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MONOPOLY AND SUSTAINABLE PRICES AS A NASH EQUILIBRIUM IN CONTESTABLE MARKETS

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The theory of perfectly contestable markets and sustainable prices, summarized in Baumol, Panzar and Willig (1982) is an extension of the ideas of Bain (1956) in which potential competition, unencumbered by frictions, entry or exit costs, affect an incumbent firm's decisions on prices, outputs and therefore profits. In particular, the theory of perfectly contestable markets studies the effect of the existence of potential entry on market structure, prices and outputs.

The purpose of this paper is to study properties of an equilibrium in a model with perfectly contestable markets. Namely, given perfectly contestable markets under what conditions, when potential entry is taken into account, would there by only one firm found producing the entire vector of outputs and operating under sustainable prices? It is shown in this paper that when technology is expressed by a joint subadditive cost function, the notion of a sustainable monopoly can be derived as a result of a Bertrand-Nash equilibrium of an economy consisting of many potential multiproduct firms.

Consider a monopoly producing n infinitely divisible goods and facing a vector $Q(p_1, \ldots, p_n) = Q(p) = (Q_1(p), \ldots, Q_n(p))$ of inverse demand functions. Here $p_j \in E^1_+$ is the market price of good j. Suppose that the monopoly uses the technology expressed by a joint subadditive cost function $C: E^n_+ \rightarrow E^1_+$ (i.e., $C(y+z) \leq C(y) + C(z)$ for each $y, z \in E^n_+$) where $C(y)$ is the minimum cost of producing the output vector $y \in E^n_+$.

Denote by $N = \{1, \ldots, n\}$ the set of all goods and let $S \subseteq N$ be a subset of N. Let s denote the number of goods in S (or the cardinality of S) then, for a given $S \subseteq N$, y^S (or similarly $Q^S(p)$) and p^S are vectors in E^s_+ denoting quantities and prices, respectively, of goods in S. For S = N the subscript S is omitted. Thus y^S and p^S are the projections of y and p, respectively, on E^s_+. For convenience the notation $z|y^S$ with $y, z \in E^n$, will sometimes be used to denote the vector $(y^S, z^{N\setminus S})$ where $N\setminus S$ denotes the complement of S with respect to N, i.e., both $z|y^S$ and $(y^S, z^{N\setminus S})$ are the vector **z** except that the coordinates in S are replaced by y^S. The convention that $C(y^S) = C(y^S, 0^{N\setminus S}) = C(0|y^S)$ will also be used.

Consider a potential entrant having access to the same technology, expressed by the cost function C(y), as possessed by the monopoly and incurring zero entry and exit costs regardless of the goods and quantities produced. The entrant may produce any vector of quantities $\hat{y}^S$ of any subset $S \subseteq N$ of the goods at price $\hat{p}^S$. Panzar and Willig (1977) (see also Baumol, Bailey and Willig (1977) considered two types of entry behavior and their corresponding sustainability concepts. The first one is partial entry sustainability.

<u>Definition</u> <u>Sustainability against partial (quantity) entry</u>. The price vector $\bar{p}$ is <u>PE sustainable</u> if for every possible triple $(S, \hat{y}^S, \hat{p}^S)$ satisfying

$$\text{(I)} \qquad \hat{p}^S \leq \bar{p}^S$$

and

(II) $$\hat{y}^S \leq Q^S(\hat{p}^S, \bar{p}^{N\setminus S})$$

then

$$\hat{p}^S \hat{y}^S - C(\hat{y}^S) \leq 0 \ .$$

Conditions (I) and (II) describe the behavior of a partial (quantity) entrant. For the goods in S, prices are offered which are not greater than those already prevailing in the market (condition I). At these prices any quantities up to those determined by the market demand functions evaluated at the new (lower) prices $\hat{p}^S$, for goods in S and the prevailing prices $\bar{p}^{N\setminus S}$, for the rest of the goods (condition II) may be sold. Thus, $\bar{p}$ is PE sustainable if a potential entrant cannot anticipate positive profits by lowering some or all of the market prices and supplying only a portion of the demand. The second sustainability concept is weaker and specifies that entrants must supply the entire market demand generated by the lower prices they offer.

Definition Sustainability against full (quantity) entry. The price vector $\bar{p}$ is FE sustainable if for every possible triple satisfying (I) and

(III) $$\hat{y}^S = Q^S(\bar{p}|\hat{p}^S)$$

then

$$\hat{p}^S \hat{y}^S - C(\hat{y}^S) \leq 0 \ .$$

Clearly, PE sustainability implies FE sustainability. In Section II conditions under which the reverse implication holds are discussed.

I. Bertrand-Nash Sustainability

In this section a simple general equilibrium model is studied. Corresponding to this general equilibrium model is a Bertrand-Nash game which is played by many potential producers. Outputs are produced by a joint subadditive cost function available to all the producers. The equilibrium points of the game are characterized by three properties: outputs are produced by a single firm, monopoly profits are zero, and equilibrium prices are FE sustainable. This characterization combined with the discussion of Section II relating FE to PE sustainability, provides a justification for the above definitions of sustainable prices.

Consider an infinite set M of producers and a finite set $N = \{1, \ldots, n\}$ of infinitely divisible outputs. The producers in M all use the same technology. They produce a subset of outputs in N using a single input (labor). The production technology is represented by the cost function

$$L = C(y_1, \ldots, y_n)$$

which measures the minimum amount of input L required to produce the vector $(y_1, \ldots, y_n)$ of outputs.

Consumers in this model play a passive role and only their aggregate demands are considered. Behind-the-scenes it is assumed that consumers are endowed with some positive amount of the input and consume n+1 goods: Leisure and the n outputs. The input is used in the model as a numeraire. Thus if $p = (p_1, \ldots, p_n)$ is the output price vector (in input units) then $Q_j(p_1, \ldots, p_n)$ is the total amount of the j-th output demanded. Let

$$Q(p) = (Q_1(p), \ldots, Q_n(p)) .$$

Let $\overline{E}^1_+ = E^1_+ \cup \{\infty\}$ and $\overline{E}^n_+ = \underset{i=1}{\overset{n}{X}} \overline{E}^1_+$. The demand function $Q(\cdot)$ is assumed to be defined on $\overline{E}^n_+$ with the convention that $p_j = \infty$ implies $Q_j(p) = 0$.

The above model is associated with the following game in strategic form. The set of players is the set M of producers. The strategy set of each producer is $\overline{E}^n_+$, i.e., the set of output price vectors. This strategy set is consistent with Bertrand's use of prices and not quantities as strategies. An M-tuple of strategies is a function $\underset{\sim}{p}$ from M to $\overline{E}^n_+$. The strategy of the i-th producer under $\underset{\sim}{p}$ is $\underset{\sim}{p}(i) \in \overline{E}^n_+$. Any M-tuple of strategies $\underset{\sim}{p}$ determines, as an outcome, the price vector $p = (p_1, \ldots, p_n) \in \overline{E}^n_+$ which is defined by

$$(1) \qquad p_j = \inf\{\underset{\sim}{p}_j(i) \mid i \in M\}, \ j \in N .$$

In (1) if the inf can be replaced by the minimum operation the price p_j is the lowest price offered for the j-th output.

The payoffs to the producers are defined as their profits. The question is how to define these profits. To answer this question additional notation is needed. For each $j \in N$, let $M_j(\underset{\sim}{p})$ be the set of all "active producers" of the j-th output under $\underset{\sim}{p}$, i.e.,

$$M_j(\underset{\sim}{p}) = \{i \in M \mid \underset{\sim}{p}_j(i) = p_j\} .$$

The set $M_j(\underset{\sim}{p})$ contains all firms willing to produce the aggregate demand $Q_j(p)$ at price p_j. If $M_j(\underset{\sim}{p})$ is not a singleton then $Q_j(p)$ must be allocated in some way among the producers in $M_j(\underset{\sim}{p})$. The main results of this section, however, do not depend on the way $Q_j(p)$ is allocated among the firms in $M_j(\underset{\sim}{p})$.

Indeed let $\alpha_j(i,\underset{\sim}{p})$ be a function which determines, for each producer $i\in M_j(\underset{\sim}{p})$ and each M-tuple of strategies $\underset{\sim}{p}$, the part of $Q_j(p)$ to be allocated to the i-th producer. It is required that, for each $j\in N$,

$$i\in M_j(\underset{\sim}{p}) \iff \alpha_j(i,\underset{\sim}{p}) > 0 \quad \text{and} \quad \sum_{i\in M_j(\underset{\sim}{p})} \alpha_j(i,\underset{\sim}{p}) = 1 \ , \quad \text{if} \quad M_j(\underset{\sim}{p}) \neq \phi \ .$$

The i-th produces, as a result of the M-tuple of strategies $\underset{\sim}{p}$, the quantity,

$$y_j^i(\underset{\sim}{p}) = \alpha_j(i,\underset{\sim}{p})Q_j(\underset{\sim}{p})$$

of the j-th output. Let

$$y^i(\underset{\sim}{p}) = (y_1^i(\underset{\sim}{p}), \ \ldots, \ y_n^i(\underset{\sim}{p})) \ .$$

The payoff $\pi^i(\underset{\sim}{p})$ is the profit under $\underset{\sim}{p}$ to the i-th producer. Namely

$$\pi^i(\underset{\sim}{p}) = py^i(\underset{\sim}{p}) - C(y^i(\underset{\sim}{p})) \ ,$$

where p is defined by (1).

<u>Definition</u> An M-tuple of strategies $\underset{\sim}{p}$ results in a monopoly if for some $i\in M$, $M_j(\underset{\sim}{p}) = \{i\}$, for all $j\in N$. In particular, if p is the price vector determined by $\underset{\sim}{p}$ then $p = \underset{\sim}{p}(i)$ and for each $k\in M$, $k\neq i$, $\underset{\sim}{p}(k) >> p$.

If an M-tuple of strategies $\underset{\sim}{p}$ results in a monopoly then its profit under the corresponding price vector p is

$$\pi(p) = pQ(p) - C(Q(p)) \ .$$

<u>Definition</u> A Bertrand-Nash equilibrium (hereafter BN equilibrium) in pure strategies in this model is an M-tuple of strategies $\overline{\underset{\sim}{p}}$ such that for each $p^i \in \overline{E}_+^n$ and for each $i\in M$,

$$\pi^i(\bar{\underset{\sim}{p}}|p^i) \leq \pi^i(\bar{\underset{\sim}{p}}) ,$$

where $\bar{\underset{\sim}{p}}|p^i$ is the M-tuple $\bar{\underset{\sim}{p}}$ with $\bar{\underset{\sim}{p}}(i)$ replaced by p^i. The price vector $\bar{p}$ determined from $\bar{\underset{\sim}{p}}$ by

$$\bar{p}_j = \inf\{\bar{\underset{\sim}{p}}_j(i)\,|\,i\epsilon M\} , \quad j\epsilon N ,$$

is called a BN equilibrium price vector.

Assumptions

(i) The aggregate demand function $Q(p)$ is continuous on $E^n_{++} = \{x\epsilon E^n | x_j > 0, j = 1, \ldots, n\}$.

(ii) The cost function $C(\cdot)$ is strictly subadditive over the set of products N. Namely if $y = y^1 + y^2$ where $y^1, y^2 \epsilon E^n_+$, $y^1 \neq 0$ and $y^2 \neq 0$ then,

$$C(y) < C(y^1) + C(y^2) .$$

Theorem 1 Under Assumptions (i) and (ii) any BN equilibrium $\bar{\underset{\sim}{p}}$ yielding a positive level of production results in a monopoly.

Theorem 2 Under Assumptions (i) and (ii), the following two conditions are necessary and sufficient for a price vector $\bar{p}$ to be a BN equilibrium price vector

I. $\bar{p}$ is FE sustainable

II. $\bar{p}$ is a cost sharing price vector, i.e.,

$$\pi(\bar{p}) = \bar{p}Q(\bar{p}) - C(Q(\bar{p})) = 0 .$$

<u>Remarks</u> 1. Notice that the payoff $\pi^i(\underset{\sim}{p}|p^i)$ is not a continuous function of p^i and thus the existence of a BN equilibrium, in this model, is not guaranteed. The sufficiency part of Theorem 2 implies, however, that any sustainable cost sharing price vector $\bar{p}$ is associated with a BN equilibrium.

2. PE sustainability cannot, in general, replace FE sustainability in condition I of theorem 2. First note that PE sustainability is inconsistent with the market mechanism described above. It seems difficult to make PE sustainability consistent with any market equilibrium in the Nash sense since it gives an entrant the opportunity to determine <u>both</u> prices and quantities. Corollary 6 below shows, however, that there are wide classes of markets for which PE and FE sustainability are equivalent.

3. If $\underset{\sim}{\bar{p}}$ is a BN equilibrium and if $\pi(\cdot)$ is differentiable then $\frac{\partial\bar{\pi}}{\partial p_j}(\bar{p})\geq 0$, for each $j\in N$. This observation follows by the FE sustainability of $\bar{p}$ and the differentiability of $\pi(\cdot)$.

4. Notice that if this model the notion of BN equilibrium is equivalent to the notion of strong Nash equilibrium. In a BN equilibrium only deviations by individual producers are considered while in a strong Nash equilibrium the deviations of groupsof producers are considered.

5. The set M is assumed to contain infinitely many producers, since the finite case is very restrictive, as is made clear in Proposition 3.

6. Using the notion of ε-equilibrium it can be shown that with no restriction on the cardinality of M, conditions I and II of Theorem 2 are necessary and sufficient for $\bar{p}$ to be an ε-BN equilibrium price vector.

Proposition 3 Under Assumptions (i) and (ii) if M is finite and if $\bar{\tilde{p}}$ is a BN equilibrium then the profit function $\pi(\cdot)$ has a local maximum at the corresponding BN equilibrium price vector $\bar{p}$ and $\pi(\bar{p}) = 0$.

In the one dimensional case Proposition 3 implies that if M is finite then the price vector $\bar{p}$ is a BN equilibrium price vector if and only if the average cost curve is tangent to the demand curve at $\bar{p}$ and for any price p below $\bar{p}$ the average cost curve is above the demand curve. This is illustrated in Figure 1 where P(y) is the inverse demand curve.

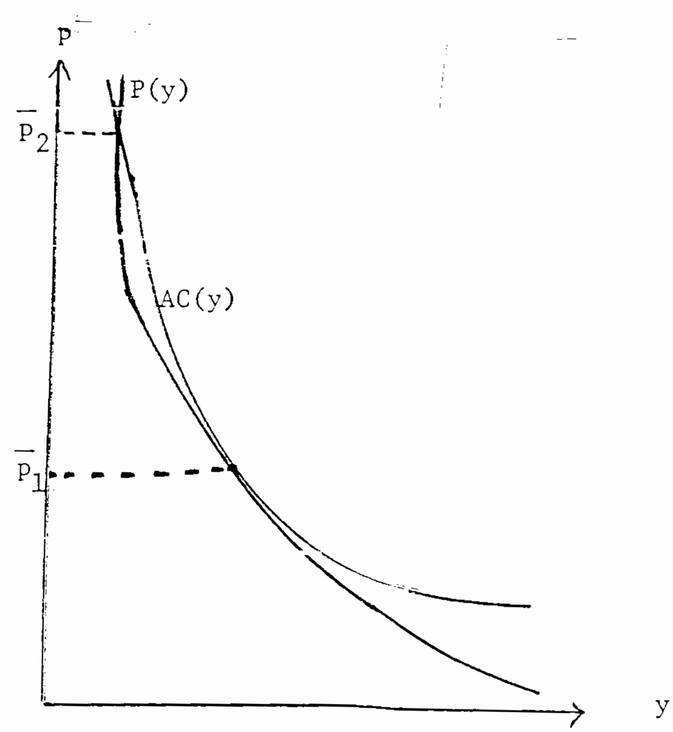

Figure 1

Both $\overline{p}_1$ and $\overline{p}_2$ in Figure 1 are PE sustainable cost sharing prices. If M is finite then $\overline{p}_1$ is a BN equilibrium price while $\overline{p}_2$ is not. If M is an infinite set then both are BN equilibrium prices. This example shows that the finite case ($|M| < \infty$) is considerably more restrictive then the infinite case.

Let $\overline{\underset{\sim}{p}}$ be an M-tuple of strategies and let $\overline{p}$ be the resulting market price vector, i.e., for each $j \in N$, $\overline{p}_j = \inf\{\overline{\underset{\sim}{p}}_j(i) \mid i \in M\}$. The following proposition states a necessary condition as well as sufficient conditions for $\overline{\underset{\sim}{p}}$ to be a BN equilibrium.

<u>Proposition 4</u> <u>Under Assumptions</u> (i) <u>and</u> (ii),

a) <u>A necessary condition for $\overline{\underset{\sim}{p}}$ to be a BN equilibrium is that for each</u> $j \in N$ <u>either $\overline{p}_j$ is a local maximum of</u> $\pi(\overline{p}|p_j)$ (<u>as a function of</u> p_j) <u>or $\overline{p}_j$ is an accumulation point of</u> $\{\overline{\underset{\sim}{p}}_j(i) \mid i \in M\}$.

b) <u>If $\overline{\underset{\sim}{p}}$ results in a monopoly,</u> $\overline{p}$ <u>is FE sustainable and</u> $\overline{p}_j$ <u>is an accumulation point of</u> $\{\overline{\underset{\sim}{p}}_j(i) \mid i \in M\}$ <u>for each</u> $j \in N$, <u>then</u> $\overline{\underset{\sim}{p}}$ <u>is a BN equilibrium</u>.

We now give the proofs of the above results.

<u>Proof of Theorem 1</u> Let $\overline{\underset{\sim}{p}}$ be a BN equilibrium. Assume that more than one firm produces positive quantities under $\overline{\underset{\sim}{p}}$, i.e.,

$$y^i(\overline{\underset{\sim}{p}}) \neq 0 \quad \text{and} \quad \sum_{k \neq i} y^k(\overline{\underset{\sim}{p}}) \neq 0 \ . \tag{2}$$

Without loss of generality it may be assumed that $\overline{p}_j > 0$, for each $j \in N$. Let $e = (1, 1, \ldots, 1)$ and let $\varepsilon > 0$ be small enough such that $\overline{p} - \varepsilon e \in E^n_+$. Consider the price vector

$$\hat{p}^i = \bar{p} - \varepsilon e .$$

Clearly

$$\pi^i(\underset{\sim}{\bar{p}}|\hat{p}^i) = \hat{p}^i Q(\hat{p}^i) - C(Q(\hat{p}^i)) \tag{3}$$

and

$$\pi^i(\underset{\sim}{\bar{p}}) = \bar{p}y^i(\underset{\sim}{\bar{p}}) - C(y^i(\underset{\sim}{\bar{p}})) . \tag{4}$$

Since $\sum_{k\in M} y^k(\underset{\sim}{\bar{p}}) = Q(\bar{p})$ we obtain by (2), together with Assumption (ii), that

$$D(\underset{\sim}{\bar{p}}) \equiv \sum_{k\in M} C(y^k(\underset{\sim}{\bar{p}})) - C(Q(\bar{p})) > 0 . \tag{5}$$

By the continuity of $Q(\cdot)$ and $C(\cdot)$, for $\varepsilon > 0$ sufficiently small there exists an $S(\underset{\sim}{\bar{p}},\varepsilon) > 0$ such that

$$\hat{p}^i Q(\hat{p}^i) - C(Q(\hat{p}^i)) > \bar{p}Q(\bar{p}) - C(Q(\bar{p})) - S(\underset{\sim}{\bar{p}},\varepsilon) \tag{6}$$

where $S(\underset{\sim}{\bar{p}},\varepsilon) \to 0$, as $\varepsilon \to 0$. By (3), (4), (5) and (6),

$$\begin{aligned}
\pi^i(\underset{\sim}{\bar{p}}|\hat{p}^i) - \pi^i(\underset{\sim}{\bar{p}}) &= \hat{p}^i Q(\hat{p}^i) - \bar{p}y^i(\underset{\sim}{\bar{p}}) + C(y^i(\underset{\sim}{\bar{p}})) - C(Q(\hat{p}^i)) \\
&> \bar{p}Q(\bar{p}) - \bar{p}y^i(\underset{\sim}{\bar{p}}) + C(y^i(\underset{\sim}{\bar{p}})) - C(Q(\bar{p})) - S(\underset{\sim}{\bar{p}},\varepsilon) \\
&= \bar{p}Q(\bar{p}) - \bar{p}y^i(\underset{\sim}{\bar{p}}) + C(y^i(\underset{\sim}{\bar{p}})) - \sum_{k\in M} C(y^k(\underset{\sim}{\bar{p}})) + D(\underset{\sim}{\bar{p}}) - S(\underset{\sim}{\bar{p}},\varepsilon) \\
&= \sum_{\substack{k\in M\\ k\neq i}} \bar{p}y^k(\underset{\sim}{\bar{p}}) - \sum_{\substack{k\in M\\ k\neq i}} C(y^k(\underset{\sim}{\bar{p}})) + D(\underset{\sim}{\bar{p}}) - S(\underset{\sim}{\bar{p}},\varepsilon) \\
&> \sum_{\substack{k\in M\\ k\neq i}} [\bar{p}y^k(\underset{\sim}{\bar{p}}) - C(y^k(\underset{\sim}{\bar{p}}))] .
\end{aligned}$$

The last inequality follows for small enough $\varepsilon > 0$ since $D(\bar{\tilde{p}}) > 0$ and $S(\bar{\tilde{p}},\varepsilon) \to 0$. as $\varepsilon \to 0$. Now since each producer makes at least zero profit (the alternative not to produce by selecting $\bar{\tilde{p}}(i) = (\infty, \ldots, \infty)$ is always possible) under the equilibrium $\bar{\tilde{p}}$ the right hand side of (9) is nonnegative. Hence

$$\pi^i(\bar{\tilde{p}}|p^i) - \pi^i(\bar{\tilde{p}}) > 0 \ ,$$

contradicting the fact that $\bar{\tilde{p}}$ is a BN equilibrium.

Proof of Theorem 2 First it will be shown that if $\bar{p}$ is a BN equilibrium price vector then it satisfies conditions I and II. Indeed let $\bar{\tilde{p}}$ be a BN equilibrium which determines the price vector $\bar{p}$. Assume first that a positive profit can be made under $\bar{p}$. Let $k \epsilon M$. The k-th firm, by offering prices below $\bar{p}$ by a sufficiently small $\varepsilon > 0$, becomes the only producer in the market. By the continuity of $Q(\cdot)$ and $C(\cdot)$ the k-th firm makes a positive profit. Consequently it pays the k-th producer to deviate from its equilibrium strategy $\bar{\tilde{p}}(k)$, which is a contradiction. Hence condition II is satisfied. Condition I follows immediately from the definition of a BN equilibrium.

Finally let us prove that if M is an infinite set then conditions I and II are sufficient for $\bar{p}$ to be a BN equilibrium price vector. Let $\bar{\tilde{p}}$ be an M-tuple of strategies such that for some $i \epsilon M$, $\bar{\tilde{p}}(i) = \bar{p}$, for each $k \neq i$, $\bar{\tilde{p}}_j(k) > p_j$, for all $j \epsilon N$ and $\bar{p}$ is an accumulation point of $\{\bar{p}(k) | k \neq i,\ k \epsilon M\}$. From condition II profits at $\bar{p}$ for firm i are zero. Hence, using condition I it is now easy to verify that $\bar{\tilde{p}}$ is a BN equilibrium. Thus the proof of Theorem 2 is complete.

Proof of Proposition 3 Since M is finite, the BN equilibrium price vector $\bar{p}$ cannot be an accumulation point of $\{\bar{\tilde{p}}(i)|i\in M\}$. By Theorem 1, there is a unique producer $i\in M$ such that $\bar{\tilde{p}}(i) = \bar{p}$ and $\bar{\tilde{p}}(k) >> \bar{p}$ for each $k\neq i$. Thus i remains the only producer even if it changes prices in a small neighborhood of $\bar{p}$. However such a change cannot yield an increase in i's profit (since $\bar{\tilde{p}}$ is a BN equilibrium). Thus $\bar{p}$ is a local maximum of π and by the necessary condition of Theorem 2 (which holds for $|M| < \infty$ as well) $\pi(\bar{p}) = 0$.

Proof of Proposition 4 a) Assume that $\bar{\tilde{p}}$ is a BN equilibrium resulting in a positive level of production. Let $i\in M$ be the resulting monopoly. By Theorem 2, $\bar{p}$ is FE sustainable. Thus

$$\pi^i(\bar{p}) = \pi(\bar{p}) \geq \pi^i(\bar{\tilde{p}}|p_j) \ , \quad \text{whenever} \quad p_j \leq \bar{p}_j \ .$$

Now assume that $\bar{p}_j$ is not a local maximum of $\pi(\bar{p}|p_j)$. Then in each neighborhood of $\bar{p}_j$ there exists p_j with $p_j > \bar{p}_j$ such that $\pi^i(\bar{\tilde{p}}) < \pi(\bar{p}|p_j)$. Thus, since $\bar{\tilde{p}}$ is a BN equilibrium, $\pi(\bar{p}|p_j) \neq \pi^i(\bar{\tilde{p}}|p_j)$ which is possible only if for some $k\in M$, $k\neq i$, $\bar{p}_j < \bar{\tilde{p}}_j(k) < p_j$. Consequently, for each neighborhood U of $\bar{p}_j$ there exists $k\in M$ such that $\bar{\tilde{p}}_j(k) \in U$ and thus $\bar{p}_j$ is an accumulation point of $\{\bar{\tilde{p}}_j(k)|k\in M\}$.

b) Let $i\in M$ be the resulting monopoly under $\bar{\tilde{p}}$. Since $\bar{p}$ is FE sustainable no other producer in M can make a positive profit. Finally it must be shown that producer i cannot increase its own profit by increasing some of the components of $\bar{p}$. If this could happen then since $\overline{p}$ is an accumulation point of $\overline{p}(k)$, $k \neq i$, the monopoly would lose the market in these components and by the FE sustainability of $\overline{p}$ the monopolist cannot increase its profits.

II. PE, FE Sustainability and BN Equilibrium

The sustainability notions of Panzer and Willig (1977) or of Baumol, Bailey and Willig (1977) aim to provide conditions which are necessary to deter a potential entrant and to sustain a monopoly through the use of prices. In Section I a simple general equilibrium model was presented consisting of a number of potential producers and a corresponding game in strategic form played by these producers in which a BN equilibrium of this game results in a monopoly sustained by the equilibrium prices. In the context of this equilibrium FE sustainability seems to be the appropriate sustainability concept. The purpose of this section is to show that if all outputs are weak gross substitutions then any FE sustainable price vector is PE sustainable. Moreover since M is an infinite set then a PE sustainable cost sharing price vector is a BN equilibrium price vector and vice versa. Namely, if all outputs are weak gross substitutes the two notions, PE sustainability and BN equilibrium prices, are equivalent.

Throughout this section we consider a monopoly operating at a cost sharing vector $\overline{p}$ (i.e., $\overline{p}Q(\overline{p}) = C(Q(\overline{p}))$).

Assumptions

(iii) The cost function C is twice differentiable on $E^n_+ \setminus \{0\}$ and $C_{\ell j} \leq 0$, for any ℓ, $j \in N$.

(iv) For each $j \in N$, $Q_j(\cdot)$ is differentiable on $E^n_+ \setminus \{0\}$.

(v) The goods in N are weak gross substitutes, namely $\frac{\partial Q_j}{\partial p_\ell} \geq 0$, for each $\ell \neq j$.

<u>Proposition 5</u> <u>Under Assumptions</u> (iii), (iv) <u>and</u> (v), <u>an entrant maximizing profits can select a subset</u> $S \subseteq N$, <u>of prices</u> $\hat{p}^S$, <u>such that</u> $\hat{p}^S \leq \bar{p}^S$ <u>and will produce the entire demand</u> $Q^S(\hat{p}^S, \bar{p}^{N \setminus S})$.

<u>Proof</u> A maximizing entrant will solve, for each $S \subseteq N$ and each p^S, with $p^S \leq \bar{p}^S$, the following problem,

$$\max_{y^S} \; y^S p^S - C(y^S) \; , \tag{10}$$

subject to,

$$y^S \geq 0 \; ,$$

and

$$Q^S(p^S, \bar{p}^{N \setminus S}) - y^S \geq 0 \; .$$

The result is the maximization of profit over $S \subseteq N$ and over $p^S \leq \bar{p}^S$. Assume that an optimal solution of the entrants problem is a set $S \subseteq N$ and a price vector $\hat{p}^S \leq \bar{p}^S$ Then, for the given S and $\hat{p}^S$, the Karush-Kuhn-Tucker necessary optimality conditions for the maximization problem (10) are

$$\begin{aligned} &\hat{p}^S - \nabla^S C(y^S) + u^S - \nu^S = 0 \\ &u^S y^S = 0 \\ &\nu^S (Q^S(\hat{p}^S, \bar{p}^{N \setminus S}) - y^S) = 0 \\ &u^S \geq 0, \; \nu^S \geq 0 \; . \end{aligned} \tag{11}$$

Now, if for some $j \in S$, $0 < y_j < Q_j(\hat{p}^S, \overline{p}^{N\setminus S})$, then by (11), $u_j = \nu_j = 0$ and hence $\hat{p}_j = \frac{\partial C}{\partial y_j}(y^S)$. In this case by shifting from y_j to $Q_j(\hat{p}^S, \overline{p}^{N\setminus S})$ the entrant cannot make less profit. Indeed in this case the profit of the entrant changes from $\hat{p}^S y^S - C(y^S)$ to

$$\sum_{\substack{\ell \in S \\ \ell \neq j}} \hat{p}_\ell \, y_\ell + \frac{\partial C}{\partial y_j}(y^S) Q_j(\hat{p}^S, \overline{p}^{N\setminus S}) - C(y^S | Q_j(\hat{p}^S, \overline{p}^{N\setminus S})) \; .$$

Thus the change Δ in the profit is

$$\Delta = \frac{\partial C}{\partial y_j}(y^S) Q_j(\hat{p}^S, \overline{p}^{N\setminus S}) - \frac{\partial C}{\partial y_j}(y^S)\, y_j - (C(y^S | Q_j(\hat{p}^S, \overline{p}^{N\setminus S})) - C(y^S))$$

$$= \frac{\partial C}{\partial y_j}(y^S)[Q_j(\hat{p}^S, \overline{p}^{N\setminus S}) - y_j] - \frac{\partial C}{\partial y_j}(y^S + \alpha(Q_j(\hat{p}^S, \overline{p}^{N\setminus S}) - y_j))[Q_j(\hat{p}^S, \overline{p}^{N\setminus S}) - y_j] \; ,$$

for some α, $0 < \alpha < 1$. Therefore by Assumption (iii), $\Delta \geq 0$. This proves that for each $j \in S$, the optimal y_j is either zero or $y_j = Q_j(\hat{p}^S, \overline{p}^{N\setminus S})$. Suppose now that S can be broken to S_1 and S_2 such that the optimal solution of (10) is

$$y^{S_1} = Q^{S_1}(\hat{p}^S, \overline{p}^{N\setminus S}) \quad \text{and} \quad y^{S_2} = 0 \; .$$

Now, by Assumption (v)

$$Q^{S_1}(\hat{p}^{S_1}, \overline{p}^{N\setminus S_1}) \geq Q^{S_1}(\hat{p}^{S_1}, \hat{p}^{S_2}, \overline{p}^{N\setminus S}) \; . \tag{12}$$

Hence the entrant by selecting S_1, instead of S, and $\hat{p}^{S_1}$ will make at least as much profit as with S and $\hat{p}^S$, since by (12), $y^{S_1} = Q^{S_1}(\hat{p}^S, \overline{p}^{N\setminus S})$ may still be selected and with $\hat{p}^S$ the same profit made. On the other hand since S together with $\hat{p}^S$ is optimal it is not possible to make more profit under S_1 and $\hat{p}^{S_1}$. Hence by selecting S_1, $\hat{p}^{S_1}$ and $Q^{S_1}(\hat{p}^{S_1}, \overline{p}^{N\setminus S_1})$ the entrant maximizes profit, as claimed. Finally, notice that S_1 might be empty in which cases the prices $\overline{p}$ are PE sustainable.

Note that a result similar to Proposition 5 was obtained by Panzar and Willig (1977) under a different assumption. Namely, assumption (iii) is replaced by declining average incremental cost (DAIC).

Corollary 6 Under Assumptions (iii), (iv) and (v) any FE sustainable price vector is PE sustainable.

Proof This is a direct consequence of Proposition 5.

It should be mentioned that the definition of PE sustainability in the case in which outputs are not gross substitutes, besides being inconsistent with BN equilibrium, suffers from a severe problem. Since the entrant is not required, under PE sustainability, to supply the entire demand resulting from the new prices, the demand might be manipulated by announcing low prices for goods not produced. For example, consider a market consisting of two complementary goods, e.g., gasoline and cars. Suppose that the cost of producing these two goods is separable (namely, there is no joint cost) and that the average cost of producing cars is declining. Clearly, if an entrant announces a near zero price for gasoline together with a minor reduction in the price of cars a higher demand for cars will result. Thus, the average cost of cars at the new demand will be lower and the entrant will make a positive profit. Hence, as long as sustainable prices are considered, it seems necessary to require that an entrant be required to produce the entire demand of the goods offered at reduced prices.

References

Bain, J. S., Barriers to New Competition, Cambridge: Harvard University Press (1956).

Baumol, W. J., E. E. Bailey and R. D. Willig, "Weak Invisible Hand Theorems on the Sustainability of Multiproduct Natural Monopoly," American Economic Review, 67 (1977), 350-365.

Baumol, W. J., J. C. Panzar and R. D. Willig, Contestable Markets and the Theory of Industry Structure, Harcourt Brace Jovanovich (1982).

Panzar, J. C. and R. D. Willig, "Free Entry and the Sustainability of Natural Monopoly," Bell Journal of Economics, 8 (1977), 1-22.