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Short-run Analysis of Fiscal Policy in a Simple Perfect Foresight Model*

by

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1. Introduction

Many recent papers have developed models which investigate the dynamic evolution of the economy with an eye towards analysis of fiscal and monetary policy. Tobin and Buiter (1976), Blinder and Solow (1973), and Turnovsky (1977) have studied dynamic versions of the Keynesian IS-LM model. The other major line of investigation has been the analysis of perfect foresight models, e.g., Hall (1971), Brock (1974,1975), Brock and Turnovsky (1981), and Abel and Blanchard (1980a,b). The major strength of the perfect foresight framework is its foundation in standard microeconomic principles and the ease of long-run analysis, whereas quantative short-run analysis has been lacking in these models. While qualitative phase diagram analysis (e.g., as in Abel and Blanchard) is instructive, it is incapable of determining the short-run response to many intertemporally complex policy shocks of interest.

This paper develops the quantative short-run analysis of a perfect foresight model. In particular, we examine how an economy initially in a steady state responds to an unanticipated and arbitrarily complex change in current and future levels of taxation and spending. Since the analysis is local, we can use basic linear techniques. The major difference between this and most other linear models is that our coefficients are derived from basic parameters if taste and technology, allowing the examination of the quantitative significance of policy shocks and their sensitivity to these parameters, using estimates of these parameters received from the empirical litereerature. Also, we add a bond market, allowing examination of policy shocks which do not have continuous budget balance.

The formulas developed below indicate the initial impact on investment, consumption, and production due to balanced-budget changes in income taxation, investment tax credit changes, and government consumption. This analysis is then applied to several issues with interesting results. For example, a permanent cut in the tax rate followed with a lag by a future spending cut, large enough to satisfy the government's budget constraint, may initiate a phase of capital decumulation and output decline, which continues until government consumption declines, after which capital accumulates until it reaches the new higher steady-state level. This possibility is realized for parameters considered representative of the U.S. economy. This is only one example of how short-run movements may differ in a quantitatively significant fashion from long-run movements, pointing out the need for tools in analyzing these short-run effects.

We then turn our attention to the issue of debt versus tax financing of government expenditures where we find that the nonequivalence of debt and income taxes is not trivial, even though the preference structure falls in the class studied by Barro (1975). This demonstrates that the distortionary character of income taxation cannot be ignored in discussing this issue. Most surprising is that outside of perverse cases a temporary balanced-budget shift to debt followed by tax increase would initially stimulate capital formation and depress consumption, contrary to the standard arguments.

The third policy issue addressed is the stimulative powers of the investment tax credit. We find that while tax credits today will stimulate investment today, future tax credits may stimulate or depress investment today, depending on rather the sum of the pure rate of time preference and the rate of depreciation is less than or exceeds the positive eigenvalue of the linearized equilibrium equations. In more intuitive terms this means that in fast-adjusting economies, future tax credits depress current investment, while they encourage current investment in slow-adjusting economies.

The paper is organized as follows. Section 2 contains a description of the basic model. Section 3 discusses a graphical analysis of one particular fiscal policy. In section 4, the basic short-run quantitative analysis of perfect foresight models is developed. Sections 5 and 6 apply these results to particular fiscal policies. Section 7 outlines extensions of the model to adjustment costs, imperfect capital markets, and elastic labor supply. Section 8 summarizes the paper's main points.

2. The Model

Assume that we have an economy of a large fixed number of identical, infinitely-lived individuals. The common utility functional is assumed to be additively separable in time with a constant pure rate of time preference, ρ:

$$U = \int_0^\infty e^{-\rho t} u(C(t))dt$$

where $C(t)$ is consumption of the single good at time t. One unit of labor is supplied inelastically at all times t by each person, for which he receives a wage of $w(t)$. This assumption is made so that we may focus on the techniques used here. The case of elastic labor supply will be discussed briefly, a complete analysis being left for a separate study.

There are two assets in this economy, government bonds and capital stock, each with the same net rate of return since they will be perfect substitutes. Let $F(k)$ be a standard neoclassical CRTS production function giving output per capita in terms of the capital-labor ration, k. At $t=0$, k_0 is the endowment of capital for each person. Capital depreciates at a constant rate of $\delta > 0$. $f(k)$ shall denote the net national product, that is, gross output minus depreciation. σ will denote the elasticity of substitution between capital and labor in the net production function.

We shall keep the institutional structure simple. Think of each agent owning his own firm, hiring labor and paying himself a rental of $r_E(t)$ per unit of capital at t, gross of taxes, credits, and depreciation. It is straightforward that the alternative assumption of value-maximizing firms would be equivalent (see Abel and Blanchard (1980a) or Brock and Turnovsky (1981) for formal demonstrations of this.) Since there will be no discussion of policies that are sensitive to institutional structure, we shall use that fact and ignore the institutional detail that firms bring. The gross return on bonds at t will be denoted $r_B(t)$.

In the future it will be convenient to use consumption defined as a function of p, the marginal utility of consumption, so define c(p):

$$(1) \qquad u'(c(p)) \equiv p$$

Also, let $\beta \equiv u''(C)C/u'(C) = c(p)p/c'(p)$ denote the elasticity of marginal utility, also called the coefficient of relative risk aversion.

The government will play no constructive role: at time t, it taxes capital income net of depreciation at a proportional rate $\tau_K(t)$, taxes labor income at a proportional rate of $\tau_L(t)$, assesses a lump-sum tax of $\ell(t)$ per capita, gives an investment tax credit on gross investment of $\theta(t)$, units of consumption per unit of investment, consumes g(t) units of output, pays interest on outstanding debt, and floats $\dot{b}(t)$ new bonds. The bonds are assumed to be continuously rolled over, allowing us to ignore effects due to the term structure of debt. The necessary adjustments for consols will be noted.

This model is consistent with two types of public consumption. First, the public consumption can be thought of as either public goods which do not affect the marginal rates of substitution among private goods, or transfers to individuals who participate in neither the capital nor labor markets. Both interpretations are modeled formally by assuming that the private utility functional is additively separable in private and such public consumption. For example, this would correspond to assuming that while there may be value to each taxpayer of transfers to the poor, the level and path of such transfers do not affect the demand of the taxpayers for their private goods. This would also correspond to assuming that public expenditures on national defense do not affect private demand for private goods. The second class of public goods consistent with this model are those which are perfect substitutes for private consumption. Being perfect substitutes, their consumption is equivalent to lump-sum transfers to the public. Therefore, our model includes both classes of public goods, g representing public goods which are additively separable with respect to private consumption and the lump-sum taxation representing those which are perfect substitutes. Since lump-sum transfers to agents who participate in capital and labor markets are equivalent to consumption of public goods which are perfect substitutes for private consumption, we shall refer to g as government consumption. With this formulation we will be able to concentrate on purely fiscal policy issues while allowing two major classes of public expenditures.

The representative agent will choose his consumption path, C(t), capital accumulation, $\dot{k}(t)$, and bond accumulation, $\dot{b}(t)$, subject to the instantaneous budget constraint, taking the wage, rental, and tax rates as given:

$$\underset{C(t),k(t)}{\text{Maximize}} \int_0^\infty e^{-\rho t} u(C(t))dt$$

$$\text{s.t.} \quad C + \dot{k} + \dot{b} = w(1-\tau_L) + ((r_E - \delta)k + r_B b)(1 - \tau_K) - \ell + \theta(\delta k + \dot{k})$$

$$k(0) = k_o$$

(Time arguments are suppressed when no ambiguity results.) It is convenient to define

$$q(t) \equiv \int_t^{\infty} e^{\rho(t-s)}((r_E - \delta)(1-\tau_K) + \delta\Theta)\, u'(c)\, ds \tag{2}$$

where q(t) is the current marginal utility value of an extra unit of capital at time t. The basic arbitrage relation which must hold is

$$(1 - \theta(t))u'(C(t)) = q(t) \tag{3}$$

This states that along an optimum path, each individual is indifferent between an extra $1-\theta(t)$ units of consumption and the extra future consumption that would result from an extra unit of investment. This expression will yield the level of consumption at any time as a function of the current q and the tax parameters:

$$C = c(q/(1-\theta)) \tag{4}$$

The arbitrage condition for investment in bonds is similar:

$$u'(C(t)) = \int_t^{\infty} e^{\rho(t-s)} u'(C(s))\, r_B(s)\, (1-\tau_K(s))\, ds \tag{5}$$

Since these equalities hold at all times t, we may conclude

$$\rho - \dot{p}/p = r_B\,(1-\tau_K) = \frac{(r_E - \delta)(1 - \tau_K) + \delta\theta - \dot{\theta}}{1 - \theta} = \rho - \dot{q}/q - \dot{\theta}/(1-\theta) \tag{6}$$

Equation (6) tells us what r_B must be in terms of r_E and the tax parameters[1]. In the foregoing, r_B will therefore be regarded as the function of r_E, θ, $\dot{\theta}$, and τ implied by (6). We shall assume that the transversality conditions at infinity hold for both assets:

$$(\mathrm{TVC}_\infty) \quad \lim_{t\to\infty} q(t)k(t)\, e^{-\rho t} = 0, \qquad \lim_{t\to\infty} p(t)b(t)\, e^{-\rho t} = 0.$$

This condition is needed to insure that p, q, and k remain bounded as $t\to\infty$ and is a necessary condition for the agent's problem if $u(\cdot)$ is bounded, which is a harmless assumption here since the net production function is bounded (see Benveniste and Scheinkman (1982)). In the case of bonds, the content of these conditions is most clear: the government is not allowed to play a Ponzi game with consumers, i.e., it cannot succeed forever in paying off interest on old bonds by floating new bonds.

To describe equilibrium, impose the equilibrium conditions

(7a) $$r_E = F'(k)$$

(7b) $$w = f(k) - kf'(k)$$

(7c) $$\dot{b} = g + \theta(\delta k + \dot{k}) - \tau_K kf'(k) + b\, r_B(1-\tau_K) - \tau_L(f(k)-kf'(k)) - \ell(t)$$

on (2) and the budget constraint, yielding the equilibrium equations

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1 Without any real loss of generality, we may assume θ to be a C^1 function of time. That is unnecessary if one interprets all the foregoing as generalized functions and uses the operational calculus.

(8a) $$\dot{q} = q\left(\rho - \frac{(1 - \tau_K)f'(k) + \delta\theta}{1 - \theta}\right)$$

(8b) $$\dot{k} = f(k) - c\big(q/(1-\theta)\big) - g$$

where $f(k) = F(k) - \delta k$, the net product. Note that these equations describe only the real activity of the economy, the path of bond holdings being determined as a residual obeying equation (7c). The transversality condition insures that

(9) $$0 < \lim_{t\to\infty} q(t),\ \lim_{t\to\infty} k(t) < \infty$$

The pair of equations, (8), describe the equilibrium of our economy at any t such that q and k are differentiable. To determine the system's behavior at points where q or k may not be differentiable, we impose the equilibrium conditions on (2), yielding

(10) $$q(t)e^{-\rho t} = \int_t^\infty e^{-\rho s} q(s)[f'(k(s))(1 - \tau_K(s)) + \delta\theta(s)]/(1-\theta(s))\, ds$$

which shows that q(t) is a continuous function of time. The system of relations given by equations (8) and (10) and the inequality (9), will describe the general equilibrium of our economy.

Why Study this Model?

Since there are many alternative models available for studying short-run effects in perfect foresight models, some being preferable on grounds of realism and/or tractability, it is a fair demand that this particular model be defended as worthy of study.

Two-period overlapping generations models, e.g., Diamond (1970), have been extensively studied, are more tractable, and easier to understand in terms of standard consumption and production theory. While two-period versions are good for understanding the qualitative features of perfect foresight analysis, they are far too rigid for meaningful quantitative short-run analysis. For purposes of application, a period in such a model would be on the order of 25-30 years, a period far longer than what would be realistically regarded as the short-run. Another alternative would be the Cass-Yaari model where time is continuous and the lifespan of a typical individual is arbitrary. However, that model is not analytically tractable. An alternative to the analytic approach here would be numerical simulation of the Cass-Yaari model, as Auerbach and Kotlikoff (1981) have done in the context of tax policy. However, the errors inherent in numerical analysis would limit us to simulating large changes in the parameters, whereas the analytical approach used here is capable of computing marginal effects of changes in the parameters. These may be substantially different due to the nonlinearities of such models. Since legislative deliberations usually concern relatively small changes, the ability to compute marginal effects is desirable.

One of the objectionable features of this model is the infinite life of the agents. While it is absurd to assume that any person has an infinite life, it is also an open question as to whether this is a bad approximation. The work of Kotlikoff and Summers (1981) indicates that substantial amounts of wealth are held for bequest purposes, in which case the true economic agent would consist of several generations of a family, having a life in excess of the roughly 50-year economic life span of an individual person. Therefore, function of current rate-of-return on capital, rendering it incapable of analysing anticipation effects which are very important in our analysis and for many of the arguments made by policymakers and analysts.

The above reasons are basically ones of theoretical soundness and realism, but not of demonstrated empirical validity. Nothing defensible on that issue will be said here, leaving each reader with his prejudices. However, the analysis below will still be of interest to those who reject this full-employment approach to macroeconomic analysis since this model is, among all models seriously considered in the literature, closest in spirit to the position of many influential policymakers. We are therefore testing their arguments for logical consistency. For example, some policymakers believe that if taxes are cut immediately to be followed later by a spending cut, the tax cut will stimulate capital formation in spite of the temporary deficit. Can they believe in their perfectly competitive philosophy and believe that there are no substantial short-run consequences of the resulting deficit for capital accumulation and production? In fact, let us now move to a graphical analysis of this issue in our model. This will serve to illuminate the basic features of this model and demonstrate how short-run effects may differ from long-run effects.

3. Graphical Analysis

One can partially analyze the impacts of policy changes on the equilibrium in a graphical fashion using phase diagrams[3]. In this section I will analyze a recent policy issue, in particular, what are the short-run consequences of a large general tax cut followed with a lag by the cut in consumption which is necessary for the government's dynamic budget to be

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3 Other examples of such graphical analysis can be found in Abel and Blanchard(1980b).

balanced? Will the deficits incurred in the short-run lead to less capital formation than if the spending cut were immediate, or if there were no cut in either taxes or spending? For the purpose of this example we will assume that there is no investment tax credit and that both capital and labor income is taxed at the rate τ; this makes the graphical analysis more transparent and later we will see that the implications are not substantially altered. In this section we will examine the more interesting case where all government expenditure is public consumption which is not perfectly substitutable for private consumption, i.e., represented by g in the equilibrium equations above.

In this case, equations 8 can be represented qualitatively by a phase diagram as in figure 1a. Note that this phase diagram is in c-k space instead of q-k space. Since labor is inelastically supplied, this representation is equally simple and clearer. It is derived from equations 8 by using the equality $q=u'(c)$, which holds since there is no investment tax credit. In this example, the $\dot{c}=0$ curve is the locus in c-k space where consumption is constant and is derived from (8a); the $\dot{k}=0$ line represents the locus where there is no investment, being derived from (8b). Within each of the four regions defined by these curves, the arrows indicate the general movement of the system described by equations 8. This differential equation system displays a saddle point structure, having a stable and an unstable manifold, the former being the set of points such that if the system starts there it converges to the steady state, point A. Note that a change in τ will affect only the $\dot{c}=0$ locus and that changes in g affect only the $\dot{k}=0$ curve. Also note that the $\dot{c}=0$ locus is vertical and that the $\dot{k}=0$ locus is upwardly sloped.

With these tools in hand we can analyze the effects of a tax cut followed with a lag by an expenditure cut sufficient to balance the dynamic budget of the government. We now examine figure 1b. In a high τ and a high g regime, the phase diagram looks like that described by the two lines intersecting at A, the corresponding steady state. If there were no lag between cuts in τ and g, consumption would jump vertically to that point on the stable manifold of the system with steady state C; this is because capital cannot move instantaneously but stability of equilibrium and the constancy of future tax rates and government consumption requires that the economy immediately move to the stable manifold. Suppose that point is D, and that the stable manifold is the curve through D and C. From D, the economy would converge to C along DC.

Now suppose that there is a lag between the cut in τ and the cut in g of T units of time. Then, in the time before the cut in g, the economy is governed by the AB-BC system with steady state at B: since τ is cut , the $\dot{c}=0$ locus moves right but the $\dot{k}=0$ locus is unchanged since g is unchanged initially. If T is small then continuity in T implies that the initial consumption level must be close to D, which is in the northwest sector of the AB-BC phase diagram where movement is northwesterly . Equation (10) implies that in equilibrium there are no jumps in c at t=T. Therefore, the system between t=0 and t=T must move from somewhere on the AD line segment to a point on DC. From this we may conclude that at t=0, the economy jumps from A to a point between A and D, say E, and that it then moves northwesterly and hits a point on the line through DC at t=T. (Note that the initial increase in consumption appears to be less due to the positive lag, and that part of the deficit must be financed by capital decumulation. This is not necessarily the case because if T were greater, the necessary cut in spending would also be greater, pushing the $\dot{k}=0$ locus upward.) For larger T, the economy may initially jump to a point like F; but since it must be on DC at T, the economy must go through some phase of capital decumulation prior to the spending cuts. The eventual stable manifold may pass below A. However, in such an economy consumption would drop immediately in response to a simultaneous permanent tax and spending cuts, a feature which most empirical analysis indicates is implausible.

This example illustrates the basic principles of the model in a transparent graphical fashion but also shows that such graphical analysis is inconclusive even in a rather simple example. We shall return to this example in section 5 below after developing the necessary analytical tools.

4. Quantitative Analysis

While the graphical analysis above is instructive, it is sometimes inconclusive in determining qualitative features of the equilibrium and it is always incapable of answering questions concerning the quantitative importance of these effects. To answer such questions we must use analytical techniques. We will concentrate on analyzing a simple perturbation of a steady state, though the analysis can be easily adjusted when the initial condition is not the steady state. Suppose that the government has been taxing at rates τ_K and τ_L, granting an investment tax credit at a rate θ, and consuming goods at a rate $\bar{g}$ for a long time, and that the economy has reached the corresponding steady state, with bonds at that level consistent with budget balance. Next suppose that at t=0, the government has announces that τ_K at t>0 will be $\varepsilon h_K(t)$ greater, τ_L at t will be $\varepsilon h_L(t)$ greater, the lump-sum tax will be $\varepsilon l(t)$ greater, the investment tax credit will be $\varepsilon z(t)$ greater, and that government consumption will be $\varepsilon g(t)$ greater. Note that if ε equals zero, the initial equilibrium will persist. To continue, it is necessary to make the following assumption.

Constancy assumption: h_K, h_L, g, l, and z are all eventually constant functions of time.

This assumption is necessary to insure the existence of a new steady state but is harmless since the date of eventual constancy is arbitrarily distant. For any fixed ε, the equilibrium of our model is therefore given by the solution to the differential equations:

(11a) $$\dot{q} = q\left(\rho - \frac{(1-\tau_K-\varepsilon h_K)f'(k) + \delta(\Theta+\varepsilon z)}{1-\Theta-\varepsilon z}\right)$$

(11b) $$\dot{k} = f - c\left(\frac{q}{1-\theta-\varepsilon z}\right) - (\bar{g} + \varepsilon g(t))$$

(11c) $$\left|\lim_{t\to\infty} k(t)\right| < \infty, \; k(0) = k_0.$$

We shall denote the solutions as $k(t,\varepsilon)$ and $q(t,\varepsilon)$, making explicit the dependence on ε. Suppose that $\varepsilon = 0$ and that the economy has reached the steady state. Now, the government announcement is essentially that ε has been increased. We would like to know the impact of this change in ε on a number of variables including capital, and capital formation at future times, i.e., we want to know the values of

$$\frac{\partial k}{\partial \varepsilon}(t,0) \equiv k_\varepsilon(t), \quad \frac{\partial}{\partial \varepsilon}\left(\frac{\partial k}{\partial t}\right)(t,0)) \equiv \dot{k}_\varepsilon(t).$$

Also of interest will be the impact on q and its time rate of change,

$$\frac{\partial q}{\partial \varepsilon}(t,0) \equiv q_\varepsilon(t) \;, \quad \frac{\partial}{\partial \varepsilon}\left(\frac{\partial q}{\partial t}\right)(t,0) \equiv \dot{q}_\varepsilon(t).$$

We are implicitly making the economically innocent assumptions which guarantee the existence of these derivatives (see Oniki (1971)). Differentiation of the equilibrium system yields a linear differential equation in the variables k_ε, q_ε:

$$(12)\qquad \begin{pmatrix} \dot{q}_\varepsilon \\ \dot{k}_\varepsilon \end{pmatrix} = \begin{pmatrix} 0 & \frac{-q(1-\tau_K)}{1-\theta} f'' \\ \frac{-c'}{1-\theta} & f' \end{pmatrix} \begin{pmatrix} q_\varepsilon \\ k_\varepsilon \end{pmatrix} + \begin{pmatrix} q\left(\frac{h_K f' - (\rho+\delta)z}{1-\theta}\right) \\ \frac{-c' qz}{(1-\theta)^2} - g(t) \end{pmatrix}$$

Since we are initially in a steady state, the matrix in (12) is actually constant. We shall call that matrix J, since it is the Jacobian of the equilibrium differential equation. Therefore, the system in (12) is actually linear with constant coefficients and we can take its Laplace transform. (The Laplace transform of a function f(t) defined for positive t, is another function F(s) defined for sufficiently large positive s, where $F(s) = \int_0^\infty e^{-st} f(t)\,dt$.) Let $Q_\varepsilon(s)$, $K_\varepsilon(s)$ be the Laplace transforms of $q_\varepsilon(t)$, $k_\varepsilon(t)$, respectively. These Laplace transforms therefore satisfy the Laplace transform of (12) which is

$$(13)\qquad \begin{pmatrix} sQ_\varepsilon(s) \\ sK_\varepsilon(s) \end{pmatrix} = J\begin{pmatrix} Q_\varepsilon(s) \\ K_\varepsilon(s) \end{pmatrix} + \begin{pmatrix} \frac{q}{1-\theta}(H_K(s)f' - (\rho+\delta)Z(s)) + q_\varepsilon(0) \\ -G(s) - c' qZ(s)/(1-\theta)^2 \end{pmatrix}$$

Solving for $Q_\varepsilon(s)$ and $K_\varepsilon(s)$ yields

$$(14)\qquad \begin{pmatrix} Q_\varepsilon(s) \\ K_\varepsilon(s) \end{pmatrix} = (sI-J)^{-1}\begin{pmatrix} \frac{q}{1-\theta}(H_K(s)f' - (\rho+\delta)Z(s)) + q_\varepsilon(0) \\ -G(s) - c' qZ(s)/(1-\theta)^2 \end{pmatrix}$$

We need to find the value of $q_\varepsilon(0)$, the initial change in the marginal utility value of an extra unit of capital. This is tied down by invoking the stability condition. We know from the stability conditions that $q(t,\varepsilon)$ and $k(t,\varepsilon)$ are bounded in t for any fixed ε; we need to prove that $k_\varepsilon(t,0)$ and $q_\varepsilon(t,0)$ are also bounded.

Lemma 1: $k_\varepsilon(t,0)$ and $q_\varepsilon(t,0)$ are bounded in t.

Proof: Since h_K, h_L, g, and ℓ are eventually constant, for some T, they are all constant for t > T. Since after T the economy must be on the stable manifold and converging to the new steady state, we conclude that

$$|k(t,\varepsilon) - k(\infty,\varepsilon)| \leq |k(T,\varepsilon) - k(\infty,\varepsilon)|$$

for t>T, where $k(\infty,\varepsilon)$ is the steady-state value of capital. This allows us to estimate $k(t,\varepsilon) - k(t,0)$, for t>T:

$$\begin{aligned} |k(t,\varepsilon) - k(t,0)| &\leq |k(t,\varepsilon) - k(\infty,\varepsilon)| + |k(\infty,\varepsilon) - k(\infty,0)| \\ &\leq |k(T,\varepsilon) - k(\infty,\varepsilon)| + |k(\infty,\varepsilon) - k(\infty,0)| \\ &\leq |k(T,\varepsilon) - k(T,0)| + 2|k(\infty,\varepsilon) - k(\infty,0)| \end{aligned}$$

Dividing the above inequality by ε and taking ε to zero, we find that

$$|k_\varepsilon(t,0)| \leq |k_\varepsilon(T,0))| + 2|k_\varepsilon(\infty,0)|$$

showing that $k_\varepsilon(t,0)$ is bounded for t>T. For t<T, we also obtain boundedness since k is jointly continuous in ε and t by continuity of solutions of differential equations with respect to parameters. Similarly, $q_\varepsilon(t,0)$ is bounded.

Let $\mu > 0 > \lambda$ be the eigenvalues of J[4]. Lemma 1 insures that $K_\varepsilon(s)$ is bounded for all s>0. In particular, it must be bounded for s=μ. At first sight, this appears to be impossible given (14) and the fact that μI-J is singular. However when we write $(sI-J)^{-1}$ in terms of the adjoint divided by its determinant, we find that $K_\varepsilon(\mu)$ is bounded if and only if

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4 That there is one eigenvalue of each sign is assured by the sign structure of J.

(15) $$\frac{q_\varepsilon(0)}{q} = \frac{1}{1-\theta}\left((\rho+\delta-\mu)\ Z(\mu) - H_K(\mu)f'\right) - \frac{\mu\ \beta}{c}\ G(\mu)$$

Combining (14) and (15), we have the solution for $K_\varepsilon(s)$ and $Q_\varepsilon(s)$. Having solved for the Laplace transforms of the adjustment paths of q and k, we can now use them to determine the impact of the shocks on economic variables, derive an expression for the government's dynamic budget constraint, and decompose the initial impact on consumption into its income and substitution effects.

(i) Impact on consumption and investment at t=0

The above determines the economy's response to a change in ε. The formulas are in terms of the Laplace transforms of the policy changes. However, it is possible to compute the value of k_ε and q_ε and their time derivatives at t=0 without solving for the inverse Laplace transforms of K_ε and P_ε. The crucial fact about Laplace transforms which we use is

(17) $$f(0) = \lim_{s\to\infty} sF(s)$$

if F(s) is the Laplace transform of f(t).

<u>Theorem 1</u>: The initial impact of the announced changes on investment is

(18) $$\dot{k}_\varepsilon(0) = \frac{c}{|\beta|(1-\theta)}\ \{z(0) + (\rho+\delta-\mu)Z(\mu) - f'\ H_K(\mu)\} + \mu G(\mu) - g(0)$$

<u>Proof</u>: From the properties of Laplace transforms, we know that

$$\dot{k}_\varepsilon(0) = \lim_{s\to\infty} s^2 K_\varepsilon(s)$$

$$= \lim_{s\to\infty} \frac{s^2}{(s-\mu)(s-\lambda)} \left\{ \frac{-c'q}{1-\theta} \left(\frac{(\rho+\delta)}{1-\theta}(Z(\mu) - Z(s)) - \frac{(H_K(\mu) - H_K(s))}{1-\theta} f' - \mu Z(\mu) \right) \right.$$

$$\left. + \mu G(\mu) - s(G(s) - \frac{sc'q}{(1-\theta)^2} Z(s) \right\}$$

$$= \frac{c}{|\beta|(1-\theta)} \left\{ z(0) + (\rho+\delta-\mu)Z(\mu) - f' H_K(\mu) \right\} + \mu G(\mu) - g(0) .$$

From the formula given in Theorem 1 for the impact on investment, we can note several aspects of the relationship between fiscal policy and capital formation. First, an increase in government expenditure at t=0, g(0), causes a dollar for dollar decrease in capital formation. This is not surprising. In a life-cycle model such as this one, a consumer endeavors to have a steady level of consumption, hence a momentary spurt in government consumption of g(0) at t=0 will be satisfied by less capital accumulation.

Second, the impact of future government consumption on capital formation is expressed in the term $\mu G(\mu)$, i.e., discount the change in government spending at the rate μ, not ρ, and multiply the result by μ. To get some intuition for this, let us first examine a plausible, but false, procedure. One may have argued that the appropriate measure of future government consumption on investment would be $\rho G(\rho)$ - i.e., take the discounted value of the expenditures, $G(\rho)$, as their capitalized value and note that a savings flow of $\rho G(\rho)$ would finance the expenditures at the existing real rate of interest. This would be an individual's response if interest rates were unaffected. However, interest rates will respond to these policy changes. Equation (18) shows that this procedure is valid for general equilibrium calculations with the proper discount rate being μ, not ρ. This fact points out the importance of general equilibrium analysis versus partial equilibrium analysis, since the positive eigenvalue is generally much larger that the pure rate of time preference for realistic values of the crucial parameters. Note that since $\mu > \rho$, $\mu G(\mu)$ puts more weight on changes in government consumption in the near term relative to the distant future changes than $\rho G(\rho)$ does, that is, the naive partial equilibrium approach overestimates the impact of government consumption in the distant future on investment today and underestimates the impact of such expenditures in the immediate future. In particular, we see that the anticipation effects of future policy changes decay very rapidly as the date of the change becomes more distant.

Third, the impact of future and present taxation on investment today is summed up in the first term. Again, note that the appropriate discount rate is μ, as expressed in $H_K(\mu)$, the Laplace transform of $h_K(t)$ evaluated at μ. From this we learn that the anticipation effects of future taxes on current investment is much smaller than one may have expected, since the positive eigenvalue is generally much larger than the pure rate of time preference. This expression has an interesting interpretation. $\frac{\rho}{1-\tau} H_K(\mu)$ is the change in revenue discounted at μ if the capital stock doesn't change, expressed as a fraction of the capital stock. Hence the change in investment is this capitalization factor times consumption divided by the elasticity of marginal utility, yielding a decomposition of the change in investment into multiplicative factors representing consumption, curvature of utility, and the value of the tax change capitalized at μ. This expression for the impact to capital formation is useful for comparative dynamic analysis and highlights two important points. First, if β is large in absolute value, the investment response to future tax changes is sluggish, since high curvature in the utility function indicates a desire for an even consumption stream and little taste for extreme changes in consumption to finance volatile investment plans. Second, investment responds much more to tax changes today and in the near future than it does to more distant tax changes.

One aspect of (18) which may initially appear to be puzzling is that an increase in future government consumption, holding current government consumption constant, encourages investment today. Since this term indicates the impact on investment today with the capital income tax rate being held constant, the spending is implicitly being financed by lump-sum taxes. Because of the bond market, the timing of these lump-sum taxes is immaterial, but their existence is essential for the government to remain within its budget constraint. Therefore, with income taxes held constant, extra spending will cause $\mu G(\mu)$ to be positive, causing investment to increase because of the consumers' needs to finance the extra lump-sum taxes.

In examining the impact of the investment tax credit changes we see that the role of timing is more crucial, for $z(0)$, the extra tax credit today, plays an important role, as well as $Z(\mu)$. Clearly, as $z(0)$ increases, so does investment at $t = 0$. This is expected since $z(0)$ is the change in today's subsidy to today's investment. The impact of the rest of the tax credit on investment today is ambiguous. The portion of the tax credit policy in effect after today causes investment today of $(\rho + \delta - \mu)Z(\mu)c/|\beta|(1-\Theta)$. Even if $z(t) > 0$, the sign of this is ambiguous, being positive for slow-adjusting economies, $\rho + \delta > \mu$, and negative for fast-adjusting economies, i.e., $\rho + \delta < \mu$. An intuitive explanation for this is that fast-adjusting economies are associated with less concave utility functions. When faced with smaller future tax credits, such investors will invest more today to take advantage of the current short-lived tax credits and when the tax credits are less generous in the future, just as rapidly decumulate, treating today's tax credit as a subsidy to future consumption. For people with more concave utility functions, such fluctuations in consumption are disliked and current investment is not treated as delayed consumption, but rather as a source of income for future consumption; hence, future tax credits are an inducement for investment today since more investment today leads to more depreciation in the future, the replacement of which is subsidized by future tax credits. Also reflected in (18) is the fact that whatever the impact of policy changes on investment today, that impact is magnified by the current investment tax credit. This result contrasts partial equilibrium analysis, e.g., Abel (1980), which argues that investment tax credits are generally stimulative whether they are permanent or temporary. These analyses do not take into account interest rate movements, ostensibly because the effects are trivial. Assuming that there would be no interest effects is odd in our context since investment tax credit policies are argued to have a macroeconomically significant impact on investment. We see that when we allow interest rate effects, the true general equilibrium result may be different than that indicated by the partial equilibrium analysis.

(ii) Balanced budget condition:

Next, we compute the relationship that must exist between the changes in taxation and expenditure due to the government's budget constraint. The differential equation governing bonds is

$$\dot{b} = \bar{g} + \varepsilon g(t) + r_B(1-\tau_K)b + (\theta+\varepsilon z(t))(\delta k + \dot{k}) - (\tau_K + \varepsilon h_K(t))kf'(k) - (\tau_L + \varepsilon h_L(t))(f(k) - kf'(k)) - \ell - l(t) \tag{19}$$

In the initial steady state when $\varepsilon = 0$, $r_B(1-\tau_K) = \rho$, hence

$$(20) \qquad 0 = \bar{g} + \rho b - \tau_K k f'(k) - \tau_L (f(k) - kf'(k)) - 1 + \theta\delta k$$

i.e., receipts equalled expenditures on goods, the investment tax credit, and interest payments. The government's dynamic budget constraint is that the present value of its obligations and expenditures must equal the present value of its revenues[5]. Differentiating that constraint with respect to ε using the definition of r_B and (20), and taking Laplace transforms, we find that

$$(21) \qquad \begin{aligned} 0 = {} & G(\rho) - \{\frac{Q_\varepsilon(\rho)}{q}\rho - \frac{q_\varepsilon(0)}{q} + \rho Z(\rho) - z(0)\}\, b \\ & - \tau_K K_\varepsilon(\rho)(f'+kf') - kf'H_K(\rho) + \tau_L kf'' K_\varepsilon(\rho) - H_L(\rho)(f-kf') \\ & - L(\rho) + \theta(\rho+\delta)K_\varepsilon(\rho) + Z(\rho)\delta k \end{aligned}$$

must hold where $Z(s)$, $H_K(s)$, $H_L(s)$, $L(s)$ are the Laplace transforms of $z(t)$, h_K, h_L, and l, respectively. If b, the initial stock of bonds were zero, this just is the expression for extra revenue equals extra spending discounted at the rate ρ, the steady state real net return. With $b > 0$, the real rate of interest which must be paid on bonds when they are rolled over changes; the net discounted value of the altered interest bill per unit of existing debt is

$$\frac{-Q_\varepsilon(\rho)}{q}\rho + \frac{q_\varepsilon(0)}{q} - \rho Z(\rho) + z(0).$$

With a nontrivial term structure, this term would be different and would disappear if bonds were actually consols. In that case, the bearer may experience a capital gain or loss at $t = 0$. This expression for budget

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[5] This can be derived from the consumers' budget constraints and their transversality conditions, as in Brock and Turnovsky(1981).

balance will prove useful later in the balanced-budget exercises.

(iii) Income and substitution effects:

To study the impact of policy changes on current consumption and investment, it will be useful to decompose such changes into substitution and income effects. Let $\bar{r}(t)$ be the after-tax return on capital and $\bar{w}(t)$ the after-tax wage. If an individual is assessed a lump-sum tax of $\bar{l}(t)$, then we can write his demand as

$$\dot{q} = q\ (\rho - (\bar{r} + \delta\theta)/(1-\theta))$$

$$(22) \qquad \dot{k} = \bar{r}k + \bar{w} - c(q/(1-\theta)) - \bar{l} + \theta(\delta k + \dot{k})$$

$$\lim_{t\to\infty} |k(t)| < \infty$$

Suppose such an individual is initially in a steady-state with his consumption level equal to $\bar{c}$ when there is an unanticipated policy change affecting $\bar{r}$, $\bar{w}$, and $\bar{l}$, such changes being denoted by $\bar{r}_\varepsilon$, $\bar{w}_\varepsilon$, and $\bar{l}_\varepsilon$. Using the same techniques as above, one discovers that the change in total lifetime utilty, U_ε, and the change in consumption at t=0, $c_\varepsilon(0)$, can be written

$$(23) \qquad \begin{aligned} U_\varepsilon &= u'(\bar{c})\ W_\varepsilon(\rho) \\ c_\varepsilon(0) &= \rho\ W_\varepsilon(\rho) + \bar{R}_\varepsilon(\rho)\ \bar{c}/\beta \end{aligned}$$

where

$$(24) \qquad \begin{aligned} W_\varepsilon(\rho) &= \int_0^\infty e^{-\rho t}\ (\bar{r}_\varepsilon(t)\ k + \bar{w}_\varepsilon(t) + \bar{l}_\varepsilon(t) + \delta k z(t))\ dt \\ \bar{R}_\varepsilon(\rho) &= \int_0^\infty e^{-\rho t}\ (\bar{r}_\varepsilon(t) + \delta z(t))\ dt\ /\ (1-\theta) + z(0)\ /\ (1-\theta) \end{aligned}$$

denote the income effect of the changes, and what we shall call the cumulative price effect, respectively. The decomposition of the effect on consumption into income and substitution effects will be particularly useful in the discussion below of the impact of debt financing.

In this section, we have derived three basic equations concerning the impact of policy changes on current consumption and investment, their decomposition into income and substitution effects, and the relation which must hold among the policy variables due to the government's dynamic budget constraing. Using these results, we can now move to the analysis of some specific macroeconomic problems.

5. Example: Cut taxes, then spending

At the present time, the U.S. economy is in the midst of a significant change in the structure and level of taxation. In particular, there has been a large reduction in taxation of income from nonresidential capital due to reduction in the general tax rates, 20% drop in the top personal tax bracket, accelerated depreciation, and the reduction in the rate of inflation. There is little disagreement in the theoretical literature that this program will result in increased capital stock and productivity in the long-run, if it is carried out.

However, the short-run effects of the Reagan program on capital formation are not as uncontroversial, in particular because the revenue losses are not being matched by cuts in government expenditure. The resulting deficit, projected to reach record peacetime levels, must be financed by government bonds. Of course, in the long run the government's budget must be balanced, or more specifically, that must be the expectation if investors are to be willing to hold bonds today. That balancing can be accomplished by reducing government consumption, g, or decreasing lump-sum transfers to those who participate in the economy. There are clearly elements of both implicit in arguments of this policy's proponents. (Recall that g includes consumption of public goods such as defense, transfers to those who do not participate in either the labor or capital markets, and the costs of the bureaucracies which manage all expenditures.) To the extent that the budget will be balanced by reductions in transfers to workers and investors, the analysis is straightforward from the foregoing graphical analysis and equation (18): only the $\dot{c} = 0$ will be affected and the economy will jump to the stable manifold associated with the new tax rate, converging monotonically to the new steady state where consumption, income, and the capital stock are all greater. Therefore, in this section we will initially address the case where the government's budget will be balanced by reductions in government consumption, g. The question we address is whether this unanticipated change in the financing and level of government consumption will crowd-out capital accumulation in the short-run, contrary to the long-run increase in capital.

For the purpose of this section, we shall assume that the tax on both labor and capital incomes are equal, that the changes in these taxes are also identical, that there is no outstanding debt initially, and that there is no investment tax credit. This description is not an accurate representation of the U.S. tax system nor of the recent changes. The purpose of this exercise is to give a rough approximation to the U.S. economy and show that short-run aspects of the perfect foresight model are nonnegligible and realistic in their magnitudes. The analysis will also indicate which parameters of taste and technology have significant impact on the answers. The results of this section turn out to be largely unaffected by the level of bonds and the investment tax credit when they are assigned reasonable values.

Suppose that the government decided to cut the tax rate to a lower level immediately, and reduce spending to a lower level at some future date $T > 0$. This can be modeled above by particular functional forms for g and h:

(25a) $h_K(t) = h_L(t) \equiv h(t) = -1$

(25b) $g(t) = -\gamma H_T(t) \equiv \begin{cases} 0, & t < T \\ -\gamma, & t \geq T \end{cases}$

where $H_T(t)$ is the Heaviside function with jump at T and where γ, the magnitude of the future cut in g, is unknown a priori because we do not know how much g must be cut at T to put the government back inside its budget constraint. The value of γ is generally determined by examining the balanced budget condition and is found to be

(26) $$\gamma = f(k)\, e^{\rho T} \frac{1 + \frac{\tau}{1-\tau} \frac{\rho}{\rho-\lambda} \frac{\rho}{\beta\mu}}{1 - \frac{\tau}{1-\tau} \frac{\rho}{\rho-\lambda} \frac{\rho}{\rho-\mu} (e^{(\rho-\mu)T} - 1)} \equiv f(k)\, \tilde{\gamma}$$

where $\tilde{\gamma}$ denotes the spending cut as a proportion of NNP. Using Theorem 1, we find that the impact on capital formation at t=0 is

(27) $$\dot{k}_\varepsilon(0) = \frac{c}{\beta\mu} \frac{\rho}{1-\tau} - \gamma e^{-\mu T} + g(0)$$

One interesting index of this impact is the general equilibrium marginal propensity to save, i.e., the portion of the extra disposable income at t=0 which is saved by individuals in equilibrium, denoted by MPS. (This is to be distinguished from the individual marginal propensity to save out of current income.) MPS_1 will be the MPS for the policy of cutting taxes now, cutting spending later to balance the budget. It is equal to

(28) $$MPS_1 = \frac{\rho}{\beta\mu} - \tilde{\gamma}\, e^{-\mu T} + 1$$

If $T > 0$, capital accumulation begins at t=0 if and only if MPS_1 exceeds unity

TABLE I

|β| (σ / τ)	.3 / .2	.3 / .4	.5 / .2	.5 / .4	1.0 / .2	1.0 / .4
.5	0.32	0.29	0.41	0.37	0.56	0.51
	0.42	0.41	0.49	0.46	0.61	0.57
	0.74	0.76	0.74	0.75	0.79	0.78
	0.98	1.00	0.96	0.98	0.97	0.97
	1.31	1.27	1.39	1.33	1.52	1.44
	(6.4)	(7.6)	(5.1)	(6.1)	(3.8)	(4.6)
1.0	0.22	0.20	0.28	0.26	0.37	0.35
	0.29	0.29	0.33	0.32	0.41	0.39
	0.53	0.56	0.52	0.54	0.54	0.54
	0.74	0.78	0.70	0.74	0.67	0.70
	1.21	1.18	1.26	1.22	1.34	1.28
	(4.8)	(5.6)	(3.8)	(4.6)	(2.9)	(3.5)
1.5	0.18	0.16	0.22	0.20	0.29	0.27
	0.23	0.23	0.26	0.26	0.32	0.31
	0.43	0.46	0.41	0.44	0.42	0.43
	0.62	0.67	0.57	0.62	0.54	0.57
	1.17	1.14	1.20	1.17	1.26	1.22
	(4.0)	(4.8)	(3.3)	(3.9)	(2.6)	(3.1)
3.0	0.12	0.11	0.14	0.14	0.18	0.18
	0.16	0.16	0.17	0.17	0.20	0.20
	0.30	0.33	0.28	0.30	0.27	0.29
	0.44	0.50	0.39	0.44	0.35	0.38
	1.11	1.09	1.13	1.11	1.16	1.13
	(3.1)	(3.7)	(2.6)	(3.1)	(2.1)	(2.5)
5.0	0.10	0.08	0.11	0.10	0.14	0.13
	0.12	0.12	0.13	0.13	0.15	0.14
	0.21	0.25	0.20	0.22	0.18	0.20
	0.31	0.39	0.27	0.33	0.24	0.28
	1.08	1.06	1.08	1.07	1.04	1.08
	(2.3)	(3.1	(2.0)	(2.7)	(1.7)	(2.3)
10.0	0.06	0.05	0.07	0.06	0.08	0.08
	0.08	0.08	0.08	0.08	0.09	0.08
	0.13	0.17	0.12	0.14	0.11	0.12
	0.20	0.27	0.17	0.22	0.14	0.17
	1.02	1.04	0.98	1.03	0.86	1.01
	(1.9)	(2.5)	(1.7)	(2.3)	(1.5)	(2.0)
20.0	0.04	0.03	0.04	0.04	0.04	0.04
	0.05	0.05	0.05	0.05	0.05	0.05
	0.08	0.10	0.07	0.09	0.06	0.07
	0.12	0.17	0.10	0.14	0.08	0.10
	0.92	1.01	0.83	0.99	0.65	0.93
	(1.6)	(2.2)	(1.5)	(2.0)	(1.4)	(1.8)

The column of numbers corresponding to a (β,σ,τ) triple are MPS_1 for T equal to 0, 2, 10, 20, and 400 periods, respectively, with the number in parantheses being μ/ρ.

since only then is there savings left over after the deficit is financed. Standard differentiation exercises for MPS_1 are tedious and inconclusive; furthermore, we really don't care about derivatives at all parameter values, just at reasonable ones, and we want some idea of the magnitudes involved. Therefore, Table I lists values of MPS_1 over a wide range of values for β, σ, T, and τ; ρ is normalized to be .01, indicating that one period of time is that duration over which utility is discounted by 1%. To those who believe that the annual rate of discount is 4%, this makes T equal to the number of quarters between the tax cut and the spending cut. Many attempts have been made to estimate the intertemporal rate of substitution between goods and the elasticity of substitution between capital and labor. The estimates turn out to vary substantially across studies (see Weber (1970) and (1975), Ghez and Becker (1975), Hansen and Singleton (1982), Berndt and Christensen (1973), Nerlove (1967), and Lucas(1969).) The values for β and σ used in Tables I and II represent the broad range of estimates. In examining both tables one should recall that σ here is the elasticity of substitution in the <u>net</u> production function, which is less than that of the gross production function. Since estimates for these parameters vary substantially, the results are reported for a large range of possible values for σ and β. Casual examination of national income accounts suggest that we take capital share to be .25 and government consumption to be .2 of net production. These are certainly acceptable values, especially since MPS_1 is insensitive to reasonable changes in these parameters compared to its sensitivity to σ and β.

From Table 1, we may conclude several things. First, the magnitudes of MPS_1 indicate that the effects on savings at t=0 of this policy shock are neither negligible, nor unrealistic. They also indicate that for most values of the parameters, capital will begin to decumulate at t=0 if there is a lag between tax cuts and spending cuts. Second, as T increases, MPS_1 also increases. This has a simple intuitive explanation: as the spending cuts are pushed further into the furture, their income effect on today's consumption decreases, resulting in less consumption and more savings today. Third, as β is less negative, i.e., the utility is less concave, MPS_1 increases. This, too, is easily explained: a more linear utility function cares more about total consumption relative to the smoothness of the consumption path; therefore, the price effect of the cheaper future goods dominates, depressing current consumption and increasing savings. Fourth, as the elasticity of substitution increases, savings out of the tax cut increases. This is because if σ is large, the marginal product of capital does not drop as rapidly during the accumulation of capital, resulting in the rate of interest declining less rapidly. The impact of the initial tax rate is ambiguous, but also not large. Finally, note that μ/ρ is substantially larger than one. Therefore, future tax and spending changes are discounted heavily in the computation of the initial impact on capital formation, equation (18).

Before ending the analysis of this policy shock we should discuss the case where the budget is eventually balanced by cutting consumption of public goods which are perfect substitutes for private goods. Recall that this is equivalent to an increase in lump-sum taxes. It is straightforward from (18), (26), (27), and the fact that $\mu > \rho$, that this case is equivalent to a tax cut with no change in g, or a change in g which occurs in the infinite future. When the elasticity of marginal utility is in the lower end of the region we are examining, 400 periods is practically infinity since the positive eigenvalue substantially exceeds the pure rate of time preference. Therefore, Table I tells us that when $|\beta|$ = .5, 1.0, 1.5, or 3.0, the MPS out of a dollar in tax cuts financed eventually by increases in lump-sum taxes is at most 1.5 and more likely about 1.2. Such balanced-budget changes in taxation and government expenditures therefore lead to capital accumulation immediately. However, note that the stimulus to capital formation due to these tax cuts, about 20¢ to 50¢ per dollar of tax cuts, is generally smaller than the capital decumulation which comes from a dollar in tax cuts which will be balanced by a cut in g, especially if the cut in g is expected to occur in the near future. Hence, we see that if the tax cuts are to be financed by roughly equal increases in lump-sum taxes (or cuts in rebates) and cuts in government consumption, g, then the capital decumulation induced by the latter will likely be the stronger influence on current investment.

6. Example: The nonequivalence of debt and taxes

An important issue in macroeconomic analysis is that of the impact of debt financing on current consumption and savings. Some have argued that replacing current taxes with debt causes consumers to feel wealthier because of their increased bond holdings, and thereby increase their current consumption. This positive wealth effect of debt on consumption plays a role in the analysis of expansionary fiscal policy, as in Blinder and Solow (1973). On the other hand, Barro has shown that if debt replaced current lump-sum taxation and if preferences were equivalent to those in our model, there would be no effect on any current or future real variable. In any practical analysis, however, the assumption of distortion-free taxation is unrealistic, especially at the margin. The case of distortionary taxation was not explicitly discussed in Barro (1974). Tobin (1980) has asserted that:

> "debt finance of government expenditure increases current consumption, reduces the savings available to purchase assets other than government securities. These conclusions are reinforced if real-world taxes are considered in place of lump-sum taxes."

In this section, we examine this issue when debt issue partially and temporarily replaces a comprehensive income tax, which must be increased later to keep the government within its dynamic budget constraint, since government consumption will remained unchanged. (Again, we assume no investment tax credits nor any bonds initially.) Since there are no changes in the consumption of public goods of any kind in this exercise, the results of this section are independent of our assumptions concerning the nature of government expenditures. We find that the effect on consumption of government debt is actually negative except in perverse cases (which do not arise for parameters representative of the US).

In the notation used above, the assumption of no change in government consumption corresponds to

$$(29) \qquad g(t) \equiv 0$$

The balanced budget condition then becomes

$$(30) \qquad \tau f'(k)K_\varepsilon(\rho) + H(\rho)f(k) = 0$$

where $h = h_K = h_L$, and H is the Laplace transform of h. The initial impact on investment becomes

$$(31) \qquad \dot{k}_\varepsilon(0) = \frac{c}{\beta}\frac{\rho}{(1-\tau)}H(\mu)$$

The case where taxes are cut today and raised to the higher permanent level at $t = T$ can be represented by

$$(32) \qquad h(t) = -1 + \zeta H_T(t)$$

where the tax increase ζ is unknown initially, being determined by the dynamic budget constraint.

TABLE II

σ	.3		.5		1.0	
τ	.2	.4	.2	.4	.2	.4
$\lvert\beta\rvert$						
.5	1.03	1.03	1.03	1.03	1.03	1.03
	1.13	1.13	1.13	1.13	1.13	1.13
	1.21	1.19	1.22	1.21	1.22	1.22
	1.31	1.27	1.39	1.33	1.52	1.44
1.0	1.02	1.02	1.01	1.02	1.01	1.01
	1.07	1.07	1.06	1.07	1.06	1.06
	1.11	1.11	1.11	1.11	1.11	1.11
	1.21	1.18	1.26	1.22	1.34	1.28
1.5	1.01	1.01	1.01	1.01	1.01	1.01
	1.04	1.04	1.04	1.04	1.04	1.04
	1.08	1.07	1.07	1.07	1.07	1.07
	1.17	1.14	1.20	1.17	1.26	1.22
3.0	1.00	1.00	1.00	1.00	1.00	1.00
	1.02	1.02	1.02	1.02	1.02	1.02
	1.04	1.04	1.04	1.04	1.03	1.03
	1.11	1.10	1.13	1.11	1.16	1.13
5.0	1.00	1.00	1.00	1.00	1.00	1.00
	1.01	1.01	1.01	1.01	1.01	1.01
	1.02	1.02	1.02	1.02	1.01	1.02
	1.08	1.06	1.10	1.07	1.11	1.08
10.0	1.00	1.00	1.00	1.00	1.00	1.00
	1.00	1.00	1.00	1.01	1.00	1.00
	1.01	1.01	1.01	1.01	1.01	1.01
	1.05	1.04	1.06	1.04	1.06	1.05
20.0	1.00	1.00	1.00	1.00	1.00	1.00
	1.00	1.00	1.00	1.00	1.00	1.00
	1.00	1.00	1.00	1.00	1.00	1.00
	1.03	1.02	1.03	1.02	1.03	1.03

Each column lists MPS_2 for T equalling 2, 10, 20, and 400, respectively. All entries of 1.00 actually exceed 1.00 but by less than .005.

If there are initially neither taxes nor bonds, the dynamic budget constraint reduces to $H(\rho) = 0$ which implies that $\zeta = e^{\rho T}$. This is intuitively clear since the debt will grow at the rate ρ and, since taxes are lump-sum in nature for small tax rates, the eventual tax rate need only be equal to the ratio of the capitalized value of the debt to the capitalized value of net national product. Then the impact on investment is found to be

$$\dot{k}_{\varepsilon}(0) = \frac{c\,\rho}{\beta} \left(\frac{e^{(\rho-\mu)T} - 1}{\mu}\right) > 0 \tag{33}$$

Hence, a temporary income tax cut financed by future taxes will always stimulate investment if the economy is initially untaxed.

If the economy is initially taxed, solving for ζ yields

$$\zeta = e^{\rho T}\left(\frac{1 + \frac{\tau}{1-\tau}\frac{\rho}{\rho-\lambda}\frac{\rho}{\beta\mu}}{1 + \frac{\tau}{1-\tau}\frac{\rho}{\rho-\lambda}\frac{\rho}{\beta\mu}\left(e^{(\rho-\mu)T} - 1\right)}\right) \tag{34}$$

The impact on capital accumulation is therefore equal to

$$\dot{k}_{\varepsilon}(0) = \frac{\rho}{(1-\tau)}\frac{c}{\beta\mu}\left(\zeta e^{-\mu T} - 1\right) \tag{35}$$

From this we immediately see that the temporary tax cut is more stimulative for large T, if ζ is positive.

Let MPS_2 be the general equilibrium impact marginal propensity to save out of extra disposable income at t=0 for this policy; then MPS_2 is

$$MPS_2 = \frac{\rho}{\beta\mu}\left(\zeta e^{-\mu T} - 1\right) + 1 \tag{36}$$

Again, comparative statics are ambiguous leading us to examine Table II, which gives values for MPS_2 over a wide range of parameter values. We again find that the impact effects are sometimes not negligible, though not really substantial unless the lag is large or the utility function is close to being linear. Most surprising is that the temporary tax cut always stimulates capital formation initially, and that its stimulative power increases as the lag betwen tax cuts and tax increases increases. In fact, we can state the following theorem.

Theorem 2: If a permanent cut in taxation would require a positive lump-sum tax for budget balance, then $MPS_2 > 1$, and capital formation at t=0 is stimulated.

Proof: Straightforward calculation shows that the value of government revenue is proportional to the numerator of ζ . Since $\rho<\mu$, if the numerator of ζ in (34) is positive, so is its denominator which is also greater than $e^{-\rho t}$ times the numerator. Hence, $\zeta e^{-\rho t}$ and $\zeta e^{-\mu t}$ are less than unity proving that MPS_2 exceeds unity.

Examination of Table II shows that MPS_2 is affected by parameter changes in the same manner as MPS_1. MPS_2 increases as capital and labor are more substitutable, as the utility function is more linear, and as the lag between tax cuts and tax increases grows. Also, MPS_2 exceeds 1 at all entries in Table 1 because the hypothesis of Theorem 2 is satisfied by all of the tabulated parameter values.

Another interesting fact to note is the size of MPS_2. Suppose that bonds were net wealth. In an intertemporal optimizing model such as this with inelastic labor supply, if an individual receives $1 more in wealth, he will save all of it, consuming the net interest income. For our model, that would have consumption per period increase by $\rho = 0.01$ since ρ is the steady-state net return on capital. We find that consumption actually drops, and usually the drop exceeds 0.01. In fact, for lags in excess of 10 periods (i.e., roughly 2-3 years) the drop is substantially larger than 0.01 when $|\beta|$ is less than 5. After allowing for multiplier effects, the magnitude of the effect of bond financing in this model is close to that in the typical IS-LM model while the sign is different. Therefore, under distortionary taxation, temporary deficit financing depresses consumption by an amount considered nontrivial.

Why is there this stimulus to capital formation and drop in consumption? We find that it is not due to an income or wealth effect but rather largely due to a price effect, that is, the temporary tax cut reduces the real price of goods tomorrow, inducing substitution away from consumption today. Straightforward calculations show that the income effect on consumption at t=0 is equal to $H(\rho)f(k)$. If there is no tax initially, then the budget balance condition reduces to $H(\rho) = 0$, implying that there is no income effect, intuitively because in an initially undistorted equilibrium there is no deadweight loss from a small tax. Hence the entire shock to investment is due to the cumulative price effect. At first this doesn't appear to be quite correct since goods in the distant future may be more expensive due to the later tax hike. However, these distant effects are heavily discounted, as seen in the following expression for the cumulative price effect:

$$(\frac{c}{\beta}) \; (\frac{\rho}{1-\tau})^2 \quad \frac{H(\rho) - H(\mu)}{(\rho-\lambda)\;(\rho-\mu)} \left(1 - \frac{\tau\theta_K\sigma}{\theta_L}\right) \; \frac{\theta_L}{\sigma}$$

which is positive since $\gamma < \rho < \mu$.

Standard public finance considerations help us see why investment is stimulated. If there is initially some positive tax, the income effect of a temporary tax cut followed by a compensating tax increase is negative (assuming that we are not in a perverse region where tax cuts raise revenue). This is because a temporary tax <u>increase</u> is partially a lump-sum tax on capital in place and any revenues thereby raised in the short-run would allow us to reduce future distortionary taxation. Hence, a temporary tax cut would reduce total utility, and the wealth effect would induce extra investment initially. Also, it is clear from the continuity of the cumulative price effect that it, too, would be substantial for small τ.

7. Some Extensions

The model studied above is an extremely unrealistic one. There are many features of tastes, technology, and market structure which are ignored above. In this section, the effects of adjustment costs and unequal access to capital markets are studied. It is not that these are considered to be the most important features left out of the simple model, but rather the aim is to demonstrate the ease with which elements of realism are added to the analysis.

(i) Adjustment Costs, Interest Rates, and Deficits

To model adjustment costs, we adopt the approach due to Uzawa:

$$\dot{k} = k\ \psi(I/k) \tag{37}$$

where I is gross investment expenses and ψ is a concave function of $I/k \equiv i$. Furthermore, we assume

$$\psi(\delta) = 0,\ \psi'(\delta) = 1 \tag{38}$$

which means that if gross investment equals depreciation, then net capital formation is zero and at that point, there are no adjustment costs at the margin. Assuming that net investment forms the depreciable base, which is depreciated for tax purposes at the true rate of capital depreciation, the arbitrage equation becomes

$$u'(c)(1-\theta)e^{-\rho t} = \psi' \int_t^\infty e^{-\rho s} u'(c)\ \left\{F'(1-\tau_K) + \frac{\psi - i\psi'}{\psi'}\ (1-\theta) + \delta\tau_K\right\} ds \tag{39}$$

The impact at t=0 on the interest rate for bonds is found to be

$$(40) \qquad r_{B\varepsilon}(0) = \frac{1}{1-\tau_K} \frac{\psi''\delta}{\psi'} \left(\rho\frac{\dot{k}_\varepsilon(0)}{I} - \frac{c}{I}\frac{f'}{|\beta|} h_K(0) + \frac{g'(0)}{I}\right) + \frac{(\rho+\delta)z(0) - z'(0)}{(1-\tau_K)(1-\theta)} + \frac{\theta\delta h_K(0)}{(1-\tau_K)^2(1-\theta)}$$

This formula decomposes the impact on interest rates today into several parts. First, capital decumulation is associated with higher interest rates. To understand this, think of the example analyzed in section 5. The deficit caused by the tax cut was usually financed by capital decumulation. With adjustment costs, this is a wasteful process. A higher interest rate will encourage more of the deficit to be financed by foregone consumption and is also necessary to make individuals willing to bear the adjustment costs of capital decumulation.

Second, the larger the tax cut today is, the lower the interest rate is. This is also clear since this instant's tax cut has no wealth effect and would therefore be invested completely unless the price of today's goods are made cheaper, that is, the interest rate declines. This cheapening occurs because the adjustment costs discourage such short-run movements in investment, causing interest rates to decline to stem these investment flows.

Third, if the policy change causes government consumption to grow, i.e., $g'(0) > 0$, then interest rates are depressed. Again, this aspect of the policy change has no wealth effect of consumers, only a price effect. Without adjustment costs, this short-run movement would be absorbed by fluctuations in investment. These movements will be dampened by adjustment costs, causing part of this growth in government expenditures to come out of current consumption. To facilitate this, today's goods must become cheaper, that is, interest rates decline.

Fourth, investment tax credit changes at t=0 have an impact on interest rates because of the change in the subsidy given investment in capital but not bonds. This follows from inspection of Equation (6).

Now that we have a model where interest rates are not tied to the capital stock, we can examine the impact of deficits on interest rates. We find in this model that we cannot draw any definite conclusions, because the interest rates and the deficits may move independently. If the deficits are due to a temporary substitution of debt for distortionary taxation, then we saw that capital accumulation would result and that interest rates would be reduced. However, if the deficits were due to a tax cut today to be followed by future reductions in government consumption, then there would initially be capital decumulation and a rise in interest rates. Since the relation between interest rates and deficits is so sensitive to the manner in which the government's budget will eventually be balanced or expected to be balanced, any empirical relation between deficits and interest rates would be due to an historical accident that one type of policy or expectation of policy occurred more frequently than the other.

One can also compute the impact of a policy change on the value of the capital stock. If there were no adjustment costs, the value, V(t), of the capital stock at t, always equals the capital stock. However, with adjustment costs, there will be a wedge between the value of a unit of existing capital and the value of a unit of investment expenditure. Therefore the value of existing capital may jump in response to a policy change. That jump is found to be

$$
\begin{aligned}
\frac{V_{\varepsilon}}{V} &= -z(0) - \frac{\psi''\delta}{\psi'}\left(\rho\,\frac{\dot{k}_{\varepsilon}(0)}{I} - \frac{c}{I}\,\frac{f'}{|\beta|}\,h_K(0) + \frac{g'(0)}{I}\right) \\
&= -z(0) - \frac{\psi''\delta}{\psi'}\,\frac{I_{\varepsilon}(0)}{I}
\end{aligned}
$$

The change today in the value of capital is the sum of two effects. First, if investment tax credits today make existing capital worth less in terms of today's goods since the investment tax credit is a subsidy to investment. Second, a drop in investment indicates that the economy wants to decumulate but such decumulation is blunted by adjustment costs through higher interest rates. Intuitively, it also leads to a drop in the value of capital since we have too much capital relative to the desired level.

(ii) Unequal Access to Capital Markets

The homogeneity assumptions of the model analyzed above are very unrealistic. One important difference among individuals is their access to capital markets. People with small amounts of savings are often forced to accept low rates of return. In particular, if the rate of return available to them is much lower than their rate of time preference, they would not accumulate substantial amounts of capital. To capture this differential access to capital markets, assume that $\alpha \leqslant 1$ is the portion of wage income which goes to workers who neither save nor lend. We choose this representation since consumers who hold substantial amounts of capital in the steady state are not likely to be the ones who are constrained in the capital markets nor are they likely to be individuals with high rates of time preference.

We shall confine our attention to the case of a comprehensive income tax equal to τ, and assume no investment tax credit. The equilibrium equations are only slightly different from the perfect capital markets case:

$$\text{(41)} \qquad \begin{aligned} \dot{p} &= p(p - (1-\tau-\varepsilon h(t))f'(k)) \\ \dot{k} &= f - c(p) - \overline{g} - \varepsilon g(t) - \alpha(f - kf') \end{aligned}$$

The only change is in the accumulation equation since the arbitrage condition for the representative investor is unchanged by the presence of noninvestors. Since p must now be interpreted as the marginal utility of consumption of investors, the income term of the accumulation equation is reduced by the wages of noninvestors, $\alpha(f - kf')$. Changes from the $\alpha = 0$ case occur in both the value of the eigenvalues and the capital accumulation equation:

$$(42)\quad \begin{aligned} \mu,\lambda &= \frac{\rho}{2(1-\tau)}\left(1 - \frac{\alpha\Theta_L}{\sigma} + \sqrt{(1 - \frac{\alpha\Theta_L}{\sigma})^2 + \frac{4(1-\alpha\Theta_L)(1-\tau)\Theta_L}{\beta\ \sigma\ \Theta_K}}\right) \\ \dot{k}_\varepsilon(0) &= \frac{c}{\beta}\ \frac{\rho}{1-\tau} H(\mu) + \mu G(\mu) - g(0) \end{aligned}$$

The form of the capital accumulation equation appears to be the same but is not because now c represents only the consumption of the capitalists which is less than the after tax national income by the net income of the noninvestors. This latter fact reduces the magnitude of the first term of the capital accumulation expression just as an increase in the curvature of the utility function would. Since α of the wages go to noninvestors the adjustment speeds are slower because capital accumulation comes from a smaller portion of national income. The slowing is accentuated by two factors - high labor share and low elasticity of substitution. The importance of labor share is clear: for any level of α, a higher θ_L means more income to noninvestors. The dependence on σ is due to the fact that for low σ, the marginal product of capital declines more rapidly as capital accumulates and that decline is borne by fewer agents as α increases.

Limited participation in the capital markets will have consequences for the macroeconomic policies discussed above. For example, the fact that μ is reduced makes it more likely that capital formation will be crowded out by a deficit incurred while waiting for a spending cut.

(iii) Elastic Labor Supply

The techniques used in this study can be extended to the case of elastic labor supply in a straightforward, though tedious, fashion. While a detailed analysis of elastic labor supply deserves a separate study, we should not close this paper without a discussion of how it would affect our central findings.

There appears to be a rough consensus among labor economists that leisure is a normal good with the compensated elasticity of labor supply in response to a wage increase being substantially reduced by the income effect. First, let's examine the case of cutting taxes now and government consumption later. For the purposes of this exercise, we assume that the long-run income and substitution effects cancel out. This assumption is consistent with the fact that per capita labor supply in the U.S. has been only slightly reduced in response to the technical change of the past four decades, that observation being relevant in this context since a tax cut on total income with the revenue loss being balanced by reduction in government consumption is equivalent to output-augmenting technical change. This cancellation of income and substitution effects is plausible because of the additional income effects due to the increase in nonlabor income and the reduction in government consumption. These observations indicate that assuming no response in long-run labor supply is a good approximation and will yield at least a partial understanding of the impact of elastic labor supply on this issue.

The long-run impact on labor supply is the sum of the long-run wealth effect and the long-run price effect. The long-run wealth effect on leisure demand is roughly the same as the short-run wealth effect. To see this, note that if an individual received an increase in wealth at $t = 0$, then he would jump immediately to the new steady-state level of consumption and leisure. However, the long-run price effect is greater since the new long-run after-tax wage is higher due to the lower tax rate and the higher gross wage due to capital accumulation, whereas in the short-run, only the lower tax rate is realized. Therefore, if price and wealth effects roughly cancel in the long-run, the wealth effect will dominate in the short-run. This initial drop in labor supply will cause a drop in total production, accentuating the drop in investment if the impact on consumption were the same as in the inelastic labor supply case. Whether the impact on consumption is greater or less in this case depends on the substitution relation between consumption and leisure. In summary, we can say that adding a variable labor supply representative of the U.S. will predict even more capital decumulation, lower employment levels, and higher interest rates at $t = 0$ relative to the inelastic labor case in response to a tax cut to be followed by a drop in government consumption, leaving the net initial impact on consumption ambiguous. (Note that all this looks like a recession.) If the tax cuts became effective a short while after being announced at $t = 0$, we would only have the wealth effects at the time of announcement, further accentuating the initial decline in production and investment.

Next, consider the temporary deficit financing example of section 6. In this case we need not make any assumption concerning the long-run effects on labor supply since income and substitution effects both work to increase labor supply in the short run. Gross wages are increased initially due to the tax cut and but are reduced in the long run due to capital decumulation and the higher tax on labor income. Also, lifetime utility will decline for the same reasons as those cited in section 6. Hence, labor supply will increase initially, resulting in higher output, further increasing investment relative to the inelastic case. Again, the impact on investment and capital is magnified when labor is elastic. We therefore see that a temporary move to deficit financing will stimulate investment and production when labor supply is elastic.

One further observation of interest to macroeconomics concerns the Phillips curve. From considerations similar to the foregoing, we see that a permanent increase in the capital income tax, with the revenue being lump-sum rebated, would cause a temporary increase in labor supply due to the negative income effect on leisure demand. Since our tax system is based on nominal income, an increase in inflation would increase the effective rate of taxation on equity-financed capital. Assuming that this tax effect dominated other real effects of inflation we would conclude that an unanticipated increase in both inflation today and expectations of future inflation will cause the effective capital income tax rate to increase, leading to a temporary increase in labor supply - i.e., a "Phillips curve" between long-run inflation expectations and current labor supply. Furthermore, the increase in labor supply is temporary since capital decumulation will lead to a fall in the wage and a substitution from goods to leisure. Hence, the "Phillips curve" so constructed will move, leaving us with a much smaller permanent effect on labor supply and presumably a long-run decline in output. Just how serious one can be about such a theory of the Phillips curve depends on the results of a more quantitative analysis of these effects. However, this approach does offer an alternative to the imperfect information theories of the Phillips curve.

8. Conclusions

The primary accomplishment of this paper was the development of analytical tools for determining short-run consequences of fiscal policy in a perfect foresight model. These tools were applied to basic macroeconomic questions with strong results. We have seen that it is likely that a program of tax cuts today followed later by cuts in government consumption will initiate a period of capital decumulation which will end only when the spending cuts are initiated. On the other hand, a temporary substitution of debt for taxes generally stimulates capital formation and depresses consumption initially. The difference in these impacts is due to the different effects of the change in government expenditures on consumer welfare, with the future spending cuts generating a positive wealth effect causing more consumption of current goods. However, when there is no future spending cuts, just tax increases, there is generally a negative impact on the consumers' lifetime utility, causing a decrease in current consumption, with this decrease being accentuated by a price effect encouraging the consumption of future goods. When an elastic labor supply was added to the model, we found that these impacts on investment were magnified. We saw that the impact of future tax and spending changes on investment is smaller than would be indicated by partial equilibrium analysis. We also found that it is unclear how future investment tax credits affect investment today, being stimulative for slow-adjusting economies and depressing current investment in fast-adjusting economies. When adjustment costs are added to the analysis we found that there was no definite relation between interest rates and deficits.

The major conclusion that follows from this analysis is that the long-run forces acting on an economy do matter in the short run in a quantitatively significant fashion. While conventional macroeconomics may be correct in arguing that other forces are important in the short run due to various rigidities, the results contained herein show that the underlying long-run real forces cannot be ignored in short-run analysis. Just as significant is that the analytical determination of these effects taking into account the dynamic adjustment process is a tractable exercise.

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Figure 1a

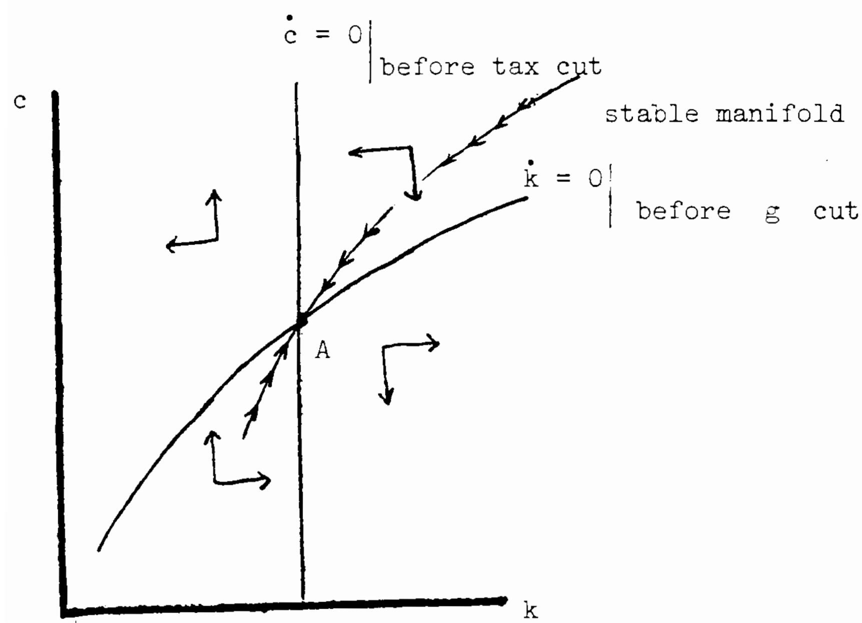
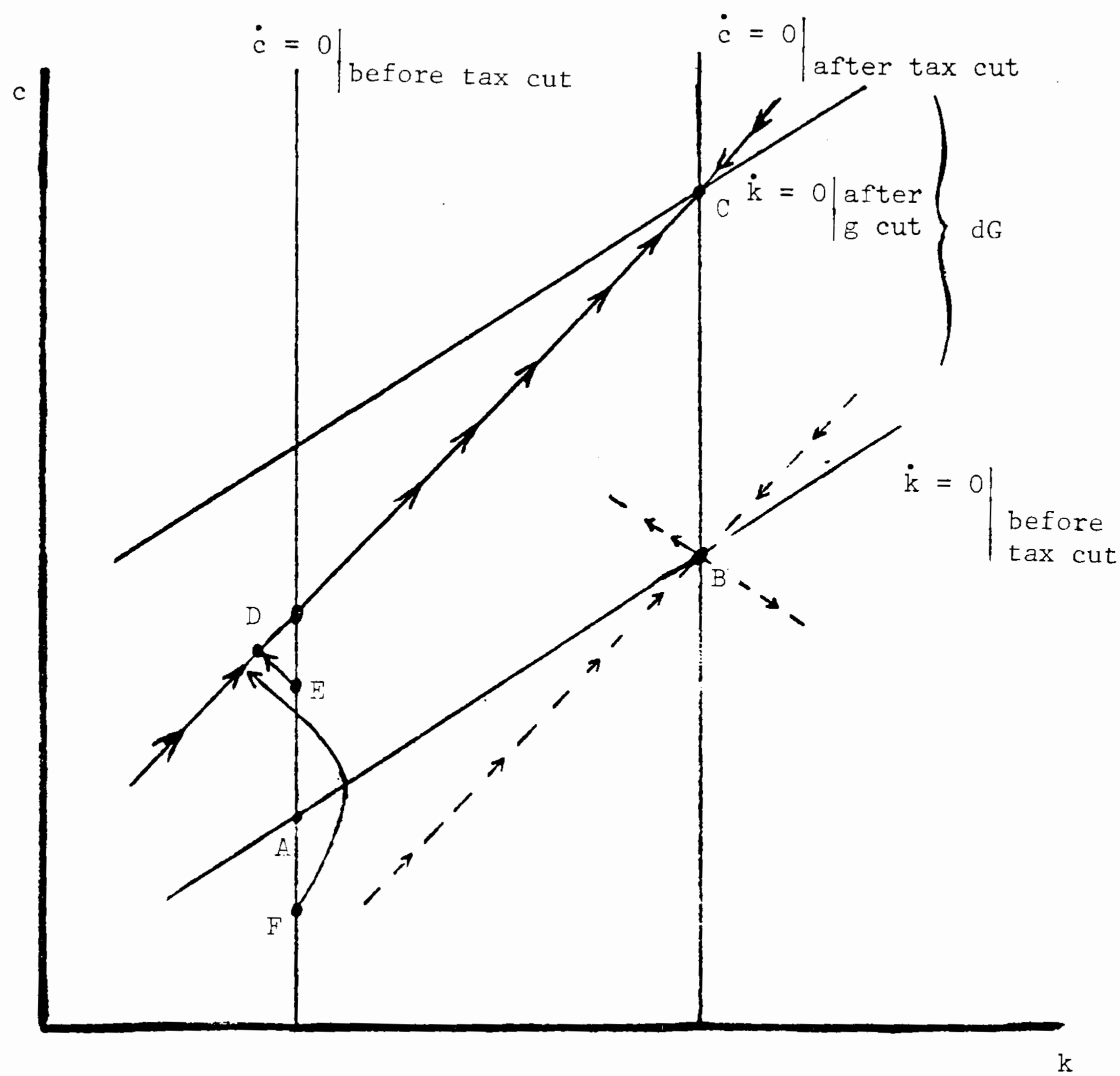


Figure 1b



Analysis of a tax cut
followed by an expenditure cut of dG