Discussion Paper #541

A NOTE ON COMPETITIVE FORESIGHT AND OPTIMUM PRODUCT DIVERSITY*

by

Larry E. Jones

November 1982

J. L. Kellogg Graduate School of Management
Department of Managerial Economics and Decision Sciences
Northwestern University
Evanston, Illinois 60201

*I would like to thank V. V. Chari and Beth Hayes for several very useful conversations. Financial support in the form of a KGSM Xerox Research Grant is gratefully acknowledged.

COMPETITIVE FORESIGHT AND OPTIMUM PRODUCT DIVERSITY

by

Larry E. Jones

Recent papers by Spence [8] and Dixit and Stiglitz [1] have been concerned with analyzing the differences between equilibrium and efficient product variety in economies with imperfectly competitive producers. They have argued that the question cannot be resolved in generality: for some economies there is (in equilibrium) too much variety (relative to efficient allocations), and for some there is too little.

The purpose of this note is to provide a simple general equlibrium example in which the first of these effects if present, viz., there is too much variety in equilibrium. Further, the example analyzed is simple enough so that it is a straightforward procedure to trace the causes of this inefficiency.

The basis for the example to be presented rests on an old idea in economics: Producers differentiate their products in order to avoid direct price competition a la Bertrand. For this to be a property of equilibrium in a strategic situation, producers must realize at the outset that this direct price competition is forthcoming and have the foresight to recognize the ramifications of this fact.

This leads naturally to the study of subgame perfect Nash equilibria of a two stage game in which product characteristics are chosen first followed by the selection of prices. (This form of competition captures the notion that prices are much more flexible than product choice, see [6] in this regard.) Since the second stage price competition leads, through Bertrand arguments, to zero profits for any two producers making the same product and producers know this at the time products are selected, producers choose to differentiate their products. This occurs even if efficiency considerations require the production of only one good.

A similar argument has appeared recently in the work of Shaked and Sutton [7] (see Prescott and Vischer [6] as well) although the efficiency implications are not discussed. (There are too many goods produced in that model as well.) The example presented here differs from that in [7] in two ways. First, the Shaked and Sulton model features indivisible consumer choice over the products of variable quality. That is, consumers are assumed to be constrained to choose one and ony one product quality of which they must choose to buy either 0 or 1 units. Second, in [7], there are infinitely many consumers with identical preferences but disparate incomes. This is an essential feature of the Shaked and Sutton model as their results fail if all consumers have identical incomes.

In contrast, the example we present here features just one consumer (and hence a degenerate income distribution) with concave preferences over a standard finite dimensional commodity space. This allows us to conclude that neither the unconventional nature of the consumers choice sets nor the distribtuion of income play an important role in the essential conclusion: Producers differentiate their products (sometimes to the point of yielding too may products from the point of view of efficiency) in order to avoid direct price competition.

Our example is presented in section 2. Some further implications of dynamic stragtetic structures such as the one considered here are discussed in section 3.

Section 2: The Example

We consider a simple economy with three goods, x, y, and m. A single consumer is endowed with money (m) only which can then be used in the production of both x and y. Money will also serve as a numeraire--we will set $P_m = 1$.

Our single consumer has preferences of a modified Stone-Geary variety (see [5]):

$$u(x,y,m) = \ell n(x + \frac{m}{10} + 1) + \ell n(y + 4).$$

Notice that, for simplicity, we have assumed that 10 dollars is a perfect substitute for one unit of x. Under the standard Stone-Geary interpretation, -4 is the subsistence level of y consumption, etc. Thus, none of the three goods is a necessity.

Our consumer is endowed with two dollars, no x and no y.

The production side of the economy consists of two firms each with the possibility of producing either x or y with constant costs of one dollar per unit of output. That is, each firm has production set

$$Y = \{(x,y,m) | x \leq m \text{ and } y = 0 \text{ or } y \leq m \text{ and } x = 0\}.$$

Notice that it is not possible for a firm to produce both goods simultaneously.

We will assume that each firm is owned by individuals caring only for money. This justifies the use of profit maximization as a goal on the part of the firms.

The firms will play a two stage game as follows:

(i) At the first stage, the firms simultaneously select products. There are three options open for each firm, x, y and n (for non-entry).

(ii) After revelation of the product choice of both firms, each firm announces a price for the product it has selected.

(iii) Our consumer, taking the announced prices as given (we will set $P_i = \infty$ if i is not selected by either firm) allocates his income across the three goods.

(iv) The payoffs for the firms are simply the revenue from sales minus production costs minus an entry fee, $\varepsilon > 0$, if x or y is chosen at the first stage. If n is chosen at the first stage, the payoff to the firm is 0.

Since the intent of this note is solely to show that certain qualitative phenomena are plausible in equilibrium, we will concentrate on a particular pure strategy equilibrium of the game just described.

As a first step, since we are interested in subgame perfect equilibria, we must first find equilibria of the second stage (price) game, contingent on the first stage choices.

(1) If the first stage choice is (n, n) there is no second stage game. Payoffs to both firms are zero.

(2) If both firms choose x(y) at the first stage, familiar Bertrand arguments can be used to show that both firms charge P_x = \$1 (P_y = \$1) hence payoffs are $-\varepsilon$ to both firms.

(3) If one firm chooses x and the other opts for y, a little more work must be done.

One can show that the consumers demand system is:

If $P_x < 10$, $m^d = 0$ so that

$$x^d = \frac{2}{P_x},\ y^d = 0 \qquad \text{if} \qquad P_x < 4P_y - 2$$

$$\left.\begin{aligned} x^d &= \frac{1 + 2P_y}{P_x} - \frac{1}{2} \\ y^d &= \frac{2 + P_x}{2P_y} - 2 \end{aligned}\right\} \qquad \text{if} \qquad 4P_y - 2 < P_x < 4P_y + 2$$

$$x^d = 0,\ y^d = \frac{2}{P_y} \qquad \text{if} \qquad P_x > 4P_y + 2$$

If $P_x > 10$, $x^d = 0$ and

$$y^d = \frac{2}{P_y},\quad m^d = 0 \qquad \text{if} \qquad P_y < 2$$

$$\left.\begin{aligned} y^d &= \frac{6}{P_y} - 2 \\ m^d &= 2P_y - 4 \end{aligned}\right\} \qquad \text{if } 2 < P_y < 3$$

$$y^d = 0,\ m^d = 2 \qquad \text{if} \qquad P_y > 3$$

Notice that if $P_x = 10$ the consumer is indifferent between m and x. For definiteness, we have assumed that no m is purchased here.

Without loss of generality, we can assume that both P_x and P_y are at least 1. Further, we can assume that $P_x < 10$.

Letting $P_x^*(P_y)$ and $P_y^*(P_x)$ denote the optimal responses to the other firms price strategy, we see that:

$$P_x^*(P_y) = \begin{cases} \sqrt{4P_y + 2} & \text{if} \quad 1 < P_y < \frac{5}{8} + \frac{\sqrt{68}}{16} \\ \min(4P_y - 2,\ 10) & \text{if} \quad P_y > \frac{5}{8} + \frac{\sqrt{68}}{16} \end{cases}$$

and

$$P_y^*(P_x) = \begin{cases} \frac{\sqrt{2 + P_x}}{2} & \text{for} \quad 1 < P_x < 4 + \frac{\sqrt{80}}{2} \\ \frac{P_x - 2}{4} & \text{for} \quad 4 + \frac{\sqrt{80}}{2} < P_x < 10 \end{cases}$$

Thus, if $P_y > \frac{5}{8} + \frac{\sqrt{68}}{16}$, it is optimal for the x producer to drive the y producer from the market and if $P_x > 4 + \frac{\sqrt{80}}{2}$, the y producer drives demand for x to zero.

A straightforward calculation gives that $P_x \approx 2.50$, $P_y \approx 1.06$ is an an equilibrium price configuration in this case. At these prices, $x^d = .748$, $y^d = .1226$, $m^d = 0$ giving profits of $1.122 - \varepsilon$ for the producer of x, $.0074 - \varepsilon$ for the producer of y.

(4) If one firm chooses x and the other n, it is straightforward to check that the equilibrium in the second stage price game has $P_x = 10$ whence $x^d = \frac{1}{5} m^d = y^d = 0$. This gives the entering producer profits of $1\frac{4}{5} - \varepsilon$. Of course, the payoff to the producer who choses not to enter is zero.

(5) If one firm chooses y and the other chooses n, the equilibrium in the price game is to set $P_y = 2$. The entering producer earns profits of $1 - \varepsilon$. Again the payoff to the nonentering producer is 0.

From this, we can see that if ε is sufficiently small (less than .0074), a subgame perfect equilibrium is for one of the firms to enter, produce x and charge $P_x = 2.50$ while the other firm enters, produces y and charges $P_y = 1.06$.

This equilibrium entails the production of too many goods, however, since by diverting the resources used in the production of y to the production of x (and leaving profits to both producers the same) utility of the consumer is increased from 0.987 to 1.006. In fact, given preferences, technology and our consumers initial resources, efficiency requires that only x be produced since $MRS_{xy} > 1 = \frac{MC_x}{MC_y}$ throughout the collection of technologically feasible alternatives.

It is easy to trace the source of this inefficiency. From point of view of the y-producer, given the strategy of the x-producer, he must choose between n, x and y. If he chooses n, his payoff is zero. If he chooses x, Bertrand price competition follows giving him (and the x-producer as well) a payoff of $-\varepsilon$. By choosing y, however, he can guarantee himself a strictly positive payoff. Thus, he avoids the direct price competition which will follow if he produces x and produces y instead.

Section 3 Complements and Comments

We close with a few brief comments.

(1) Notice that even if ε (entry costs) is zero, the strategies described in Section 2 still constitute an equilibrium configuration. Further, if there are more firms and free entry is allowed, two firms playing the strategies of Section 2 and all other firms playing n is still an equilibrium. This follows by the usual Bertrand argument--if a third firm enters, price competition drives the price of the chosen good to marginal (and hence average as well) cost. Thus, there is no benefit from entering (the potential entrant is indifferent if $\varepsilon = 0$) and so the original configuration

is an equilibrium.

If $\varepsilon = 0$ there are many other equilibria as well however. For example, any configuration with at least two firms producing each good and the price of each at marginal cost (= \$1) is an equilibrium. (Note that no y is in fact produced in this situation).

If $\varepsilon > 0$ (but still less than .0074) these other equilibria vanish - there are no subgame perfect equilibria in pure strategies in which more than 2 firms choose other than n at the first stage. Again, this follows since the inpending second stage price competition guarentees payoffs of $-\varepsilon$ to any firm in an industry with at least one other firm. Hence no entry occurs.

Technically, since the comptetitive solution is an equilibrium of the game with $\varepsilon = 0$, the equilibrium correspondence (as parametrized by ε) is not lower-hemicontinuous.

(2) Applying similar reasoning to the classical Cournot-Bertrand situation, we arrive at a surprising result: As above, consider a game in two stages with one good. Firms first make an entry decision (simultaneously) then compete by setting prices. Firms are charged $\varepsilon > 0$ if they enter. Payoffs from the second stage are calculated from a straight line demand curve as in Bertrand -- $Q^d = \frac{P - a}{-b}$ $(a,b > 0)$ — assuming constant marginal costs of c. $\frac{a}{b}$ is then the "size" of the market. Note that for prices higher than a, demand is zero. Assume $c < a$.

As above, the only pure strategy equilibrium in the second stage is:

$$P = c \quad \text{if more than two firms enter,}$$

$$P = \frac{a + b}{2} \quad \text{if only one firm enters.}$$

Thus, for $\varepsilon > 0$, the only subgame perfect equilibrium in pure strategies is the monopoly solution. Again, when $\varepsilon = 0$, both the competitive ($P = c$) and the monopoly solutions are equilibria.

At this stage, we should point out that these equilibrium inefficiencies do not arise as the result of idle threats which would not be rational to carry out (as is often the case in dynamic strategic models). It is exactly for this reason that only subgame perfect equilibria are considered. There are equilibria in the example of Section 2 which are not subgame perfect and do arise as the result of dynamically inconsistent threats on the part of the producers. One such equilibrium is for one producer to enter choosing x and threaten $P_x = 1$ if the second firm enters. Given this strategy, the second firm will not enter leaving only one firm with a monopoly in x. This equilibrium is not subgame perfect since if the second producer does enter and produce y, $P_x = 1$ is not part of an equilbrium price configuration in the second stage game.

(3) Notice that the monopoly solution is an equilibrium in the game described in (2) independent of the size of the market (no matter how large b is) even if $\varepsilon = 0$. Given the economic plausibility of the strategic form, this example suggests that the possibilities for the extension of the results in Dubey, Mas-Colell and Shubik (2) and Green (3) on the asymptotic Pareto optimality of games in large economies is limited. No matter how large we make the consumption sector or how many firms we allow, monopoly remains an equilibrium.

Of course, the results in (2) and (3) do not apply in our case due to the discontinuities in payoffs which accompany price competition. Further, the monopoly solution is only an equilibrium for this game in large economies because there is no bound on the possible output the firms. If we alter the technological specifications to bound output by any individual firm independent of the size of the market, monopoly will no longer be an equilibriium for sufficiently large economies (i.e., for b sufficiently large).

(4) There are several other "ways out" of the conclusions in (2) above that are suggested by the recent work on rational cooperation in repeated games (e.g. (4)). That is, it is plausible to assume that price competition is repeated for many periods after the initial entry decisions are made. If this is the case and there is some uncertainty about (e.g) the other firms costs, an equilibrium configuration might look like (this speculation is based on extrapolation of the results in (4):

Both firms enter and something like the monopoly price is charged repeatedly by until very near the end of the game.

Of course, product differentiation provides another way out.

(5) Finally, it is possible that the equilibrium outlined in Section 2 is not unique. In particular, we have not even considered the possibility of mixed strategies. Since the aim of this note is solely to show that certain quilitative phenomena are possible in equilibrium this seems a justifiable approach.

Bibliography

1. A. Dixit and J. Stiglitz, Monopolistic competition and optimum product diversity, Am. Econ. Review 67 (1977), 297-308.

2. P. Dubey, A. Mas-Colell, and M. Shubik, Efficiency properties of strategic market games: an axiomatic approach, J. Econ. Theory 22 (1980), 339-362.

3. E. Green, Continuum and finite-player noncooperative models of competition, Cal Tech Working Paper (1981).

4. D. Kreps, P. Milgrom, J. Roberts and R. Wilson, Rational cooperation in the finitely repeated Prisoners' Dilemma, J. Econ. Theory 27 (1982), 245-252.

5. L. Phlips, Applied Consumption Analysis, North Holland, Amsterdam, 1974.

6. E. Prescott and M. Visscher, Sequential location among firms with foresight, Bell J. Econ. 8 (1977), 378-393.

7. A Shaked and J. Sutton, Relaxing price competition through product differentiation, R. Econ. Stud. 49 (1982), 3-13.

8. M. Spence, Procduct selection, fixed costs and monopolistic competition, R. Econ. Stud. 43 (1976), 217-235.