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A MONOTONIC SOLUTION TO GENERAL COOPERATIVE GAMES*

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1. Introduction

The cooperative games that are discussed here are multiperson coalitional form games where utility is not assumed to be transferable. These games are also often refered to as characteristic function games without sidepayments and as nontransferable utility games. In such games a set of feasible utility allocations (vectors) is described for every coalition of players. The main question is to determine the final utility allocation that the players will agree to or that an arbitrator will recommend.

We follow an established tradition of game theory by seeking an axiomatic solution to the questions above (see, for example, Nash [1950] and Shapley [1953]). That is, we postulate conditions, or axioms, which we feel are desirable for a solution to satisfy and investigate the logical and mathematical consequences of these axioms. As it turns out, the axioms disucssed in this paper are strong enough to determine essentially a unique solution.

Two subclasses of this general class of games have been studied extensively. The first is the class of games with transferable utility. In this class the feasible sets of the coalitions are such that if a utility allocation is feasible then every other allocation which yields the same total utility (summed over all the players in the coalition) is also feasible. Thus, it is implicitly assumed that it is feasible for players to transfer utility from one to another. The most prominent and established axiomatic solution for this subclass is the Shapley [1953b] value.

The second subclass of games that has been studied extensively consists of the bargaining games. In this subclass utility is not assumed to be transferable but there is a restriction that only the coalition of all players, the grand coalition, has profitable feasible utility allocations. Here the most prominent axiomatic solution is the one proposed by Nash [1950].

Until recently there were no axiomatic solutions to the general class of coalitional form games. What researchers tried to do was to define solutions for the general class that coincide with the prominent solutions on the two subclasses mentioned above. Solutions of this type were proposed by Shapley [1969], Harsanyi [1963], Owen [1972] and others. Recently Aumann [1983] succeeded in axiomatizing for the first time a value of this type, the one proposed by Shapley, which is called the NTU (nontransferable utility) value. With a similar set of axioms Hart (from verbal communications) characterized the Harsanyi solution.

One of the appealing properties that the Shapley value exhibits on the subclass of transferable utility games is one of monotonicity. This condition states that if the feasible set of one of the coalitions increases, and the feasible sets of all other coalitions remain the same, then none of the members of this coalition should become worse off because of this change. This condition is appealing as a fundamental principle for cooperation but it must also hold for many cooperative games from noncooperative principles of individual utility maximization. In many such games an alternative is feasible for a given coalition if and only if every member of the given coalition supports it. In other words, every player can veto every feasible alternative of a coalition to which he belongs. In such situations, if contrary to the monotonicity condition, a player of a given coalition stands to lose because of the the availability of new alternatives, he would veto these new alternatives, reduce the situation back to the old one and lose nothing. With this veto option available to the players, and under a utility maximization assumption, it follows from the above argument that a solution must be monotonic. Conversely, if a solution is monotonic, then none of the players has incentives to veto any alternative, destroy or misrepresent his resources, and in this sense a maximal level of cooperation should result (see Section 10 for a formal discussion of these ideas).

Turning to the subclass of bargaining games, we know that the Nash solution does not satisfy the monotonicity condition (for references see Roth [1979] and Kalai [1983]). The only solution which is monotonic (in the presence of other standard conditions) is the egalitarian solution introduced by Kalai [1977] (under the name of proportional solution).

In this paper we introduce an axiomatic solution to the general class of coalitional form game. This solution generalizes the Shapley value on the subclass of transferable utility games and the egalitarian solution on the subclass of bargaining games. We call it the egalitarian solution. The egalitarian solution is monotonic and we show that in the presence of other weak and standard conditions it is the only monotonic solution.

Studies of the monotonicity axiom and related conditions have been numerous. For some of these studies and further references we refer the reader to Luce-Raiffa [1957], Owen [1968b], Roth [1979], Megiddo [1974], Kalai-Smorodinsky [1975], Thomson-Myerson [1980], Thomson [1982], and Young [1982].

We defer further discussion of the egalitarian solution, its relationship to other solutions, and its properties, for the later sections in the paper after presenting the above ideas formally.

2. An Example

We consider three players, called 1, 2 and 3, faced with the following situation. Every player acting alone can secure a payoff of zero utility for himself. Cooperation of any two players does not change outcomes and thus when any two player coalition cooperates the result will still be a payoff of zero to every one of its members. However, the cooperation of all three players is potentially profitable. When all three players cooperate they can bring about any one of the following three payoff vectors:

$$(4,4,4) \qquad (7,0,0) \qquad (0,12,0).$$

We assume also that every convex combination of these three payoffs is feasible for the players. We refer to this situation as game A.

When we apply the Harsanyi extended solution or the Shapley NTU extension to the game A it follows that

$$\text{the outcome of A} = (4,4,4).$$

Assume now that players 1 and 2 found a new vehicle with which they can cooperate. Under cooperation the two of them can now bring about any of the two payoffs

$$(7,0) \qquad (0,12)$$

and any convex combination of them. We now face a new cooperative game, which we call B. B is the same as A except for the coalition of players 1 and 2. If we apply the same solutions as before to the game B, we observe that

$$\text{the outcome of B} = (3.5,\ 6,\ 0).$$

This example illustrates the lack of monotonicity discussed in the introduction. The new ability of players 1 and 2 to cooperate improved the outcome of player 2 by 2 units but brought about a loss: -0.5 units to player 1. If the underlying situation that gave rise to the games A and B is such that player 1 has control over his own cooperation then the outcome of game B should be at least as good for him as the outcome of game A because player 1 can reduce the game B back to the game A.

The symmetric egalitarian solution presented in this paper will choose the

outcome of A = (4,4,4), and the

outcome of B = (4.421, 4.421, 0),

and will satisfy the monotonicity condition.

3. Notations and Definitions

We let $N = \{1,\ldots,n\}$ denote the set of players ($n \geq 1$). A coalition is a subset of N. The n-dimensional Euclidean space is denoted by $\mathbb{R}^N$. For x, $y \in \mathbb{R}^N$ and a coalition S, $x \geqq_S y$ means $x_i \geqq y_i$ for each $i \in S$, $x \geq_S y$ means $x \geqq_S y$, and for some $i \in S$, $x_i > y_i$, and $x >_S y$ means $x_i > y_i$ for each $i \in S$. For S = N we omit the subscript N. For each coalition S, we denote $\mathbb{R}^S = \{x \in \mathbb{R}^N | x_i = 0,\ i \notin S\}$, $\mathbb{R}^S_+ = \{x \in \mathbb{R}^S | x \geqq 0\}$ and $\mathbb{R}^S_{++} = \{x \in \mathbb{R}^S | x > 0\}$. For a vector $x \in \mathbb{R}^N$ we denote by x_S the projection of x on $\mathbb{R}^S$, i.e., $(x_S)_i = x_i$ for $i \in S$ and $(x_S)_i = 0$ for $i \notin S$. We use the notation $A \subset B$ to denote that A is a proper subset of B and $A \subseteq B$ to denote that A is a subset of B.

An n-person characteristic function game without side payments v (a game for short) is a function from the set of all coalitions to subsets of $\mathbb{R}^N$ such that for every $S \subseteq N$ the following conditions are satisfied.

1. $v(\emptyset) = \{0\}$
2. $v(S)$ is a closed, nonempty subset of $\mathbb{R}^S$.
3. $v(S)$ is comprehensive, i.e., if $x, y \in \mathbb{R}^S$, $x \in v(S)$ and $y \leq x$, then $y \in v(S)$.
4. $v(S)$ is bounded in the sense that there exists no monotonically increasing unbounded sequence of points in $v(S)$, i.e., if $\{x_t\}_{t=1}^{\infty}$ is a sequence of points in $v(S)$ with $x_{t+1} \geq x_t$ for $t=1,2,\ldots$, then there is a point $y \in \mathbb{R}^S$ such that $x_t \leq y$ for each t.

We let Γ denote the set of games satisfying the conditions stated above. For every player i we let $\theta_i = \max\{x_i | x \in v(\{i\})\}$ and we denote by θ the vector $(\theta_i)_{i\in N}$. A point $x \in \mathbb{R}^N$ is <u>individually rational for the coalition</u> S in the game v if $x \geq_S \theta$. An individually rational point for N is said to be <u>individually rational</u>. A point $x \in \mathbb{R}^N$ is (weakly) <u>Pareto optimal for a coalition</u> S (or in v(S)) in a game v if $x \in v(S)$ and there is no $y \in v(S)$ with $y >_S x$. The point $x \in \mathbb{R}^N$ is <u>strongly Pareto optimal for</u> S if $x \in v(S)$ and there is no $y \in v(S)$ with $y \geqslant_S x$.

4. <u>The Axioms</u>

A solution for Γ is a function $\phi: \Gamma \rightarrow \mathbb{R}^N$.

Next we discuss five axioms that we impose on solutions for Γ.

We first introduce a condition under which a solution of a game v should be individually rational. The condition is that each individual player while joining coalition S does not hurt it by eliminating alternatives that S had without him. More specifically, we say that a <u>game</u> v <u>is</u> <u>montononic</u> if for each coalition S and $j \notin S$, if x is individually rational for S, then there exists an x' in $v(S \cup \{j\})$ such that $x'_i = x_i$ for each $i \in S$ and $x_j \geq \theta_j$.

(1) Individual Rationality

If a game v is monotonic then $\phi(v)$ is individually rational.

Observe that this axiom resembles an analogous condition which guarantees individual rationality of the Shapley value for games with side payment, namely that the contribution of each player to each coalition is nonnegative.

The next axiom is analogous to the carrier axiom which is used in axiomatic characterizations of the Shapley value. A coalition S is called a carrier of the game v if for each coalition T, $v(T) = v(T \cap S) - R^T_+$.

(2) Carrier. If S is a carrier of the game v then $\phi(v)$ is Pareto optimal for S.

For a vector $a \in \mathbb{R}^N$ we denote by $\hat{a}_S$, the game in which the coalition S bargains over the vector a_S. Formally, for every coalition T, $\hat{a}_S(T) = 0 - R^T_+$ if $T \not\supseteq S$ and $\hat{a}_S(T) = a - R^T_+$ if $T \supseteq S$. A vector a is called acceptable to S if $\phi(\hat{a}_S) = a_S$.

The next axiom is a generalization of the translation invariance which most of the solutions for bargaining problems and cooperative games have (see Aumann [1983] for additional discussion of this condition). A solution has the translation invariance property if by adding a constant to the utility of a player the payoff of this player in the solution changes by the same constant. Formally, it means that $\phi(v + \hat{a}_{\{i\}}) = \phi(v) + a_{\{i\}}$ for each game v, player i, and $a \in R^N$. This property can also be interpreted as saying that if player i has a personal endowment and he adds it to the game then his payoff in the solution will be changed exactly by his personal endowment. We generalize this axiom by considering personal endowments given to the players of a coalition S when these players are cooperating. Here again we require that by adding endowments to the game the solution will change just by the addition of these endowments--i.e., $\phi(v + \hat{a}_S) = \phi(v) + a_S$, but since the endowments S depend on the cooperation of all members of S we restrict the requirement only to those cases in which S considers the vector a_S as a "fair" alloction--i.e., when $\hat{a}_S$ is acceptable.

(3) Translation Invariance. If the vector a is acceptable to S then $\phi(v + \hat{a}_S) = \phi(v) + a_S$ for each game v.

Alternatively, the axiom can be viewed as a weak version of the additivity axiom of the Shapley value. Notice that this axiom by itself does not imply that there are vectors which are not acceptable to S.

(4) Monotonicity. If for the games v and w, v(T) = w(T) for each $T \neq S$ and $v(S) \supseteq w(S)$ then $\phi(v) \geq_S \phi(w)$.

This axiom is an obvious extension of the monotonicity axiom used in Kalai [1977] to characterize the egalitarian solution for bargaining problems. It is also a property of the Shapley value.

We define a topology on Γ as follows. A sequence of games $\{v_t\}_{t=1}^{\infty}$ converges to v iff for each coalition S, $\{v_t(S)\}_{t=1}^{\infty}$ converges to v(S) in the Housdorff topology. Given this topology on Γ we require:

(5) Continuity. ϕ is continuous on Γ.

Finally, a solution for Γ which satisfies the axioms (1) individual rationality, (2) carrier, (3) translation invariance, (4) monotonicity, and (5) continuity is called a value.

5. The Egalitarian Solution

We define the symmetric egalitarian solution for a given game v, E(v), by inductively constructing two functions Z and D defined from the set of all coalitions to $\mathbb{R}^N$.

We first define

$$D(v,\phi) = 0 \quad \text{and} \quad Z(v,\phi) = 0.$$

Then for each coalition S,

$$Z(v,S) = \sum_{T \subset S} D(v,T)$$

and

$$D(v,S) = e_S \max\{t \mid (Z(v,S) + te_S) \in v(S)\}$$

where e is the vector $\mathbb{R}^N$ with $e_i = 1$ for each $i \in N$. Finally, define

$$E(v) = \sum_{S \subseteq N} D(v,S)$$

Observe that the existence of t for which $(Z(v,S) + te_S) \in v(S)$ is due to the comprehensiveness of v(S) and that the finiteness of the maximum is guaranteed by the boundedness condition on v(S). D(S) can be described as a vector of dividends allocated by S to its members. All the members of S receive from S the same dividend, and the total amount of the dividend vectors allocated by all subcoalitions of S (as well as S) is a Pareto optimal point in v(S).

The nonsymmetric egalitarian solution is obtained when dividends are allocated not equally but according to some prescribed positive weights. For $\lambda \in \mathbb{R}^N_{++}$, the egalitarian solution E^λ is defined by $D^\lambda(v,\phi) = 0$, $Z^\lambda(v,\phi) = 0$, $Z^\lambda(v,S) = \sum_{T \subset S} D^\lambda(v,T)$, $D^\lambda(v,S) = \lambda_S \max\{t \mid (Z^\lambda(v,S) + t\lambda_S) \varepsilon v(S)\}$ and $E^\lambda(v) = \sum_{S \subseteq N} D^\lambda(v,S)$. Observe that the strict positivity of λ is required and sufficient in order that the set over which the max is taken is not empty.

An equivalent way of computing E^λ is by rescaling the utilities of the players and then applying to the rescaled game the symmetric egalitarian solution. In other words, the egalitarian solution is determined uniquely up to individual rescaling of utilities. This is done as follows.

For every vector $\lambda \in \mathbb{R}^N_{++}$ define

$$\lambda * x = (\lambda_1 x_1, \lambda_2 x_2, \ldots, \lambda_n x_n) \quad \text{and} \quad \lambda^{-1} * x = (\frac{x_1}{\lambda_1}, \frac{x_2}{\lambda_2}, \ldots, \frac{x_n}{\lambda_n}).$$

For a game $v \in \Gamma$ define the game $\lambda^{-1} * v$ by

$$(\lambda^{-1} * v)(S) = \lambda^{-1}(v(S)) = \{\lambda^{-1} * x \mid x \in v(S)\}.$$

It is easy to verify that $E^\lambda(v) = \lambda * E(\lambda^{-1} * v)$.

6. A Characterization of the Egalitarian Solution

Theorem 1. A solution ϕ defined on Γ is a value (i.e., satisfies axioms 1-5) if and only if it is egalitarian.

We first prove that the symmetric egalitarian solution E is a value. It is easy to check that this implies that E^λ is also a value for each $\lambda \in \mathbb{R}^N_{++}$. We leave it to the reader to verify that the carrier and translation invariance axioms are satisfied by E and prove it for the remaining three axioms. We denote $E(v,S) = \sum_{T \subseteq S} D(v,T)$. When the game v is clear from the context we write instead E(S), z(S) and D(S).

The following equality will be used in the sequel:

(*) For every coalition S and every $i,j \in S$:

$$Z_i(v,S) - Z_j(v,S) = \sum_{i \in T \subset S} D_i(v,T) - \sum_{j \in T \subset S} D_j(v,T) =$$

$$= \sum_{i \in T \subset S-\{j\}} D_i(v,T) - \sum_{j \in T \subset S-\{i\}} D_j(v,T) =$$

$$= E_i(v,S-\{j\}) - E_j(v,S-\{i\}).$$

Lemma 1. E satisfies the individual rationality axiom.

Proof. Let v be a monotonic game. We prove by induction on the coalition size, k, that $E(S) \geq_S \theta$ for each S. This is clearly true for S of size 1. Assume that this inequality holds for all coalitions of size k and let S be a fixed coaltion of size k+1. Let $Z_j(S) - \theta_j = \min_{i \in S}(Z_i(S) - \theta_i)$, and define $F = Z(S) - (Z_j(S) - \theta_j)e_S$. Obviously, $F \geq \theta$, $F_j = \theta_j$ and by (*) $F_i = E_i(S-\{j\}) - E_j(S-\{i\}) + \theta_{\{j\}}$ for each $i \in S-\{j\}$.
By the induction hypothesis $F_i \leq E_i(S-\{j\})$ for all $i \neq j$, and therefore by the monotonicity of v there exists $F' \in v(S)$ such that $F'_{S-\{j\}} = F_{S-\{j\}}$ and $F'_j \geq \theta_j$. Since $F \leq_S F'$, F is in V(S) and by the definition of E(S), $E(S) \geq_S F \geq_S \theta$. Q.E.D.

Lemma 2. The egalitarian solution E satisfies the monotonicity axiom.

Proof. Let v and v' be two games and S a coalition such that $v(S) \subseteq v'(S)$ and $v'(T) = v(T)$ for each $T \neq S$. Observe first that for each $T \not\supset S$, $Z(v',T) =$

$Z(v,T)$ and for each $T \not\supseteq S$ $D(v,T) = D(v',T)$ and $E(v,T) = E(v',T)$. We prove now by induction on the size of T that $E_i(v',T) \geq E_i(v,T)$ for each $T \supseteq S$ and $i \in S$. Since $Z(v',S) = Z(v,S)$ and $v'(S) \supseteq v(S)$, clearly $E_i(v',S) \geq E_i(v,S)$ for each $i \in S$. Suppose now that the inequality is proved for all $T \supseteq S$ of size k, and let $T \supseteq S$ be of size k+1. We show that

$$(**)\quad Z_i(v',T) - Z_i(v,T) \geq Z_j(v',T) - Z_j(v,T)$$

for each $i \in S$ and $j \in T$. Indeed, by equality (*) and since $T-\{i\} \not\supseteq S$

$$Z_i(v',T) - Z_j(v',T) = E_i(v',T-\{j\}) - E_j(v',T-\{i\}) = E_i(v',T-\{j\}) - E_j(v,T-\{i\})$$

and

$$Z_i(v,T) - Z_j(v,T) = E_i(v,T-\{j\}) - E_j(v,T-\{i\}).$$

Therefore, subtracting the left hand side of the last two equalities and using the induction hypothesis

$$[Z_i(v',T) - Z_j(v',T)] - [Z_i(v,T) - Z_j(v,T)] = E_i(v',T-\{j\}) - E_i(v,T-\{j\}) \geq 0$$

and (**) follows readily. Next we show that from (**) follows $E_i(v',T) \geq E_i(v,T)$ for each $i \in S$. Let $E(v,T) = Z(v,T) + \alpha e_T$. Let $i \in S$ and denote $\beta = \alpha + Z_i(v,T) - Z_i(v',T)$ (by (**) the difference in the right hand side is the same for all $i \in S$), and let $F = Z(v',T) + \beta e_T$. By (**) for each $j \in T$ and $i \in S$

$$\beta + z_j(v',T) - [\alpha + z_j(v,T)] \leqslant \beta + z_i(v',T) - [\alpha + z_i(v,T)] = 0$$

and thus $F \leqq_T E_T(v,T)$ and by the comprehensiveness of $v(T)$, $F \varepsilon v(T)$, so also $F \varepsilon v'(T)$. By the definition of $E(v',T)$, $E(v',T) \geqq F$; moreover $F_i = E_i(v,T)$ for each $i \varepsilon S$, and therefore $E(v',T) \geqq_S E(v,T)$. Q.E.D.

Lemma 3. E^λ is continuous for every $\lambda \varepsilon \mathbb{R}^N_{++}$.

Proof. Since the operations of individual utility rescaling are continuous it suffices to show that the symmetric egalitarian solution is continuous. Recall that since $E(v) = \sum_{S \subseteq N} D(v,S)$ it suffices to show that for every coalition S, $D(v,S)$ is continuous in v. This can easily be shown by induction on the size of S. Recall that $D(v,S) = Z(v,S) + e_S \max\{t \mid (Z(v,S) + te_S) \varepsilon v(S)\}$. $Z(v,S)$ is continuous by the induction hypothesis since $Z(v,S)$ is a finite sum of dividends of coalitions of strictly smaller size. The max is then seen to be continuous by the comprehensiveness of v. Q.E.D.

We turn now to prove that each value is an egalitarian solution

A game v is said to be inessential if 0 is Pareto optimal for each coalition.

Lemma 4. If v is an inessential game and ϕ is a value then $\phi(v) = 0$.

Proof. Assume first that 0 is strongly Pareto optimal for each S. In this case the game is monotonic. By individual rationality $\phi(v) \geqq 0$, and therefore $\phi(v) = 0$. An inessential game in which 0 is weakly Pareto optimal for some S can be approximated as follows. For a real number $\varepsilon > 0$ define $A_\varepsilon = \{x \varepsilon \mathbb{R}^N \mid x_i > -\varepsilon \text{ for each } i \varepsilon N, \sum_{i \varepsilon N} x_i > 0\}$, and $B_\varepsilon = \mathbb{R}^N - A_\varepsilon$. Define a game v^ε by $v^\varepsilon(S) = v(S) \quad B_\varepsilon$ for each coalition S. Clearly in v^ε zero is strongly Pareto optimal for each S since $v^\varepsilon(S) \cap \mathbb{R}^N_+ = \{0\}$, and therefore

$\phi(v^{\varepsilon}) = 0$. But $v_{\varepsilon} \underset{\varepsilon \to 0}{\longrightarrow} v$ and therefore by the continuity of ϕ, $\phi(v) = 0$.

Q.E.D.

We call a game v a <u>bargaining game for the coalition</u> S if for each coalition T, $v(T) = 0 - \mathbb{R}^T_+$ whenver $T \not\supseteq S$ and $v(T) = v(S) - \mathbb{R}^T_+$ whenever $T \supseteq S$. We let B^S be the set of all bargaining games for the coalition S.

<u>Lemma 5</u>. If ϕ is a value then for each coalition S there exists a unique positive vector, up to a multiplication by positive constants, $\lambda^S \in \mathbb{R}^S_{++}$ such that for each $v \in B^S$, $\phi(v) = \lambda^S \max\{t \mid t\lambda^S \in v(S)\}$.

<u>Proof</u>. Let S be a fixed coalition and define for each number t a game u^t_S by $u^t_S(T) = 0 - \mathbb{R}^T_+$ for $T \not\supseteq S$ and $u^t_S(T) = \{x \in \mathbb{R}^N \mid \sum_{i \in S} x_i \leqslant t\} - \mathbb{R}^T_+$ for $T \supseteq S$. Let $\lambda^t = \phi(u^t_S)$. By the carrier and the individual rationaltiy axioms, $\sum_{i \in S} \lambda^t_i = t$, $\lambda^t \in \mathbb{R}^S$, and for $t \geqslant 0$, $\lambda^t \geqq_S 0$. Let t be fixed. For a real number $\varepsilon > 0$ define $\mu^{\varepsilon} = \lambda^t + \varepsilon e_S$ and consider the game $\hat{\mu}^{\varepsilon}_S$ and the game v^{ε} which is defined by $v^{\varepsilon}(T) = u^t_S(T) \cap \hat{\mu}^{\varepsilon}_S(T)$ for each T. By the Pareto optimality of $\phi(v^{\varepsilon})$ in $v^{\varepsilon}(S)$ either $\sum_{i \in S} \phi_i(v^{\varepsilon}) = t$ or $\phi_i(v^{\varepsilon}) = \lambda^t_i + \varepsilon$ for some $i \in S$. On the other hand, by monotonicity $\lambda^t = \phi(u^t_S) \geq_S \phi(v^{\varepsilon})$ and therefore $\phi(v^{\varepsilon}) = \lambda^t$. Since $v^{\varepsilon} \underset{\varepsilon \to 0}{\longrightarrow} \hat{\lambda}^t_S$ it follows that $\phi(\hat{\lambda}^t_S) = \lambda^t$, i.e., λ^t is acceptable to S. By the translation invariance axiom for each t and s, $\lambda^{t+s} = \phi(u^{t+s}_S) = \phi(u^t_S + \hat{\lambda}^s_S) = \lambda^t + \lambda^s$, i.e., λ^t is additive in t. By the monotonicity, if $t > s$, $\lambda^t \geqslant \lambda^s$. Therefore, λ^t is homogeneous of degree one in t and with the notation $\lambda^S = \lambda^1$ we conclude that $t\lambda^S (=\lambda^t)$ is acceptable to S. To see that $\lambda^S >_S 0$ observe that if for some $i \in S$, $\lambda^S_i = 0$ then $\hat{\lambda}^S_S$ is inessential and thus by Lemma 4 $\lambda^S = \phi(\hat{\lambda}^S_S) = 0$ which contradicts $\sum_{i \in S} \lambda^S_i = 1$.

Now let $v \in B^S$ and let $t_0 = \max\{t \mid t\lambda^S \in v(S)\}$. The max is well defined since v(S) is comprehensive and $\lambda^S >_S 0$. by the boundedness condition on

$v(S)$, $t_0 < \infty$. Consider now the game w defined by $w(T) = 0 - \mathbb{R}^T_+$ for $T \not\supseteq S$ and $w(T) = v(T) - t_0\lambda^S$ for $T \supseteq S$. The game is inessential and $w + t_0\hat{\lambda}^S_S = v$. Therefore, $\phi(v) = \phi(w) + \phi(t_0\hat{\lambda}^S_S) = t_0\hat{\lambda}^S_S$ as required. Q.E.D.

The next lemma relates the λ^S's of different coalitions to each other. The vector λ^N_S in the following is the projection of λ^N on $\mathbb{R}^S$.

<u>Lemma 6.</u> Let ϕ be a value and for each coalition S let λ^S be the vector described in Lemma 4. Then for every coalition S there exists $k > 0$ such that $\lambda^N_S = k\lambda^S$.

<u>Proof.</u> Since λ^S is determined up to a multiplicative positive constant we may assume that $\min_{i \in S} \lambda^N_i/\lambda^S_i = 1$. In particular, $\lambda^N \geq_S \lambda^S$. Consider now the game v defined by $v(T) = \hat{\lambda}^S_S(T)$ for each $T \neq N$ and $v(N) = \lambda^N - \mathbb{R}^N_+$. Observe that the game v is obtained from the game $\hat{\lambda}^N_N$ by successively increasing $v(T)$ for coalitions T containing S and therefore by monotonicity $\phi(v) \geq_S \phi(\hat{\lambda}^N_N) = \lambda^N$ which implies $\phi_i(v) = \lambda^N_i$ for each $i \in S$. On the other hand, consider the game w defined by $w(T) = 0 - \mathbb{R}^T_+$ for $T \not\supseteq S$ and $w(T) = v(T) - \lambda^S$ for $T \supseteq S$. Clearly, by the choice of λ^S, w is inessential and $w + \hat{\lambda}^S_S = v$ and therefore $\phi(v) = \lambda^S$, i.e., $\lambda^N_i = \lambda^S_i$ for each $i \in S$. Q.E.D.

<u>Lemma 7</u> Let ϕ and ψ be two values on V. If $\phi(v) = \psi(v)$ for each $v \in B^N$ then $\phi = \psi$.

<u>Proof.</u> By Lemmas 5 and 6 there exists a vector $\lambda \in \mathbb{R}^N_{++}$ such that for each $v \in B^S$, $\phi(v) = \psi(v) = \lambda_S \max\{t \mid t\lambda_S \in v(S)\}$. Moreover, for any constant k and each coalition S, $k\lambda$ is aceptable to S with either ϕ or ψ.

We denote by Γ_k (k=0,...,n) the set of all games in V for which 0 is Pareto optimal for all coalitions of size not greater than k. Clearly $\Gamma_0 = \Gamma$. We will show by backwards induction on k that ϕ and ψ coincide on each Γ_k. For Γ_n this was proved in Lemma 4. Suppose ϕ and ψ coincide on Γ_k for some $1 \leq k \leq n$, and let $v \in \Gamma_{k-1}$. For each S of size k define $m^S = \lambda_S \max\{t \mid t\lambda_S \in v(S)\}$. Consider the game w defined by $w(T) = v(T)$ if T is of size less than k and

$$w(T) = v(T) - \sum_{\substack{S \subseteq T \\ |S|=k}} m^S$$

for T of size greater than or equal to k. Observe that $v = w + \sum_{|S|=k} \hat{m}^S_S$, $w \in \Gamma_k$ and that m^S is acceptable for S with both ϕ and ψ. Therefore

$\phi(v) = \phi(w) + \sum_{|S|=k} m^S = \psi(w) + \sum_{|S|=k} m^S = \psi(v)$. Q.E.D.

To finish the proof of Theorem 1 we observe that if ϕ is a value then by Lemma 5 for some $\lambda \in \mathbb{R}^N_{++}$ ϕ and E^λ coincide on B^N. By Lemma 7 it follows that $\phi = E^\lambda$.

7. The Harsanyi and Myerson Solutions

Harsanyi [1965] used the egalitarian solution as part of his solution for general cooperative games. Harsanyi's solution tries to capture two notions of "fairness" based on interpersonal utility comparison; one requires equality of utility, and the other requires transfers that increase total welfare. Formally a vector u in v(N) is Harsanyi's solution to v if there exists $\lambda \in \mathbb{R}^N_{++}$ such that $u = E^\lambda(v)$ and $\sum_{i \in N} \lambda_i^{-1} u_i = \max_{x \in v(N)} \sum_{i \in N} \lambda_i^{-1} x_i$.

We observe that while the Harsanyi solution of a game v is of the form $E^\lambda(v)$ it is different from the egalitarian solution. In the Harsanyi solution the λ depends on the game under consideration and it changes as we vary the game. In the egalitarian solution λ is fixed over all the games (as long as the players do not change their utility scale) which is essential in order to obtain the monotonicity property.

Another solution which is related to the egalitarian solution is the fair allocation rule of Myerson [1980] which is defined as follows. Consider a fixed game v. A set Q of coalitions is called a conference structure. For a given conference structure Q, define the equivalence relation $\sim$ as follows. For $i,j \in N$, $i \sim_Q j$ if $i=j$ or if there are players $i = i_1, i_2, \ldots, i_m = j$ such that for each k $(k = 1,\ldots,m)$ there exists $S \in Q$ such that $i_k, i_{k+1} \in S$. We denote by N/Q the set of all equivalence classes defined on N by $\sim_Q$. A fair allocation rule for a a game v is a function X which assigns to each conference structure Q a vector $X(Q) \in \mathbb{R}^N$ such that:

(1) For each Q and $S \in N/Q$, $X_S(Q)$ is Pareto optimal for S.

(2) For each Q, $S \in Q$ and $i,j \in S$,

$$X_i(Q) - X_i(Q-\{S\}) = X_j(Q) - X_j(Q-\{S\}).$$

Myerson proved for each game v the existence and uniqueness of a fair allocation rule. In the following theorem, we identify this rule with an extension of the egalitarian solution, and provide as a byproduct a short proof for the existence of the fair allocation rule. For a conference structure Q denote $\tilde{Q} = \bigcup_{Q' \subseteq Q} (N/Q')$. The conference structure $\tilde{Q}$ can alternatively be described as the set which contains all individual players $(N/\{\phi\})$ and all the unions of coalitions in Q which are Q-connected coalitions. Clearly $\tilde{Q} \supseteq (N/Q)$, and moreover each $S \in N/Q$ is a maximal element in $\tilde{Q}$. We define now inductively two functions Z and D from $\tilde{Q}$ to $\mathbb{R}$.

$$Z(\phi) = D(\phi) = 0$$

and for each $S \in \tilde{Q}$,

$$Z(S) = \sum_{\substack{T \subset S \\ T\varepsilon \tilde{Q}}} D(T)$$

$$D(S) = e_S \max\{t \mid (Z(S) + te_S) \; \varepsilon \; v(S)\}$$

and finally define

$$X(Q) = \sum_{S\varepsilon \tilde{Q}} D(S).$$

Obviously for the conference structure Q which contains all the subcoalitions of N, X(Q) is the symmetric egalitarian solution for v.

Theorem 2. X as defined above is the unique fair allocation rule for v.

Proof. Since by Theorem 1 in Myerson [1980] a fair allocation rule for v is unique it is enough to show that X satisfies the two requirements (1) and (2). Observe that for each coalition $S \; \varepsilon \; \tilde{Q}$, $\sum_{T \subseteq S} D_S(T)$ is Pareto optimal for S. It follows then from the fact that all the coalitions of N/Q are maximal in $\tilde{Q}$ that (1) is satisfied. Next, for $S \; \varepsilon \; Q$ denote $Q' = Q - \{S\}$. Observe that $\tilde{Q} \supseteq \tilde{Q}'$, and that the coalitons which are in $\tilde{Q}$ and are not in $\tilde{Q}'$, are exactly those coalitions in Q which contain S. Therefore

$$X_S(Q) - X_S(Q') = \sum_{S \subseteq T\varepsilon \tilde{Q}} D_S(T).$$

But the right hand side is a multiple of e_S by a constant and thus all members of S lose or gain the same by eliminating S from Q. Q.E.D.

8. Further Properties of the Egalitarian Solution

Consider the restriction of the game v to a coaltion S and its subsets. By ignoring the coordiantes of players outside S (which are anyway zero), this restriction is a game for which S is the grand coalition. We denote the restriction of v to S by v_S. The symmetric egalitarian solution for v_S is a byproduct of the inductive construction of the solution for v--i.e.,

$$E(v_S) = E(v,S) = \sum_{T \subseteq S} D(v,T)$$

The first property we discuss is strong independence of irrelevant alternatives (SIIA). This is a generalization of Nash's independence of irrelevant alternatives axiom (IIA) which is used to characterize Nash's solution to bargaining problems. SIIR requires that the solution for v depends on the alternative available for S and its subcoalitions, only through the solution for the game played by S, v_S. In other words, changing v_S while keeping the solution of v_S does not change the solution for v.

Proposition 1. The egalitarian solution has the SIIR property--i.e., for games v and w and a coalition S, if $v(T) = w(T)$ for each $T \neq S$, $v(S) \subseteq w(S)$, and $E(v_S) \in w(S)$ then $E(v) = E(w)$.

Proof. This is an immediate consequence of the inductive definition of E.

Proposition 2. If $v(N) = W(N)$ and for each $i \in N$ $E(v,N-\{i\}) = E(w,N-\{i\})$ then $E(v) = E(w)$.

Proof. From property (*) of section 5 we observe that

$$E_i(v) - E_j(v) = E_i(v,N-\{j\}) - E_j(v,N-\{i\}) \text{ for every } i,j \in N.$$

Thus, we obtain n-1 independent conditions on the vector E(v) from the values of the n-1 players' coalitions. The fact that E(v) is Pareto optimal yields an additional indepedent condition which shows that E(v) is determined by v(N) and the $E(v, N-\{i\})$'s. Q.E.D.

We observe that Proposition 2 makes the inductive computation of E(v) much easier since the computation of E(v,S) depends on the values E(v,T) for S's subcoalitions T, consisting of s-1 members only. Thus we avoid the repetitive addition of dividends and remembering dividends for all the subcoalitions of S (as given in the definition of E).

Another interesting property of the egalitarian solution is what we may describe as equality among partners. We call a coalition S a coalition of partners if for every $T \subset S$ and for every $M \subseteq N - T$, $v(M \cup T) = v(M) - \mathbb{R}_+^{M \cup T}$. In other words, a coalition of partners is one in which no subset of the partners can contribute anything to any of the coalitions unless all the partners are present.

Proposition 3. If S is a coalition of partners in the game v then for every two partners i and j in S, $E_i(v) = E_j(v)$.

Clearly the properties discussed in Propositions 1, 2, and 3 can be formulated and proved mutatis mutandis for the general egalitarian solution E^λ.

Our last remark concerns the behavior of the solutions E^λ on the family of games with side payments. A game v in Γ is said to be with side payment if there exists a function $\hat{v}$ which assigns to each coalitions a real number $\hat{v}(S)$ such that for each S, $v(S) = \{x \in \mathbb{R}^S | \sum_{i \in S} x_i \leqq \hat{v}(S)\}$. (When only games with sidepayments are considered the function $\hat{v}$ rather than v is considered as the game.) The set of games with side payments Γ_0 is a finite dimension linear where for each two games v and w, $(v + w)(S) = v(S) + w(S)$ and for each scalar α, $(\alpha v)(S) = \alpha v(S)$. It is well known that the set of unanimity games $\{u_S\}_{S \subseteq N}$ is a basis for Γ_0, where the game u_S is defined by the function $\hat{u}_S$ which is defined by $\hat{u}_S(T) = 1$ for $T \supseteq S$ and $u_S(T) = 0$ otherwise. We cosnider the restriction of E^λ to Γ_0.

Proposition 3. The egalitarian solution E^λ is linear on Γ_0.

Proof. Without loss of generality we assume that $\sum_{i\in N} \lambda_i = 1$.

Now let $v = \Sigma\alpha_S u_S$ be a game in Γ_0. It can be easily shown that $v = 0 + \sum_{\phi \neq S \subseteq N} \alpha_S \hat{\lambda}^S$ where 0 is the game defined by $\hat{0}$ which is the function that vanishes for all coalitions. By the translation invariance and since 0 is an inessential game $E^\lambda(v) = \Sigma\alpha_S\lambda_S$. The linearity of E^λ on Γ_0 follows readily. Q.E.D.

As for the egalitarian solution for unanimity games one can easily show that for each S, $E^\lambda(u_S) = \lambda_S$. This last fact with the linearity of E^λ on Γ_0 shows that E^λ restricted to Γ_0 is exactly the weighted Shapley valued (Shapley [1953a]). Observe that when all the components of λ are the same, E^λ is the Shapely value. An axiomitization of the weighted Shapley value using the equality of partners' property is discussed by the authors elsewhere (see Kalai-Samet [1983]). A special meaning of λ in the context of games with sidepayment is discussed in the following section.

9. Discussion of the Weights

The usefullness of the egalitarian solution depends crucially on the choice of the vector λ. If we consider an arbitrator arbitrating a game then λ is a parameter left to the arbitrator's discretion. However, the egalitarian solution does supply him with a substantial simplification. The arbitrator may decide on the λ's for an imiginary simple game (maybe the simplest bargaining game) and then use these λ's to compute the dividends and to determine the egalitarian solution for the game being considered.

A similar simplification is possible when we try to predict the outcome of a game. If the players have played some games in the past then the λ's are available from these past games. If no previous games have been played in the past then we need to predict the outcome of a simple imaginary game and again use these resulting λ's to predict the outcome for the more complicated game.

For a fixed λ, the egalitarian solution, E^λ, does depend on the scale of the utilities chosen to represent individual preferences. More specifically, given λ, a game v and its solution $E^\lambda(v)$, let us consider a different game $\bar{v}$ which is obtained when, say, player 1 multiplies his scale of utility by a factor of, say 2,

$$\bar{v}(S) = \{(2x_1, x_2, \ldots, x_n) : x \varepsilon v(S)\}.$$

For the solution to be invariant under multiplicative scale changes we should have

$$E^\lambda(\bar{v}) = (2E_1^\lambda(v), E_2^\lambda(v), \ldots, E_n^\lambda(v)).$$

This can easily be shown not to be the case for E^λ by almost every nondegenerate example of a game v and seems to present a fundamental difficulty with the egalitarian solution. However this difficulty disappears if we observe that with a change of scale for player 1's utility we should carry a corresponding change of scale in his λ. Thus if player 1's scale was changed by a multiplicative factor of 2 then we should use $\bar{\lambda} = (2\lambda_1, \lambda_2, \ldots, \lambda_n)$ and indeed

$$E^{\bar{\lambda}}(\bar{v}) = (2E_1^{\lambda}(v),\ E_2^{\lambda}(v), \ldots, E_n^{\lambda}(v)).$$

Thus, we do obtain invariance for the general process which includes the choice of λ (to depend on the utility scales) in addition to the application E^{λ}.

To illustrate how the λ's may be chosen to be in accordance with the discussion above we present the following example. We emphasize that this is an example of how the mechanics of the procedure may work and not an endoresement of the particular choices of the parameters.

The arbitrator decides that a unit of leisure time is a fair unit to compare the utility gains of the players. Faced with a game v he chooses λ with λ_i being player i's utility for a unit of liesure at the present status quo. With this vector λ he then applies E^{λ} to the game v. We observe that if a player's utility scale is changed by some multiplicative factor, then so does his utility for leisure and therefore his λ is changed by the same multiplicative factor as discussed above. Thus the real outcome chosen by the arbitrator is not affected by the individual choices of scales.

An equivalent way of describing this procedure is the following. The arbitrator rescales the utility of the players in such a way that in the rescaled version every player's utility for a unit of leisure time is 1. He then applies the symmetric egalitarian solution to the rescaled game in order to determine the final choice. It is obvious that whatever initial choice of scale was done by a player, its effect is washed away when the arbitrator does the rescaling.

From the above discussion it is apparent that a good interpretation of the λ_i's are as interpersonal weights to compare the utility of the individuals for fixed scales used by them. Before we proceed with other possible interpretations we discuss two examples of simple games and their egalitarian solutions.

The first game is the simplest 3-person bargaining game, sometimes refered to as divide-the-dollar game. Formally, we define it by

$$d(S) = 0 - \mathbb{R}_+^S \text{ if } S \neq \{1,2,3\} \text{ and}$$

$$d(\{1,2,3\}) = \{x \in \mathbb{R}^3 : \textstyle\sum_{i=1}^3 x_i \leqq 1\}.$$

The second game we consider is the 3-person majority game with sidepayments. It is defined by

$$m(i) = 0 - \mathbb{R}_+^{\{i\}} \text{ for } i=1,2,3,$$

$$m(i,j) = \{x \in \mathbb{R}^{\{i,j\}} : x_i + x_j \leqq 1\} \text{ for } i \neq j, \text{ and}$$

$$m(1,2,3) = \{x \in \mathbb{R}^3 : x_1 + x_2 + x_3 \leqq 1\}.$$

In the first game, d, the consent of all three players is required in order to "receive the dollar" while in the second game, m, any majority can receive the dollar. The symmetric egalitarian solution, coinciding with the Shapley value for the game with transferable utility, allocates (1/3, 1/3, 1/3) in both games. We are interested in the allocation of the nonsymmetric egalitarian solution for these two games.

Consider $\lambda = (1,1,M)$ where we think of M as a large positive number. It is obvious that

$$E^{\lambda}(d) = \frac{1}{2 + M}(1,1,M)$$

and the player with the large λ receives most of the dollar. To understand this with the illustration given above in mind we observe that in the present scale of utility of player 3, a unit of leisure is comparable to many utiles. Since the arbitrator uses a unit of leisure as a fair comparison it follows that in his present scale the third player should recieve a relatively large payoff.

To compute the solution of the second game, $E^{\lambda}(m)$, we compute the dividends inductively.

$$D(\{i\}) = (0,0,0,) \quad \text{for} \quad i=1,2,3.$$

$$D(\{1,2\}) = (.5, .5, 0)$$

$$D(\{1,3\}) = \frac{1}{1 + M}(1,0,M)$$

$$D(\{2,3\}) = \frac{1}{1 + M}(0, 1, M)$$

Now

$$\sum_{S \subset N} D(S) = \frac{1}{1 + M}(1.5 + .5M, 1.5 + .5M, 2M) = Z(N).$$

To compoute D(N) we should find t such that $Z(N) + t\lambda$ would have coordinates adding up to 1.

$$Z(N) + t(1,1,M) = \frac{1}{1+M}(1.5 + .5M + t[1+M],\ 1.5 + .5M + t[1+M],\ 2M + tM[1+M])$$

Solving for t we obtain

$$t = -2(1 + M)/(2 + 3M + M^2).$$

Substituting for t we obtain

$$E^{\lambda}(m) = Z(N) + t(1,1,M)$$

$$= \frac{1}{1 + M}(1.5 + .5M - \frac{2[1 + M]}{2 + M},\ 1.5 + .5M - \frac{2[1 + M]}{2 + M},\ 2M - \frac{2M[1 + M]}{2 + M}$$

and by simple inspection we see that

$$E^{\lambda}(m) \to (.5,\ .5,\ 0) \text{ as } M \to \infty.$$

Thus what we observe is that while in the bargaining game the player with the big λ receives a high payoff--in the majority game he receives a small payoff. We find this outcome to be quite intuitive. In the bargaining game his participation is necessary and given his λ he must be highly paid. In the majority game on the other hand, the other two players can do without him and including him is very costly. Therefore in such a game the high λ person is likely to be left out yielding him a low payoff.

It was suggested to us that the different λ's may also indicate nonsymmetric bargaining ability on the part of the players. This, however, does not seem to be consistent with the examples above. We feel that if a big λ indicated a high level of bargaining ability then such a player would be wise enough to lessen his demands in the majority game and improve his outcome.

It turns out that when we restrict our attention to games with transferable utility, the (possibly nonsymmetric) egalitarian solution, E^{λ}, coincides with Shapley's [1953] generalization of his value to weighted value. For this generalization Owen [1968a] exhibited an interesting interpretation to the weights λ_i as rates of slowness to arrive to the bargaining. Quoting from Owen's [1968a] paper (with some change in symbols to be consistent with ours) we read:

> "Let us consider the following model: suppose the n players agree to meet some place at a given time. Their individual times of arrival will be random variables; assume that player i's arrival time is a random variable X_i with distribution
>
> $$p_r\{X_i \leq x\} = x^{\lambda_i}$$
>
> for $x \varepsilon [0,1]$. If player i is preceded by the members of S, he receives the payoff $v(S \cup \{i\}) - v(S)$ [these are the real numbers v(T) from the transferable utility representation of the game]. Then, we shall see that E_i^{λ} is the expectation of this payoff."

10. The Necessity and Sufficiency of Monotonicity

It was argued in the previous sections that monotonicity of a solution is a necessary and sufficient condition in order to bring about full cooperation. In this section we present one model of a noncooperative prebargaining game to illustrate this point.

The analysis of this noncooperative prebargaining game is necessary in situations under which players can control their level of cooperation with different coalitions. In other words, if we have a given cooperative game, v, the players can choose to alter the feasible sets v(S)'s by manipulating parts of the environment that they individually control, changing the game v to a game $\bar{v}$ in which their individual payoffs may be better. These types of manipulations can be observed when players choose to destroy some initial resources at their disposal, breaking lines of communications with other players, or vetoing some of the alternatives available to a coalition by threatening to break cooperation.

When such manipulations are available a game $\bar{v}$ would be played rather than the original game v. Of course, the game $\bar{v}$ that the players individually choose to play depends on the cooperative solution that an arbitrator, or the group of players, chooses to impose. Thus we must analyze the combination of the noncooperative prebargaining choices simultaneously with the cooperative solution that we apply.

To make the analysis possible we take this ability to manipulate to an extreme. We assume that every player can veto any feasible alternative of a coalition to which he belongs.

We start with a given cooperative solution ϕ and a cooperative game v. A strategy for player i in the prebargaining game is a list $(\bar{v}^i(S))_{S:i\varepsilon S}$ where each $\bar{v}^i(S) \subseteq v(S)$ and $\bar{v}^i(S)$ is required to satisfy the conditions in the definition of a game. Our interpretation is that for a given i and S, $\bar{v}^i(S)$ contains precisely the alternatives in v(S) that i is willing to support if he bargains with S. Equivalently, we could think that player i chooses to veto all thealtneratives in $v(S) - \bar{v}^i(S)$ when he bargains with the coalition S.

Given n strategies of this type we define a resulting remaining game $\bar{v}$ by

$$\bar{v}(S) = \bigcap_{i \in S} \bar{v}^i(S).$$

$\bar{v}(S)$ contains precisely all the alternatives that have a unanimous support by all the members of S.

We define the outcome of the prebargaining game by $\phi(\bar{v})$.

Proposition 4. If ϕ is a monotonic solution then for every player i

$$(\bar{v}^i(S))_{S:i\in S} = (v(S))_{S:i\in S}$$

is a dominant strategy in the prebargaining game.

The proof of this proposition is immediate and it shows that if we use a monotonic solution ϕ, then in the prebargaining game all the players have a strong incentive to cooperate and not to veto any feasible alternative.

A stronger version of the converse to this proposition also holds.

Proposition 5. Let ϕ be a solution such that for every game v the strategies $(\bar{v}^i(S))_{S:i\in S} = (v^i(S))_{S:i\in S}$ are a Nash equilibrium of the prebargaining game, then ϕ is monotonic.

The proof of this proposition is also obvious. It shows that if we want our players to fully cooperate and keep the game v as it is, without reducing feasible alternatives by vetoing then we must use monotonic solutions.

It follows from the above two propositions that when we apply monotonic solutions to cooperative games then prebargaining manipulations will not take place. Also, for no manipulation to take place, and assuming a noncooperative Nash equilibrium behavior, we must use monotonic solutions. If we apply nonmonotonic solutions to cooperative games and assume noncooperative Nash behavior in the prebargaining game it is hard to predict what properties the outcome will have. One very plausible guess is that Pareto optimality will be violated.

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