=0}^{D'} (1+r)^{i\gamma/(1-\gamma)}(1+\gamma)^{-i/(1-\gamma)}(1-p_i)\right]^{1-\gamma}.$$ [10]

The fraction of lifetime earnings _a_ which an individual would pay to participate just solves

$$(13) \qquad V_{Ut}((1 + a)\ (w_o)) = V_{At}(w_o).$$

That rate will depend on the individual's risk aversion and probabilites of survival in different periods, as well as on the rates of interest and time preference.

If all individuals were identical in terms of their probabilities of survival,[11] then (with risk-neutral insurers) a competitive equilibrium in the provision of fair annuities would be possible. The existence of a competitive equilibrium may be precluded by asymmetries of information between individuals and insurers. This is, of course the familiar "adverse selection" phenomenon discussed by Rothschild and Stiglitz (1976).[12] There may be additional "moral hazard" or "free-rider" barriers to the existence of an annuities market. If individuals conjecture that the state will support them in deprivation, the need to purchase annuities is diminished. A rigorous development of optimal second-best provision of annuities is beyond the scope of this paper.

Public provision of the annuities through social security is one possibility for correcting the market failure. Moral hazard problems still make voluntary participation difficult. Consider, though, a compulsory public pension system ("social security") of the following form. Individuals are compelled to pay a payroll tax at rate $t_s$ on gross wages. During retirement they receive annuity benefits $S_t$ in each period t until death. The budget constraint in (8) becomes

$$(14) \qquad \sum_{t=o}^{D'} C_t(1+r)^{-t} = (1 - t_s) \sum_{t=o}^{Q} w_o\left(\frac{1 + g}{1 + r}\right)^t + \sum_{t=Q+1}^{D'} S_t(1+r)^{-t}.$$

If benefits are set according to a replacement rate of the terminal wage, then the economy-wide actuarially fair benefit $\hat{S}$ satisfies the condition that

$$(15) \qquad \hat{S} \sum_{t=Q+1}^{D'} (1 + p_t)(1 + r)^{-t} = t_s \sum_{t=o}^{Q} w_o \left(\frac{1+g}{1+r}\right)^t .$$

Substituting the actuarially fair social security benefit into the budget constraint in (14) yields

$$(16) \quad \sum_{t=o}^{D'} C_t(1 + r)^{-t} = (1-t_s) \sum_{t=o}^{Q} w_o\left(\frac{1+g}{1+r}\right)^t + t_s\left(\frac{\sum_{t=o}^{Q} w_o \left(\frac{1+g}{1+r}\right)^t}{\sum_{t=Q+1}^{D'} (1 - p_t)(1 + r)^{-t}}\right) \sum_{t=Q+1}^{D'} (1+r)$$

$$+ [1 + t_s(\omega - 1)] \sum_{t=o}^{Q} w_o\left(\frac{1+g}{1+r}\right)^t ,$$

where $\omega$ arises because of the difference in discount rates under certainty and uncertainty and is equal to

$$\left(\sum_{t=Q+1}^{D'} (1 + r)^{-t}\right) / \left(\sum_{t=Q+1}^{D'} (1 - p_t)(1 + r)^{-t}\right).$$

Since $\omega$ is greater than unity, the social security system generates an increase in lifetime resources. Note that this increase in resources occurs even in a system which is actuarially fair and fully funded.[13] To the extent that a cohort receives a greater return on social security than the actuarially fair return, the impact on saving is magnified. This analysis follows in spirit the examination of access to a private (fair) annuities market in Kotlikoff and Spivak (1981). In the case of the imposition of the social security annuities, initial consumption must rise because of an increase in lifetime resources. Precisely to the extent that it acts like Yaari's "actuarial notes," an actuarially fair social security system reduces the accumulation of wealth by life-cycle consumers.[14]

Table 1 shows the percentage increase in lifetime resources generated by an actuarially fair social security system under various assumptions about the real rate of interest and the social security payroll tax rate.[15] For example, when r=0.04 and $t_s$ = 0.14, a 32 percent increase in lifetime wealth is afforded by an actuarially fair social security system.[16]

**TABLE 1**

**PERCENTAGE INCREASE IN LIFETIME RESOURCES GENERATED BY ACTUARIALLY FAIR SOCIAL SECURITY**

| | | $t_s$ | | |
|---|---|---|---|---|
| | | 0.10 | 0.12 | 0.14 |
| r | 0.02 | 29 | 35 | 41 |
| | 0.04 | 21 | 26 | 32 |
| | 0.06 | 16 | 19 | 23 |

What is the impact of a system whose effective tax rates and replacement rates are nonlinear in labor income? Again, let $\bar{w}$ represent the ceiling on taxable income; the growth rate of the taxable wage base and the determination of the replacement rate are as before. The budget constraint in (14) then becomes

$$(17) \qquad \sum_{t=o}^{D'} C_t(1 + r)^{-t} = \sum_{t=o}^{Q} (1 + \tilde{t}_s(\omega - 1))w_o(\frac{1 + g}{1 + r})^t,$$

where $\tilde{t}_s$ is equal to $t_s(\frac{\overline{w}}{w_o})$. The impact of social security on an individual's saving rate depends on his income. As an annuity, social security administered in this way generates a smaller reduction in saving for high-income people than for low-income people.

An actuarially fair social security system generates both an income effect and a substitution effect on consumption and saving. The income effect arises because of the increase in lifetime resources made possible by access to a fair annuity and the reduction in "accidental" bequests; the substitution effect comes from the implied reduction in the price of retirement consumption. The former points to a reduction in the level of individual (or household) wealth accumulation and the latter to a shift in the age-consumption profile, allowing for greater wealth decumulation in old age. The balance between the income and substitution effects depends on the individual's risk aversion.

## III. SOCIAL SECURITY AND DYNAMIC WEALTH ACCUMULATION

### A. Individual Saving Behavior

We can use the derivation from the previous section of the impact of mandatory actuarially fair social security on saving to study individual patterns of dynamic wealth accumulation and wealth-age profiles. We can write for any time t the present value (at time 0) of an individual's accumulated stock of wealth. That amount $K_t$ would be the present value of the "accidental bequest" of an individual who died in period t. That is,

$$(18) \qquad K_t = \sum_{i=0}^{t} (1 + r)^{-i}((1 - t_s)w_i + S_i - C_i).$$

Wages and social security benefits are the sources of income to the individual. $w_t$ is zero in the interval [Q + 1, D'], and $S_t$ is zero in the interval [0,Q]. Using the expressions derived before for $w_t$, $S_t$, and $C_t$, we can rewrite (18) as

$$(19a)\qquad K_t = (1 - t_s)w_o \sum_{i=0}^{t} (\frac{1+g}{1+r})^i - [1 + t_s(\omega-1)](w_o \sum_{i=0}^{Q} (\frac{1+g}{1+r})^i)$$

$$(\frac{\sum_{i=0}^{t} (1+r)^{\frac{i\gamma}{1-\gamma}} (1 + \delta)^{\frac{-i}{1-\gamma}} (1-p_i)^{\frac{1}{1-\gamma}}}{\sum_{i=0}^{D'} (1+r)^{\frac{i\gamma}{1-\gamma}} (1 + \delta)^{\frac{-i}{1-\gamma}} (1 - p_i)^{\frac{1}{1-\gamma}}}),\ t \ \varepsilon\ [0,Q], \text{ and}$$

$$(19b)\qquad K_t = (1 - t_s)w_o \sum_{i=0}^{Q} (\frac{1+g}{1+r})^i + t_s(w_o \sum_{i=0}^{Q} (\frac{1+g}{1+r})^i) \frac{\sum_{i=Q+1}^{t} (1+r)^{-i}}{\sum_{i=Q+1}^{D'} (1 - p_i)(1 + r)^{-i}}$$

$$- [1 + t_s(\omega-1)](\sum_{i=0}^{Q} w_o (\frac{1+g}{1+r})^i) \frac{\sum_{i=0}^{t} (1 + r)^{\frac{i\gamma}{1-\gamma}} (1 + \delta)^{\frac{-i}{1-\gamma}} (1 - p_i)^{\frac{1}{1-\gamma}}}{\sum_{i=0}^{D'} (1+r)^{\frac{i\gamma}{1-\gamma}} (1 + \delta)^{\frac{-i}{1-\gamma}} (1 - p_i)^{\frac{1}{1-\gamma}}},\ t \ \varepsilon\ [Q+1,D$$

To provide an intuitive framework for considering an individual's wealth accumulation over the life cycle, note that if we denote the present values of lifetime labor income and social security taxes by $V_L$ and $V_S$, respectively, we can rewrite (19a) and (19b) as:

(20a) $$\frac{K_t}{V_L} = \frac{(1 - t_s)w_o \sum_{i=0}^{t} (\frac{1+g}{1+r})^i}{V_L}$$

$$- (1 + \frac{V_S}{V_L} (\omega-1)) (\frac{\sum_{i=0}^{t} (1+r)^{\frac{i\gamma}{1-\gamma}}(1 + \delta)^{\frac{-i}{1-\gamma}} (1-p_i)^{\frac{1}{1-\gamma}}}{\sum_{i=0}^{D'} (1+r)^{\frac{i\gamma}{1-\gamma}}(1 + \delta)^{\frac{-i}{1-\gamma}}(1 - p_i)^{\frac{1}{1-\gamma}}}), \text{ and}$$

(20b) $$\frac{K_t}{V_L} = 1 \frac{V_S}{V_L} + \frac{V_S}{V_L} \frac{\sum_{i=Q+1}^{t} (1 + r)^{-i}}{\sum_{i=Q+1}^{D'} (1 - p_i)(1 + r)^{-i}}$$

$$- (1 + \frac{V_S}{V_L} (\omega - 1)) (\frac{\sum_{i=0}^{t} (1 + r)^{\frac{i\gamma}{1-\gamma}}(1 + \delta)^{\frac{-i}{1-\gamma}}(1 - p_i)^{\frac{1}{1-\gamma}}}{\sum_{i=0}^{D'} (1 + r)^{\frac{i\gamma}{1-\gamma}} (1 + \delta)^{\frac{-i}{1-\gamma}}(1 - p_i)^{\frac{1}{1-\gamma}}}) .$$

The ratio $K_t/V_L$ tracks an individual's accumulated stock of assets relative to lifetime earnings. In a world of no uncertainty over longevity, $K_t/V_L$ is simply a function of age, as long as the social security system is fair. That is, the impact of social security annuties is to displace non-pension wealth dollar for dollar. With lifetime uncertainty, wealth is still built up relative to earnings during the working period, but the rate at which consumption draws down accumulated wealth depends on survival probabilities and relative risk aversion. Because an actuarially fair social security system generates an increase in individual lifetime resources, lifetime consumption rises. Much of this increase in consumption comes during an individual's working life, as the need to save for retirement is reduced. Depending on risk aversion, while retirement consumption is higher in the presence of social security, dissaving in retirement is likely to be less than in the certainty case.[17]

Figure 1 below illustrates the dynamics of wealth accumulation for an individual in the absence of and in the presence of an actuarially fair social security system with an exogenously set payroll tax rate. The associated wealth-age profiles are given in Figure 2.

**FIGURE 1**

**INDIVIDUAL WEALTH ACCUMULATION OVER THE LIFE-CYCLE**

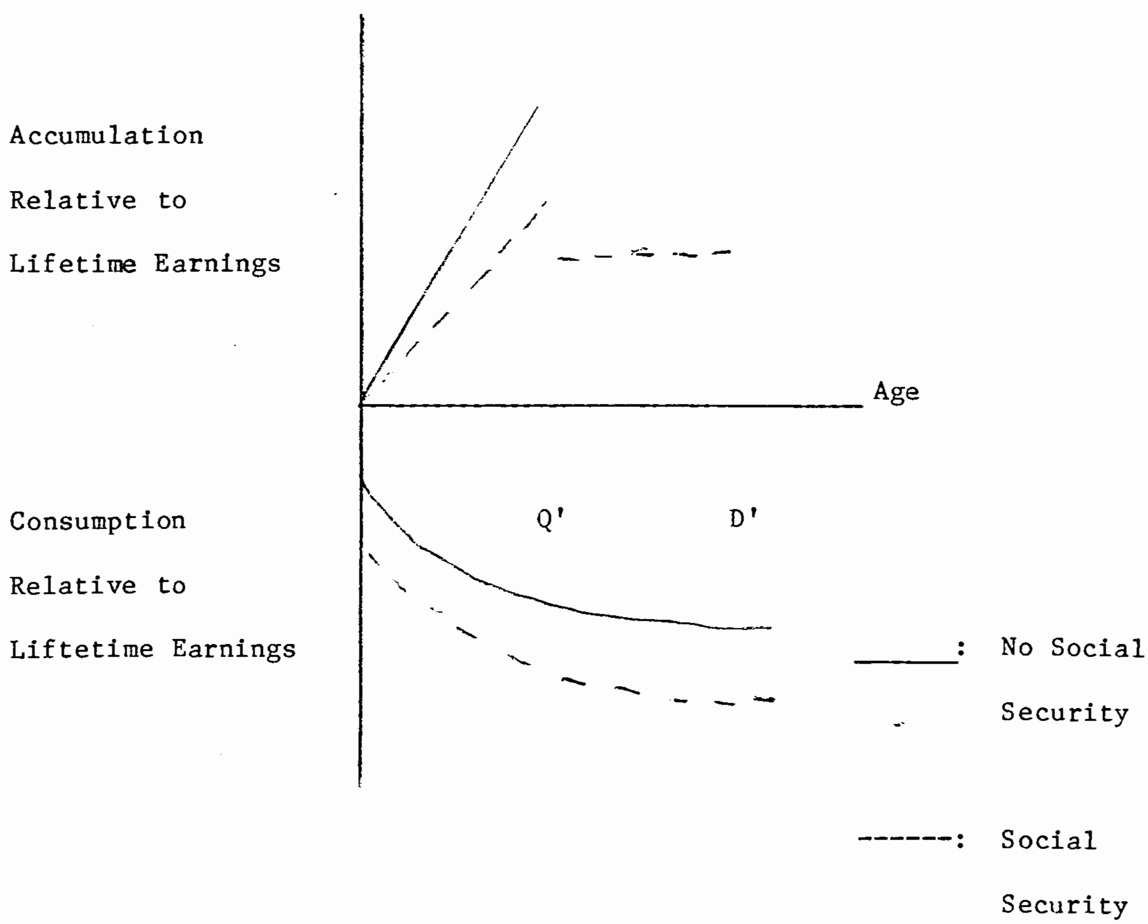
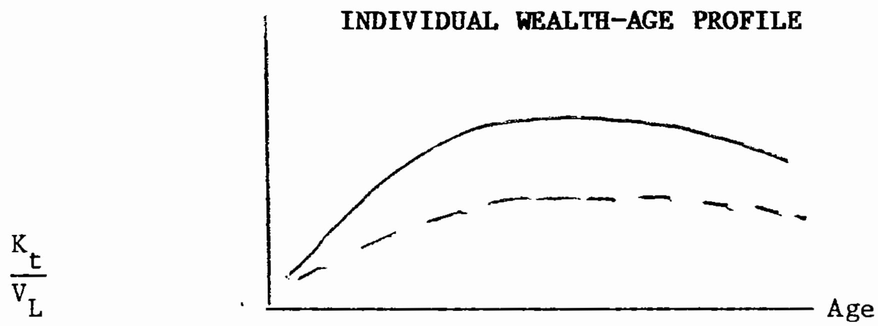


**FIGURE 2**




The problem becomes more complicated when the insurance coverage provided by social security is not the same across individuals. Suppose (as discussed in Section II) that there is a ceiling on the level of earnings against which payroll tax rates and replacement rates are calculated. If that ceiling is $\bar{w}$ in period 0 and grows at the same rate as the wage base, then the effective tax rate is not $t_s$, but $\tilde{t}_s = t_s(\frac{\bar{w}}{w_o})$. In that situation, equation (20) reveals that the ratio of wealth to lifetime earnings rises with the level of lifetime earnings, though at a decresing rate.[18] This nonlinearity of saving rates with respect to lifetime earnings occurs in the absence of any explicit bequest motive. The implications of this effect for studies of the relationship between bequests and lifetime resources will be discussed later.

A related problem surfaces in the consideration of received bequests which augment lifetime resources. If we let $A_\theta$ represent the initial bequest, then we can rewrite equation (20) as

(21a) $$\frac{K_t}{V_L} = \frac{A_0}{V_L} + \frac{(1 - t_s)w_o \sum_{i=0}^{t} (\frac{1+g}{1+r})^i}{V_L}$$

$$- (1 + \frac{V_S}{V_L} (\omega-1)) (\frac{\sum_{i=0}^{t} (1+r)^{\frac{i\gamma}{1-\gamma}}(1 + \delta)^{\frac{-i}{1-\gamma}} (1-p_i)^{\frac{1}{1-\gamma}}}{\sum_{i=0}^{D'} (1+r)^{\frac{i\gamma}{1-\gamma}}(1 + \delta)^{\frac{-i}{1-\gamma}}(1 - p_i)^{\frac{1}{1-\gamma}}}), \text{ and}$$

(21b) $$\frac{K_t}{V_L} = (1 + \frac{A_0}{V_L}) - \frac{V_S}{V_L} + \frac{V_S}{V_L} \frac{\sum_{i=Q+1}^{t} (1 + r)^{-i}}{\sum_{i=Q+1}^{D'} (1 - p_i)(1 + r)^{-i}}$$

$$- (1 + \frac{V_S}{V_L} (\omega - 1)) (\frac{\sum_{i=0}^{t} (1 + r)^{\frac{i\gamma}{1-\gamma}}(1 + \delta)^{\frac{-i}{1-\gamma}}(1 - p_i)^{\frac{1}{1-\gamma}}}{\sum_{i=0}^{D'} (1 + r)^{\frac{i\gamma}{1-\gamma}} (1 + \delta)^{\frac{-i}{1-\gamma}}(1 - p_i)^{\frac{1}{1-\gamma}}}) .$$

As in the case of labor income, the rate at which lifetime resources are consumed depends on survival probabilites and risk aversion. The initial capital endowment $A_0$, which must come in this model from an accidental bequest from the previous generation, raises the individual's lifetime resources, increasing the consumption out of the present value of labor income and reducing the ratio of accumulated wealth to lifetime earnings. In the case in which participation in social security annuities is rationed by income, initial wealth endowments may smooth the nonlinearity in earnings of saving rates brought about by such a social security system.

To quantify the impact of social security and bequests on individual consumption and wealth-age profiles, the model embodied in equation (21) can be simulated for plausible parameter values. Simulations were performed over a set of different values of r, g, $\delta$, and $\gamma$. The following relationships among the parameters are assumed: $r > g$, $r > \delta$, and $\delta > 0$.[19] There is some evidence on the value of $\gamma$ in the literature. In their study of household portfolio allocation, Friend and Blume (1975) estimated the coefficient of relative risk aversion to be in excess of 2.0, implying a value of $\gamma$ of at most -1.0. Farber's (1978) estimation of preferences of United Mine Workers from collective bargaining agreements yielded estimates of the coefficient of relative risk aversion of 3.0 and 3.7. Here we use three alternative values of $\gamma$: 0.25, - 1.0, and -3.0. g is assumed to equal 0.02, while $r = 0.04$, and $\delta = 0.03$.[20]

Table 2 reports $K_t/V_L$ for selected ages. The optimization begins at age 20; individuals are assumed to retire at age 65. Figures are expressed as differences from the non-social security case. Column 1 reports values in the absence of social security, but with an initial bequest equal to 25 percent of lifetime earnings. Column 2 reports the reduction in $K_t/V_L$ when the individual participates in a social security system in which $\tilde{t}_s = t_s = 0.14$. The third column shows the reduction in $K_t/V_L$ for an individual whose effective tax rate (participation) in the system is only half of the nominal rate. Finally, the fourth column shows the change in the wealth-age profile for an individual with an initial bequest equivalent to 25 percent of his lifetime earnings and for whom $\tilde{t}_s = t_s = 0.14$.

# TABLE 2

## SOCIAL SECURITY AND $K_t/V_L$

$K_t/V_L$

(DIFFERENCE FROM THE NO SOCIAL SECURITY CASE)

| AGE | $t_S = 0,\ \frac{A_0}{V_L} = .25$ | $\tilde{t}_s = t_s = .14$ | $\tilde{t}_s = .07,\ t_s = .14$ | $\tilde{t}_s = t_s = .14,\ \frac{A_0}{V_L} = .25$ |
|---|---|---|---|---|
| | | $\gamma = 0.25$ | | |
| 40 | .150 | -.208 | -.104 | -.057 |
| 50 | .104 | -.296 | -.148 | -.192 |
| 65 | .041 | -.393 | -.196 | -.337 |
| 70 | .025 | -.336 | -.168 | -.311 |
| 75 | .014 | -.273 | -.137 | -.260 |
| 80 | .006 | -.222 | -.111 | -.215 |
| | | $\gamma = -1.00$ | | |
| 40 | .155 | -.201 | -.101 | -.046 |
| 50 | .114 | -.282 | -.141 | -.168 |
| 65 | .060 | -.386 | -.193 | -.326 |
| 70 | .044 | -.330 | -.175 | -.306 |
| 75 | .031 | -.321 | -.161 | -.290 |
| 80 | .020 | -.298 | -.150 | -.279 |
| | | $\gamma = -3.00$ | | |
| 40 | .160 | -.195 | -.097 | -.035 |
| 50 | .122 | -.272 | -.136 | -.150 |
| 65 | .072 | -.362 | -.181 | -.307 |
| 70 | .057 | -.333 | -.177 | -.276 |
| 75 | .044 | -.304 | -.152 | -.260 |
| 80 | .033 | -.282 | -.141 | -.250 |

Several interesting patterns emerge. As expected, higher values of relative risk aversion (lower values of $\gamma$) encompass higher wealth in all periods, particularly in old age. Given uncertainty over longevity with no social security, an initial bequest of 25 percent of lifetime earnings is almost completely consumed by age 75 when $\gamma = 0.25$. When $\gamma = -1.0$, however, about 13 percent remains; nearly 20 percent remains in the case in which $\gamma = -3.0$.

The next two columns, which address the implied resource gains made possible by access to actuarially fair social security, display the reduction in $K/V_L$ attributable to social security (when $t_s = 0.14$). When the effective tax rate is less than the nominal tax rate, the reduction in $K/V_L$ is smaller. Hence, rationing participation in social security by income, *ceteris paribus*, leads to saving rates which rise with earnings (and, *a fortiori*, stocks of wealth which rise with earnings). As $\gamma$ is decreased (higher relative risk aversion), the social security system permits greater wealth decumulation in old age. In other words, the more risk-averse the individual, the less of the "income effect" of social security participation consumed prior to retirement. Those findings are intuitive, since the value of annuity is highest for very risk-averse individuals.

We can now consider the issue of the consumption patterns of the elderly, addressed earlier by Mirer (1979) and by Davies (1981). Given uncertainty over length of life, the rapid reduction in consumption (relative to lifetime resources) in old age confirms the findings in Davies (1981) that positive net worth may continue indefinitely after retirement. The resulting slow decline (or possible increase) in net worth in retirement ignores, however, the decline in the value of the social security annuity. As long as individuals acknowledge the actuarial value of their social security holdings, that dissaving must take place.

For each year t in retirement, withdrawals to finance consumption relative to lifetime earnings can be expressed as

$$(22) \qquad \frac{C_t}{V_L} = \frac{(1 + t_s(\omega-1)) \, (\frac{1+r}{1+\delta})^{\frac{t}{1-\gamma}} (1-p_t)^{\frac{1}{1-\gamma}}}{\sum_{i=0}^{D'} (1+r)^{i\gamma/(1-\gamma)} (1 + \delta)^{-i/(1 - \gamma)} (1 - p_i)^{1/(1-\gamma)}} \, .$$

Correspondingly, in each year t, the decline in the annuity value of social security relative to lifetime earnings is

$$(23) \qquad \frac{\Delta V_{St}}{V_L} = \frac{S(1 - p_t)}{V_L} = \frac{t_s \, (1 - p_t)}{\sum_{i=Q+1}^{D'} (1 - p_i)(1 + r)^{-i}} \, .$$

The relationship between these two uses of total (pension plus nonpension wealth) depends on $\gamma$ and the distribution of survial probabilities. To see the importance of considering the "dissaving" of annuity wealth, Table 3 contrasts consumption and annuity revaluations in retirement of the case of $\gamma = -1$, $t_S = 0.14$, $r = 0.04$, and $\delta = 0.03$.[21]

TABLE 3

ANNUITY AND NONANNUITY DISSAVING IN RETIREMENT

| AGE | $C_t/V_L$ | $\Delta V_{St}/V_L$ | $\Delta V_{St}/C_t$ |
|---|---|---|---|
| 66 | .033 | .084 | 2.55 |
| 70 | .032 | .072 | 2.25 |
| 75 | .029 | .055 | 1.90 |
| 80 | .026 | .038 | 1.46 |
| 85 | .020 | .021 | 1.05 |

The last column of Table 2 shows the combined impact on the wealth-age profile of the combination of effective participation in social security at the nominal rate (14 percent here) and the receipt of an initial bequest. From the information in the first column of Table 2 and from a comparison of the second and fourth columns, most of the impact of initial bequests on consumption occurs prior to retirement. That is, the differences in $K/V_L$ in old age (with respect to the no-social security case) are almost invariant to the initial bequest (at least in the range examined here). This is an important finding. To correctly estimate the net effect of social security on individual consumption and wealth accumulation after the commencement of the system, we must also consider its impact on intergenerational transfers (here, accidental bequests). By affecting the accidental bequests of previous generations, social security further influences individual consumption patterns. It is to this issue which we now turn.

**B. Long-run Effects on Individual Saving Behavior**

An actuarially fair social security system in the context of the model presented here reduces individual saving by more than the amount of the taxes paid. The amount of that reduction depends on individual characteristics, such as relative risk aversion. For plausible underlying assumptions about individual discount rates, survival probabilities, and the intertemporal elasticity of substitution in consumption, the magnitude of that reduction is substantial. The partial equilibrium conclusion is clear — estimates of the reduction in individual saving brought about by social security which focus only the extent to which the system delivers a present value of anticipated benefits greater the present value of taxes paid are, if anything, an underestimate. Before discussing general equilibrium interpretations of this finding (in the sense that the wage rate and real interest rate are endogenous and respond to changes in the saving rate), it is important to address the issue raised in the simulation exercises of the links among generations provided by accidental bequests.

An initial bequest from an "early death" of a member of the previous generation raises beneficiaries' consumption relative to lifetime earnings. In the model, the size of that bequest depends on the testator's coverage by social security and his age at death. By facilitating greater consumption out of lifetime earnings, social security reduces the accidental bequest. On that account, the initial resources available to the heir (and, from Table 2, consumption when young) are lower. Even within the partial equilibrium analysis, the impact of social security on the consumption and saving patterns of individuals in a given generation depends on the balance between the effective increase in lifetime resources made possible by access to a fair annuity and the reduction in inheritances because of that impact on the saving of the previous generation.[22]

To see this more clearly, note that for an individual receiving an accidental bequest from a "parent" who died at age t in the interval [Q + 1, D'], the reduction in the bequest because of the parent's participation in social security is

$$(24)\qquad \frac{dA_0}{dV_S} = (1 + r)^t \{-1 + \frac{\sum_{i=Q+1}^{t} (1+r)^{-i}}{\sum_{i=Q+1}^{D'} (1 - p_i)(1 + r)^{-i}}$$

$$-(\omega-1)\frac{\sum_{i=0}^{t} (1 + r)^{\frac{i\gamma}{1-\gamma}}(1 + \delta)^{\frac{-i}{1-\gamma}}(1-p_i)^{\frac{1}{1-\gamma}}}{\sum_{i=0}^{D'} (1+r)^{\frac{i\gamma}{1-\gamma}} (1 + \delta)^{\frac{-i}{1-\gamma}}(1 - p_i)^{\frac{1}{1-\gamma}}}\}.^{23}$$

We know from the individual's optimization problem that social security generates an increase in lifetime resources of $V_S(\omega-1)$. If the "parent" and "child" have the same lifetime earning potential (i.e., the same $w_o$), then the next effect of social security is to increase lifetime resources by the amount E, where

$$(25)\qquad E = V_s(\omega-1) \{1 - (1+r)^t (\frac{\sum_{i=0}^{t} (1 +r)^{\frac{i\gamma}{1-\gamma}}(1 + \delta)^{\frac{-i}{1-\gamma}}(1 - p_i)^{\frac{1}{1-\gamma}}}{\sum_{i=0}^{D'} (1+r)^{\frac{i\gamma}{1-\gamma}} (1 + \delta)^{\frac{-i}{1-\gamma}}(1 - p_i)^{\frac{1}{1-\gamma}}})$$

$$+ (1+r)^t(\omega-1)^{-1} (\frac{\sum_{i=Q+1}^{D'} (1+r)^{-i}}{\sum_{i=Q+1}^{D'}(1 - p_i)(1 + r)^{-i}} - 1)\}.$$

Note that if the parent lived to the maximum age, then $E = 0$. In general, the net increment to lifetime resources E made possible by social security depends on the age at which the parent died (magnitude of the accidental bequest).[24] To consider the net effect of social security on saving n generations after its introduction, an n-generational analogue to equation (25) could be constructed given the ages of death of previous testators.

### C. General Equilibrium Effects of Social Security on the Capital Stock

The partial equilibrium effects of social security on individual saving will be dampened in a general equilibrium analysis of the impact of social security on aggregate capital formation.[25] The reduction in individual wealth accumulation brought about by social security will induce changes in factor returns, exhibiting both income and substitution effects on consumption. The higher real interest rate decreases lifetime resources; in addition, a higher rate of interest reduces the price of consumption in old age.

Kotlikoff (1979a), using a life-cycle model with no uncertainty over longevity and a Cobb-Douglas production technology, considered the impact of a pay-as-you-go social security system on the capital stock in a general equilibrium. For plausible parameter values, he found that the positive lifetime wealth increment traceable to social security (because of growth of the wage base) caused a twenty-percent steady-state reduction in the capital stock in the general equilibrium.[26] While this is certainly substantial, it is roughly half of his partial equilibrium effect, which is directly related to the extent to which benefits are unfair (i.e., to the extent that the present value of benefits exceeds the present value of social security taxes paid).

While general equilibrium calculations of the impact of social security on aggregate saving are not performed here, it is useful to differentiate the effects implied by the life-cycle model under uncertain lifetimes from those implied by the perfect certainty life-cycle model. First the partial equilibrium effect in the uncertain lifetime framework must be larger; the commencement of even an actuarially fair, fully funded social security system will substantially reduce individual saving. When the effect of social security on accidental bequests (which arose from the life-cycle model under uncertain lifetimes) is considered, however, the partial equilibrium impact is reduced over time. Smaller changes are induced in factor returns, so that the divergence between the general equilibrium impact and the partial equilibrium impact (including the intergenerational component) is reduced. Thus, while evaluating the effect of social security on wealth accumulation under uncertain lifetimes leads to partial equilibrium impacts well in excess of those found earlier in the literature, the general equilibrium effects may well be smaller than those earlier findings suggest.

## IV. CONCLUSIONS AND EXTENSIONS

Assessing the impact of the social security system on individual wealth accumulation is important for many analyses of welfare, capital formation, and equity in the distributions of income and wealth. The life-cycle model of consumption offers an intuitive starting point for examination. Most treatments of the life-cycle model in this context have assumed perfect certainty over length of life, so that the only influences of social security on saving come through the collection of the payroll tax used to finance it and through any change in lifetime resources accruing to an individual to the extent the system is not actuarially fair.

The first part of the paper focuses on the consumption-saving choice in a life-cycle model under an uncertain lifetime. In such a world, even an actuarially fair social security system can reduce individual saving, though individual welfare is improved. Hence, partial equilibrium estimates of the impact of social security on saving which rely solely on the extent to which certain individuals earn a more than fair return on social security are underestimates of the true effect. The intuition here traces back to that in Yaari's (1965) seminal paper, in which the availability of indexed annuities reduced the need for other forms of saving.

Unlike Yaari's "actuarial notes," the purchase of future social security benefits is not a choice variable for individuals. Within the framework of the model used in the paper, it is useful to examine the impacts on consumption (saving) over time of alternative social security systems. Of particular interest are systems in which the effective income replacement rate of benefits is nonlinear in income. For example, in the U.S., taxes and benefits are calculated only up to an earnings ceiling. High-income individuals have incomplete access to the "social security annuity" system. Hence, even in the absence of an explicit bequest motive, the ratio of wealth to lifetime earnings should rise with the level of lifetime earnings. Empirical implementation of the model of section II is left as a task for future research.

Including a discussion of private pension annuities is an important extension of the model discussed above. Moreover, econometric estimates of the impact of private pension annuities on non-pension wealth accumulation as well as on links between changes in social security annuities and the extent of private pension participation are necessary for a thorough consideration of the impact of the social security system on individual saving. The latter link is both important and not often noted. That annuity markets are extremely imperfect in the real world is not evidence per se of a severe market failure, as individuals have some control over their participation in private pension annuities either explicitly (for participants in defined-contribution plans) or implicitly (through choice of employer). To the extent that individuals adjust their pensions for variation in social security annuities, the effective annuity market may be large. The magnitude of that adjustment must be resolved empirically.

Section III uses the results derived in section II of the impact of social security on saving under uncertain lifetimes to construct individual patterns of dynamic wealth accumulation over the life-cycle. Those individual patterns differ substantially from their "certainty" analogues. The impact of an actuarially fair social security system on saving depends on risk aversion, survival probabilities, and the extent to which the purchase of social security annuities is rationed by earnings class. That impact is reduced when initial endowments are considered. Specifically, accidental bequests, which arise in the model because of lifetime uncertainty, provide an intergenerational link for saving decisions. To the extent that the introduction of social security reduces the size of accidental bequests, the net effect of social security on the consumption of succeeding generations is mitigated. In addition, general equilibrium considerations, primarily the endogeneity of factor returns, can be expected to reverse part of the partial equilibrium impact. Because of these two considerations, the impact of social security on the steady-state capital stock is likely to be substantially smaller than the partial equilibrium impact.

The approach taken here suggests some extensions in analyzing individual wealth accumulation over time. One finding of the paper is that when participation in pension annuities is not a choice variable for individuals, the precise influence of those annuities on saving depends on the way in which participation is determined. Rationed access to publicly provided pension annuities may provide an impetus to the growth of private pension annuities, for example.[27] The implications of "pension annuity rationing" for the relationship between bequests and lifetime resources may shed some light on the findings of some recent empirical studies of bequests.[28]

The debate over the influence of pensions on individual saving brings together questions of consumer choice under uncertainty, and the effectiveness of fiscal policy. Researching the relationships among social security, private pensions, annuity markets, and bequests facilitates close empirical scrutiny of models of individual and aggregate saving, permitting consideration of the welfare effects of compulsory pensions. Further empirical study of saving behavior in the context of uncertain lifetimes is an important step in an economic analysis of government retirement saving and income policy.

**FOOTNOTES**

[1]The intergenerational consequences of this point are taken up by Abel (1983), with the implication that the insurance feature of the social security system may act to reduce inequality in the distribution of wealth.

[2]Other studies have addressed the possibility of other motives for saving. Kotlikoff and Summers (1981) reject the ability of the life-cycle model to explain wealth accumulation in the U.S., as their constructed measure of "life-cycle holdings" fell far below total wealth.

[3]Social Security wealth is just the present value of anticipated social security benefits. Net social security wealth is exclusive of the present value of payroll taxes paid.

[4]These results have by no means gone unchallenged. Later studies by Barro (1978) and by Darby (1978) using the social security wealth variable were inconclusive. See also the debate between Leimer and Lesnoy (1982) and Feldstein (1982).

[5]Kotlikoff and Summers (1981), for example, find that bequests may account for an important share of the capital stock.

[6]The models of Kotlikoff and of Summers were done in continuous time, while the exercise herein is done in discrete time.

[7]The linear expansion path implied by (3) and (4) comes about because of the assumption of constant relative risk aversion implicit in the isoelastic utility function.

[8]To the extent that the system is unfair, so that some cohorts earn an excess rate of return on social security, the impact of the system on an individual's saving will depend on the relationship of $\bar{w}$ to $w_o$.

[9]Note that the introduction of uncertainty here is equivalent to changing the discount rate from $\delta$ to $(\delta + p_t)/(1-p_t)$. When $p_t = 0$, this is the same as before. When $p_t = 1$, the new discount rate is infinite.

[10]Using a similar framework, Kotlikoff and Spivak (1981) examined the value to an individual of access to a fair annuity market. Actuarially fair social security benefits constitute a fair annuity, but their availability is rationed; social security tax and benefit levels are not choice variables for the individual.

[11]Note that this does not require that they actually die at the same time.

[12]Rothschild and Stiglitz (1976) show that there will be no "pooling equilibrium," where all buy the same contract. They illustrate conditions under which a "separating equilibrium" occurs, in which different contracts are purchased by different risk groups. Following their argument and that of Riley (1979), if there is a continuous distribution of survival probabilities, there is little hope even for a separating equilibrium.

[13]While the imposition of the social security system increases lifetime resources, nothing has been said about the optimal tax rate. Current law prohibits the explicit leverage of anticipated social security benefits. The ability to implicitly borrow against future benefits will depend on differences in $w_o$ (differences in ability to procure "unsecured" loans). Under the assumption of complete (explicit and implicit) nomarketability of benefits, we can demonstrate that there is an interior solution ( $0 < t_s < 1$) for the individuals's optimal tax rate (a sufficient statistic of participation as long as benefits are actuarially fair). The intuition is that while the purchase of "social security retirement annuities" increases resources available in old age, it decreases the resources available for current consumption. The optimal tax rate $t_s$ in such a world is just

$$\hat{t}_s = \frac{1}{\omega+1}\left[\frac{\sum\limits_{i=Q+1}^{D'} (1+r)^{i\gamma/(1-\gamma)}(1+\delta)^{-i/(1-\gamma)}(1-p_i)^{1/(1-\gamma)}}{\sum\limits_{i=o}^{D'} (1+r)^{i\gamma/(1-\gamma)}(1+\delta)^{-i/(1-\gamma)}(1-p_i)^{1/(1-\gamma)}}\right]$$

which is equal to zero for individuals who "know" that they will die prior to retirement.

[14]Note that the actuarially fair benefit is constructed with respect to economy-wide survival probabilities. Recalling footnote 13 above, an individual's optimal tax rate will differ from an optimal tax rate chosen with respect to economy-wide life tables to the extent that his life expectancy differs from the average. It is true, then, that individuals who believe that they will die "young" will want to purchase less than the "average optimal" amount of social security annuities, while those who expect to live a long time will want more. Both groups are better off, however, with the mandatory social security system than without it, since in its absence, adverse selection effectively foreclosed the possibility of a market in actuarially fair annuities. A discussion of the potential separating equilibria in the private provision of annuities which may arise after the imposition of mandatory social security is given Eckstein, Eichenbaum, and Peled (1983).

[15]A retirement age of 65 was assumed. Probabilities for survival were taken from Faber (1982).

[16]To the extent that the present value of benefits exceeds the present value of taxes paid for an individual or household, the potential offset could still be higher.

[17]This effect is most pronounced in the absence of explicit capital market restrictions. With no initial endowment (and, hence, binding restrictions on the nomarketability of social security when young), relative impacts on "working-period" and "retirement-period" consumption will depend on the relationship of the individual's actual and optimal tax rate (participation). The importance of (accidental) bequests as intergenerational links will be discussed later in the paper.

[18]This nonlinearity has surfaced in some recent studies of the impact of social security on saving. See for example Diamond and Hausman (1982) and Hubbard (1983).

[19]For a more complete discussion of the implications of the choice of parameter values, see Levhari and Mirman (1977) or Davies (1981).

[20]As in Table 1, survival probabilities are taken from Faber (1982).

[21]Note that if participation in social security is rationed by income, low-income individuals have more of their retirement dissaving in the form of reduction in the value of their social security annuity than do high-income individuals. This analysis assumes that annuity and non-annuity holdings are perfect substitutes in dissaving. The studies cited in the beginning of the paper have found good but not perfect substitutability of social security for non-pension wealth in accumulation. Empirical evidence in Hubbard (1983) suggests that the substitutability is greatest for high-income individuals.

[22]In a world with capital market restrictions, then, a social security system of this type may increase saving, since received initial bequests are more liquid than anticipated social security benefits. The impact of social security on intergenerational transfers is an important component of the system's net effect on individual saving.

[23]The implicit assumption, of course, is that the parent dies at the beginning of the child's (optimizing) life, age twenty here. This assumption is made to highlight the point that the existence of social security for the previous generation mitigates the impact of the present generation's participation in social security on its own wealth accumulation. More general assumptions about the timing of a testator's death would complicate expressions like (22) in the text, but the qualitative point would remain.

[24]This damping through intergenerational transfers of the impact of social security on wealth accumulation is mitigated if "children" earn more on average than their "parents" (because of productivity growth).

[25]Methodologically, the consumption of individuals of each age can be calculated from equations (9) and (10), given the initial wage. The growth rate of the population will determine the relative number of persons at each age. Aggregate consumption can be calculated by summing consumption over ages, weighted by the relative population size.

[26]Kotlikoff's (1979a) analysis also incorporates the influence of social security on retirement age, which is taken as exogenous here. To the extent that social security lowers the desired retirement age, the partial equilibrium wealth replacement effect of social security on saving is dampened.

[27]The rationing of social security annuities leaves excess demand for fair annuities on the part of high-income individuals. Adverse selection and the possibility of multiple insurance still render unlikely the provision of such annuities by competitive insurance markets. Employer-sponsored private pension funds may act to fill this gap. Employers are likely to have better information on individual workers' life expectancies than would a disinterested insurance company. Second, by definition, such annuities can only be purchased at an individual's place of work; multiple insurance is not possible. Finally, the pension instrument may provide an added degree of freedom for the firm in influencing workers behavior (as in Lazear, 1983).

[28]Despite serious data limitations, there have been several recent efforts to estimate empirically the relationship between bequests and lifetime resources, generally finding that the ratio of bequests to earnings rises with the level of earnings. This finding is corroborated in the careful empirical study of Menchik and David (1983), who found the "marginal propensity to bequeath" to be five times higher for individuals in the top quintile of the earnings distribution than for those in the lower four quintiles. To the extent that participation in social security (or private pensions) is rationed and lifetime is uncertain, omitting pension "wealth" variables biases upward the measured influence of the level of earnings on bequests. Whether bequests are accidental is important here. Reduced-form estimates of the bequests-resources relationship may be flawed, in that a change in pension (particularly social security) policy (i.e., benefit levels or replacement rates for different income groups) changes the individual's optimal intertemporal consumption allocation.

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