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FEES VERSUS ROYALTIES AND THE PRIVATE VALUE OF A PATENT

by

Morton I. Kamien*
and
Yair Tauman**

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*Department of Managerial Economics and Decision Sciences, J. L. Kellogg Graduate School of Management, Northwestern University, 2001 Sheridan Road, Evanston, Illinois 60201.

**Currently visiting, Faculty of Management, Tel Aviv University, Israel.

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1. Introduction

In 1962 Arrow addressed the question of whether a cost reducing innovation of a given magnitude was of greater value to a perfectly competitive industry or to a monopoly. To determine the value of a cost reducing innovation to a perfectly competitive industry, he asked how much profit could an innovator, facing no competition from similar innovations, realize by licensing it. Payment for the license was to be in the form of a linear royalty, that is, a royalty per unit output independent of the number of units. To determine the value of the identical innovation to a monopolist, Arrow assumed that the innovation was developed by the monopolist himself. This supposition avoids the bilateral monopoly problem arising from the single innovator facing a single buyer, and the associated bargaining problem.

Arrow's analysis was confined to the two polar market structures, perfect competition and monopoly, and to a single licensing arrangement, a linear royalty. Kamien and Schwartz (1982) extended the analysis of the private value of a patent to market structures intermediate between perfect competition and monopoly and allowed for both a fee and a royalty. They assumed that the patent was to be licensed to firms of an n-firm Cournot oligopoly. Their analysis was limited, however, by the assumption that a firm will buy the license to the patent only if its net profit will remain the same as it was before the innovation. This does not allow for the possibility that some firms, as a result of competition to use the patent, will buy the right to use it even if it results in profits lower than their profits before the innovation. The relevant comparison for the firm is between its profit with the use of the patent and its profit with the use of the old technology. Both of these profits depend on the action taken by the other firms and the amount charged by the patent holder. This conflict can be naturally analysed as a noncooperative game in strategic form. The game involves $n + 1$ players--the n firms in the industry and the patent holder. The strategies available to the patent holder is the amount to charge each firm for the license. The strategies available to each of the firms is whether to buy or not buy the license as a function of its price. When all the strategies are selected the price of the license and the number of licensees are uniquely determined. The payoff to each firm can then be determined as its net profit under the resulting Cournot equilibrium. The profit to the patent holder is the total amount he extracts from the licensees. The Nash equilibrium of this game determines the value of the patent as well as the number of licensees.

We follow Kamien and Schwartz by assuming that the potential buyers of a cost reducing innovation are the firms of a Cournot oligopoly, of which there are $n > 2$. The possibility of one potential buyer is ruled out to avoid the issue of bilateral monopoly and the associated bargaining problem. Similarly, the possiblity of cooperative behavior--say, through coalition formation--among the buyers, is not allowed. The firms in the Cournot oligopoly have identical linear costs functions that pass through the origin. Thus, their marginal cost functions are horizontal. The firms produce identical products and the industry as a whole faces a linear demand function.

The patent holder has an invention that will lower the marginal cost of producing the item sold by the industry. There are no similar inventions that will lower the industry's cost, so the patent holder faces no competition from other inventors. The patent holder may license his invention by means of a fee only, a royalty only, a fee and a royality, or by receiving a share of a licensee's profits. In the present analysis we also allow for the possiblity of nonzero contracting costs associated with the licensing of the patent. The importance of contractual costs, especially in the form of costs incurred in teaching licensee how to use the new technology, is pointed out by Taylor and Siberston (1973). They also observe that some of the firms in the sample they studied employed a profit-sharing arrangement in licensing a patent. Our analysis in this paper is confined largely to the cases of a fee only, a royalty only, contracting costs, and profit sharing. An extensive analysis of the mixed fee and royalty case is conducted in our companion paper, Kamien and Tauman (1983).

Our analysis discloses that use of a linear royalty alone is inferior to the use of a fee alone in terms of profits to the patent holder when the number of firms in the Cournot oligopoly is finite. Licensing via a fee is also superior for consumers as compared to licensing via a royalty as the former method does not lead to a restriction of production while the latter does. As the number of firms approaches infinity, so that the Cournot equilibrium approaches the perfectly competitive equilibrium, the two licensing arrangements yield the same profit for the patent holder. This profit is exactly the magnitude of the cost reduction times the perfectly competitive output under the original technology. The same is true for the case when the patent holder uses a combination of a royalty and a fee to license the invention. Our analysis discloses, however, that in the presence of contracting costs, for sufficiently large number of firms, employment of a fee alone is optimal for the patent holder, regardless of how small contracting costs are. We also show that the use of a fee alone or profit sharing alone are equivalent in the sense of yielding the same profit to the patent holder and the same market equilibrium. Finally, we show that it is more profitable for the patent holder to license his innovation than enter the industry as a producer.

Since in our analysis the use of a fee alone is a superior means of licensing than the use of a royalty alone from the standpoint of the patent holder, we summarize some of the results that obtain in this case. First, it is possible that the Nash equilibrium of the game between the patent holder and the firms of the Cournot oligopoly results in not all of them purchasing the license to the innovation. In this case the firms that do not purchase the license may continue producing with the old technology or they may cease producing. We characterize both situations as a function of industry size and the magnitude of the innovation. In either event, we have a situation where technological advance determines market structure in the sense of leading to possibly different number of firms and different output levels from those that existed previously. Moreover, this impact of technological advance on market structure is not a consequence of a change in economies of scale but of the patent holders effort to realize as large a profit from his invention as possible. (Changes in economies of scale as a result of technological advance, diseconomies leading to more firms and economies leading to fewer firms in an industry, have long been recognized as sources of changes in market structure). Second, as a result of the innovation, the Cournot oligopoly degenerates into a monopoly if and only if the innovation is drastic. A drastic innovation is one where the monopoly price with the new technology does not exceed the perfectly competitive price with the old technology. Moreover, as expected, a drastic innovation yields a higher profit to the patent holder than a nondrastic innovation. Third, regardless of whether the innovation is drastic or not, the Cournot equilibrium ouput of the product will increase relative to its level prior to the innovation. Thus, the Cournot equilibrium price will decline and consumers will be better off. The firms of the Cournot oligopoly will be worse off, however, as their profits after the innovation will be lower than they were before it. This, of course, is a consequence of the patent holder's ability to exploit competition among the potential licensees of the patent for its use.

These results hold under the further assumptions that: there is no uncertainty regarding the efficacy of the invention, everyone knows the magnitude by which the invention will lower marginal costs, there is no possiblility that a new invention will come along that is superior to this one, and there is no possiblity that demand for the industry's product will change. We could allow for uncertainty with regard to the last two possibilities, providing everyone had identical expectations. The absence of uncertainty regarding the efficacy of the patent avoids the possibility that some potential buyers will postpone purchasing it until others have tried it and found that it works. Also, the absence of uncertainty regarding the development of a superior invention avoids the possibility that potential licensees will postpone purchasing this patent in anticipation of buying a better one. It also eliminates the preference that buyers might have for a royalty over a fee. For under a royalty, payment for the use of the patent would cease when the right to use the new patent was purchased, Finally, the absence of uncertainty regarding changes in demand for the product also eliminates possible differences in preferences for a royalty over a fee by the licensees and the licensor of the patent. For example, if demand were expected to increase the licensor might prefer a royalty while the licensees might prefer a fee, and the preferences would be reversed if demand were expected to decline. In reality, all these uncertainties do exist and impact on the choice of licensing arrangements and the private value of the patent. By abstractng from these realities we are seeking the pure private value of the patent. Our analysis also abstracts from any issues involving time. That is, profits are expressed in present value terms. The patent is licensed at the moment it has been granted and runs concurrently with the duration of the patent.

In the next section we analyze the case where the patent holder is restricted to licensing via a fee only. In Section 3 we consider the case of royalty only. In Section 4 we take up the mixed case, where the patent holder can employ a fee and a royalty. We then turn to the affect of the presence of contracting costs on the patent holder's choice of licensing arrangement in Section 5. Finally, we consider licensing via profit sharing in Section 6. The proofs of the results appear in Section 7. Our concluding remarks are the content of Section 8.

2. Licensing by Means of a Fee Only

2.1 The Model

Consider an industry consisting of $n \geq 2$ firms all producing the same good with an identical technology expressed by the linear cost function

$$f(q) = cq$$

where q is the quantity produced and $c > 0$ is the constant marginal cost of production. The aggregate demand for this good is given by

$$p = a - Q$$

where $a > c$ and Q is the total production level. In addition to the n firms there is a patent holder who has a cost reducing innovation that lowers marginal cost form c to $c - \varepsilon$, where $\varepsilon > 0$.

The patent holder licenses the patent to all or some of the n firms so as to maximize his profit. The amount that a firm is ready to pay for the license to the patent depends on the incremental profit it will realize by the use of the superior technology. This increment, however, clearly depends on the number of other licensees. This interaction can be naturally described as a noncooperative game G_1 played by n + 1 players: the patent holder and the n firms of the industry. In this game the patent holder moves first. He chooses for each firm i a lump-sum fee α_i for the license to the patent. Let $\alpha = (\alpha_1,\ldots,\alpha_n)$. The firms, all of whom are informed of α, react simultaneously and each decides whether to purchase the license under α. Thus, a strategy of the patent holder is an element α of E^n_+. A strategy of the i^{th} firm is a decision rule τ_i that determines for each $\alpha \in E^n_+$ its reaction regarding the purchase of the license. Thus, τ_i is a function from E^n_+ to $\{0,1\}$ with the convention that $\tau_i(\alpha) = 1$ iff i purchases the license under α, and otherwise $\tau(\alpha) = 0$.

Any (n + 1) tuple of strategies $(\alpha, \tau_1,\ldots,\tau_n)$ uniquely determines a set S of k licensees ($S \subseteq \{1,\ldots,n\}$) who will produce with the cost function

$$F_i(q_i) = \alpha_i + (c - \varepsilon)q_i, \quad i \in S.$$

The other firms will use the old technology $f(q) = cq$. Assuming that total industry output is at a Cournot equilibrium, it can be easily verified that the production level q_i of each firm i is uniquely determined for each $(\alpha, \tau_1,\ldots,\tau_n)$. The profit of each firm is then given by

$$\pi_i(\alpha, t_1,\ldots,\tau_n) = \begin{cases} q_i(p - c + \varepsilon) - \alpha_i & i \in S \\ q_i(p - c) & i \notin S \end{cases}$$

where $p = a - \sum_{j=1}^{n} q_j$ and $\alpha = (\alpha_1,\ldots,\alpha_n)$. The profit π_{PH} of the patent holder is then defined by

$$\pi_{PH}(\alpha, \tau_1,\ldots,\tau_n) = \sum_{i\varepsilon S} \alpha_i .$$

Choosing the functions $\pi_{PH}, \pi_1,\ldots,\pi_n$ as the payoff functions of the n + 1 players one obtains a well defined game G_1 in strategic form.

We now proceed to study the subgame perfect Nash equilibrium (in pure strategies) of this game. In our context an (n + 1)-tuple of strategies $(\alpha^*, \tau_1^*,\ldots,\tau_n^*)$ is a subgame perfect equilibrium in pure strategies of G_1 if (i) the strategy α^* is the best reply strategy of the patent holder for the strategies $\tau_1^*,\ldots,\tau_n^*$ of the n firms and (ii) for each i, $1 \leq i \leq n$, $\tau_1^*(\alpha)$ is the best reply choice of firm i given the choices $\tau_1^*(\alpha),\ldots, \tau_{i-1}^*(\alpha)$, $\tau_{i+1}^*(\alpha),\ldots, \tau_n^*(\alpha)$ of the other n - 1 firms, for <u>each</u> $\alpha \in E_+^n$ and not only for $\alpha = \alpha^*$.

Notice that for the definition of a Nash equilibrium the requirement (ii) should hold only for $\alpha = \alpha^*$. We choose the notion of perfect equilibrium to avoid undesirable equilibrium points. For example, note that $\tau_1^* = \ldots = \tau_n^* \equiv 0$ together with $\alpha_i^* = (\frac{a - c + \varepsilon}{2})^2 + 1$, which is a unit above the monopoly profit with the new technology, defines a Nash equilibrium (with a high fee and no licenses) but not a subgame perfect equilibrium.

2.2 <u>Statement of Results</u>

Our results depend naturally on the magnitude ε of the innovation. We use Arrow's definition of a drastic innovation.

<u>Definition</u>: An innovation is <u>drastic</u> if the monopoly price p_m' under the new technology does not exceed the competitive price c under the old technology. An innovation is nondrastic if the reversed inequality holds, i.e., if $c < p_m'$.

Notice that if $c = p_m'$ the innovation is both drastic and nondrastic. This allows better representation of the results. Also notice that since $p_m' = \frac{a + c - \varepsilon}{2}$ an innovation is drastic if and only if $\frac{a - c}{\varepsilon} \leq 1$.

<u>Proposition 1</u>: The game G_1 has a unique[1] subgame perfect equilibrium in pure strategies.

<u>Proposition 2</u>: If the innovation is not drastic, then the equilibrium[2] number, k^* of licensees, the profit π^*_{PH} of the patent holder and the profit π^*_i of the i^{th} firm are given by

$$k^* = \begin{cases} n & n \leq \frac{2}{3}(\frac{a - c}{\varepsilon} + 1) \\ \frac{a - c}{2\varepsilon} + \frac{n + 2}{4} & \frac{2}{3}(\frac{a - c}{\varepsilon} + 1) \leq n \leq 2(\frac{a - c}{\varepsilon} - 1) \\ \frac{a - c}{\varepsilon} & 2(\frac{a - c}{\varepsilon} - 1) \leq n \end{cases}$$

$$\pi^*_{PH} = \begin{cases} \frac{2n^2}{(n + 1)^2}\, \varepsilon^2 [\frac{a - c}{\varepsilon} + 1 - \frac{n}{2}] & n \leq \frac{2}{3}(\frac{a - c}{\varepsilon} + 1) \\ \frac{2n}{(n + 1)^2}\, \varepsilon^2 [\frac{a - c}{2\varepsilon} + \frac{n + 2}{4}]^2 & \frac{2}{3}(\frac{a - c}{\varepsilon} + 1) \leq n \leq 2(\frac{a - c}{\varepsilon} - 1) \\ \frac{n(n + 2)}{(n + 1)^2}\, \varepsilon(a - c) & 2(\frac{a - c}{\varepsilon}) \leq n \end{cases}$$

and

$$\pi^*_i = \begin{cases} \frac{1}{(n + 1)^2}[(a - c)^2 - \varepsilon(k^* - 1)(2(a - c) - \varepsilon(k^* - 1))] & i \in S \\ (\frac{a - c - k^*\varepsilon}{n + 1})^2 & i \notin S \end{cases}$$

<u>Proposition 3</u>: If the innovation is not drastic then:

(1) The number k^* of licensees does not exceed $\frac{a - c}{\varepsilon}$ regardless of the number of firms $(n > \frac{a - c}{\varepsilon})$.

(2) The number $k^* = k^*(\varepsilon)$ of licensees is a nondecreasing function of ε.

(3) For n sufficiently large (namely $n > 2(\frac{a - c}{\varepsilon} - 1)$) only the $\frac{a - c}{\varepsilon}$ licensees produce in equilibrium.

(4) The profit of each firm decreases with ε while the profit of the patent holder increases with ε. If the industry is purely competitive, i.e., when $n \to \infty$, the patent holder's profit is $\varepsilon(a - c)$.

(5) The most profitable size, n, of the industry for the patent holder to license his innovation to depends on the magnitude ε of the innovation. For small ε, the optimal n is proportional to $\frac{1}{\sqrt{\varepsilon}}$. However, when comparing the two extreme cases, a monopoly versus a purely competitive industry, the patent holder is always better off licensing to a competitive industry. For example, assume that the magnitude ε satisfies $\frac{a - c}{\varepsilon} = 5$. Then the profits of the patent holder in a monopolistic market and in a competitive market are $2.75\varepsilon^2$ and $5\varepsilon^2$, respectively. However, in an industry of size $n = 4$, his profit is $5.12\varepsilon^2$.

Remark: Notice that for $n > 2(\frac{a - c}{\varepsilon} - 1)$ the equilibrium number of firms in the industry after the introduction of a nondrastic innovation is $k^* = \frac{a - c}{\varepsilon}$. Each firm then produces with the new technology the quantity (see Lemma 1, below)

$$q_i^* = \frac{a - c + (n - k^* + 1)\varepsilon}{n + 1} = \varepsilon$$

Thus, total production is $Q = a - c$ and the market price is c. Also, each firm pays a fee of

$$\alpha_i = \frac{n(n+2)}{(n+1)^2}\,\varepsilon^2,$$

and its net profit is then

$$\pi_i = q_i^*(p - c + \varepsilon) - \alpha_i = \frac{\varepsilon^2}{(n+1)^2}$$

The profit of the patent holder is

$$\pi_{PH} = \frac{n(n+2)}{(n+1)^2}\,\varepsilon(a - c)$$

In this context a natural question[3] to ask is why the patent holder does not produce the quantity a - c and realize the entire oligopoly profit of the $\frac{a-c}{\varepsilon}$ licensees (in this case he obtains in addition to $\sum_{i\varepsilon S} \alpha_i$ also $\sum_{i\varepsilon S} \pi_i$). The increment in the patent holder's profit is then

$$\Delta = \frac{a-c}{\varepsilon} \cdot \frac{\varepsilon^2}{(n+1)^2} = \frac{\varepsilon(a-c)}{(n+1)^2}$$

There are two cases to consider. If the game in quantities is played simultaneously (the Cournot version) with the n + 1 potential producers then given that the original n firms do not produce, it pays the patent holder to reduce the production level below a - c and consequently to increase the market price above c. But then under the new higher price each firm can make positive profit and thus will produce. Hence an industry consisting of n + 1 firms, a patent holder who is able to produce with a marginal cost of c - ε and n other firms each able to produce with a marginal cost of c, does not yield a Cournot equilibrium with quantities $q_{PH} = a - c$ and $q_i = 0$ for each i, i=1,...,n, where q_{PH} is the production level of the patent holder. In fact, the Cournot equilibrium quantity and profit of the patent holder in this industry is given by (see Lemma 1, below)

$$q_{PH} = \frac{a - c + (n + 1)\varepsilon}{n + 2}$$

$$\pi_{PH} = \left(\frac{a - c + (n + 1)\varepsilon}{n + 2}\right)^2.$$

Since the magnitude $\frac{a - c}{\varepsilon}$, when applied to the number k^* of licensees, is assumed to be an integer and since a strictly nondrastic innovation satisfies $\frac{a - c}{\varepsilon} > 1$, we have $\frac{a - c}{\varepsilon} \geq 2$. Under this inequality it is easy to verify that

$$\left(\frac{a - c + (n + 1)\varepsilon}{n + 2}\right)^2 < \frac{n(n + 2)}{(n + 1)^2}\,\varepsilon(a - c)$$

and thus the patent holder is better off licensing his patent to the other firms than using it himself.

Now, if the game is played sequentially and the patent holder is the first to choose price and quantity, then $Q = a - c$ with $p = c$ will deter any entry and he will be better off producing exclusively. However, the fact that the patent holder was not a producer initially can be explained only by an entry cost of at least $\left(\frac{a - c}{n + 2}\right)^2$ (that is the Cournot oligopoly profit in an industry of $n + 1$ firms in which each is producing with a marginal cost of c). With the new technology, by producing exclusively, the patent holder realizes $\frac{\varepsilon(a - c)}{(n + 1)^2}$ over and above the fee that he could have extracted by just licensing. Thus, he will indeed produce only if

$$\frac{\varepsilon(a - c)}{(n + 1)^2} \geq \left(\frac{a - c}{n + 2}\right)^2.$$

But again for $\frac{a - c}{\varepsilon} > 2$, $\frac{\varepsilon(a - c)}{(n + 1)^2} < (\frac{a - c}{n + 2})^2$ and he is again better off licensing the patent than using it himself.

<u>Proposition 4</u>: The subgame perfect equilibrium of G_1 results in a monopoly if and only if the patent is drastic. In this case

$$\pi^*_{PH} = (\frac{a - c + \varepsilon}{2})^2 - (\frac{a - c}{n + 1})^2$$

and

$$\pi^*_m = (\frac{a - c}{n + 1})^2$$

where π^*_m is the profit of the resulting monopoly.

Thus, in the case of drastic innovation the profit of the patent holder again increases with ε while the profit of each firm does not depend on ε. Also, we conclude that the patent holder has the greatest incentive to invest in the development of an innovation when it can be licensed to a purely competitive industry. Let us try to provide an intuitive explanation for the last proposition. The most profit a patent holder can expect to realize is the monopoly profit under the new technology. Since a fee is a fixed cost it does not affect the level of production of a firm (as long as its profit covers fixed cost). Thus, licensing more than one firm for a total fee equal to the monopoly profit will generate a Cournot equilibrium wherein the total profit of the firms does not cover their total fee. In the case of a drastic innovation (where $\frac{a - c}{\varepsilon} < 1$) licensing the patent to just one firm will result in a monopolistic industry (since the other firms will not produce; see the proof of Proposition 2 for the case where $1 = k > \frac{a - c}{\varepsilon}$). In this case the patent holder still cannot realize the entire monopoly profit of $(\frac{a - c + \varepsilon}{2})^2$ since by deviating from his strategy the only licensee can realize $(\frac{a - c}{n + 1})^2$. Thus, the profit to the patent holder is $(\frac{a - c + \varepsilon}{2})^2 - (\frac{a - c}{n + 1})^2$, as asserted in Proposition 4. The situation is different in the case of a nondrastic innovation. There, by choosing $k = 1$--namely, by selling the license just to one firm--the other firms in equilibrium will all produce positive quantities since now $k < \frac{a - c}{\varepsilon}$; see the proof of Proposition 1. Thus, compeititon from the unlicensed firms cannot be avoided and the licensee would be unable in equilibrium to realize a profit exceeding the monopoly profit under the new technology.

Finally, competition among firms to purchase the license to the patent lowers their profits and benefits consumers.

Proposition 5: For any $\varepsilon > 0$, each firm is worse off relative to its profit level prior to the innovation, unless the innovation is drastic and then the resulting monopoly breaks even. However, in both cases total production increases, the market price falls, and the consumers are better off.

3. Licensing by Means of a Royalty Only

In this section we examine the case where the patent holder licenses only via a per unit royalty. This case has been studied by Arrow without employing game theory. It turns out, however, that the results obtained by Arrow coincide with those obtained here. We show further that both the patent holder and consumers are worse off with the use of royalty as compared to the use of a fee only.

3.1 The Model

The model is analogous to the previous one with a fee only. Here we consider the game G_2 played by the same $n + 1$ players of the game G_1. The patent holder chooses an element $\beta \in E^n_+$, $\beta = (\beta_1,\ldots,\beta_n)$ where β_i is the royalty that the i^{th} firm is required to pay per each unit it produces with the new technology. The firms then decide independently and simultaneously whether to pay this royalty or to continue to produce nonnegative quantities with the old technology. Any (n + 1)-tuple of strategies determines an industry with possibly two technologies. Let S be the set of licensees and let $k = |S|$. The industry structure resulting from an (n + 1)-tuple of strategies consists of k licensees, producing with

$$F_i(q_i) = (c - \varepsilon + \beta_i)q_i, \quad i \in S$$

and n − k firms producing with the old technology, $f(q) = cq$. Let $q_1, \ldots, q_n$ be the Cournot equilibrium of the industry. The payoffs are then given by

$$\pi_i = \begin{cases} (p - c + \varepsilon - \beta_i)q_i & i \in S \\ (p - c)q_i & i \notin S \end{cases}$$

where $p = a - \sum_{j=1}^{n} q_j$. Also

$$\pi_{PH} = \sum_{j \in S} \beta_j q_j .$$

The analysis of the game G_2 becomes much simpler if we restrict the discussion to nondiscriminatory royalties only, i.e., where $\beta_i \equiv b$, $i=1,\ldots,n$. The general case with different royalties is more complicated since it is possible then that a firm loses even with the use of the new technology. For example, if β_1 is sufficiently close to ε while β_i for $i > 1$ is close to zero, then in a large industry the first firm would be unable to recover its own costs. Nevertheless, it can be shown that the restriction

$\beta_i \equiv b$, $i=1,\ldots,n$ does not result in a loss of generality (although the proof of this statement involves many computations). To simplify the analysis we deal with the same per unit royalty b for all firms.

<u>Proposition 6</u>: There exists a unique subgame perfect equilibrium for the game G_2. This equilibrium satisfies the following (independent of the magnitude ε of the innovation):

(1) All n firms purchase the license to use the patent (i.e., $k^* = n$). The royalty for a nondrastic innovation is ε and for a drastic innovation is $\frac{a - c + \varepsilon}{2}$.

(2) For each finite n the patent holder realizes a lower profit using a royalty than using a fee.

(3) In the case of pure competition (where $n \to \infty$) the patent holder's profits using a fee or using a royalty coincide. In the case of a nondrastic innovation they both equal $\varepsilon(a - c)$ and in the case of a drastic innovation they both equal $(\frac{a - c + \varepsilon}{2})^2$.

(4) The profit of each firm is at least as high as it was prior to the innovation. In fact, its profit does not change as a result of a nondrastic innovation and increases as a result of a drastic innovation.

(5) Consumers are better off when a fee is used than when a royalty is used. This is because output is not restricted when a fee is used but is when a royalty is used.

(6) The profit of the patent holder increases with n and thus, when a royalty is used, a purely competitive market generates the greatest incentive to develop an improved technology.

Thus, from society's point of view as well as from the point of view of the inventor, use of a fee is preferred to use of a royalty. In the course of the proof of Proposition 6 it is shown that when the innovation is not drastic, the Cournot equilibrium production level and thus the market price does not change relative to its level prior to the innovation. In this case the patent holder charges each producer the entire magnitude ε of the improvement as a per unit of production royalty. This, however, is not the case if the innovation is drastic. In this case both the Cournot equilibrium profit and the production level of each firm increases and thus consumers do benefit from a drastic innovation (although they benefit less relative to the when a fee is used). Also, as expected, the profit of the patent holder is higher under a drastic innovation than under a nondrastic innovation--but in both cases he makes less profit than when a fee is used.

4. Licensing with a Combination of Fee and Royalty

In this section the case where the patent holder is allowed to use both a lump-sum fee and a per unit royalty to license the patent is examined. For simplicity we will restrict the analysis just to a nondiscriminatory fee and royalty. Thus, in the game G under consideration, a strategy of the patent holder is a pair $(\alpha, b) \in E^2_+$, where α is the fee and b is the per unit royalty that each licensee is required to pay for use of the patent. The strategy sets of the firms are defined similarly to those of the games G_1 and G_2.

Proposition 7: The game G has a unique subgame perfect equilibrium in pure strategies. This equilibrium satisfies:

(1) In a purely competitive industry (i.e., $n \to \infty$), the patent holder makes the same profit as in the other two games G_1 and G_2. Namely, for a nondrastic innovation this profit is $\varepsilon(a - c)$ while for a drastic innovation it is $(\frac{a - c + \varepsilon}{2})^2$.

(2) For n sufficiently large ($n > 2(\frac{a - c}{2} - 1)$ the game G_1 (with fee only) yields a higher level of consumers' benefit than the equilibrium levels

of the other two games, G_2 and G.

It should be mentioned that while the second part of the proposition is demonstrated only for $n > 2(\frac{a - c}{\varepsilon} - 1)$ it is likely to hold for every n. We were, however, unable to prove this. Also, notice that the first part of the proposition deals only with a purely competitive industry. The general case appears to be very complicated. Moreover, even this special case requires many computations and thus is separately handled in a companion paper (Kamien and Tauman (1983)). In this regard let us mention that if the profit of the patent holder, in G, could be shown to be bounded from above by $\varepsilon(a - c)$, for any n and ε, then the first part of the proposition follows immediately. Unfortunately, this assertion fails to be true even in the game G_1 (see Proposition 2, parts 4 and 5).

5. Contracting Costs

In this section we consider the possibility of an additional cost, namely, a cost incurred by the patent holder in executing contracts with the various licensees and providing them the technical assistance and training required in the use of the patent. The game we consider is similar to G where both the use of a fee and royalty are allowed.

By contracting cost we mean a real valued function $d(\cdot)$ on N, the set of positive integers. The value d(k) is the cost to the patent holder of dealing with k licensees. We deal with increasing contracting cost functions only. Namely, we consider functions $d(\cdot)$ that satisfy $d(k + 1) > d(k)$ for each $K \varepsilon N$. Clearly the interesting case is the one where

$$d(\infty) = \lim_{k \to \infty} d(k) < \infty$$

For an increasing function $d(\cdot)$ this is equivalent to

$$\sum_K [d(k + 1) - d(k)] < \infty$$

We show that in the presence of contracting costs the patent holder employes, for n sufficiently large, only a fee and not a royalty.

Proposition 8: Let $d(\cdot)$ be an increasing contracting cost function. Then if n is sufficiently large, the subgame perfect equilibrium G satisfies:

(1) Each licensee is charged only a fee and not a royalty;

(2) The number of licensees does not exceed $\frac{a - c}{\varepsilon}$

6. Profit Sharing

In the previous sections we dealt with three methods of licensing: fee only, royalty only, and a combination of fee and royalty. Another observed method is profit sharing, in which the patent holder receives a certain share of each licensee's profit. The resulting game is defined similarly to the game G_1 or G_2. A strategy for the patent holder is an element $t = (t_1,\ldots,t_n) \in E^n_+$ with $0 \leq t_i \leq 1$ for each i, $i=1,\ldots,n$. The profit π_{PH} of the patent holder associated with the (n + 1)-tuple of strategies $(t, \tau_1,\ldots,\tau_n)$ is defined by

$$\pi_{PH} = \sum_{i\varepsilon S} t_i q_i^* [p^* - c + \varepsilon]$$

where S is the set of licensees, t_i, $i \varepsilon S$, is the proportion of the i^{th} firm's profit to be allocated to the patent holder in return for the right to use the patent, q_i^* is Cournot equilibrium production level of the i^{th} firm and $p^* = a - \sum_{j=1}^{n} q_j^*$. The profits of the firms are defined as follows

$$\pi_i = \begin{cases} (1 - t_i)q_i^*(p^* - c + \varepsilon) & i \in S \\ \\ q_i^*(p^* - c) & i \notin S \end{cases}$$

Proposition 9: The profit sharing method is equivalent to the use of a fee only.

Proofs of the Results

Proof of Proposition 1: The proof of this proposition is obtained in the course of proving Proposition 2 and Proposition 4 below.

Before proving the next proposition we state the formula for the Cournot equilibrium outputs for an industry with n different technologies.

Lemma 1: In an industry with n technologies $f_1,\ldots,f_n$ where

$$f_i(q_i) = c_i q_i$$

the i^{th} firm's Cournot production level q_i^* is given by

$$q_i^* = \begin{cases} [a - (n + 1)c_i + \sum_{j=1}^{n} c_j]/(n + 1) & c_i < C \\ \\ 0 & \text{otherwise} \end{cases}$$

where $c = \frac{a + \Sigma c_j}{n + 1}$. Its profit π_i^* is given by

$$\pi_i^* = \begin{cases} [a - (n + 1)c_i + \Sigma c_j]^2/(n + 1)^2 & c_i < C \\ \\ 0 & \text{otherwise} \end{cases}$$

<u>Proof of Proposition 2</u>: Let $\sigma = (\alpha, \tau_1, \ldots, \tau_n)$ be an $(n + 1)$-tuple of strategies. Let S be the corresponding set of licensees and let $K = |S|$. Then, by Lemma 1, the Cournot equilibrium quantities and profits are given by

(1) $k \leq \frac{a - c}{\varepsilon}$ implies

$$q_i^* = \begin{cases} [a - c + (n - k + 1)\varepsilon]/(n + 1) & i \in S \\ [a - c - k\varepsilon]/(n + 1) & i \notin S \end{cases}$$

$$\pi_i^* = \begin{cases} [a - c + (u - k + 1)\varepsilon]^2/(n + 1)^2 - \alpha_i & i \in S \\ [a - c - k\varepsilon]^2/(n - 1)^2 & i \notin S \end{cases}$$

(2) $k \geq \frac{a - c}{\varepsilon}$ implies

$$q_i^* = \begin{cases} (a - c + \varepsilon)/(k + 1) & i \in S \\ 0 & i \notin S \end{cases}$$

$$\pi_i^* = \begin{cases} (a - c + \varepsilon)^2/(k + 1)^2 - \alpha_i & i \in S \\ 0 & i \notin S \end{cases}$$

Suppose now that $\sigma^* = (\alpha^*, \tau_1^*, \ldots, \tau_n^*)$ is a subgame perfect equilibrium of G_1 in pure strategies. For each $\alpha \in E_+^n$, $(\tau_1^*(\alpha), \ldots, \tau_n^*(\alpha))$ uniquely determines the number $k = k(\alpha)$ of licensees. Define for each k, $1 \leq k \leq n$,

$$A(k) = \{\alpha \in E_+^n | k = k(\alpha)\}.$$

The equilibrium strategy α^* of the patent holder is computed as follows: for each k find the optimal strategy of the patent holder in A(k). Then choose the best over all k, $1 \leq k \leq n$. We will first compute it for $k \leq \frac{a - c}{\varepsilon}$. Let $\alpha \in A(k)$ and let S be the set of licensees determined by $(\alpha, \tau_1^*, \ldots, \tau_n^*)$. Then for each $i \in S$ $\tau_i^*(\alpha) = 1$, which means that the incremental profit of the i^{th} firm due to the use of the patent covers the fee α_i. Namely,

$$\alpha_i \leq [\frac{a - c + (n - k + 1)\varepsilon}{n + 1}]^2 - [\frac{a - c - (k - 1)\varepsilon}{n + 1}]^2,$$

(otherwise it is better off under $\tau_i^*(\alpha) = 0$). Thus

$$(3) \qquad \alpha_i < \frac{n\varepsilon}{(n+1)^2}\,[2(a-c) + (n-2k+2)\varepsilon], \qquad i \in S.$$

If $i \notin S$ then $\tau_i^*(\alpha) = 0$, that means

$$\alpha_i > \left[\frac{a-c+(n-k)\varepsilon}{n+1}\right]^2 - \left[\frac{a-c-k\varepsilon}{n+1}\right]^2, \qquad i \notin S,$$

or equivalently

$$(4) \qquad \alpha_i > \frac{n\varepsilon}{(n+1)^2}\,[2(a-c) + (n-2k)\varepsilon], \qquad i \notin S.$$

By (3) and (4) the optimal $\alpha \in A(k)$ satisfies (4) and

$$(5) \qquad \alpha_i = \frac{n\varepsilon}{(n+1)^2}\,[2(a-c) + (n-2k+2)\varepsilon]$$

for each $i \in S$. Hence all the licensees pay the same fee, that is given by (5). Using the same fee for $i \notin S$ will satisfy (4) and will yield the unique nondiscriminatory equilibrium fee. By (5) the profit π_{PH} of the patent holder is

$$(6) \qquad \pi_{PH} = \frac{kn\varepsilon}{(n+1)^2}\,[2(a-c) + (n-2k+2)\varepsilon], \qquad k < \frac{a-c}{\varepsilon},$$

and this magnitude should be maximized over $k < \frac{a-c}{\varepsilon}$.

$$\frac{\partial \pi_{PH}}{\partial k} = \frac{n\varepsilon}{(n+1)^2}\,[2(a-c) + (n-2k+2)\varepsilon - 2\varepsilon k],$$

and hence

$$\frac{\partial \pi_{PH}}{\partial k} \geq 0 \quad \text{iff} \quad k \leq \frac{a - c}{2\varepsilon} + \frac{n + 2}{4}.$$

Thus, the optimal number of licensees under $k \leq \frac{a - c}{\varepsilon}$ is

$$(7) \qquad k = \min\left(\frac{a - c}{\varepsilon}, \frac{a - c}{2\varepsilon} + \frac{n + 2}{4}, n\right),$$

that is equivalent to

$$(8) \quad k = \begin{cases} n & n \leq \frac{2}{3}\left(\frac{a - c}{\varepsilon} + 1\right) \\ \frac{a - c}{2\varepsilon} + \frac{n + 4}{4} & \frac{2}{3}\left(\frac{a - c}{\varepsilon} + 1\right) \leq n \leq 2\left(\frac{a - c}{\varepsilon} - 1\right) \\ \frac{a - c}{\varepsilon} & 2\left(\frac{a - c}{\varepsilon} - 1\right) \leq n \end{cases}$$

In this case, by (6) and (8) we obtain for $k \leq \frac{a - c}{\varepsilon}$

$$\pi_{PH} = \begin{cases} \frac{2n^2}{(n + 1)^2}\, \varepsilon^2\left(\frac{a - c}{\varepsilon} + 1 - \frac{n}{2}\right) & n \leq \frac{2}{3}\left(\frac{a - c}{\varepsilon} + 1\right) \\ \frac{2n}{(n + 1)^2}\, \varepsilon^2\left(\frac{a - c}{2\varepsilon} + \frac{n + 2}{4}\right)^2 & \frac{2}{3}\left(\frac{a - c}{\varepsilon} + 1\right) \leq n \leq 2\left(\frac{a - c}{\varepsilon} - 1\right) \\ \frac{m(n + 2)}{(n + 1)^2}\, \varepsilon(a - c) & 2\left(\frac{a - c}{\varepsilon} - 1\right) \leq n \end{cases}$$

Let us proceed to the case where $k > \frac{a - c}{\varepsilon}$. To simplify computations let us assume that $\frac{a - c}{\varepsilon}$ is an integer. Thus, $k > \frac{a - c}{\varepsilon}$ implies that $k - 1 \geq \frac{a - c}{\varepsilon}$. Here $\tau_i^*(\alpha) = 1$ implies that the incremental profit due to the use of the patent is less than the fee α_i. By (2), and since $k - 1 \geq \frac{a - c}{\varepsilon}$, we have

$$(10) \qquad \tau_i^*(\alpha) = 1 \quad \text{iff} \quad \alpha_i < \left(\frac{a - c + \varepsilon}{k + 1}\right)^2$$

Also,

$$(11) \qquad \tau_i^*(\alpha) = 0 \quad \text{iff} \quad \alpha_i > \left(\frac{a - c + \varepsilon}{k + 2}\right)^2.$$

Hence the fee

$$\alpha_i = \left(\frac{a - c + \varepsilon}{k + 1}\right)^2$$

for each i, i=1,...,n satisfies both (10) and (11) and is the best strategy for the patent holder in A(k) for $k > \frac{a - c}{\varepsilon}$.

In this case

$$\pi_{PH} = \frac{k(a - c + \varepsilon)^2}{(k + 1)^2},$$

that is to be maximized over $k > \frac{a - c}{\varepsilon} + 1$. Since $\frac{k}{(k + 1)^2}$ is a decreasing function of k we have

$$(12) \qquad \text{Max } \pi_{PH} = \frac{\left(\frac{a - c}{\varepsilon} + 1\right)(a - c + \varepsilon)^2}{\left(\frac{a - c}{\varepsilon} + 2\right)^2} = \frac{\varepsilon(a - c + \varepsilon)^3}{(a - c + 2\varepsilon)^2}$$

where the max ranges over $k > \frac{a - c}{\varepsilon} + 1$. This profit should be now compared with (9). For this purpose we need the following two lemmas.

<u>Lemma 2</u>: For any two real numbers x and y, $x > 1$, $y > 1$

$$\frac{(x + y)(3x + y + 2)^2}{(x + y + 1)^2} > \frac{8(x + 1)^3}{(y + 2)^2}.$$

Proof: Denote $z = x + 1$ and $w = y - 1$. Then we need to prove that

$$\frac{(z + w)(3z + w)^2}{(z + w + 1)^2} \geq \frac{8z^3}{(z + 1)^2}.$$

Let

$$A = \frac{(z + 1)^2(z + w)(3z + w)^2}{(z + w + 1)^2 z^3},$$

and let us show that $A \geq 8$.

$$A = \frac{(1 + \frac{w}{z})(3 + \frac{w}{z})^2}{(1 + \frac{w}{z + 1})^2} \geq \frac{(1 + \frac{w}{z})(3 + \frac{w}{z})^2}{(1 + \frac{w}{z})^2} = \frac{(3 + \frac{w}{z})^2}{1 + \frac{w}{z}}.$$

To complete the proof we show that

$$(3 + \frac{w}{z})^2 \geq 8(1 + \frac{w}{z}).$$

Indeed

$$(3 + \frac{w}{z})^2 - 8(1 + \frac{w}{z}) = (\frac{w}{z} - 1)^2 \geq 0.$$

Lemma 3: Let $g: [0,\infty) \to \mathbb{R}$ be defined by

$$g(x) = \frac{x(x + 2)}{(x + 1)^2}$$

then g is a nondecreasing function of x.

Proof: The proof is easy to verify.

We return now to the proof of Proposition 2. We have to compare the three cases of (9) with (12). Since $k \leq n$, the case where $k \geq \frac{a-c}{\varepsilon} + 1$ cannot be obtained for $n < \frac{a-c}{\varepsilon} + 1$. Hence only two cases need be considered.

<u>Case 1</u>: $\frac{2}{3}(\frac{a-c}{\varepsilon} + 1) \leq n \leq 2(\frac{a-c}{\varepsilon} - 1)$.

In this case we compare the profit

$$\pi^1_{PH} = \frac{2n\varepsilon^2}{(n+1)^2}\left(\frac{a-c}{2\varepsilon} + \frac{n+2}{4}\right)$$

with the profit

$$\pi^2_{PH} = \frac{\varepsilon(a-c+\varepsilon)^3}{(a-c+2\varepsilon)^2}$$

Let $x = \frac{a-c}{\varepsilon}$ and let $y = n - x$. Then

$$\pi^1_{PH} = \frac{2(x+y)\varepsilon^2(3x+y+2)^2}{16(x+y+1)^2} = \frac{1}{8}\,\frac{(x+y)(3x+y+2)^2}{(x+y+1)^2}$$

By Lemma 2

$$\pi^1_{PH} \geq \frac{(x+1)^3}{(y+2)^2} = \pi^2_{PH}$$

<u>Case 2</u>: $n \geq 2(\frac{a-c}{\varepsilon} - 1)$.

In this case we compare the profit

$$\pi^1_{PH} = \frac{n(n+2)}{(n-1)^2}\,\varepsilon(a-c)$$

with the profit

$$\pi^2_{PH} = \frac{\varepsilon(a - c + \varepsilon)^3}{(a - c + 2\varepsilon)^2}$$

that obtains when $k > \frac{a - c}{\varepsilon} + 1$. Thus, this comparison is relevant when $n > \frac{a - c}{\varepsilon} + 1$. By Lemma 3 we then have

$$\pi^1_{PH} = \frac{(\frac{a - c}{\varepsilon} + 1)(\frac{a - c}{\varepsilon} + 3)}{(\frac{a - c}{\varepsilon} + 2)^2}\varepsilon(a - c) = \frac{(a - c)(a - c + \varepsilon)(a - c + 3\varepsilon)\varepsilon}{(a - c + 2\varepsilon)^2}.$$

To prove that $\pi^1_{PH} > \pi^2_{PH}$ we show that

$$(a - c)(a - c + 3\varepsilon) > (a - c + \varepsilon)^2.$$

Indeed,

$$(a - c)(a - c + 3\varepsilon) - (a - c + \varepsilon)^2 = \varepsilon^2(\frac{a - c}{\varepsilon} - 1).$$

We now use the fact that the innovation is not drastic to obtain $\frac{a - c}{\varepsilon} - 1 > 0$. Thus, $\pi^1_{PH} > \pi^2_{PH}$ and the profit of the patent holder is then given by (9). Also, the number k^* of licensees is bounded from above by $\frac{a - c}{\varepsilon}$ and is given by (8). Finally, the formula for π^*_i follows immediately by (1) and (5).

<u>Proof of Proposition 4</u>: Assume first that the innovation is drastic, i.e., $\frac{a - c}{\varepsilon} < 1$. If $k = 1$ then since $k > \frac{a - c}{\varepsilon}$, only this licensee will produce and be charged a fee of

$$(\frac{a - c + \varepsilon}{2})^2 - (\frac{a - c}{n + 1})^2,$$

that is the difference between the monopoly profit under the new technology and the oligopoly profit under the old technology (the term $(\frac{a - c}{n + 1})^2$ is substracted to guarantee no deviation of the only licensee). If $k > 2$ then $k > \frac{a - c}{\varepsilon}$ and it is shown in the proof of Proposition 2 that

$$\alpha_i + (\frac{a - c + \varepsilon}{k + 1})^2, \quad i \varepsilon S.$$

Hence, the profit π^*_{PH} of the patent holder is

$$\pi^*_{PH} = \text{Max}[\underset{k>2}{\text{Max}} \frac{k(a - c + \varepsilon)^2}{(k + 1)^2}, (\frac{a - c + \varepsilon}{2})^2 - (\frac{a - c}{n + 1})^2]$$

$$= \text{Max}[\frac{2}{9}(a - c + \varepsilon)^2, (\frac{a - c + \varepsilon}{2})^2 - (\frac{a - c}{n + 1})^2].$$

It is easy to verify that for $n > 2$

$$\pi^*_{PH} = (\frac{a - c + \varepsilon}{2})^2 - (\frac{a - c}{n + 1})^2$$

and thus $k^* = 1 > \frac{a - c}{\varepsilon}$ and the industry becomes a monopoly.

Assume now that the industry becomes a monopoly. Then $k^* = 1$ and $k^* > \frac{a - c}{\varepsilon}$. Hence $\frac{a - c}{\varepsilon} < 1$ and the innovation is drastic.

<u>Proof of Proposition 5</u>: If the innovation is drastic then the industry becomes a monopoly (Proposition 4) realizing revenue equal to $(\frac{a - c + \varepsilon}{2})^2$ but paying a fee of $(\frac{a - c + \varepsilon}{n + 1})^2$. Hence its net profit is $(\frac{a - c}{n + 1})^2$, that coincides with its profit prior to the innovation. In this case, total production is $\frac{a - c + \varepsilon}{2}$ while the Cournot equilibrium production level prior to the innovation is $\frac{n}{n + 1}(a - c)$. Now

$$\frac{a - c + \varepsilon}{2} > \frac{n}{n + 1}(a - c) \quad \text{iff} \quad \frac{a - c}{\varepsilon} < \frac{n + 1}{n - 1}$$

Since $\frac{a - c}{\varepsilon} < 1$ and $\frac{n + 1}{n - 1} > 1$ we obtain that the postinnovation production level is strictly greater than the Cournot equilibrium preinnovation production level, and thus consumers are better off.

Assume now that the innovation is not drastic, i.e., $\frac{a - c}{\varepsilon} > 1$. By Proposition 2, $k^* < \frac{a - c}{\varepsilon}$, and

$$\pi_i^* = [(a - c)^2 - \varepsilon(k^* - 1)(2(a - c) - \varepsilon(k^* - 1))]/(n + 1)^2, \quad i \varepsilon S.$$

Now since, $2(a - c) - \varepsilon(k^* - 1) > 0$ and since $k^* - 1 > 0$,

$$\pi_i^* < (\frac{a - c}{n + 1})^2$$

Thus each licensee's profit falls below its preinnovation level. If $i \notin S$ then

$$\pi_i^* = (\frac{a - c - k^*\varepsilon}{n + 1})^2 + (\frac{a - c}{n + 1})^2,$$

and we obtain the same result. Let us now find the total production level. Since $k^* < \frac{a - c}{\varepsilon}$, we have by (1)

$$\sum_{i=1}^{n} q_i^* = \frac{k^*(a - c + (n - k^* + 1)\varepsilon)}{n + 1} + \frac{(n - k^*)(a - c - k^*\varepsilon)}{n + 1}$$

$$= \frac{n(a - c) + k^*\varepsilon}{n + 1} > \frac{n}{n + 1}(a - c).$$

Thus, in this case too, production increases and consumers are better off.

Proof of Proposition 6: Consider first an industry of n firms producing a single good with technologies

$$f_i(q_i) = c_i q_i, \quad i=1,\ldots,n.$$

By Lemma 1 each firm's profit is a decreasing function of c_i unless $c_i > \frac{a + \Sigma c_j}{n + 1}$. In the latter case the i^{th} firm produces nothing and hence makes zero proift. In our model, c_i is either $c - \varepsilon + b$ or c depending on whether i is a licensee or not. Suppose now that $b < \varepsilon$. Then each licensee's Cournot equilibrium quantity and profit are positive. A nonlicensee produces either a positive or zero quantity. In the latter case he is better off purchasing the license to the patent since he then will make positive profit. In the former case his profit is a strictly decreasing function of his marginal cost of production. Therefore, this firm is again better off purchasing the license. Hence, for $b < \varepsilon$ we have $k^* = n$ and thus by Lemma 1

$$q_i^* = \frac{a - c + \varepsilon - b}{n + 1}, \quad i=1,..,n.$$

Consequently

$$\pi_{PH} = \underset{0<b<\varepsilon}{\text{Max}} \; b(a - c + \varepsilon - b)\frac{n}{n + 1},$$

and the maximum is attained at

$$b^* = \min(\frac{a - c + \varepsilon}{2}, \varepsilon)$$

Now, if the innovation is not drastic $\frac{a - c}{\varepsilon} > 1$ and hence $b^* = \varepsilon$. In this case $\pi_{PH} = \frac{n}{n+1}\varepsilon(a - c)$. If the innovation is drastic then $\frac{a - c}{\varepsilon} < 1$ and $b^* = \frac{a - c + \varepsilon}{2}$. In this case

$$\pi_{PH} = \frac{n}{n+1}\left(\frac{a - c + \varepsilon}{2}\right)^2.$$

Let us compare these profits with those obtained when only a fee is used. We begin with the nondrastic case. Consider the following three cases:

(a) $n < \frac{2}{3}\left(\frac{a - c}{\varepsilon} + 1\right)$

(b) $\frac{2}{3}\left(\frac{a - c}{\varepsilon} + 1\right) < n < 2\left(\frac{a - c}{\varepsilon} - 1\right)$

(c) $n > 2\left(\frac{a - c}{\varepsilon} - 1\right)$

Let $x = \frac{a - c}{\varepsilon}$ and apply (9) of the proof of Proposition 2.

<u>Case a</u>: $x > \frac{3}{2}n - 1$. The difference in the two profits is given by

$$\Delta = \frac{2n^2\varepsilon^2}{(n+1)^2}\left(x + 1 - \frac{n}{2}\right) - \frac{n}{n+1}\varepsilon(a - c) = \frac{n\varepsilon^2}{n+1}\left[\frac{n-1}{n+1}x + \frac{2n}{n+1}\left(1 - \frac{n}{2}\right)\right]$$

$$> \frac{n\varepsilon^2}{n+1}\left[\frac{(n-1)(\frac{3}{2}n - 1)}{n+1} - \frac{2n}{n+1}\left(1 - \frac{n}{2}\right)\right]$$

$$= \frac{n\varepsilon^2}{(n+1)^2}\left(\frac{1}{2}n^2 - \frac{1}{2}n + 1\right) > 0.$$

<u>Case b</u>: $\frac{1}{2}n + 1 < x < \frac{3}{2}n - 1$.

$$\Delta = \frac{2n\varepsilon^2}{(n+1)^2}\left(\frac{a - c}{2\varepsilon} + \frac{n+2}{4}\right) - \frac{n\varepsilon(a - c)}{n+1}$$

$$= \frac{n}{(n+1)}\varepsilon\left[\frac{2\varepsilon}{n+1}\left(\frac{a - c}{2\varepsilon} + \frac{n+2}{4}\right)^2 - (a - c)\right].$$

Define

$$g(n) = \frac{2\varepsilon}{n + 1}(\frac{a - c}{n + 1} + \frac{n + 2}{4})^2.$$

It is easy to verify that $g'(n) < 0$. Therefore

$$g(n) > g(2(\frac{a - c}{\varepsilon} - 1)) = g(2x - 2).$$

Since the innovation is nondrastic $x > 1$ and hence

$$\frac{2\varepsilon}{n + 1}(\frac{a - c}{2\varepsilon} + \frac{n + 2}{4})^2 - (a - c) > \frac{2\varepsilon}{2x - 1}(\frac{x}{2} + \frac{2x - 2 + 2}{4}) - \varepsilon x$$

$$= \frac{\varepsilon x}{2x - 1} > 0.$$

Thus $\Delta > 0$.

<u>Case c</u>: $x > \frac{1}{2} n + 1$.

In this case

$$\Delta = \frac{n(n + 2)}{(n + 1)^2}(a - c) - \frac{n\varepsilon}{n + 1}(a - c) = \frac{n}{n + 1}(a - c)(\frac{n + 2}{n + 1} - 1) > 0.$$

Consequently, the patent holder's profit is greater with the use of a fee only than with the use of a royalty only.

Assume now that the innovation is drastic, i.e., $x < 1$. The increment in profit is

$$\Delta = (\frac{a - c + \varepsilon}{2})^2 - (\frac{a - c}{n + 1})^2 - \frac{n}{n + 1}(\frac{a - c + \varepsilon}{2})^2$$

$$= \frac{(a - c + \varepsilon)^2}{4(n + 1)} - \frac{(a - c)^2}{(n + 1)^2} = \frac{\varepsilon^2}{4(n + 1)^2}[(x + 1)^2(n + 1) - 4x^2]$$

$$= \frac{\varepsilon^2}{4(n + 1)^2}[(n - 3)x^2 + 2(n + 1)x + n + 1].$$

It is now easy to verify that $\Delta > 0$ for each n and each $x < 1$.

To establish part (3) of the proposition, notice that

$$(13) \qquad \pi_{PH} = \begin{cases} \frac{n}{n + 1}\,\varepsilon(a - c) & \frac{a - c}{\varepsilon} > 1 \\ \frac{n}{n + 1}(\frac{a - c + \varepsilon}{2})^2 & \frac{a - c}{\varepsilon} < 1 \end{cases}$$

Hence

$$\lim_{n\to\infty}\pi_{PH} = \begin{cases} \varepsilon(a - c) & \frac{a - c}{\varepsilon} > 1 \\ (\frac{a - c + \varepsilon}{2})^2 & \frac{a - c}{\varepsilon} < 1 \end{cases}$$

Part (4) of the proposition follows easily from the fact that $b^* = \varepsilon$ in case of a nondrastic innovation and $b^* = \frac{a - c + \varepsilon}{2} < \varepsilon$ in case of a drastic innovation.

To establish part (5) of the proposition notice that in case of a nondrastic innovation total production with the use of a fee is $\frac{n(a - c) + k^*\varepsilon}{n + 1}$ while with the use of a royalty it is only $\frac{n}{n + 1}(a - c)$. If the innovation is drastic, total production with the use of a fee is $\frac{a - c + \varepsilon}{2}$, while with the use of a royalty it is only $\frac{n}{n + 1}(\frac{a - c + \varepsilon}{2})$. Thus, in both cases total production obtained with the use of a fee only exceeds total production obtained with the use of a royalty only.

The last part of the proposition follows immediately by (13).

<u>Proof of Proposition 7</u>: Part (1) of the proposition is established in Kamien-Tauman (1983). For the second part of the proposition, notice first that along the lines of the proof of the first part, the number k^* of licensees in

a nondrastic innovation satisfies

(14) $$k^* < \frac{a - c}{\varepsilon - b^*}$$

In this case total production is given by (see (15) of Kamien-Tauman (1983))

$$Q = \frac{n(a - c) + k^*(\varepsilon - b^*)}{n + 1}$$

On the other hand, in the case of fee only total production Q_1, under a nondrastic innovation and $n > 2(\frac{a - c}{\varepsilon} - 1)$, is given by

$$Q_1 = a - c$$

Thus $Q > Q_1$ is equivalent to $k^* > \frac{a - c}{\varepsilon - b^*}$, that by (14) holds only for $k^* = \frac{a - c}{\varepsilon - b^*}$. This, together with (4) of Kamien-Tauman (1983) implies that

$$\pi_{PH} = b[\frac{a - c}{(n + 1)^2}] + \frac{n(n + 2)}{(n + 1)^2}\,\varepsilon(a - c)$$

that is maximized at $b^* = \varepsilon$, contradicting $k^* = \frac{a - c}{\varepsilon - b^*}$. Hence, $Q_1 > Q$ as claimed. Finally, notice that if the innovation is drastic both games G and G_1 yield the same level of production, namely, $Q = Q_1 = \frac{a - c + \varepsilon}{2}$.

<u>Proof of Proposition 8</u>: Denote by $\pi^f(k^*)$ the patent holder's equilibrium profit with the use of a fee only and under zero contracting cost (i.e., $d \equiv 0$). Denote by $\pi^{fb}(k_n^*, b_n^*) - d(k_n^*)$ and by $\pi^{fb}(\hat{k}_n^*, \hat{b}_n^*)$ the patent holder's equilibrium profit with contracting cost $d(\cdot)$ and with zero contracting cost, respectively, (b_n^* and $\hat{b}_n^*$ are the equilibrium royalties of these two cases, respectively.) The index n of k_n^* and b_n^* refers to an industry of n firms.

This index has been omitted from $\pi^f(k^*)$ since in this case by Proposition 2 for n sufficiently large $k^* = \frac{a-c}{\varepsilon}$ and k^* does not depend on n.

Lemma 4: For n sufficiently large the sequence $(k_n^*)_{n=1}^{\infty}$ is bounded above by $\frac{a-c}{\varepsilon}$.

Proof: Notice first that the proposition is obviously correct if $d(\infty) = \infty$. Thus, let us assume that $d(\infty) < \infty$. Let $(k_n^*)_{j=1}^{\infty}$ be a subsequence of $(k_n^*)_{n=1}^{\infty}$ such that $n_j \to \infty$ as $j \to \infty$ and let $\sigma = \lim_j k_{n_j}^*$ (possibly $\sigma = \infty$). Assume now that $\sigma > \frac{a-c}{\varepsilon} = k^*$.

For each $\delta > 0$ there exists an integer n_0 such that $n > n_0$ implies

$$\pi^{fb}(k_{n_j}^*, b_{n_j}^*) - d(k_{n_j}^*) \leq \pi^{fb})k_{n_j}^*, b_{n_j}^*) - d(\sigma) + \delta$$

$$\leq \pi^{fb}(\hat{k}_{n_j}^*, \hat{b}_{n_j}^*) - d(\sigma) + \delta.$$

By Proposition 7

$$\lim_{j\to\infty} \pi^{fb}(\hat{k}_{n_j}^*, \hat{b}_{n_j}^*) = \pi^f(k^*)$$

Hence for j sufficiently large

$$\pi^{fb}(k_{n_j}^*, b_{n_j}^*) - d(k_{n_j}^*) \leq \pi^f(k^*) - d(\sigma) + 2\delta$$

$$= \pi^f(k^*) - d(k^*) + 2\delta - (d(\sigma) - d(k^*)).$$

Since $d(\cdot)$ is increasing function and since $\sigma > k^*$ we have $d(\sigma) > d(k^*)$ and hence, for $0 < \delta < \frac{1}{2}(d(\sigma) - d(k^*))$

$$\pi^{fb}(k^*_{n_j},b^*_{n_j}) - d(k^*_{n_j}) < \pi^{f}(k^*) - d(k^*).$$

This means that with the use of a fee only, the patent holder can extract more profit than he could under any combination of fee and royalty, which is a contradiction. Hence, $\sigma \leq \frac{a - c}{\varepsilon}$ and the proof of the lemma is complete.

We return now to the proof of Proposition 8. By Lemma 4, $k^*_n \leq \frac{a - c}{\varepsilon}$ and hence for any $b^*_n > 0$ $k^*_n < \frac{a - c}{\varepsilon - b^*_n}$. Thus, by expression (4) of Kamien-Tauman (1983)

$$\pi^{fb}(k^*_n,b^*_n) = \max_{0\leq b\leq\varepsilon} \pi^{fb}(k^*_n,b) \tag{15}$$

$$= \max_{0\leq b\leq\varepsilon} \Big\{\frac{nk^*_n}{n + 1}[2(a - c)(\varepsilon- b) + (n - 2k^*_n + 2)(\varepsilon - b)^2]$$

$$+ \frac{k^*_n}{n + 1}[b(a - c) + (n - k^*_n + 1)b(\varepsilon - b)]\Big\}$$

Since

$$\frac{d\pi^{fb}}{db}(k^*_n,b) = \frac{nk^*_n}{(n + 1)^2}[-2(a - c) - 2(n - 2k^*_n + 2)(\varepsilon - b)]$$

$$+ \frac{k^*_n}{n + 1}[a - c + (n - k^*_n + 1)(\varepsilon - 2b)]$$

we have that $\frac{d\pi^{fb}}{db} > 0$ if and only if

$$b < - \frac{(n - 2)(a - c) + (n^2 - 3nk^*_n + 2n + k^*_n - 1)\varepsilon}{2nk^*_n - 2k^*_n + 2} \equiv b^0_n.$$

Using $k^*_n \leq \frac{a - c}{\varepsilon}$ we obtain for n sufficiently large that $b^0_n < 0$.

Consequently, the maximizer of (15) is $b_n^* = 0$. This means that the patent holder uses, for n sufficiently large, only a fee and no royalty.

Proof of Proposition 9: Assume first that $k < \frac{a-c}{\varepsilon}$. Without loss of generality $t_1 = t_2 = \ldots = t_n \equiv T$. To avoid the deviation of a nonlicensee it is required that

$$(1 - T)\left(\frac{a - c + (n - k + 1)\varepsilon}{n + 1}\right)^2 = \left(\frac{a - c - (k - 1)\varepsilon}{n + 1}\right)^2.$$

Thus

$$1 - T = \left(\frac{a - c - (k - 1)\varepsilon}{a - c + (n - k + 1)\varepsilon}\right)^2$$

Consequently the profit of the patent holder is

$$\pi_{PH} = Tk\,\pi_i, \quad i \in S \tag{16}$$

Since

$$T = 1 - \left(\frac{a - c - (k - 1)\varepsilon}{a - c + (n - k + 1)\varepsilon}\right)^2 = \frac{n\varepsilon[2(a - c) + (n - 2k + 2)\varepsilon]}{[a - c + (n - k + 1)\varepsilon]^2}$$

$$= \frac{n\,\varepsilon\,k}{(n + 1)^2}\,[2(a - c) + (n - 2k + 2)\varepsilon]$$

The last expansion coincides with (6) which is the profit of the patent holder under a fee only. The case where $k > \frac{a-c}{\varepsilon}$ is treated similarly.

8. Conclusion and Remarks

We have analyzed the determinants of the private value of a patent. We find that the private value of a patent depends on the method of licensing employed and, of course, on the magnitude of the innovation as measured by how much it reduces the marginal cost of production. We find that the different means of licensing a patent are not equivalent either from the standpoint of the patent holder, the licensees, or consumers. Licensing by means of a fee appears to be the most attractive from the standpoint of the patent holder, especially in the presence of contracting costs, a very plausible assumption. A fee is also desirable from the consumers' standpoint, as it leads to lower prices. Our analysis also discloses that the means of licensing a patent affects the structure of an industry. Thus, a royalty alone does not affect the final number of firms in an industry, while a fee either alone or in combination with a royalty may.

There are a large number of possible extensions of this work. The most obvious involve relaxing of the assumptions of complete information, no uncertainty, linearity of the cost function and of the demand function, and the abscence of competing innovations. In addition it would be interesting to know if these results carried over to the licensing of product innovations.

Notes

[1]It should be clarified in what sense the equilibrium is unique. the number k^* of licensees and the fee α_i^* of each licensee are uniquely determined. However, any subset of k^* firms of the total n firms can result in equilibrium as the set S of licensees. Also the fee α_i^* of each nonlicensee $i \notin S$ is not uniquely determined since any $\alpha_i > \alpha_i^*$, $i \notin S$, will deter i from purchasing the license. Nevertheless, there is a unique nondiscrimatory equilibrium fee.

[2]We use the short term equilibrium for the subgame perfect equilibrium of Proposition 1.

[3]This question was asked in various seminars on this topic.

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