UNCERTAIN LIFETIMES, SOCIAL SECURITY, AND INDIVIDUAL SAVING

by

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Uncertain lifetimes create a demand for annuities to provide for consumption in old age. Because such markets may be imperfect or missing, compulsory participation in social security system may fulfill some of this demand. A simple life-cycle model is used to show that a fair, fully funded social security system can reduce individual saving by more than the tax paid. Moreover, if access to the social security annuities is rationed by income because of tax and replacement rates which are nonlinear in earnings, saving rates will rise with earnings, even in the absence of a bequest motive. While the partial equilibrium impact of social security on individual saving is larger than that found in studies in which lifetime is uncertain, long-run effects may be smaller, because of the impact of social security on intergenerational transfers ("accidental bequests").

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INTERNATIONAL STOCKPILING AGREEMENTS AS RESPONSES TO
COMMODITY SHOCKS

by

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ABSTRACT

This paper investigates the potential of buffer stock schemes for reducing the price impacts of transitory supply shocks in commodity markets. We focus on the oil market, both as an archetype of international commodity markets subject to supply shocks, and because its enormous size and importance to the well-being of industrialized economies has made it a concern for public policy. The issue is viewed from the standpoint of the large importing countries; since the market is integrated worldwide, the actions of one have spillover effects on the others. We formulate the optimization problem of national stockpile authorities and contrast the cooperative and noncooperative solutions to the international stockpile game. In general, whether the response for a given country is greater under a collusive agreement than in a noncooperative equilibrium depends on the oligopoly potential generated by collusion and on the persistence of a transitory shock’s effects on prices. The paper examines the foundations of “persistence” in the contract structure of the market and shows how changes in "persistence" affect the desirability of cooperation. Finally, the principal results are reviewed to assess the viability of currently proposed international agreements.

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I. INTRODUCTION

The past decade has been one of violent fluctuations in international commodity markets. The "commodity boom" has been most pronounced in the oil market, where transitory supply shocks in 1973-1974 and in 1979 have dislocated established trading patterns, inflicted substantial damage on the OECD economies, effected enormous wealth transfers to oil-exporting countries, and resulted in large and persistent increases in oil prices. The average price of a barrel of crude oil stood at roughly $2 in 1970, $12 in 1975, and $30 in 1980.

The large macroeconomic costs of oil supply shocks have elicited studies along several dimensions. Detailed analyses of the economic transmission of the first oil shock can be found in Gordon (1975) or in the volume edited by Fried and Schultz (1975). Sachs (1979) has focused on the role of factor price responses (of the real wage rate and real interest rate). Bruno (1982) and Bruno and Sachs (1982) have considered the role of the increase in the relative price of energy in explaining the slowdown in productivity experienced by most industrial nations in the middle and late 1970s. The extent to which oil price increases may be responsible for current account movements has been discussed by Sachs (1981). Krugman (1983) looked at links between oil price increases and appreciation of the U. S. dollar. The Energy Modeling Forum at Stanford University is conducting a study of the economic costs of oil supply shocks using a set of macroeconometric models. (See Hickman and Huntington, 1982).

Economists have proposed and examined policies to alleviate the adverse effects of supply shocks (Gordon, 1975; Gramlich, 1979; Mork and Hall, 1980), but have had relatively little to say about mitigating the
shocks themselves. Inasmuch as macroeconomic costs are related to the magnitude of the shocks, the use of policies designed to act directly on oil prices suggests itself. Our focus here is on one such policy—the use of a national oil stockpile. The United States has had such a stockpile (known as the Strategic Petroleum Reserve) since 1975.

Since the oil market is internationally integrated, the use of a buffer stock by one country has spillover effects on others. The possibility of international policy coordination thus becomes important in attempting to reduce the transitory oil shocks. Our investigation parallels a recent strand of the macroeconomics literature (Johansen, 1982; Canzoneri and Gray, 1983; Sachs, 1983) that compares international coordination of stabilization policies with noncooperative (Cournot-Nash) equilibria.

While issues of the optimal size of public oil stockpiles have been discussed and empirical estimates of the impact of stockpile releases on world oil prices have been made (see Hubbard and Weiner, 1983b; and U.S.G.A.O., 1983), virtually no attention has been paid to optimizing public stockpile behavior. Second, while empirical studies have found merits in terms of lower prices of international stockpile coordination, issues of whether such an outcome would occur in the absence of an agreement and of what types of institutional mechanisms might facilitate cooperation have been largely ignored.

The goal here is to characterize optimal stockpiling behavior of consuming countries in a market subject to transitory supply shocks and to examine the ability of coordinated agreements to enhance the benefits of such behavior. Toward those ends, the paper is organized as follows. Section II develops intertemporal optimizing models of private
and public stockpiling, comparing their motivations and implications. Since our focus is on market behavior in the short run, our formulation of supply neglects the exhaustible nature of the resource. Stockpiles held by consumers, however, are similar to reserves, lending a Hotelling flavor to the problem.

Of particular importance is the role of "persistence" of the price impacts of transitory shocks. Knowledge of the intertemporal correlation of the impacts of shocks is central to the analysis of optimal stockpiling schemes. Treating public stockpile behavior as an international game, noncooperative and cooperative solutions are contrasted. In general, whether the response for a given country is higher under a collusive agreement than in a Cournot-Nash equilibrium depends on the oligopoly potential generated by collusion and on the persistence of a transitory shock's effects on prices. With no intertemporal correlation of prices, the noncooperative response will be too large. At high levels of persistence, the noncooperative solution is Pareto inferior to cooperation, and, in particular, stockpile responses to shocks will be "too small."

Section III takes up the issue of persistence by examining its foundations in the contract structure of the oil market, which is characterized by trades on both "spot" and "contract" markets. Here persistence is shown to depend on, inter alia, the price elasticity of demand and the fraction of trades carried out through contracts. Changes in these parameters alter the relationship between the noncooperative and cooperative solutions of section II. The fourth section reviews the principal results of the paper in assessing the viability of proposed international agreements. Conclusions and directions for future research are given in section V.
II. INTERTEMPORAL OPTIMIZING MODELS OF PRIVATE AND PUBLIC STOCKPILING

A. Optimal Private Stockpiling

As a point of reference, we begin with a model of private stockpile behavior. The emphasis is on speculative stockpiling, that is, inventory acquisition for the purpose of a future sale on the anticipation of substantial price appreciation. We assume that firms maximize the present value of expected future profits. Inventory adjustment is assumed to be costly—in fact, increasingly costly—in the size of the adjustment due to rising payments to factors fixed in the short run, in this case storage facilities, tankers, and pipelines. Thus changes in price expectations cannot be fully acted upon instantaneously. We follow the literature in modeling these costs as quadratic, the simplest specification of "diminishing returns."

The notation is as follows: Stock levels are measured at the end of period \( t \). Firms (assumed to be price-takers) buy oil in period \( t \) at price \( p_t \), and expect to sell at price \( p_{t+1} \) next period. The stream of expected future profits is discounted at rate \( c \). Holding costs have a linear component at a constant rate \( c \); the cost of adjustment parameter is denoted by \( a \).

The firm's optimization problem can be written:

\[
(1) \quad \max_{I_t} \mathbb{E} \left\{ (1 + \delta)^{-t} \left( (1+\delta)^{-1} p_{t+1} I_{t+1} - p_t I_t - c I_t + \frac{b}{2} (I_t - I_{t-1})^2 \right) \right\},
\]

subject to \( I_{t+1} = I_{t+1} - R_t + X_{t+1} \), where \( R \) and \( X \) are the firm's sales and purchases respectively. \( \mathbb{E} \) denotes the expectation operator (conditional on the information set in period \( t \)).
Differentiation of (1) with respect to $I_t$ yields the Euler equations:

\[ E_t ((1 + \delta)^{-1}[(1+\delta)^{-1}p_{t+1} - p_t - c - h(I_t - I_{t-1}) = 0. \]

This is a second-order linear inhomogeneous difference equation in inventories, with boundary conditions given by the size of the initial stock and the requirement that

\[ \lim_{t \to \infty} ((1 + \delta)^{-1}[p_{t+1} - p_t - c - h(I_t - I_{t-1})] = 0. \]

The two roots are 1 and $1 + \delta$, so the particular solution is given by:

\[ I_t - I_{t-1} = h^{-1} \sum_{k=0}^{\infty} \frac{((1 + \delta)^{-1}p_{t+k+1} - p_{t+k} - c). \]

Thus, inventory changes are a function of adjustment costs (through the parameter $h$) and of expected profits, with geometrically declining weights into the future.

### B. Optimal Government Stockpiling

Understanding the motives for public stockpiling is important not only for a realistic analysis of the response of public reserves to price movements in the world oil market, but also for evaluating the viability of particular international agreements. Agreements whose provisions run in opposition to the optimizing behavior of the various nations involved are unlikely to prove workable and successful in a crisis.
The public stockpile is to be used in accord with each country $j$'s assumed economic policy of maximizing the present discounted value of real income (output less imported intermediate goods). In each country $j$, output ($y_{jt}$) of a single final good is produced from oil ($Q_j$) and other factors ($X_j$) according to the production function

$$y_{jt} = f_j(j_t, X_j); f_{j1}, f_{j2} > 0, \text{ and } f_{j11}, f_{j22} < 0.$$  

Each nation imports all of its oil, which is the only imported intermediate input. Non-oil factor supplies are fixed, so that $f$ is separable. Oil use depends negatively on its relative price $p/P$, where $p$ and $P$ are the prices of (imported) oil and output. For simplicity, we make the produced good the numeraire, so that $P$ equals unity. Hence,

$$Q_j = Q_j(p), \quad Q_j' < 0.$$  

At first, we assume that only one oil price, $p$, prevails in the market and that it is determined according to

$$p_t = p\left(\sigma - \frac{c_t}{\sum_m S_m}\right) + c_t', \quad p' > 0,$$

where $S$ represents the net release of stockpiled oil from the public inventory.

The stockpile authority's objective is to maximize the discounted present value of real income (by minimizing oil price increases) less the cost of carrying out the stockpile program and of adjusting
stockpile levels, subject to the constraint that stockpile releases not exceed the amount of oil held in the reserve. The problem for each country \( j \) is to choose the stockpile level \( I_j \) (or, equivalently, the net stockpile release \( S_j \)) in each period \( t \) so as to:

\[
(7) \quad \max_{I_j, t+1} \sum E_t [(1 + \delta_j)^{-1} (y_j, t+1 - p_{t+1} Q_j, t+1 + (1 + \delta_j)^{-1} (p_{t+1} I_j, t+1 - p_{t+1} I_j, t+1 - c I_j, t+1 - \frac{b_j}{2} (I_j, t+1 - I_j, t+1 - 1)^2)],
\]

subject to the constraint that

\[
(8) \quad I_j, t+1 = I_j, t+1 - S_j, t+1 ,
\]

where \( \delta_j \) is the discount rate in the \( j \)th country. Again, the quadratic term is a proxy for the cost of adjusting stock levels.

There are clear distinctions between the optimization problem for the public stockpile authority and the problem for the private firm stated earlier. Most obvious is the attention paid by the public authority to aggregate output. Private firms do not consider the macroeconomic effects of their stockpiling behavior; that is, they do not consider the impact of their transactions on the world oil price. Second, the behavior of other countries is important. Because the stockpiling decisions of other countries affect the oil price, they can affect the optimal release strategy of the domestic authority.

Because the market for oil is a world market, price outcomes from one country's stockpile movement depend on the actions of other countries. The problem is inherently game-theoretic. As a base case, we can consider the noncooperative solution, wherein players do not
consider the beneficial impact of their own actions on the others. Each country takes the stockpiling decisions of the others as given, then selects its own stockpile level. As a result, this solution does not fully exploit the positive externalities associated with stockpile policy.

If the discount rates and stock adjustment parameters are the same across countries, so that \( \delta_j = \delta, \Psi_j, \) and \( h_j = h, \Psi_j, \) then the solution to (7) can be written as:

\[
I_{j,t} = \lambda_j^{i} I_{j,t-1} + h^{-1}(1+\delta) \lambda_j^{-1} \sum_{k=0}^{\infty} \lambda_j^{-k} t^k p_{t+k+1} - p_{t+k} - c l
\]

\[
- h^{-1}(1+\delta) \lambda_j^{-1} \sum_{k=0}^{\infty} \lambda_j^{-k} \frac{dp}{dI_j} I_{j,t+k}^k,
\]

where \( \lambda_j < 1, \Psi_j, \) and where \( \lambda_2 j > 1+\delta, \Psi_j, \) \( \frac{dp}{dI_j} \) is written with respect to the conjecture about the reaction of other countries; \( \frac{dp}{dI_j} I_{j,t} \) is evaluated for changes in \( I_{j,t} \) alone.

Examining (9), we can consider the impacts of changes in expected future prices on public stockpiling behavior. The expected impact of shocks in the current period on future oil prices plays an important role here. Note that we can rewrite (9) as

\[
I_{j,t} = \lambda_j^{i} I_{j,t-1} + h^{-1} \lambda_j^{-1} (1+\delta) p_t + (1-\lambda_j^{-1})(1+\delta) \sum_{k=1}^{\infty} \lambda_j^{-k} t^k p_{t+k}
\]

\[
- h^{-1}(1+\delta) \lambda_j^{-1} \sum_{k=0}^{\infty} \lambda_j^{-k} \frac{dp}{dI_j} I_{j,t+k}^k.
\]
An explicitly one-period increase in the price will induce a stockpile release since \( \frac{\partial I_j}{\partial p} e^{-h(1+i)^{-1}} + \frac{\partial q_j}{\partial I_j} q_j < 0 \). However, as in the case of a profit-maximizing firm, higher expected future prices, ceteris paribus, lead to larger stockpiles (smaller releases) today. That impact is smaller in the "public" case than in the "firm" case because the former takes into account the fact that the stockpile release affects current-period oil prices, i.e., because of the benefits of a stockpile release in terms of inframarginal oil consumption (imports).\(^{10}\)

What is required to determine the optimal stockpile policy is knowledge of the intertemporal correlation of prices. If \( \frac{\partial I_j}{\partial p} e^{-h(1+i)^{-1}} \) is small, then transitory shocks exhibit little persistence, and the optimal stockpile response is a drawdown. The greater is the intertemporal correlation, the less likely is a release at the onset of a shock.

C. EFFECTS OF COOPERATION ON OPTIMAL PUBLIC STOCKPILE LEVELS

To evaluate the benefits of cooperation, we analyze the case of perfect collusion, where a single stockpile authority maximizes the joint benefits of reserve management. This case provides a measure against which alternative solutions ("agreements") can be judged. Using the assumptions which generated (9), Pareto-optimal policies are given by sequences \( [I_{1t}], [I_{2t}], \ldots, [I_{Mt}] \) that maximize \( \sum_j \mathcal{G}_j \), where

\[
\mathcal{G}_j = \sum_{t=0}^{\infty} (1+i)^{-t} \left[ y_{j,t+1} - P_{t+1} q_{j,t+1} - (1+i)^t \left( p_{t+1} I_{j,t+1} - P_{t+1} I_{j,t+1} - c I_{j,t+1} - \frac{h}{2} (I_{j,t+1} - I_{j,t+1})^2 \right) \right].
\]

That is,
(11) \[
\max \sum_{i_1, \ldots, i_m} \sum_{j=1}^n \left(1 + \delta \right) \cdot \left[ y_{j,t+i-1} + \frac{1}{2} \left( p_{t+i} - p_{t+i+1} \right) \right] + \sum_{i=1}^n \sum_{t=1}^i \left( 1 + \delta \right) \cdot \left[ t_{j,t+i-1} - c_i \right] + \sum_{t=1}^i \sum_{j=1}^n \left( \frac{1}{2} \left( I_{j,t+i} - I_{j,t+i-1} \right)^2 \right)
\]
subject to the constraint that

(12) \[
\sum_{j=1}^n I_{j,t+i} = \sum_{j=1}^n I_{j,t+i-1} - \sum_{j=1}^n S_{j,t+i}.
\]

The solution to (11) can be expressed for each country \( j \) as:

(13) \[
I_{j,t} = \sum_{k=0}^{\delta-1} \frac{\lambda_{j,t-1}^k}{h_{j,k}} \left( \frac{dp_{j,j,t-1}}{dt_{j,j,t-1}} \right) \cdot q_{j,t+k}.
\]

Because \( \frac{dp_{j}}{dt_{j}} \) in the cooperative case is evaluated considering all stock changes (i.e., because \( \frac{dp_{j}}{dt_{j}} > \frac{dp_{j}}{dt_{j}} \)), \( \lambda_{j,t}^* \) and \( \lambda_{j,t}^* \) are used.

The question is thus the following. Given an oil shock, how will the sizes of the stockpiles (stockpile releases) under the noncooperative and cooperative solutions diverge? From (9), the sum of the stocks in the noncooperative solution is:

(14) \[
\sum_{j=1}^n \sum_{t=1}^i \sum_{j=1}^n \lambda_{j,t-1}^k \cdot \left( \frac{dp_{j,j,t-1}}{dt_{j,j,t-1}} \right) \cdot q_{j,t+k} - c_{t+k} - \sum_{j=1}^n \sum_{t=1}^i \sum_{j=1}^n \lambda_{j,t-1}^k \cdot \left( \frac{dp_{j,j,t-1}}{dt_{j,j,t-1}} \right) \cdot q_{j,t+k}.
\]

Under the cooperative solution, the sum can be expressed as:
\[
(15) \quad \frac{\delta N}{\delta p} = \sum_{j=1}^{M} \frac{b^j}{1 - \lambda_2^j \cdot (1 + \delta)^{-1} \cdot \frac{\delta p}{\delta p_{k+1}} \cdot p_{k+1} - c} \]

\[
- h^{-1} (1 + \delta)^{-1} \lambda_2^j \sum_{k=0}^{\infty} \frac{\delta p}{\delta p_j} \cdot (\lambda_2^j \cdot 1)^{j+1} \cdot 0 \cdot t + k, \]

where the superscripts \( N \) and \( C \) denote noncooperative and cooperative, respectively.

Comparing (14) and (15) reveals the importance of intertemporal correlation of prices ("persistence") and the oligopoly power of the importing countries in the world oil market, since \( \frac{\delta p}{\delta p_j} \leq \frac{\delta p}{\delta p_j} \cdot 1 \). Our empirical work in an earlier paper (Hubbard and Weiner, 1987b) corroborated this finding. Using a simulation model of oil price determination, we found substantial increasing returns to coordination in terms of lower world oil prices.

The solutions presented here make clear that whether the stockpile release in response to a shock in the current period is larger under coordination or under the noncooperative solution depends on certain underlying parameters—the slope of the supply function \( (p') \), the price elasticities of demand in consuming countries \( (\{q_i\}) \), and persistence \( \{1 + \delta\} \). The difference between the cooperative and noncooperative solutions can be ascertained from

\[
(16) \quad \frac{\delta N}{\delta p} = h^{-1} (-2 \lambda_2^j (1 + \delta) \left( 1 + \delta \frac{\delta p}{\delta p_j} \cdot q_i^{j+1} \right) + (1 - \lambda_2^j (1 + \delta)) + \sum_{k=1}^{\infty} \lambda_2^j \delta p_{k+1} / \delta p_j) \\
- (1 + \delta) \left( \frac{\delta p}{\delta p_j} \cdot q_i^{j+1} \sum_{k=1}^{\infty} \lambda_2^j \delta E_t p_{k+1} / \delta p_j),
\]

and
\[
(17) \quad \frac{dI_t^C}{dP_t} = -l_2^{s-1}(1+\delta)(1 + \frac{dP_t}{\delta}) \left( I_j \right)^Q_j + (1-\lambda_2^{s-1} (1+\delta)) \sum_{k=1}^{m} \lambda_2^{s-k} \left( dE_t P_{t+k}/dP_t \right).
\]

Ignoring all periods but the current period, the optimal stockpile response under cooperation is actually smaller than that which would prevail in the Cournot-Nash equilibrium. This is true because, as countries perceive their collective influence on the price, the stockpile response required to achieve a given stabilization of prices is smaller.

Persistence of the effects of shocks complicate matters, however. As the intertemporal correlation of prices is increased, the current-period response is dampened more in the noncooperative solution than in the cooperative solution, so that at very high levels of persistence, the stockpile responses in the Cournot-Nash equilibrium will be suboptimal.

What factors could account for persistence? In a textbook neoclassical market, a transitory shock should not affect expected future prices. In the next section, we abandon the neoclassical supply assumption and consider the evolution of persistence from the existence of contracts in the oil market. We investigate the effect of contract structure on optimal stockpile coordination, then proceeding to discuss the implications of the conditions described above for the viability of various international stockpile agreements in section IV.
III. SUPPLIER BEHAVIOR AND THE FOUNDATIONS OF PERSISTENCE

As the macroeconomic costs of an oil supply shock depend in part on the increase in the oil price, the ultimate benefits of public stockpile "rules" or their international coordination must be measured in terms of oil price reduction. Again, the intertemporal dimensions of the problem surface. From equation (9), we see that the extent of the optimal public stockpile response depends on, inter alia, the course of expected future prices. That is, it is important to evaluate the intertemporal correlation of current and expected future prices (that is, the persistence effects on oil prices of oil shocks).

In this section, we discuss the dynamic response of oil prices to shocks in order to more realistically evaluate equations (14) or (15) and the ability of coordinated policies to reduce either the initial fillip to oil prices or its persistence. Here, the structure of the oil market (particularly with respect to contracts) becomes relevant. An important characteristic of the world oil market is its "two-price" structure, with a slowly adjusting contract price and market-clearing spot price. If there were a single spot market price of oil, then transitory shocks could exert no persistence; there would be a one-period change in the price. Unless the shocks were themselves correlated, there would be no reason to expect persistence. The existence of long-term contracts implies, however, that the persistence of transitory supply fluctuations depends on the ability of contract provisions to adjust to market conditions. Shifts in either the term-structure of contracts or in the mix of spot and contract trades can alter the short-run and long-run impacts of shocks on prices.15
Consider the following organization of the oil market. There are two types of oil producers: OPEC and non-OPEC. Non-OPEC producers are assumed to be "price takers," supplying along a marginal cost curve. OPEC acts as a dominant-firm monopolist, satisfying residual demand at its profit-maximizing price. As noted above, we have simplified producer behavior in order to focus on buffer stocks held by consumers. A more complex model would allow suppliers to act as dynamic optimizers. We introduce the two-price system into the model by allowing a fraction $\alpha$ of world trade to be accomplished through long-term contracts with producers; the remaining portion is traded on the spot market. The system is subject to demand shocks ($\epsilon_{D_t}$) and supply shocks ($\epsilon_{S_t}$), which are assumed to be independently and identically distributed with mean zero and variance $\epsilon_t^2 = \delta_{D,S}$.

Given this market structure, one can easily construct a demand-price relationship in the long-run equilibrium in which the spot and long-term contract prices are equal. The demand curve faced by OPEC ($q^D_{OPEC}$) is obtained by subtracting non-OPEC supply ($q^S_{NO}$) from total demand ($q^D$). The optimal contract price ($p^*$) and quantity ($q^*$) are obtained by equating OPEC's marginal revenue and marginal cost. At that price, OPEC supplies $q^*_{OPEC}$ The balance, $q^* = q^*_{OPEC}$, is supplied competitively. That sequence of price determination is an equilibrium story. Below, attention is focused on the adjustment of spot and contract prices to shocks.

Consider the impact of a partial interruption in the supply of oil from OPEC on spot and contract prices. Given a negative supply shock $\epsilon_{St}$ and an unchanged contract price, there is excess demand in the contract market, i.e.,
where a caret over a variable denotes its value in the initial equilibrium. Demand then spills over into the spot market. As before the price of crude oil to consumers and refiners is

\[ p_t^* = a \rho_{ct}^* + (1 - a) p_t^S, \]

where \( p^* \) and \( p^S \) are the long-term contract price and the spot price, respectively. Then the spot price solves

\[ (1 - a)q^D(p_t^*) + \epsilon_{D_t} - \epsilon_{St} = q^S(p_t^S), \]

where the willingness of producers to supply oil on the spot market depends positively on the spot price. Under the simplifying assumption of linear responses of supply and demand to price, (20) can be rewritten as

\[ -(1 - a)f(c_p^* + (1 - a)p_t^S) + \epsilon_{Dt} - \epsilon_{St} = \epsilon_{St}, \]

so that

\[ p_t^S = (1 - a)q_t^* + \epsilon_{Dt} - \epsilon_{St}. \]

Holding \( a \) fixed in the short run, we can totally differentiate (22) to evaluate the impact of an oil supply interruption on the spot price in the current period, i.e.,

\[ \frac{\Delta p_t^S}{\Delta St} = \beta - f_a(1 - a) \frac{dp_t^*}{\Delta St}, \]
where

\[
\theta = (g + (1 - a)^2 \beta)^{-1}.
\]

A supply interruption raises the spot price in the short run. This reaction to shocks in the oil market goes in the other direction for a negative demand shock in the contract market, indicating that, to the extent that contracts are honored (i.e., that the contract price does not change), the effects of shocks on the spot price are destabilizing. The ultimate impact on the spot price of negative supply shocks depends on the way in which contract prices adjust to market imbalances.

Formalizing the adjustment process of long-term contract prices is a crucial step toward understanding price dynamics in the oil market and evaluating the merits of government policy responses. The previous discussion of OPEC's setting the contract price \( p^* \) as a dominant-firm monopolist was in the framework of a static equilibrium. Over time, adjustments in the contract price must be able to distinguish between transitory shocks to demand or supply and changes in the underlying parameters, such as the price elasticity of demand. Suppose that at time \( t \), the contract price is determined according to

\[
p^*_t = \gamma (y_t^+ X_t + \gamma X_t^+ X_{t+1}),
\]

where \( X \) represents net excess demand for contract oil in the market.\(^{19}\)

In particular,
(26) \[ x_t = q_t^D(p_t^*) + \epsilon_{Dt} - \epsilon_{St}. \]

Under the simplifying assumptions from before,

(27) \[ x_t = -fp_t + \epsilon_{Dt} - \epsilon_{St}. \]

The contract price is set after considering the history of prices and the expectation of the price to prevail next period. Excess demand factors are also important, since they carry information about short-run and long-run market conditions. Finally, demand and supply shocks are introduced.

Combining (25) and (27), we have that

(28) \[ p_t^* = \tau_1 p_{t-1} + \tau_2 p_{t+1} + \gamma[\tau_3(-fp_t + \epsilon_{Dt} - \epsilon_{St}) + \tau_4(-fp_t p_{t+1})]. \]

Using the definition of the composite price and assuming that the weights on past and expected future prices sum to unity, it is possible to combine terms, simplifying (28) as

(29) \[ p_t^*(1 + \gamma p_t a(1 - f(1 - a)^2)) \]

\[ = \tau_1 p_{t-1} + \tau_2 p_{t+1} + (1 - \tau_1)(1 - \gamma a(1 - f(1 - a)^2))\epsilon_{Dt}^* \]

\[ + \gamma \epsilon_{1}(1 - f(1 - a)^2)(\epsilon_{Dt} - \epsilon_{St}). \]
If expectations are rational, solving the second-order inhomogeneous difference equation by standard methods yields

\[ p_t^* = \left[ (1 + \gamma (1 - f(1 - a)^2 \beta)) - \psi (1 - \epsilon) (1 - f(1 - a)^2 \beta) \right]^{-1} \times \left[ \epsilon (p_{t-1}^* + \gamma (1 - f(1 - a) \beta) (c_{dt} - c_{ge}) \right], \]

where

\[ \psi = \frac{(1 + \gamma (1 - f(1 - a)^2 \beta))}{\gamma (1 - \epsilon) (1 - f(1 - a)^2 \beta)} \]

\[ \times \left[ (1 + \gamma (1 - f(1 - a)^2 \beta))^2 - 4 \gamma (1 - \epsilon) (1 - f(1 - a)^2 \beta)) \right]^{1/2} \]

\[ \frac{2(1 - \gamma) (1 - f(1 - a)^2 \beta))}{2(1 - \epsilon) (1 - f(1 - a)^2 \beta))}. \]

Consider first the case in which \( c_{ge} < 0 \), i.e., an oil supply interruption. The immediate impact of the supply reduction on the contract price depends on the sensitivity of contract prices to excess demand in the oil market, on the extent to which price determination is "backward-looking," and on the price elasticity of demand. The shock not only generates an immediate increase in the price, but the effect of the shock persists even when the shocks are in no way serially correlated because of the backward lookingness of price setting.²¹

The greater the extent to which price setting depends on the information contained in past prices, the greater is the persistence of the shock, since \( \phi / \partial_t > 0 \). The more elastic is the demand for oil, the smaller is the initial increase in price and the lower is the persistence. When the future is not considered at all in price setting \( (\epsilon = 1) \), the reaction to transitory shocks is just as great as the reaction to a permanent shift in demand.
The equations governing the evolution of prices illustrate the role of persistence. The responses of spot and contract prices to negative supply shocks in the current period and in previous periods are

\[
\frac{dp^*_t}{d(-\varepsilon_{st-1})} = \beta, \quad i = 0
\]

\[
= -fa(1 - \alpha)\psi^t(\gamma\psi(1 - f(1 - \alpha)\delta)), \quad i \neq 0.
\]

and

\[
\frac{dp^*_t}{d(-\varepsilon_{st-1})} = \psi^t(\gamma\psi(1 - f(1 - \alpha)\delta)), \quad v_t.
\]

The cumulative effects of a negative supply shock \( i \) periods ago on current prices are

\[
\frac{dp^*_t}{d(-\varepsilon_{st-1})} \bigg|_{\text{cumulative}} = \beta[(1 - f(1 - \alpha)(\gamma\psi(1 - f(1 - \alpha)\delta)) \sum_{j=0}^{\infty} \psi^j],
\]

and

\[
\frac{dp^*_t}{d(-\varepsilon_{st-1})} \bigg|_{\text{cumulative}} = (\gamma\psi(1 - f(1 - \alpha)\delta)) \sum_{j=0}^{\infty} \psi^j.
\]

The responses of the two prices to shocks can be ascertained from equations (32), (33), (34), and (35). The spot price jumps discontinuously, then decreases over time as the contract price adjusts. Long-term contract prices follow a smoother path in response to a shock. These responses depend on the price elasticity of supply on the spot market, the price elasticity of demand, the share of total trades carried out on the spot market, the extent to which producers are "backward-looking" in setting long-term prices, and the sensitivity of
long-term prices to excess demand factors. Figures 1-4 show the within-period and cumulative effects of a negative supply shock on spot and contract prices under two conditions of persistence, $v_0$ and $v_1$, where $v_1 < v_0$. 
RESPONSES OF OIL PRICES TO TRANSITORY NEGATIVE SUPPLY SHOCKS

Figure 1
Effect of a Transitory Shock on the Spot Price

Figure 2
Effect of a Transitory Shock on the Contract Price

Figure 3
Cumulative Effect of a Transitory Shock on the Spot Price

Figure 4
Cumulative Effect of a Transitory Shock on the Contract Price

\[ \gamma = \gamma_o \text{ (High Persistence)} \]
\[ \gamma < \gamma_o \text{ (Low Persistence)} \]
We can now examine the impact of different levels of persistence (occurring as a result of changes in the degree of forward-lookingness of contract price setting, changes in the price elasticity of demand, or changes in the mix between spot trades and contract trades) on the optimal stockpile rules derived in section II. Let the persistence parameter $\bar{\psi}$ serve as a measure of intertemporal correlation in the composite price $p_t$, so that $dE_t p_{t+k}/dp_t = \bar{\psi}^k, \bar{\psi}_t$. Then, the responses of stockpiles in the noncooperative and cooperative solutions for each country $j$ to a change in the current-period price can be gleaned from (10) and (13) as

$$
\frac{dL}{dp_t} = -h^{-1}(1+\delta)[1-\lambda_j]^{-1} + \frac{d\bar{\psi}}{d\bar{\psi}}[Q_j^i] 
+ h^{-1}(1-\lambda_j)\bar{\psi}_t^{-(1+\delta)} \left( \frac{\bar{\psi}}{1-\lambda_j} \right) - h^{-1}(1+\delta)\bar{\psi}_t^{-(1+\delta)} \left( \frac{\bar{\psi}}{1-\lambda_j} \right) 
= h^{-1} \left( \frac{\bar{\psi}}{1-\lambda_j} \right)^{-(1+\delta)} \left[ \bar{\psi} - (1+\delta)(1 + \frac{d\bar{\psi}}{d\bar{\psi}}[Q_j^i]) \right],
$$

and

$$
\frac{dC}{dp_t} = -h^{-1}(1+\delta)[\lambda_j]^{-1} + \frac{d\bar{\psi}}{d\bar{\psi}}[Q_j^i] + h^{-1}(1 - \lambda_j)\bar{\psi}_t^{-(1+\delta)} \left( \frac{\bar{\psi}}{1-\lambda_j} \right) 
- h^{-1}(1+\delta)\bar{\psi}_t^{-(1+\delta)} \left( \frac{\bar{\psi}}{1-\lambda_j} \right) 
= h^{-1} \left( \frac{\bar{\psi}}{1-\lambda_j} \right)^{-(1+\delta)} \left[ \bar{\psi} - (1+\delta)(1 + \frac{d\bar{\psi}}{d\bar{\psi}}[Q_j^i]) \right],
$$

where
(39) \( \hat{\nu} = a [1 - \pi(1 - a)^2] \psi. \)

Considering just the first two components (from the impacts on the margin) of (36) and (37), increasing \( \hat{\nu} \) decreases both responses absolutely, but increases the cooperative response relative to the noncooperative response. Higher levels of persistence also magnify the cooperative response relative to the noncooperative response through the third term (which captures the response to deriving benefits of price reduction on the inframarginal consumption). On balance, \( d(\nu^N - \nu^C) > 0. \)

Before discussing the policy implications for particular international agreements, it is useful to review the principal findings of the analysis. The examination of intertemporal optimizing models of private and public stockpiling illustrated the role of "persistence" of the price impacts of transitory shocks. Noncooperative and cooperative solutions for individual countries revealed that whether responses were higher under collusion depended on the persistence described above and on the oligopoly potential provided by the coordinated buyer behavior. Given oligopoly power, higher levels of persistence made the Cournot-Nash equilibrium response "too small" relative to the collusive solution. By examining the structure of contracts in the oil market, the persistence was shown to depend on (among other things) the fraction of trades carried out through contracts and on the price elasticity of demand in consuming countries. As these factors changed the persistence effects of transitory shocks, the relationship between the noncooperative and cooperative stockpiling solutions also changed.
IV. IMPLICATIONS FOR EXISTING PETROLEUM AGREEMENTS

The last two sections set out the causes of persistence in the impacts of transitory supply shocks and illustrated their importance in viewing buffer stock policy as a dynamic international game. This section offers a brief description of the existing agreement and an interpretation in light of the preceding analysis.

The relevant regulations are codified in the International Energy Program, signed by the United States in 1974 and administered by the OECD. The details are too involved to present here (see U.S. Senate, 1974), but the salient points are three. First, countries are required to hold buffer stocks in proportion to their imports. Second, the agreement is dormant until a determination of emergency is made. The emergency is signalled as a quantity shock, which must be sufficiently large in absolute value to reduce supply by seven percent compared to its pre-shock value. (In practice, the time unit is the quarter, and the pre-shock value is a moving average of the previous four quarters.) Third, the agreement calls for countries to "restrain demand" by seven percent (through taxes, tariffs, regulation, exhortation, etc.) and substitute buffer stock releases in making up any remaining loss in supply (e.g., a ten percent reduction in quantity supplied calls for three percent to be made up by stockpile releases in addition to the seven percent demand restraint). The scheme's monopsonistic intent is clear.

That cooperation can reap benefits begs the question of how it might be achieved. Regulation at the international level is difficult to enforce; since there is no regulator with the power to require compliance, the incentive question naturally arises. While import
restriction is clearly in the interest of the group as a whole, the
effectiveness of the regulatory rules in attaining the cooperative
outcome is not evident.

Experience with oil supply shocks is (fortunately) insufficient for
econometric tests, but casual empiricism allows some observations.
First, in the past, as negative supply shocks have been unanticipated
(Hamilton, 1983), and have been accompanied by large and sustained
differences between spot and contract prices. The fraction of trade
conducted on a spot basis has been small. Thus, a reasonable
expectation would be that the effects of unanticipated shocks on prices
would not disappear suddenly, i.e., persistence. As noted above, this
implies that the noncooperative release is insufficient. In fact, most
countries accumulated inventories during the 1979 shock.

Second, the use of current quantity loss as a regulatory signal is
misdirected, since it ignores the critical influence on national
optimizing behavior of crisis dynamics. Loosely speaking, whether the
shock is anticipated to "improve" or "worsen" determines the
relationship between the cooperative and noncooperative solutions.

Of course, any policy not in effect at all times requires a
"trigger" to activate it. A natural candidate, used in buffer stock
schemes for other commodities, is price. In a market characterized by
short-run contract rigidities, however, a supply shock leads to at least
two prices prevailing at any given time. Treating the spot price as the
marginal cost of acquiring oil, however, is in general unwarranted; the
usefulness of this price as a signal depends on the fraction of trades
carried out in the spot market. It is the refiners' marginal
acquisition cost (denoted by p above) which is relevant.
Finally, scattered evidence suggests that the market is becoming more flexible (i.e., the share of contract trade is declining), implying that noncooperative behavior will be less costly in the future than in the "high persistence" regime of the past.\textsuperscript{23} If such is the case, we should concentrate on developing guidelines for using the Strategic Petroleum Reserve and not be preoccupied with other nations' incentives to cooperate.

V. CONCLUSIONS

During the past decade, transitory oil supply shocks have caused significant economic damage in terms of lost output and increased inflation in the industrial countries of the OECD. To the extent that the macroeconomic costs of shocks are a function of the magnitude of the oil price increases, domestic policies or internationally coordinated efforts to restrain oil price increases during disruptions can be beneficial. One such policy initiative is the release of oil held in public stockpiles.

In the second section of the paper, we addressed the motivations for private and public stockpiling behavior in an intertemporal optimizing model. The benefits to one country from public stockpile releases during an oil shock depend on the stockpiling behavior of other countries. By contrasting noncooperative and cooperative solutions to the optimization problem, we illustrate the conditions for beneficial cooperation and develop conditions under which public stockpile authorities are likely to release oil during a crisis.

Uncertainty over the path of future oil prices plays an important role in explaining inventory behavior. \textit{Cateris paribus}, after a
negative supply shock, the anticipation of higher oil prices in the future (i.e., serial correlation of the shock) leads to a higher rate of public inventory accumulation (lower optimal stockpile release) in the current period. This is true even when the objective of the stockpile authority is to maximize real income (i.e., to minimize the oil price increase). During a crisis in which the (now higher) oil price is expected to decline, countries are willing to draw down their stockpiles at the onset of a shock, even in the absence of a coordinating agreement. If the oil price is expected to increase further, however, a drawdown in the current period mandated by a stockpile coordination agreement is not in the interests of the individual members.

The cooperative solution of section II illustrates that international stockpile coordination can raise the optimal stockpile release in response to an oil supply shock. Optimal stockpile rules, however, are shown to depend on the persistence effects (on oil prices) of supply shocks. To explain that persistence, section III discusses the structure of contracts in the oil market and puts forth a simple theoretical model of oil price determination, focusing on the dynamic response of oil prices to shocks and on the ability of stockpile coordination to reduce the initial fillip to oil prices and its persistence.

In the fourth section of the paper, we reviewed our results in the context of current energy policy, particularly with respect to the agreements by the International Energy Agency. The optimal stockpile rules of section II indicate that the focus on the IEA agreement on the period in which a shock occurs ignores the more important role of the uncertainty over future oil prices.
A clear direction for future work is to analyze the role of simultaneous coordination of macroeconomic and stockpile policies in reducing the costs of large oil price increases. (Even within the context of examining stockpiling policies, the exchange rate is obviously a factor in determining the "price of oil" outside the U.S.). The benefits of coordinated fiscal and monetary policies probably greatly exceed those generated from stockpile cooperation, though achieving the former is likely to be even more difficult than achieving the latter.
1 Surprisingly, the substantial recent literature devoted to modeling the oil market has virtually ignored short-run issues and has been characterized as ill-suited to capturing the effects of supply disruptions (Stanford University Energy Modeling Forum, 1982).

2 Some earlier analyses of the size and management of public stockpiles can be found in Hogan (1983), Teisberg (1981), Wright and Williams (1982) and Hubbard and Weiner (1983a). Nicholas and Zeckhauser (1977) review some of the motivations for establishing public strategic stockpiles.

3 This finding is related to Blinder's (1982) study of optimal inventory behavior in the presence of serially correlated demand shocks, in which he determined that shocks elicit smaller (in absolute value) responses of investment inventory. Here, we are modeling commodity inventory responses of price-taking firms to transitory shocks in the presence of contracts. It will still be true that persistence reduces stockpile responses, but persistence here comes from contracts, rather than from serial correlation of the actual quantity shocks.

4 That is, represents the firm's opportunity costs of holding stocks.

5 Sargent (1979) contains a discussion of linear-quadratic optimization problems and solution techniques.

6 Note that countries do not increase only their own oil supplies by releasing stored oil. The oil market is a world market; the effect of a stockpile release will be on the world price.

7 We can see that a stockpile release by the jth country, ceteris paribus, lowers the world oil price, increasing domestic output because

$$\frac{dp}{dt} = \omega \sum_{m} \frac{dL_{m}}{dt} > 0 \quad \text{and} \quad \frac{dy_{j}}{dt} = f' \omega \sum_{m} \frac{dL_{m}}{dt} < 0,$$

where

$$\omega = \frac{p}{1-p} \sum_{n} Q_{n}.$$ 

Note the importance of the conjectured stockpile movements of other players.

8 The countries need not recognize the game-theoretic structure of the problem to land at the suboptimal Cournot-Nash solution. They may just employ "reduced-form estimates" of the impact of their demand for oil on the world price. This point is made in a different context in Sachs (1983).
Note that the \( \lambda \)'s differ across countries because each country evaluates \( dp/dL_j \) given its change in stockpile levels. In all cases, \( \lambda_{1j} \) and \( \lambda_{2j} \) differ from the values of the roots in the "private firm" case. Specifically, \( \theta_{ij}, \lambda_{1j} < 1 \) and \( \lambda_{2j} > (1 + \delta) \), so that both the impact and long-run effects of higher expected future oil prices on oil inventories are muted. The dampening arises from countries' (like "very large firms") taking into account the impact of their inventory decisions on the world oil price. For the firm,

\[
\lambda_1 + \lambda_2 = \frac{h(1 + (1 + \delta)^{-1})\delta}{h(1 + \delta)^{-1}}, \quad \lambda_1 \lambda_2 = (1 + \delta),
\]

implying that \( \lambda_1 = 1 \) and that \( \lambda_2 = (1 + \delta) \). Now, for each country \( j \),

\[
\lambda_{1j} + \lambda_{2j} = \frac{\frac{dp}{dL_j} + \frac{\delta \theta_{ij}}{(1 + \delta)^{-1}}}{h(1 + \delta)^{-1}} = \lambda_{1j} \lambda_{2j} = (1 + \delta).
\]

So as illustrated in the diagram,

\[
\lambda_{1j} < 1 \text{ and } \lambda_{2j} > 1 + \delta.
\]

If an initial "oil shock" at time \( t \) is expected to worsen at time \( t+1 \), then oil consumption at time \( t+2 \) falls relative to oil consumption at time \( t \). This effect is scaled by the extent to which movements in public inventory accumulation affect the world price. For example, because of their size, small countries are unlikely to have much effect on world oil prices through their stockpiling. In the limit, they may behave like private firms, taking \( dp/dL_i \) as zero. In that case, equation (9) reduces exactly to the "firm" case.

The argument is the same as in footnote 8. Cooperation dampens price-smoothing inventory response further as each player assumes that the others will reinforce its action. Dropping the assumption of separate costs of adjustment for each country (i.e., letting a single agent economize on these costs) would reinforce the statements that for all \( j \), \( \lambda_1 < \lambda_{1j} \) and \( \lambda_2 > \lambda_{2j} \).

The difference between the cooperative and noncooperative solutions arises in similar problems of coordination across sovereign nations given economic integration, as pointed out by Cooper (1968). Cooper noted three significant consequences of interdependence for national economic policy, namely (i) an
increase in the number of disturbances with which national economic policy must cope, (ii) a reduction in the speed with which the impacts of stabilization policy are felt, and (iii) the notion that competition in the use of national policies can leave the community of nations worse off.

To see this, note that

\[ \frac{d\lambda}{dt} = -h^{-1} \lambda \frac{d}{dt} \left( 1 + \lambda \right) \left( 1 + \lambda \right) \frac{d}{dt} \left( \frac{1}{\lambda} \right), \]

and

\[ \frac{d\lambda}{dt} = -h^{-1} \lambda \left( 1 + \lambda \right) \frac{d}{dt} \left( \frac{1}{\lambda} \right). \]

The intuition behind this last point is as follows. High persistence implies that price reduction today has a continuing payoff. Each country would like to release oil, but would have little impact by itself and would reduce its future stockpile availability in periods of even higher prices. Coordination would bring about greater responses.

A more detailed discussion of the "two-price" phenomenon in the world oil market can be found in Hubbard (1983). Another example in the literature is the world copper industry; see for example Mc Nicol (1975). Other international markets in metals, such as aluminum, nickel, vanadium, and molybdenum also have or have had multiple-price regimes.

That is, non-OPEC producers are competitive, so that their profit maximization yields the usual supply representation of "price equals marginal cost." Such a description need not imply that OPEC acts as a monolith. For example, a division into "optimizing members" and "fringe members" is equivalent to assigning the "fringe members" to the other price-takers.

In this paper, we take the parameterization of the the two-price system as given. Risk aversion is the motivation usually given for such a scheme (see Carlen, 1979; and Roberts, 1980). Our focus here is its role in information revelation in the absence of futures markets.

That p is the relevant price for decisionmaking requires some justification. Consider the case in which a negative supply shock initially raises the spot price relative to the contract price. The spot market price is not the cost of a marginal unit. A buyer could purchase oil at the lower contract price but would have to commit himself to buy oil at that price for the duration of the contract. The resulting tradeoff involves the mean and variance of the distribution of expected prices, which underlie the optimal value of \( \alpha \), which is taken parametrically here.

This approach is similar to that used in Taylor's (1979, 1980) analysis of aggregate wage and price behavior.
Past prices could be important in setting prices in the current period because of transactions costs involved in always gathering new estimates of demand. For example, contract prices may not respond immediately to excess demand or supply because of the inability to distinguish between temporary and permanent changes. Expected future prices may reflect expectations about changing market structure, new technologies, etc. A rigorous microeconomic foundation for (25) is not given here. The values of the parameters $c_t$ and $\gamma$ may certainly vary over time.

Shocks today would have no impact on the contract price if price setting were entirely forward-looking ($\eta_t = 0$).

Technically, the solutions come from maximizing (7) and (11) subject to the further constraint that

$$p_t = a_0 p_t^* + (1-a)p_t^S,$$

where $p_t^*$ is determined from equation (3) and $p_t^S$ is determined from equation (22). Note that the difference between $\eta$ and $\gamma$ varies positively with the fraction of trades carried out through contracts.

Thinking about the original form of (9) and (13) the intuition behind these findings is straightforward. The persistence of high prices into the future after a shock makes the use of stocks in later periods more attractive, reducing the release in the current period. However, as noted in footnote 14 above, cooperation which exploits oligopoly power raises the cooperative drawdown relative to the noncooperative drawdown, as consuming countries can blunt prices in future periods by lowering the price in the current period.

The agreement has been signed by the major OECD members except France (there is some indication that France is a de facto signatory), which consume 75 to 85 percent of the petroleums traded in the non-communist world and hold nearly 100 percent of the stocks. It is up for a ten-year review in 1986, making this a propitious time to evaluate it.

A stabilization policy analogue is that monetary policy coordination is unnecessary in a world of perfectly flexible prices.
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