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Exercises in Voodoo Economics*

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Exercises in Voodoo Economics

I. Introduction

In recent years there has been an intensive political debate concerning the proper level and mix of income taxation and investment incentives. One of the more provocative claims made by current policymakers is that cutting income taxation and increasing investment incentives through more generous depreciation allowances and investment tax credits will result in *increased* tax revenues due to the great increase in capital stock and output which would result. Even among many who reject this notion, there is a feeling that the efficiency cost of capital taxation is high. Another claim of some conservative policymakers is that the workers gain due to the increase in wages which will accompany capital accumulation. In this paper we examine these issues in a simple equilibrium growth model.

This paper is similar in spirit to several other contributions to the literature on capital income taxation, but differs in substance and methodology. We adopt an intertemporal maximization approach to modeling asset accumulation versus the neoclassical growth approach of Feldstein (1974,1975) and Bernheim (1981). We adopt a continuous time model as opposed to the two-period overlapping generations model of Diamond (1971) and Feldstein (1978). This is done to avoid the intertemporal aggregation of the latter approach. We adopt a representative infinitely-lived agent approach instead of the finite-lived agents assumed in Auerbach and Kotlikoff (1983). Since there is evidence (e.g., Kotlikoff and Summers (1981) that substantial amounts of capital are held for bequest purposes, it is important that the representative agent model be examined. In order to concentrate on capital accumulation we assume a fixed labor supply. In a methodological vein, we adopt a local linearization approach instead of the quadratic approximation approach of Chamley (1981) and the numerical analysis approach of Auerbach and Kotlikoff. This allows us to compute the exact marginal revenue and welfare effects of a change in tax rates and investment incentives and compare them with the corresponding average changes computed by Chamley.

When we evaluate the model for reasonable parameters of taxes, tastes and technology, we can make several interesting conclusions, some obvious and general, but some dependent on those parameter values. First, an equal cut in both capital and labor taxation will almost surely cause a decline in revenue. Second, if only the tax on capital is cut, the capital tax revenues will decline with much of that revenue loss being offset by increased labor tax revenues due to the induced wage increases. However, if capital and labor are somewhat more substitutable or if effective taxation of capital is greater than commonly thought, total revenue would rise. Third, if the investment tax credit is increased, the direct revenue loss is possibly covered by increased labor and capital tax revenues. Fourth, whereas capitalists always gain from a cut in the capital income tax, they may either lose or gain from an investment tax credit increase, with the result remaining ambiguous when we assume reasonable parameter values. Fifth, for moderate levels of capital taxation, the marginal revenues of a capital income tax increase are less than the decline in wages, indicating that workers may gain from capital income tax cuts even if the revenue losses were covered by increase labor taxation. Sixth, the efficiency gain from an unanticipated cut in capital income taxes is substantial; for example, if capital income is taxed at 50 per cent, investors receive a five per cent tax credit, and taste and technology parameters are assigned reasonable parameters, the gain is at least 50 cents per dollar of lost capital income tax revenues, net of investment tax credit, and possibly close to \$2.00, gains which are substantially greater than the average gains computed by Chamley. Also, this gain is much larger when one takes into account the induced gains in wage tax revenues. Seventh, the efficiency gain of an anticipated future cut is somewhat, but not substantially, higher. Eighth, the efficiency gain of an unanticipated permanent increase in the investment tax credit financed by an increase is even greater, ranging between \$1.00 and possibly \$5.00 per dollar loss in net capital income tax for the example cited above, with the gain being substantially greater for temporary investment tax credit increases. From this we may conclude that investment incentives are substantially superior to tax cuts, with investment tax credit increases financed by capital tax increases even yielding substantial efficiency gains.

II. The Model

Since the model is fully described in several other papers, e.g., Brock and Turnovsky (1981), we shall only review its essential elements here.

Assume that we have an economy of a large fixed number of identical, infinitely-lived individuals. The common utility functional is assumed to be additively separable in time with a constant pure rate of time preference, ρ:

$$U = \int_0^\infty e^{-\rho t} u(C(t))dt$$

where $C(t)$ is consumption of the single good at time t. One unit of labor is supplied inelastically at all times t by each person, for which he receives a wage of $w(t)$. This assumption is made so that we may focus on aspects of capital taxation, the case of elastic labor supply deserving a separate study.

There are two assets in this economy, government bonds and capital stock, each with the same net rate of return since they will be perfect substitutes. Let $F(k)$ be a standard neoclassical CRTS production function giving output per capita in terms of the capital-labor ration, k. At $t=0$, k_o is the endowment of capital for each person. Capital depreciates at a constant rate of $\delta > 0$. $f(k)$ shall denote the net national product, with σ being the elasticity of substitution between capital and labor.

We shall keep the institutional structure simple. Think of each agent owning his own firm, hiring labor and paying himself a rental of $r_E(t)$ per unit of capital at t, gross of taxes, credits, and depreciation. The gross return on bonds at t will be denoted $r_B(t)$.

It is convenient to let $c(p)$ represent consumption defined as a function of p, the marginal utility of consumption:

$$(1) \qquad u'(c(p)) \equiv p$$

Also, let $\beta \equiv -u''(C)C/u'(C) = -c(p)p/c'(p)$ denote the elasticity of marginal utility, also called the coefficient of relative risk aversion.

To focus on the efficiency issues of taxation, we will assume that the government will play no constructive role: at time t, it taxes capital income net of depreciation at a proportional rate $\tau_K(t)$, taxes labor income at a proportional rate of $\tau_L(t)$, assesses a lump-sum tax of $\ell(t)$ per capita, gives an investment tax credit on gross investment of $\theta(t)$ units of consumption per unit of investment, pays interest on outstanding debt, floats $\dot{b}(t)$ new bonds, and returns the net receipts to agents in a lump-sum and uniform fashion. The bonds are assumed to be continuously rolled over, allowing us to ignore effects due to the term structure of debt.

The representative agent will choose his consumption path, $C(t)$, capital accumulation, $\dot{k}(t)$, and bond accumulation, $\dot{b}(t)$, subject to the instantaneous budget constraint, taking the wage, rental, and tax rates and the lump-sum transfer, ℓ, as given:

$$\underset{C(t),k(t)}{\text{Maximize}} \int_0^\infty e^{-\rho t} u(C(t))dt$$

$$\text{s.t.} \quad C + \dot{k} + \dot{b} = w(1-\tau_L) + ((r_E - \delta)k + r_B b)(1 - \tau_K) + \ell + \theta(\delta k + \dot{k})$$

$$k(0) = k_o$$

(Time arguments are suppressed when no ambiguity results.) We define

$$(2) \qquad q(t) \equiv \int_t^\infty e^{\rho(t-s)}((r_E - \delta)\ (1-\tau_K) + \delta\Theta)\ u'(c)\ ds$$

where $q(t)$ is the current marginal utility value of an extra unit of capital at time t. The basic arbitrage relation which must hold is

$$(3) \qquad (1 - \theta(t))u'(C(t)) = q(t) = \int_t^\infty e^{\rho(t-s)} u'(C(s))\ r_B(s)\ (1-\tau_K(s))\ ds$$

This states that along an optimum path, each individual is indifferent between an extra $1-\theta(t)$ units of consumption and the extra future consumption that would result from an extra unit of investment in either capital or bonds. This expression will yield the level of consumption at any time as a function of the current q and the tax parameters:

$$(4) \qquad C = c(q/(1-\theta))$$

We shall assume that the transversality conditions at infinity hold for both assets:

$$(\text{TVC}_\infty) \quad \lim_{t\to\infty} q(t)k(t)\ e^{-\rho t} = 0, \qquad \lim_{t\to\infty} p(t)b(t)\ e^{-\rho t} = 0.$$

This condition is needed to insure that p, q, and k remain bounded as $t\to\infty$. It is a necessary condition for the agent's problem if $u(\cdot)$ is bounded, which is a harmless assumption here since the net production function is bounded (see

Benveniste and Scheinkman (1982)). In the case of bonds, the content of these conditions is most clear: the government is not allowed to play a Ponzi game with consumers, i.e., it cannot succeed forever in paying off interest on old bonds by floating new bonds.

To describe equilibrium, impose the equilibrium conditions

$$(5a) \qquad r_E = F'(k)$$

$$(5b) \qquad w = f(k) - kf'(k)$$

$$(5c) \qquad \dot{b} = g + \theta(\delta k + \dot{k}) - \tau_K kf'(k) + b\, r_B(1-\tau_K) - \tau_L(f(k)-kf'(k)) + \ell(t)$$

on (2) and the budget constraint. Differentiation of the result yields the equilibrium equations

$$(6a) \qquad \dot{q} = q\left(\rho - \frac{(1-\tau_K)f'(k) + \delta\theta}{1-\theta}\right)$$

$$(6b) \qquad \dot{k} = f(k) - c\left(q/(1-\theta)\right)$$

Note that these equations describe only the real activity of the economy, the path of bond holdings being determined as a residual obeying equation (5c). The transversality condition insures that

$$(7) \qquad 0 < \lim_{t\to\infty} q(t),\ \lim_{t\to\infty} k(t) < \infty$$

The pair of equations, (6), describe the equilibrium of our economy at any t such that q and k are differentiable. To determine the system's behavior at points where q or k may not be differentiable, we impose the equilibrium conditions on (2), yielding

$$(8) \qquad q(t)e^{-\rho t} = \int_t^\infty e^{-\rho s} q(s)[f'(k(s))(1-\tau_K(s)) + \delta\theta(s)]/(1-\theta(s))\, ds$$

which shows that q(t) is a continuous function of time. The system of relations given by equations (6) and (8) and the inequality (7), will describe the general equilibrium of our economy.

To examine revenue and efficiency impacts of capital taxation, we analyze a simple perturbation of a steady state, though the analysis remains valid as long as the linear approximation to (6) is acceptable. Suppose that the government has been taxing at constant rates τ_K and τ_L, granting an investment tax credit at a constant rate θ, with all agents expecting these tax instruments to be constant forever, and that the economy has reached the corresponding steady state, with bonds at that level consistent with budget balance. Next suppose that at t=0, the government has announces that τ_K at $t>0$ will be $\varepsilon h(t)$ greater, the lump-sum rebate will be $\varepsilon \ell(t)$ greater, and the investment tax credit will be $\varepsilon z(t)$ greater. Note that if ε equals zero, the initial equilibrium will persist. (We do not examine changes in τ_L since it is a lump-sum tax.) To continue, it is necessary to make the following assumption.

Constancy assumption: h, ℓ, and z are all eventually constant functions of time.

This assumption is necessary to insure the existence of a new steady state but is harmless since the date of eventual constancy is arbitrarily distant. For any fixed ε, the equilibrium of our model is therefore given by the solution to the differential equations:

$$\text{(9a)} \qquad \dot{q} = q\left(\rho - \frac{(1-\tau_K-\varepsilon h)f'(k) + \delta(\theta+\varepsilon z)}{1-\theta-\varepsilon z}\right)$$

$$\text{(9b)} \qquad \dot{k} = f - c\left(\frac{q}{1-\theta-\varepsilon z}\right)$$

(9c) $$\left|\lim_{t\to\infty} k(t)\right| < \infty, \ k(0) = k_0.$$

We shall denote the solutions as $k(t,\varepsilon)$ and $q(t,\varepsilon)$, making explicit the dependence on ε. Suppose that $\varepsilon = 0$ and that the economy has reached the steady state. Now, the government announcement is essentially that ε has been increased. We would like to know the impact of a small change in ε on the crucial economic variables, i.e., we want to know the values of

$$\frac{\partial k}{\partial \varepsilon}(t,0) \equiv k_\varepsilon(t), \quad \frac{\partial}{\partial \varepsilon}\left(\frac{\partial k}{\partial t}\right)(t,0)) \equiv \dot{k}_\varepsilon(t).$$

$$\frac{\partial q}{\partial \varepsilon}(t,0) \equiv q_\varepsilon(t), \quad \frac{\partial}{\partial \varepsilon}\left(\frac{\partial q}{\partial t}\right)(t,0) \equiv \dot{q}_\varepsilon(t).$$

We are implicitly making the economically innocent assumptions which guarantee the existence of these derivatives (see Oniki (1971)).

Differentiation of the equilibrium system yields a linear differential equation in the variables k_ε, q_ε:

(10) $$\begin{bmatrix} \dot{q}_\varepsilon \\ \dot{k}_\varepsilon \end{bmatrix} = \begin{pmatrix} 0 & \frac{-q(1-\tau_K)}{1-\theta} f'' \\ \frac{c}{\beta q} & f' \end{pmatrix} \begin{bmatrix} q_\varepsilon \\ k_\varepsilon \end{bmatrix} + \begin{bmatrix} q\left(\frac{h\ f' - (\rho+\delta)z}{1-\theta}\right) \\ \frac{c}{\beta}\frac{z}{1-\theta} - g(t) \end{bmatrix}$$

Since we are initially in a steady state, the matrix in (10), which we shall call J, is constant. Therefore, the system in (10) is actually linear with constant coefficients, suggesting the use of Laplace transforms. (The Laplace transform of a function $f(t)$ defined for positive t, is a function $F(s)$ defined for sufficiently large positive s where $F(s) = \int_0^\infty e^{-st} f(t)\,dt$.) Let $Q_\varepsilon(s)$, $K_\varepsilon(s)$ be the Laplace transforms of $q_\varepsilon(t)$, $k_\varepsilon(t)$, respectively. These Laplace transforms therefore satisfy the Laplace transform of (10):

(11) $$\begin{pmatrix} sQ_\varepsilon(s) \\ sK_\varepsilon(s) \end{pmatrix} = J\begin{pmatrix} Q_\varepsilon(s) \\ K_\varepsilon(s) \end{pmatrix} + \begin{pmatrix} \frac{q}{1-\theta}(H(s)f' - (\rho+\delta)Z(s)) + q_\varepsilon(0) \\ -G(s) - cZ(s)/(1-\theta)\beta \end{pmatrix}$$

where H, G, and Z are the Laplace transforms of h,g, and z, respectively. Solving for $Q_\varepsilon(s)$ and $K_\varepsilon(s)$ yields

$$(12) \qquad \begin{pmatrix} Q_\varepsilon(s) \\ K_\varepsilon(s) \end{pmatrix} = (sI-J)^{-1} \begin{pmatrix} \frac{q}{1-\theta}(H(s)f' - (\rho+\delta)Z(s)) + q_\varepsilon(0) \\ -G(s) + cZ(s)/(1-\theta)\beta \end{pmatrix}$$

We need to find the value of $q_\varepsilon(0)$, the initial change in the marginal utility value of an extra unit of capital. Let $\mu > 0 > \lambda$ be the eigenvalues of J[1]. From the stability conditions (see Lemma 1 in Judd (1982b)), we may conclude

$$(13) \qquad \frac{q_\varepsilon(0)}{q} = \frac{1}{1-\theta}\Big((\rho+\delta-\mu)\ Z(\mu) - H(\mu)f'\Big) - \frac{\mu\ \beta}{c}\ G(\mu)$$

Combining (12) and (13), we have the solution for $K_\varepsilon(s)$ and $Q_\varepsilon(s)$. Having solved for the Laplace transforms of the adjustment paths of q and k, we can now use them to determine the impact of the shocks on economic variables, derive an expression for the government's dynamic budget constraint, and decompose the initial impact on consumption into its income and substitution effects.

(i) Balanced budget condition:

Next, we compute the relationship that must exist between the changes in taxation and expenditure due to the government's budget constraint. The differential equation governing bonds is

$$(14) \qquad \begin{aligned} \dot{b} = {} & r_B(1-\tau_K)b + (\theta+\varepsilon z(t))(\delta k + \dot{k}) - (\tau_K + \varepsilon h(t))kf'(k) \\ & - \tau_L(f(k) - kf'(k)) - (\ell + \varepsilon l(t)) \end{aligned}$$

In the initial steady state $r_B(1-\tau_K) = \rho = f'(1-\tau_K) + \delta\theta$. Also, we will assume that b=0 initially, allowing us to concentrate on the essential issues

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1 That there is one eigenvalue of each sign is assured by the sign structure of J.

of revenues and efficiency. Hence, in the initial steady state

$$\tau_K kf'(k) + \tau_L(f(k) - kf'(k)) = 1 + \theta\delta k \tag{15}$$

i.e., receipts equal expenditures, the investment tax credit and the rebates. The government's dynamic budget constraint is that the present value of its obligations and expenditures must equal the present value of its revenues[2]. Differentiating that constraint with respect to ε, using (15), and taking Laplace transforms, we find that

$$\tau_K K_\varepsilon(\rho)(f'+kf') + kf'H(\rho) + \tau_L kf''K_\varepsilon(\rho) = L(\rho) + \theta(\rho+\delta)K_\varepsilon(\rho) + Z(\rho)\delta k \tag{16}$$

must hold where $Z(s)$, $H(s)$, $L(s)$ are the Laplace transforms of $z(t)$, $h_K(t)$, and $l(t)$, respectively. This expression states that extra revenue equals extra spending discounted at the rate ρ, i.e., when valued at the initial intertemporal prices. Equation (16) must be interpreted carefully. $L(\rho)$ is the value, using the prices which held before the perturbation, of the change in transfers. It is <u>not</u> the change in total revenue. Since relative prices change, the present value of old tax receipts will change. However, that change in present value is not interesting since its magnitude and sign are sensitive to the choice of the $t=0$ commodity as the numeraire. $L(\rho)$ is the appropriate measure of the real change in government revenue since the extra revenue from $l(t)$ allows the government to increase transfers permanently and immediately by $\rho L(\rho)$ units of consumption per period[3]. In general, the flow of transfers at $t>0$ may be raised by $h_T(t)$ if $\int_0^\infty h_T(t)\, e^{-\rho t} dt$ equals $L(\rho)$.

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2 This can be derived from the consumers' budget constraints and their transversality conditions, as in Brock and Turnovsky(1981).

3 See Judd (1982b).

(ii) Income effects:

To study the impact of policy changes on lifetime utility, we need to express it in terms of the change in tax rates and prices. Let $\bar{r}(t)$ be the after-tax return on capital and $\bar{w}(t)$ the after-tax wage. If an individual is given a lump-sum transfer of $\bar{l}(t)$, then we can write his demand as

$$\dot{q} = q\,(\rho - (\bar{r} + \delta\theta)/(1-\theta))$$

$$\dot{k} = \bar{r}k + \bar{w} - c(q/(1-\theta)) + \bar{l} + \theta(\delta k + \dot{k}) \tag{17}$$

$$\lim_{t\to\infty} |k(t)| < \infty$$

Suppose such an individual is initially in a steady-state with his consumption level equal to $\bar{c}$ and capital holdings equal to k when there is an unanticipated policy change affecting $\bar{r}$, $\bar{w}$, and $\bar{l}$, such changes being denoted by $\bar{r}_\varepsilon$, $\bar{w}_\varepsilon$, and $\bar{l}_\varepsilon$. Using the same techniques as above, one discovers that the change in total lifetime utility, U_ε, is

$$U_\varepsilon = u'(\bar{c})\, W_\varepsilon(\rho) \tag{18}$$

where

$$W_\varepsilon(\rho) = \int_0^\infty e^{-\rho t}\,(\bar{r}_\varepsilon(t)\,k + \bar{w}_\varepsilon(t) + \bar{l}_\varepsilon(t) + \delta k z(t))\,dt \tag{19}$$

denotes the income effect of the changes.

In this section, we have derived the two basic equations concerning the impact of policy changes on real income and the relation which must hold among the policy variables due to the government's dynamic budget constraint. Using these results, we can now move to the analysis of the revenue and welfare effects of taxation.

III. Revenue Changes

In this section we examine the impact of a decrease in the capital income tax rate and an increase in the investment tax credit on the discounted value of the government revenue stream.

a) Reduce τ_K

Using the revenue expression of equation (16), and the solution for $K_\varepsilon(\rho)$ from equations (12) and (13), we find that if the capital income tax rate drops immediately and permanently ($h(t) = 1$ and $z(t) = 1$), then

$$(20) \qquad dR = -\left[\left\{\tau_K(1-\theta_L/\sigma) + \tau_L\theta_L/\sigma - \theta\,\frac{\rho+\delta}{f'}\right\}\frac{\lambda}{\lambda-\rho}\,\frac{\sigma}{\theta_L}\,\frac{1}{1-\tau_K} - 1\right]\cdot\frac{\theta_K f}{\rho}\,d\tau_K$$

expresses the value of the change in revenue as a fraction of capital's before tax share of the present value of output, $\theta_K f/\rho$.

The formula for the discounted change in revenue can be decomposed intuitively into its separate components. First, if there were no change in capital stock, then a 1% increase in the tax rate would increase discounted revenues by 1% of $\theta_K f/\rho$, capital's share of the net product.

However the capital stock is affected, causing a change in capital income tax revenues, in wages and wage tax revenues, and in investment tax credit outlays. A change in the capital stock of dk will cause the capital income tax base, kf', to change by $(f' + kf'')dk = f'(1-\theta_L/\sigma)dk$, resulting in revenues from existing capital changing by $\tau_K f'(1-\theta_L/\sigma)dk$. Similarly, wages are changed by $(\theta_L/\sigma)f'dk$ and wage taxes increase by $\tau_L\theta_L f'dk$. An increase in the capital stock by dk will increase the present value of investment tax credit outlays in two ways: replacement investment will increase by δdk, causing the flow of investment tax credit outlays to increase by $\theta\delta dk$, and there will also be investment tax credits paid on the extra capital, the present value of that outflow being Θdk. Therefore, the value of the net change in revenues at some time due to the induced changes in the capital stock is

$$(21) \qquad \left[f'\{(1 - \theta_L/\sigma)\tau_K + (\theta_L/\sigma)\tau_L\} - \theta(\rho+\delta)\right] \frac{dk}{\rho}$$

Since τ_K and τ_L are substantially larger than θ and θ_L/σ is not large (almost surely less than 2.0) and $(\rho+\delta)/f'$ is also not much different than unity for reasonable values of τ_K and θ, it is most likely that capital accumulation will raise revenues.

Next we need to know how large the increase in k, dk, is as a function of the change in τ_K, $d\tau_K$. Differentiation of the steady-state formula for $f'(k)$,

$$f'(k) = \frac{\rho(1-\theta) - \delta\theta}{1 - \tau_K}$$

shows that

$$(22) \qquad \frac{dk}{k} = \frac{-\sigma}{\Theta_L} \frac{d\tau_K}{1-\tau_K}$$

demonstrating that the relative change in capital stock due to a change in τ_K is greater in magnitude as capital and labor are more substitutable, as labor share is less, and as the current tax rate on capital income is greater.

If the change in capital stock were instantaneous, then we could combine equations (21) and (22) to determine the impact of the induced capital formation on income tax revenues and investment tax credit outlays. However, the capital stock converges gradually to the new steady state according to a linear stock adjustment process

$$(23) \qquad \dot{k} = \lambda(k-k^{\infty})$$

where k^{∞} represents the new steady state capital stock. Since the steady-state rate of discount is ρ, the discounted change in income tax revenues and investment tax credits is $\lambda/(\lambda-\rho)$ times that which would be the case if adjustment were instantaneous, i.e., if $\lambda = -\infty$. The sum of these effects is (20).

Comparative static exercises for the change in revenue are cumbersome due to the complex dependence of λ on σ, θ_L, τ_K, and ϑ. The primary concern here is the sign of dR, and more generally, a feeling as to how much of the loss in revenue which would occur if there were no change in capital stock is eliminated by the extra revenue which results from the capital accumulation. Therefore, we have tabulated the results of some calculations in Table 1. In Table 1 we assume that τ_K=.5, θ=.05, and θ_K=.25, a parameterization fairly representative of the U.S. economy. The .5 capital tax rate is a compromise between the higher effective nonfinancial corporate tax rate (see Feldstein and Summers) and the lower noncorporate rate. Note that we are ignoring the intersectoral distortions due to unequal treatment caused by the corporate tax and the structure of depreciation allowances, concentrating solely on intertemporal issues. The .25 capital share implicitly means that we are ignoring consumer durables. The relatively low effective marginal capital income tax rate and capital share are chosen for close examination because they both bias the results against the main points of this paper.

The first column of Table 1 gives the change in capital tax revenue, net of investment tax credits, as a portion of the steady-state present value of capital income, i.e., $(dR/d\tau_K)/(\theta_K f/\rho)$, for various plausible values for σ, β, and τ_K. (See Berndt and Christensen, Ghez and Becker, Hansen and Singleton, Lucas, and Weber.) Note that if there were no induced capital accumulation, this number would be unity. In comparing column with column 2, which expresses the discounted change in wage income also as a multiple of discounted capital income, we see that with even just moderate labor taxation much of the direct loss of revenue due to the cut in τ_K is offset by the increased revenue due to either higher capital or labor income[4], however, only for unrealistically high rates of labor taxation would a decrease in τ_K actually lead to greater revenue in present value terms. The extent to which increased labor tax revenues offset capital tax losses is increased as β decreases because a small β indicates a small desire for a constant rate of consumption, implying that agents are more willing to save today in order to raise lifetime consumption and leading to a more rapid adjustment, causing the accumulation effects on revenue to be more important. The same is true as σ increases since the adjustment is greater and the net capital tax loss is less.

For higher rates of taxation and capital-labor substitutability, it is increasingly likely that a capital tax cut would result in higher total revenues. For example, if τ_K were .6, τ_L were .4, the production function were Cobb-Douglas, and utility were logarithmic, then a capital income tax cut would leave total tax revenues unchanged. If τ_K were .7, this would hold if σ were only .8. For τ_K=.8, we find that capital income tax revenues alone may increase with a tax cut. While these parameters are on the fringe of what is considered reasonable, it does point out how close we may be to this perverse possibility, especially if we were to add other realistic elements such as the nonuniform taxation of capital.

In comparing these results with Fullerton(1982) we find that this intertemporal maximizing growth model is more likely to yield the perverse revenue movements compared to his neoclassical savings specification.

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4 When $\sigma < \theta_K$ capital accumulation results in a lower rate of return for capital and depresses revenues per unit of capital, explaining why many entries in column 1 exceeds unity, i.e., the loss of revenue due to a tax cut is accentuated by the induced capital formation.

Fullerton finds that revenues go up with the tax rate even when τ_K is over .8, something which would not happen in this model for reasonable values of β and σ. This reflects the incentive effects of future tax cuts on savings today which exist in our model, whereas with a savings rate function formulation, individuals aren't allowed to save currently in response to high future returns.

b) Increase θ

We next examine the impact of an increase in the investment tax credit on discounted revenue. Letting $z(t) = 1$ and $h(t) = 0$ represent this policy change and using (12) and (16), we find that

$$(24)\quad dR = \left\{\left[\tau_K(1-\theta_L/\sigma) + \tau_L\theta_L/\sigma - \theta\,\frac{\rho+\delta}{f'}\right]\frac{\sigma}{\theta_L}\,\frac{1}{1-\tau_K}\,\frac{\lambda}{\lambda-\rho}\,\frac{\rho+\delta}{\delta} - 1\right\}\frac{\delta k}{\rho}\,d\theta$$

if we assume that θ is increased immediately and permanently, i.e., $z(t) \equiv 1$. Here we express the change in revenue as a fraction of the present value of economic depreciation. The direct cost of greater tax credits on replacement investment is substantially offset by the greater income tax revenues due to the capital accumulation. We immediately note that an increase in the investment tax credit is more likely to result in greater revenues than a cut in capital income tax for reasonable parameter values, since comparing (20) and (24) shows that the revenue change due to capital formation induced per dollar in subsidy to replace existing capital is $(\rho + \delta)/\delta$ times the revenue change induced by a one dollar cut in taxation of existing capital.

Column 4 in Table 1 displays the net revenue loss as a fraction of that which would occur if there were no capital accumulation, i.e., $(dR/d\theta)/(\delta k/\rho)$, which would be unity if there would be no induced capital accumulation. In comparing column 4 with column 5, which expresses the present value of the increase in the wages also as a fraction of $\delta k/\rho$, we see that an increase in θ would lead to only small revenue losses even for moderate rates of wage taxation. Actual revenue increases are substantially more likely in response to an increase in the investment tax credit. Moderately higher tax rates would make perverse revenue movements even more likely.

We have seen that whereas reductions in τ_K have a small likelihood of raising revenues unless τ_K and τ_L are very high, increases in θ are much more likely to increase total revenues. One final point concerning revenues that must be made is that an equal cut in both τ_K and τ_L will surely lead to a recution in revenue. Since $\theta_K = .25$, the loss of revenue flow due to a cut in τ_L of $d\tau_L$ will be $3\theta_K f d\tau_L$. Adding this loss to (2) yields the total revenue change. It is straightforward to see that when $d\tau_K = d\tau_L$, tax cuts will increase revenues only for unrealistically high values of τ_K and σ; for example, if $\sigma = 1$ and adjustment were instantaneous, τ_K would have to exceed .75.[5]

IV. Welfare Effects

Also of interest to policymakers is the efficiency cost of taxation since, for example, it necessary to determine appropriate cost-benefit criteria. In this section, we compute the impact on welfare of a cut in τ_K or an increase in θ. We find that the marginal efficiency cost at representative tax rates in this model is substantial. Recall that we are assuming that the revenues are used to make lump-sum subsidies to agents. Therefore, this exercise is equivalent to measuring the efficiency gain if we cut τ_K or raised

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[5] The effect of an elastic labor supply on all of these calculations would be ambiguous since a capital income tax cut would raise both wage and nonwage income and price and income effects shift labor supply in opposite directions.

the investment tax credit and replaced the lost revenues with lump-sum taxes.

a) Reduce τ_K

Since consumption equals output, from our expression for $K_\varepsilon(\rho)$ we can compute the change in utility in terms of the t=0 commodity:

$$dy \equiv \frac{dU}{u'(\bar{c})} = (f' - \rho)K_\varepsilon(\rho)\ d\varepsilon \tag{25}$$

From our solution for $K_\varepsilon(\rho)$, we find that the change in real income when τ_K is cut immediately and permanently is

$$dy = \frac{\lambda}{\rho-\lambda}\ \frac{f'-\rho}{f'}\ \frac{\sigma}{\theta_L}\ \frac{\theta_K f}{\rho}\ d\tau_K \tag{26}$$

which is positive for positive $d\tau_K$ whenever the effective tax rate, $(f' - \rho)/f'$, is positive.

We immediately make some intuitive observations concerning the welfare gain from a cut in τ_K. As the rate of adjustment is greater, i.e., as $|\lambda|/\rho$ increases, the welfare impact is greater. Also the welfare impact is greater as the marginal product of capital diverges from ρ, which occurs as τ_K is greater. A small elasticity of substitution implies that the demand curve for capital is less elastic, as does a large labor share. Therefore, as σ/θ_L is greater, the welfare impact is greater, corresponding to our standard static intuition.

The index of the cost of taxation we choose to examine is the welfare cost of taxation as a fraction of the revenue change

$$\text{MDWL} = -\frac{dy}{dR} \tag{27}$$

The marginal deadweight loss of taxation, MDWL, is the dollar loss in utility per dollar of revenue raised from a small change in the tax structure. This definition of MDWL is equivalent to the welfare gain from a balanced-budget switch to a lump-sum tax. It is also the appropriate measure for cost-benefit analysis if utility is separable in the private and public good in that the agent will gain from increasing capital income taxation and expenditures by one dollar if and only if the marginal utility of the public good exceeds the marginal utility of private consumption by a factor of MDWL. A negative MDWL would indicate that an increase in revenues is associated with an increase in utility, and occurs exactly when a decrease in tax rates causes an increase in tax revenue. In column 3 of Table 1 we display values for MDWL when the capital income tax is immediately and permanently increased. Equivalently, if the government were to impose $l(t)$ in new lump-sum taxes at $t>0$, and use the revenues to finance a cut in τ_K or an increase in θ, then the change in consumption, valued at the original prices, would be MDWL dollars per dollar raised by the new lump-sum taxes, valued again at the old prices. For example, if the tax change reduced τ_K so that capital income tax flow were reduced by 1% of old consumption at each $t>0$, and MDWL were .5, then the increase in utility be equivalent to .5% increase in consumption at each $t>0$.

From column 3 of Table 1 we see that the efficiency cost of capital taxation is substantial for the parameter values represented, ranging from 40¢ to \$2.00, with \$1.00 being a central value. Recall that this is the marginal efficiency gain per dollar of marginal net capital tax revenues. Taking into account the impact on labor tax revenues would increase the marginal efficiency gains by 50% in the central cases even if the marginal wage tax were only .33.

In comparing these results with other computations of average deadweight loss, we find important differences. For example, the marginal deadweight loss results we find here are substantially larger than those obtained by Chamley. Assuming $\sigma = 1 = \beta$, $\theta_L = .33$, and a 2 percent rate of outputaugmenting technical change, he shows that the average deadweight loss of capital taxation per dollar of revenue at $\tau_k = .5$ is 26¢, and therefore that the marginal loss is somewhat greater than 52¢ per dollar of revenue, using the rule of thumb that marginal losses are roughly double average losses. Using our techniques, one finds that the true MDWL for that case is over a dollar per dollar of extra capital tax revenue. These differences highlight the need to compute the exact marginal deadweight loss of capital taxation.

b) Increase θ

Next, we examine the marginal welfare gain of an immediate and permanent increase in θ, which is computed to be

$$(28) \qquad dy = \frac{\rho+\delta}{f'} \frac{f'-\rho}{f'} \frac{\lambda}{\lambda-\rho} \frac{\sigma}{\theta_L} \frac{1}{1-\tau_K} \frac{\theta_K f}{\rho} d\theta$$

Again, the welfare impact of the tax change is greater as σ/θ_L is greater, as f' diverges from ρ, and as the rate of adjustment to the steady state, λ, is greater in magnitude. As long as the effective tax rate is positive, an increase in the investment tax credit will increase total welfare.

Column 6 in Table 1 displays the MDWL for an immediate and permanent increase in the investment tax credit. Each entry is the welfare gain per dollar of lost revenue. These gains are substantially larger. This of course is not surprising since an income tax cut is partially a lump-sum rebate to owners of current capital, whereas an increase in the investment tax credit goes to current capital only as a subsidy to replacement investment. What we see in Table 1 is just how much more efficient investment subsidies are compared to tax cuts. Therefore, the decision between income tax cuts and investment subsidies has a strong redistributional element. We will see this more precisely below.

In summary, we have seen in this section that the efficiency gains associated with unanticipated and permanent cuts in capital taxation or increases in the investment tax credit are substantial in a perfect foresight representative agent model, being much higher than the losses calculated in other models.

V. Anticipation Effects

A major advantage of our general solution for $K_\varepsilon(\rho)$ is that we can analyze temporally complex changes in τ_K and θ by allowing arbitrary functions for h and z, allowing us to determine the welfare effects of temporary policy changes and policy changes which are announced to occur in the future. We find that the welfare effects of temporary and anticipated policies may differ substantially from those of permanent policy changes. In this section we continue to assume that $\tau_L = 0$ and concentrate net capital taxation.

It is well known that a temporary increase in τ_K will result in practically no efficiency loss due to the fixed nature of capital in the short run. However, the efficiency cost of even relatively short-lived capital taxation may be nontrivial. If we fix a period to be that unit of time in which utility is discounted by 1%, computations show that if $\tau_K = .5$ and $\theta = 0$, even an 8-period temporary increase will have efficiency losses of 9 to 23 cents per dollar of revenue raised as σ varies between .5 and 1.0 and β varies between 1.0 and 3.0. A 20-period (roughly 4-5 years) tax increase yields losses of 20 to 53 cents over the same range of parameters.

On the other hand, announced future tax increases in τ_K will have only slightly higher efficiency costs than immediate increases. In the central range of parameters that we are studying, a lag of 8 periods between announcement and implementation raises efficiency losses by at most 10%, and a lag of 20 periods raises it by 15-25%.

Anticipation effects are much more important in the consideration of investment tax credits. When $\tau_K = .5$ and $\theta = 0$, even a 4-period lag between announcement and implementation of an investment tax credit will reduce the efficiency gain per dollar of revenue losses by 10%. This buttresses the standard wisdom that investment incentives should take place immediately, otherwise the anticipation of future investment tax credits may cause current investment to drop (see Judd(1982b). Such a drop would cause greater efficiency losses since the capital income tax causes the marginal product of capital to exceed the supply price.

Temporary investment tax credit increases turn out to be much more effective than permanent increases. For the examples studied in Table 1 when $\sigma = .7$, the efficiency gain per dollar of lost revenue of a short-term (4-period) increase in θ is almost double that of a permanent increase. As σ increases, this advantage of temporary over permanent increses in θ rises rapidly.

If we finance investment tax credits with capital income taxation, it is obvious and well-known that we would achieve efficiency gains. From our analysis, we can estimate the magnitude of such a program. Using a permanent increase in τ_K to finance a permanent increase in θ will result in an efficiency gain per dollar of new tax credits equal to the difference between columns 6 and 3 in Table 1. This gain generally exceeds 50¢ for the tabulated cases and if the production is locally Cobb-Douglas, it may be well over $2.00. Since temporary tax increases and temporary credits are both more potent, a balanced budget temporary increase in capital income tax and investment tax credit will result in substantially larger gains.

VI. Distributional Effects

Another nickname attached to the current policies is trickle-down economics because of the claim that the tax cuts will so stimulate capital formation that the increase in wages will leave workers better off even if they are taxed to finance the program, either directly, or indirectly through lower provision of public goods. Neoclassical growth models, such as in Grieson and Boadway, have been used to argue that this is unlikely. In this section we examine a disaggregated interpretation of our model and examine distributional impacts of a cut in τ_K and an increase in θ. We assume that all agents inelastically supply one unit of labor per unit of time but own varying amounts of capitals. If we assume that the elasticity of marginal utility of consumption, $-u''(c)c/u'(c)$, is equal to a constant, β, then it is straightforward to show that the system aggregates and that the general equilibrium movement of per capita consumption, c, and aggregate capital per worker, k, is given by the solutions to

$$\text{(29a)} \qquad \dot{c} = -\frac{c}{\beta}\left(\rho - \frac{(1-\tau_K)f'(k) - \delta\theta}{1-\theta}\right)$$

$$\text{(29b)} \qquad \dot{k} = f(k) - c$$

The change of variable

$$\text{(30)} \qquad q = u'(c)(1-\theta)$$

converts (29) into our equations (9), showing that (9) can be given a disaggregated interpretation. For the purposes of this exercise, we assume that all revenues are redistributed lump-sum to all in a uniform fashion, hence, equal to $\tau_K f(k) - \theta\dot{k} - \theta\delta k$ per person. Since the wage tax is effectively a uniform lump-sum tax, we set it to 0.

a) Cut τ_K

First assume that the capital income tax is decreased instantly and permanently. The impact on the discounted value in the change in wages, dW, is given by:

$$(31) \qquad dW \equiv -kf''K_{\varepsilon}(\rho) = \frac{\theta_L}{\sigma} f'K_{\varepsilon}(\rho) = \frac{\lambda}{\rho-\lambda} \frac{1}{1-\tau_K} \frac{\theta_K f}{\rho} d\tau_K$$

which is always positive for a tax cut, being greater as λ, τ_K, and θ_K are greater in magnitude, and as σ and β are smaller. Column 2 in Table 1 gives values of the discounted wage change as a fraction of capital income, i.e., $(dW/d\tau_K)/(\theta_K f/\rho)$, for an unanticipated permanent cut in τ_K. We see that β is the crucial parameter determining whether the wage gain exceeds the revenue loss, with the wage gain being greater if β is smaller than (approximately) 2.0. This is because a small β implies a rapid adjustment process. The empirical literature has not yet determined whether β exceeds 2.0, with the recent work of Hansen and Singleton giving estimates on both sides of 2.0.

The impact on an investor holding one unit of capital is $d\Pi$, the discounted value of the change in the net-of-tax return on the existing capital stock, and is expressed in:

$$(32) \qquad d\Pi = \frac{f'}{\lambda - \rho} d\tau_K$$

We immediately see that whereas the holder of capital always gains from the tax cut since λ, the rate of adjustment, is negative, the induced capital accumulation substantially reduces the gain and is smaller as λ is greater in magnitude.

To focus on the trickle-down aspects of capital tax cuts, separate from the benefits of less taxation, it is natural to add wage changes and rebate changes to measure the net impact on an individual holding no capital. This net change in worker welfare is

$$(33) \qquad dy^W = \frac{\theta_K f}{\rho} \left[[(\tau_K(1 - \frac{\theta_L}{\sigma}) - \theta \frac{\rho+\delta}{f'}) \frac{\sigma}{\theta_L} + 1] \frac{1}{1-\tau_K} \frac{\lambda}{\lambda-\rho} - 1 \right] d\tau_K$$

Since this disaggregated model is equivalent to the representative agent model we examined earlier, we may use Table 1 in assessing these impacts. Note in Table 1, where $\tau_K = .5$ and $\theta = .05$, that the increase in wages substantially exceed the loss in revenue from a cut in τ_K for most reasonable values of σ and β. An interesting question is how high τ_K can be before the revenue gains from increases in τ_K are offset by the loss in wages. In Table 2, we show the tax rate τ_K such that $dy^w = 0$ when $\theta = 0$ for various values of σ and β. If τ_K exceeds this value, then <u>all</u> agents will benefit from a permanent unanticipated decrease in τ_K, since a cut in τ_K benefits all to the extent they hold capital. Note that these rates are relatively low unless the utility function is very concave, demonstrating the weakness of even unanticipated permanent capital taxation as an instrument of redistribution.

b) Increase θ

Next we consider the welfare impact of increasing the investment tax credit. When θ is changed, the change in discounted value of wages is

$$(34) \qquad dW = -kf''K_\varepsilon(\rho) = \frac{\rho+\delta}{\delta}\,\frac{\lambda}{\lambda-\rho}\,\frac{1}{1-\tau_K}\,\frac{\delta k}{\rho}\,d\theta$$

which is positive whenever the tax credit is increased.

When θ is increased, the change in investor welfare per unit of capital is the discounted value of the change in net income on one unit of capital:

$$(35) \qquad d\Pi = \frac{\lambda + \delta}{\rho - \lambda}\,d\theta$$

The changes in investor welfare due to increased profits needs to be distinguished from the change in the present value of his investment. The value of the capital stock at any time is expressed in terms of the commodity good at that time. In our model with no adjustment costs, the capital and the good are perfect substitutes and hence the value of k units of capital in place will be k as long as $\theta = 0$. However, when $\theta > 0$, then new investment is subsidized and one is indifferent between one unit of consumption and $1-\theta$ units of capital in place, so the value of k units of capital is $k(1-\theta)$. Therefore changes in τ_K leave value unchanged but changes in θ affect the value of the capital stock. These values do not reflect the welfare changes of a tax change because the induced capital formation changes the relative prices of goods across time. Our formulas (32) and (35) give the true welfare impact. The crucial feature of (35) to note is the ambiguity of the sign of $d\Pi$ since $\delta > 0 > \lambda$. Even after using reasonable values for taste, technology, and depreciation of capital stock, the sign of $\delta + \lambda$ is ambiguous, being negative for low values of β; roughly speaking, $\lambda + \sigma$ is negative when $\beta < 2$, positive when β is larger. When $\beta = 1$ and $\sigma = .7$, our central case, $d\Pi$ is about $d\theta/3$, that is, a 1% increase in the investment tax credit rate will cause the utility value of a unit of capital to drop by about 1/3%, not a large effect.

When we add the increase in wages to the loss in rebate income, the change in a worker's utility is

$$(36) \qquad dy^w = \frac{\delta k}{\rho}\left[[\tau_K(1 - \frac{\theta_L}{\sigma}) - \theta \, \frac{\rho+\delta}{f'}) \, \frac{\sigma}{\theta_L} + 1] \, \frac{\rho+\delta}{\delta} \, \frac{1}{1-\tau_K} \, \frac{\lambda}{\lambda-\rho} - 1\right] d\tau_K$$

In comparing (33) to (36) we see that a worker is more likely to gain from increasing θ than from decreasing τ_K, reflecting again the fact that an investment tax credit subsidizes only investment whereas a tax cut is partially an investment incentive but also a lump-sum tax cut to holders of the current capital stock. Column 5 of Table 1 expresses the change in wages due to an unanticipated permanent increase in wages due to an unanticipated permanent increase in θ as a fraction of depreciation, i.e., $(dW/d\theta)/(\delta k/\rho)$. Note that the increased tax credit expense due to a larger θ is always much less than the wage gain in Table 1. In particular, for the log utility cases, the wage gain is two to three times the revenue loss. Only when β is unrealistically large does the revenue loss come close to the wage gain.

In this section, we have examined a disaggregated version of this model. We have seen that for reasonable values of the parameters, wage gains may exceed revenue losses when τ_K is cut and β is small or when θ is increased. This is a relevant calculation when the lost revenues are balanced by either an increased tax burden on labor or a lower level of provision of public goods which are good substitutes for private consumption. This shows that there may be some validity to the "trickle-down" claims of current policy.

VII. Conclusions

In this paper we have analyzed the impacts of capital income tax cuts and increases in an investment tax credit on both revenue and the welfare of both investors and workers in a perfect foresight representative agent model of equilibrium growth. We've seen that when there is a moderate labor income tax rate, and moderately high capital income tax rate, a cut in the capital income tax rate would probably not increase the discounted value of government revenue. However, this is much more likely if instead the investment tax credit is increased. Both possibilities are plausible, however, when we assume tax, taste and technology parameters on the fringe of what is considered representative of the U.S. economy. These revenue calculations are sensitive to the parameterizations used, and we cannot make any robust claim concerning them.

On the other, our welfare analysis indicates that for moderately high rates of capital income taxation, a permanent and unanticipated cut in the capital income tax rate can be a Pareto improvement even when the revenues are distributed uniformly. We also found that permanent investment tax credits financed by capital income tax increases could yield substantial increases in welfare, at the margin, the net benefit being between 50¢ to $3.50 per dollar of new investment tax credits. The performance of such a substitution is substantially better if both were temporary increases. This argues for a much greater reliance on investment incentives on tax reform as opposed to tax cuts.

Table 1

		Reduce τ_K			Increase θ		
σ	β	-dR (1)	dW (2)	MDWL (3)	-dR (4)	dW (5)	MDWL (6)
.5	.5	1.31	1.58	.80	1.68	3.39	1.34
	1.0	1.28	1.42	.74	1.61	3.05	1.26
	1.5	1.26	1.30	.69	1.56	2.80	1.20
	3.0	1.21	1.08	.59	1.46	2.31	1.05
	5.0	1.18	.89	.50	1.38	1.90	.92
	10.0	1.13	.63	.37	1.27	1.35	.71
.7	.5	1.12	1.51	1.25	1.26	3.24	2.39
	1.0	1.11	1.33	1.12	1.23	2.85	2.16
	1.5	1.10	1.20	1.02	1.21	2.58	1.98
	3.0	1.08	.95	.82	1.17	2.05	1.64
	5.0	1.06	.76	.67	1.13	1.63	1.34
	10.0	1.04	.51	.46	1.09	1.10	.94
1.0	.5	.86	1.42	2.20	.70	3.05	5.79
	1.0	.88	1.21	1.84	.74	2.61	4.67
	1.5	.89	1.08	1.60	.77	2.31	3.98
	3.0	.92	.82	1.18	.83	1.76	2.82
	5.0	.94	.63	.89	.87	1.35	2.07
	10.0	.96	.40	.56	.92	.86	1.26

$\theta_K = .25$, $\tau_K = .5$, $\theta = .05$, $\delta k/f = .12$

Table 2

β \ σ	.3	.5	.8	1.0
.5	.34	.30	.27	.25
1.0	.42	.38	.35	.33
1.5	.48	.44	.41	.39
3.0	.57	.54	.52	.51
10.0	.75	.74	.73	.73

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