DISCUSSION PAPER NO. 540 S

QUALITATIVE CHOICE AND THE BLENDING OF DISCRETE ALTERNATIVES

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August, 1982

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Distribution of this paper was supported by the Sloan Foundation Grant for a workshop in applied microeconomics at Northwestern University.

ABSTRACT

In discrete choice theory a decision maker is assumed to choose one of a number of mutually exclusive and indivisible options. In this paper it is argued that mutually exclusive and indivisible options can be smoothed by choosing to blend them over some period of time. Such blending results in a more continuous mix of the qualitative attributes of the options. The multinomial logit model is redeveloped under these assumptions and it is shown that the resulting new model is not subject to the "independence from irrelevant alternatives "restriction. Using a travel diary for a week on the travel mode choices of a number of commuters from Seoul, Korea, the new model is estimated and shown to yield substantially different results from the traditional binary logit model estimated from the same data. It has generally been recognized that the traditional model tends to overestimate elasticities. It is argued that it does so because the smoothing that can be achieved by blending is ignored. The price elasticity estimated with the traditional approach exceeds that obtained when blending over time is permitted by 54%. The travel cost coefficient of the traditional model exceeds that of the blending model by a factor of eight.

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1. Introduction

Microeconomic theory deals with problems of quantitative choice such as what quantity of a commodity a consumer should purchase, what quantity of a product a firm should produce or how much labor, capital or land a firm should utilize in its production process. Beginning with McFadden's contribution [11], econometricians became interested in problems of discrete choice. In this context the consumer, firm or other entity chooses one of a number of discrete and indivisible but substitutable alternatives. Each alternative may represent a course of action, an activity such as travel, recreation, employment or a particular service or commodity. In the spirit of Lancaster's [8] consumer theory, each discrete alternative is described by a number of attributes which measure various qualities of that alternative. The alternatives available for choice are compared by weighting their qualitative attributes and the most attractive alternative is chosen. Given a sufficiently strong improvement in the attributes of an unchosen alternative, one will switch to that alternative.

Models of discrete choice (also called qualitative or quantal choice models) deal, not with questions of "how much", but with questions of "which", "when" or "where". The best known applications of discrete choice models are in the area of travel demand analysis developed and successively refined by Warner [16], Ben-Akiva [2], McFadden [11], Domencich and McFadden [4] and

1 The authors are indebted to Ja Hong Ku of the Economic Planning Board of Korea for helping to design and conduct the travel diary survey, and to Chausie Chu who helped with the econometric estimation.

others. In this literature a traveler decides how to commute to work (by auto, bus, train) or how to go shopping. Another area of application is the choice of housing or residential location (Quigley [14], Lerman [10], McFadden [12], Anas [1], Ellickson [6]). In these applications households choose the location or community in which to rent or buy housing or the type of housing to occupy. In most cases housing choice is viewed as involving the simultaneous determination of automobile ownership or commuting mode. Thus, the above mentioned empirical studies by Quigley, Lerman and Anas are studies of joint travel and location decisions.

The papers by Carlton [3] and Miller and Lerman [13] have applied discrete choice models to the problems of business and retail store location: a firm decides in which of a number of possible locations (regions, suburban municipalities or shopping centers) to establish business. The study by Miller and Lerman [13] is an example of the hybrid choice model developed by Heckman [7] and Duncan [5]. In hybrid models, choices consist of discrete (qualitative) and continuous (quantitative) dimensions. For example, a firm must decide where to locate its new plant and also how large a plant it should set up. A shopper may choose a supermarket in which to do shopping and also how much shopping to do there. A prospective homeowner must decide which of a number of available homes to bid on and what amount of money to bid. In Miller and Lerman's model [13] clothing retailers decide in which shopping center to set up store and also the square feet of floor space and the number of employees in the store if it is to be set up optimally at that location.

Car ownership (how many cars should a family own?) or trip frequency (how many shopping trips per month should a person make?) can be examined by a model of choice among nested alternatives (Sheffi [15]). In this formulation the decision to own a second car or to make a second trip implies that the decision to own the first car or to make the first trip has already been made: the choice of one alternative implies that all lower ranked alternatives have been chosen previously. In this type of model it is assumed that decision makers behave sequentially, ignoring the possibility that families can deliberately plan to own a number of cars or have a number of children. The fact that cars may be purchased sequentially does not in any way disprove the possibility that a sequence of choices is the result of deliberate and foresightful planning. Another example of the application of discrete choice models to sequential decisions are contexts in which decisions are semi-Markovian in character. An application of this is Lerman's study of multi purpose trip-chaining behavior [10]. In this context a traveler who visits a number of destinations makes a decision at each destination of whether to go on to the next destination or terminate his travel by returning to the origin. The decision to continue depends on the preceding travel experience as well as the utility of continuing to the next destination.

The purpose of the present paper is to reconsider the fundamental rationale underlying all of the above applications of discrete choice theory. Our point of departure is that in discrete choice models decision makers are treated as myopic. In the travel demand problem, for example, each commuter is assumed to decide how to commute to work by making a new decision each day. In the problem of destination choice a shopper is assumed to choose one of a number of shopping destinations, making a new decision for each shopping trip. Which choice is made on any given day depends on the characteristics of the decision maker, the attributes of the choices on that day and on random effects.

A fresh look at discrete choice theory hinges on recognizing that more than one occurence of the same decision can be planned simultaneously. There are many contexts in which such planning is possible, rational and necessary. A commuter may not be able to afford or like to travel by car everyday and can plan travel in advance for a week or a month by deciding how many trips to make by car, how many by bus, etc. A shopper can plan a month's shopping trips in advance by deciding how many trips to make to each of a number of destinations in order to achieve an optimal combination of the commodity attributes offered at each destination. Wealthy families can choose to own and live in several homes in different locations at different times in order to optimally combine urban and rural amenities. Multiplant firms can, and in fact must, plan the locations of several of their plants simultaneously in order to insure an optimal overall proximity to labor sources and output markets. A family with certain driving requirements per year may choose to own two cars: a large car for comfort and safety on long trips, and a small car which saves gasoline for urban travel. Such a family can adjust the amount of driving it does by each car to find its optimal mix of comfort, safety and gasoline expenditure over the year. There is even the following amusing example: given two brands of tooth paste one of which offers cavity prevention, the other of which offers teeth whitening, there is nothing to prevent a consumer who brushes twice a day from using one brand in the morning and the other at night or even combining various amounts of the two pastes in the same brushing. To cast this as a discrete choice problem in which one or the other brand is chosen is erroneous because consumers can plan to avoid exclusive brand loyalty.

The above examples amply demonstrate that many problems in discrete choice can be effectively viewed as problems in continuous choice if we first realize that decision makers have the freedom and the need to plan their choices over an extended period by blending the discrete choices available to them and optimizing the degree of the blend in a way that maximizes their utility or profit. In many situations decision makers who behave as if discrete choices are mutually exclusive are suboptimizers and their choices cannot be truly utility or profit maximizing. If decision makers act with perfect information about the environment and the choice attributes, then their plans can be carried out over the time horizon without adjustment. The presence of uncertainty or the emergence of new unanticipated trends in the environment, on the other hand, will induce decision makers to reevaluate and adjust their previously chosen plans. For example, a commuter who normally chooses to commute three times a week by car and two times a week by transit may decide to commute by car five times a week during an unusually cold or rainy winter.

Of crucial importance in planning choices over time is the length of the time horizon within which the discrete alternatives will be blended. There are contexts in which the time horizon is so long that decision makers will discount utility significantly within the horizon and will be sensitive to the sequence as well as the blend of the discrete alternatives. In other contexts the appropriate time horizon may be brief so that discounting and sequencing within the horizon can be effectively ignored.

This new perspective of blendable discrete choices leads to a substantial departure from traditional formulations and suggests improved specifications of discrete choice models. New data collection instruments and the possibility of a new generation of discrete choice models with interesting applications is suggested. These issues are explored in the following sections. In the next section we examine the most commonly applied discrete choice model: that of binary work travel mode choice and we recast this into a model of planning commuting choices over the week. The appropriate specification of the multinomial logit model (MNL) for this context is then discussed. It is shown that the new MNL model differs from the traditional binary choice model in that the relative odds of choosing any two modes are not subject to the well known property of "independence from irrelevant alternatives." This improvement in empirical realism is achieved without making any assumptions about intercorrelated random utilities. In section three we describe the results of a travel diary survey conducted in Seoul, Korea and model the choice of commuters who are able to combine bus and taxi over the week. We report on the estimates obtained with our new MNL model and traditional binary hoice models are estimated in section four. The traditional binary choice model yields a price elasticity which is 54% higher than that obtained with the new model. By ignoring the possibility that discrete choices can be deliberately blended, econometricians can seriously overestimate the responsiveness of demand to changes in the explanatory attributes. This results from underrepresenting the subtle degrees of substitution available through blending, and the associated smoothing of the qualitative contents of the alternatives.

2. The Case of Travel to Work

The best known example of the discrete binary choice problem is that of mode choice in work travel [4]. In this context, each commuter is assumed to have to make a choice between two travel modes, say auto and public transit, during each work day. The choice made during a given work day is assumed to be independent of the choices made during previous work days. To put it another way, the commuter is treated as a memoryless and myopic utility maximizer. On any given day there is a probability that a randomly selected commuter will choose to travel by auto or by public transit. These daily choice probabilities are determined by the relative utilities and are functions of travel attributes such as travel cost, travel time, and comfort, of that day.

To introduce the possibility of blending, we assume that the commuter is indifferent to the sequencing of auto and transit trips within the week and does not discount utility within the week. Under these assumptions the following six choices comprise the weekly choice set,

$$(A,T) = \{ (5,0), (4,1), (3,2), (2,3), (1,4), (0,5)\} \tag{1}$$

where A is the number of round trips by auto and T is the number of round trips by public transit with A+T=5. In the traditional formulation the time horizon is a single day and the choice set is,

$$(A,T) = \{ (1,0), (0,1)\}. \tag{2}$$

Utility is derived from a number of attributes which may include travel cost, in and out of vehicle travel times, and measures of noise, risk, discomfort, etc., which describe each trip. We let $\bar{q}_a^h$ and $\bar{q}_t^h$ be the vectors of attributes for a two-way auto and transit trip respectively for commuter h. For simplicity, we will assume that these unit attributes are stable from day to day within the week. The "total quantity" of the attributes experienced by the hth commuter over the week can be expressed as the sum of the daily unit attributes. In vector form this is,

$$\bar{Q}^h(A,T) = A\,\bar{q}_a^h + T\,\bar{q}_t^h . \tag{3}$$

The total weekly utility of the hth commuter is,

$$\hat{U}^h(A,T) = U[\bar{S}^h , \bar{Q}^h(A,T)] + \xi(A,T). \tag{4}$$

It consists of a systematic part (or strict utility), U[ ], which is common to all commuters in the population, and a random part ξ (A,T) which varies among commuters for each travel plan (A,T) and depends on unobserved attributes of the travel plan and the commuter. The vector $\bar{S}^h$ contains the observed socioeconomic attributes of the commuter such as income, age, etc. The six travel plans in (1) may be indexed as i = 1 . . . 6. The probability that commuter h will choose the weekly travel plan i is,

$$\pi_i^h = \text{prob}\ [U(\bar{S}^h,\ \bar{Q}_i^h) + \xi_i > U(\bar{S}^h,\ \bar{Q}_j^h) + \xi_j; \forall_{j \neq i}],\ i = 1\ .\ .\ .\ 6. \quad (5)$$

The most tractable choice model, multinomial logit (MNL), is derived by assuming that the elements of $\bar{\xi}$ are independently distributed according to the Gumbel distribution. These probabilities have the form,

$$\pi_i^h = \exp\ U(\bar{S}^h,\ \bar{Q}_i^h)\ /\ \sum_{j=1}^{6}\ \exp\ U(\bar{S}^h,\ \bar{Q}_j^h). \quad (6)$$

The expected probability that a commuter h will choose auto or transit on any given day can be computed as,

$$P_a^h = \frac{1}{5} \sum_{j=1}^{6} (6-j)\ \pi_j^h \quad (7)$$

$$P_t^h = \frac{1}{5} \sum_{j=1}^{6} (j-1)\ \pi_j^h \quad . \quad (8)$$

In the traditional model of daily binary choice the daily choice probabilities are,

$$\pi_a^h = \text{prob}\ [U(\bar{S}^h,\ \bar{q}_a^h) + \xi_a > U(\bar{S}^h, \bar{q}_t^h) + \xi_t] \text{ and } \pi_t^h = 1 - \pi_a^h, \quad (9)$$

and for the binary logit model they are,

$$\pi_a^h = \exp\ U(\bar{S}^h,\ \bar{q}_a^h)\ /\ \sum_{k=a,t}\ \exp\ U(\bar{S}^h,\ \bar{q}_k^h). \quad (10)$$

In these binary choice models $P_a^h = \pi_a^h$ and $P_t^h = \pi_t^h$: i.e. the binary choice probabilities and the expected daily choice probabilities are identical.

A well-known weakness of the logit model is the independence from irrelevant alternatives (IIA) property which occurs when the number of alternatives are three or more. With m = 1 . . . M modes the probability of choosing the kth mode in a daily MNL choice model is,

$$\pi_k^h = \exp U(\bar{S}^h, \bar{q}_k^h) \;/\; \sum_{m=1}^{M} \exp U(\bar{S}^h, \bar{q}_m^h); \; k = 1 \; . \; . \; . \; M. \tag{11}$$

For any two modes k and n,

$$\pi_k^h \;/\; \pi_n^h = \exp \; [U(\bar{S}^h, \bar{q}_k^h) - U(\bar{S}^h, \bar{q}_n^h)], \tag{12}$$

which states that the relative odds of choosing k and n are independent of the attributes of the other modes, which are irrelevant to these relative odds. If commuting is planned over a five day work week and there are three modes (auto, transit and bus) available for a round trip on any given day, then the weekly choice set is (A,T,B) and contains twenty-one alternatives such that A+T+B = 5. In this case the expected probabilities that a round trip will take place by auto , transit or bus are P_a^h, P_t^h and P_b^h respectively. The relative odds computed from these expected trip probabilities depend on the attributes of all three modes and the IIA property does not hold. Indeed in the weekly choice model the relative odds of choosing two weekly choice plans depend on the weekly utilities of those two choice plans alone. However, the weekly utilities of these choice plans depend on the attributes of all modes included in the plans. The relative odds of two choice plans depend on the attributes of only a subset of the modes, if and only if both choice plans consist of the modes in this same subset. As noted above, however, the IIA property will never hold for the relative odds computed from the expected trip probabilities, whereas in the traditional model which does not allow blending IIA will always hold for such relative odds. This circumvention of IIA is a natural result of introducing an interdependence among the choice alternatives by allowing consumers to choose blends of these alternatives within a longer time horizon. This method of circumventing the IIA property contrasts sharply with the traditional remedy of the nested logit model [2,12]. In that model the IIA restriction is overcome by assuming that the unobserved attributes of different alternatives are correlated. In the model with blending no such assumption is necessary and interdependence among the relative odds comes entirely from the observed attributes.

Given a population of commuters, the traditional MNL model predicts the expected proportion of commuters choosing each mode. Different commuters choose different modes only because of random influences, i.e. unobserved attributes. Blending of modes occurs in the extensive margin (or across the population) due entirely to stochastic variations among commuters. In the weekly MNL model, commuters blend modes deliberately in order to avoid the possibility of suboptimal choice patterns. Thus blending of modes can occur in the intensive margin (or for each commuter) and need not be stochastic in origin.

3. The Data Set: Choice of Mode to Work in Seoul, Korea

The traditional choice model is typically estimated from a data set derived from a one-day survey of commuters' mode choices: A number of commuters are sampled and asked to report their chosen mode on a given day (typically the morning trip) and the values of each of a number of attributes for that mode on that day. The attribute values for other (unchosen) modes are also reported by the commuter or measured independently by the analysts.

From this information the choice model is estimated. To estimate the new model developed here, commuters must be asked to report their mode choices and travel attributes over a number of work days. The survey instrument takes the form of a travel diary.

Such a travel diary survey was used to sample the heads of households whose place of residence is a suburb in the southern part of the Seoul metropolitan area in Korea.[2] The respondents reported their commuting experiences for six working days from July 10, to July 16, 1981. In order to estimate the model a subset of 148 respondents who reported choices of bus and/or taxi were chosen for the analysis. Since a commuter can choose bus to travel to work and taxi to return or vice versa and since there are six works days, there are twelve one way trips and thriteen possible weekly travel plans. These plans, the bus/taxi blend of each and their sample market shares are reported in Table 1. None of the commuters in the sample had access to a private automobile. The reported trip costs were very nearly the same for each day during the week. The majority (67%) did all their commuting by bus and only about 5% used taxi everyday. 28% of those surveyed blended the two modes, and of the 1776 one-way trips during the work week, 87% were by bus and 13% by taxi. All these respondants confirmed that commuting was the sole purpose of their chosen mode during each trip. Since taxi is more expensive than bus, blending reduces weekly expenditure of a commuter but increases discomfort due to the crowded nature of bus travel in Seoul. The high percentage of commuters using only bus is due to the low real incomes of

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2. The survey questionnaire was distributed to (and returned by 400 respondents and returned via their daughters who attended the Jinsun women's high school. 340 of the returned questionnaires were found suitable for inclusion in the analysis.

Korean commuters, or correspondingly the high cost of taxi travel. Yet, the sample reveals substantial blending of the two modes and can be used to look for empirical differences between the traditional binary choice model of daily commuting, and the weekly choice model proposed here.

4. Empirical Estimation and Results

The traditional binary choice model for the daily work trip has the form

$$\pi^h_{bus} = \exp(u^h_{bus}) \;/\; \exp(u^h_{bus}) + \exp(u^h_{taxi});\; \pi^h_{taxi} = 1 - \pi^h_{bus}, \quad (13)$$

where h denotes the individual commuter and u^h_{bus} and u^h_{taxi} are the utilities of a trip by bus and taxi respectively. The simplest model is the one which specifies these utilities as linear functions of travel cost and a constant. Thus,

$$u^h_{bus} = \alpha c^h_{bus};\; u^h_{taxi} = \alpha c^h_{taxi} + \beta \quad (14)$$

where c^h_{bus} and c^h_{taxi} are the travel costs, α is the marginal utility of travel cost and β is a constant for the taxi mode. The above model was estimated fifteen times. First, it was estimated for the morning trip for each of the six days using the reported morning costs. Second, it was estimated for the evening reverse trip for each of the same six days using the reported evening costs. Each of these estimations used the 148 cases in the sample. Next, the six morning trips were pooled treating each day independently and the model reestimated with 888 cases. A similar pooling and reestimation was done for the evening trip. Finally, the six morning and six evening trips were pooled and the model reestimated using the resulting 1776 cases. Table 2 reports the mean coefficient and t-score estimates for the six morning and evening models and the pooled model. These are contrasted with the weekly mode choice model reported in the last column. This model has the form

$$\pi_j^h = \exp(U_j^h) \;/\; \sum_{k=1}^{13} \exp(U_k^h), \tag{15}$$

where π_j^h is the probability that the jth weekly travel plan will be chosen and U_j^h is the weekly utility of the jth plan given as,

$$U_j^h = \alpha C_j^h + \gamma_j \tag{16}$$

where C_j^h is the weekly cost under travel plan j and γ_j the constant of the for travel plan j. The weekly cost C_j^h is,

$$C_j^h = (13-j)c_{bus}^h + (j-1)c_{taxi}^h. \tag{17}$$

The expected probabilities that a bus or taxi trip will be chosen on any given day during the week are,

$$P_{bus}^h = \frac{1}{12} \sum_{j=1}^{13} (13-j)\; \pi_j^h \tag{18}$$

$$P_{taxi}^h = \frac{1}{12} \sum_{j=1}^{13} (j-1)\; \pi_j^h\;. \tag{19}$$

Table 1 also reports the elasticities of demand with respect to travel cost. For the daily choice models the sample mean elasticities are

$$E_{mode} = \sum_{h=1}^{N} \varepsilon_{mode}^h \;\pi_{mode}^h \;/ \sum_{h=1}^{N} \pi_{mode}^h \tag{20}$$

where N is the sample size and mode = (bus, taxi). The elasticity of the hth commuter is ε^h_{mode} and is computed as,

$$\varepsilon^h_{mode} = \alpha c^h_{mode} \, [1-\pi^h_{mode}]. \tag{21}$$

The total sample elasticity is,

$$E_{total} = [(\sum_{h=1}^{N} \pi^h_{taxi}) \, E_{taxi} + (\sum_{h=1}^{N} \pi^h_{bus}) \, E_{bus}]/N \tag{22}$$

In the weekly mode choice model the individual elasticities are,

$$\varepsilon^h_{mode} = - \alpha[P^h_{mode} - \frac{W^h_{mode}}{P^h_{mode}}]12 \; c^h_{mode} \tag{23}$$

$$W^h_{bus} \triangleq \frac{1}{144} \sum_{j=1}^{13} (13-j)^2 \, \pi^h_j \tag{24}$$

$$W^h_{taxi} \triangleq \frac{1}{144} \sum_{j=1}^{13} (j-1)^2 \, \pi^h_j \quad . \tag{25}$$

The sample mean elasticities are,

$$E_{mode} = \sum_{h=1}^{N} \varepsilon^h_{mode} \, P^h_{mode} \; / \sum_{h=1}^{N} P^h_{mode} \tag{26}$$

$$E_{total} = [E_{bus}(\sum_{h=1}^{N} P^h_{bus}) + E_{taxi}(\sum_{h=1}^{N} P^h_{taxi})] \;/N, \tag{27}$$

The results of Table 2 are instructive in several respects. First, we see that preferences for the evening trip are subtantially different than for the morning. In the evening, travelers are less sensitive to cost. The evening price elasticities of bus and taxi commuters are 65% and 63% lower than the respective morning price elasticities. Not surprisingly, pooling morning and evening trips results in elasticities near the midpoint of the morning-evening range. Second, the weekly choice model yields a cost coefficient close to eight times lower than those of the daily models and an associated drop in all elasticities. The bus, taxi and total elasticities of the model with blending are 38%, 15% and 35% lower than the corresponding elasticities of the morning-evening pooled model.

The model with blending yields lower elasticitics because it entails an increase in the availability of closely substitutable choices. In the traditional daily binary choice model a commuter reacts to a sufficiently large increase in the price of a mode by abandoning that mode and switching to the other mode. In the weekly choice model the adjustment to the same price increase for a mode need not be as drastic: one can reduce the weekly frequency of trips by that mode without completely abandoning it.

The reduction in the total trips by that mode is not as great and this results in lower elasticities. Choice models which allow the blending of discrete alternatives ought to be good theoretical candidates for estimation in those contexts where the estimated elasticities from traditional discrete choice models are known or strongly believed to be too high. In such contexts consumers can avoid drastic reactions to price changes by extending the time horizon over which they plan the blending of the discrete alternatives available to them. The longer the time horizon the lower the sensitivity to a given change in an attribute.

5. Conclusions

We have argued that for almost all of the problems in the discrete choice literature, the "discreteness" of alternative choices has been overstated. Decision makers can blend mutually exclusive and indivisible alternatives over time. Combinations of discrete alternatives form choice plans which are still discrete in nature but provide a more continuous blend of options and their qualitative attributes. This smoothing of available options allows decision makers to react with reduced sensitivity to changes in prices and attributes and to avoid extreme adjustments in their habits. We were able to empirically demonstrate this general principle by examining the weekly blend of taxi and bus commuters in a suburb of Seoul. It has generally been recognized that the traditional model tends to overestimate elasticities. Even though only 13% of the sampled trips were by taxi and even though only 28% of the commuters chose to blend the two modes during the week, ignoring the possibility of blending resulted in price elasticity which is 54% higher and an estimated travel price coefficient which is eight times larger. Similar results can be obtained for many other contexts in which the blending of discrete alternatives is a rational strategy with obvious benefits to decision makers.

TABLE 1

CHOICES AND SAMPLE SHARES FOR BUS AND TAXI BLENDERS

IN THE SEOUL DATA

TRAVEL PLAN	NUMBER OF TRIPS BUS	NUMBER OF TRIPS TAXI	NUMBER CHOOSING	SAMPLE SHARE(%)	WEEKLY ONE-WAY TRIPS BUS	WEEKLY ONE-WAY TRIPS TAXI
1	12	0	99	66.89	1188	0
2	11	1	8	5.40	88	8
3	10	2	10	6.76	100	20
4	9	3	8	5.40	72	24
5	8	4	5	3.38	40	20
6	7	5	5	3.38	35	25
7	6	6	2	1.35	12	12
8	5	7	2	1.35	10	14
9	4	8	0	0.00	0	0
10	3	9	1	0.68	3	9
11	2	10	0	0.00	0	0
12	1	11	1	0.68	1	11
13	0	12	7	4.73	0	84
		Total	148	100.00	1549	227
		Share in sample (%)			87.22	12.78
		Total one-way trips				1776

TABLE 2
LOGIT ESTIMATION RESULTS AND ELASTICITIES
(T - STATISTICS IN PARENTHESES)

	DAILY CHOICE MODELS MEANS[1]		POOLED MODELS[5]			WEEKLY CHOICE MODEL[6]
	MORNING	EVENING	MORNING	EVENING	MORNING/ EVENING	
α	-0.001469	-0.000459	-0.001352	-0.00039	-0.0007791	-0.00009452
	(-2.762)	(-1.159)	(-6.779)	(-2.784)	(-6.682)	(-1.730)
β	-0.1637	-1.2185	-0.2412	-1.270	-0.8318	----
	(-0.328)	(-3.548)	(-1.063)	(-6.325)	(-5.547)	
γ_2	----	----	----	----	----	-2.376
						(-6.28)
γ_3	----	----	----	----	----	-2.018
						(-5.480)
γ_4	----	----	----	----	----	-2.112
						(-4.870)
γ_5	----	----	----	----	----	-2.457
						(-4.500)
γ_6	----	----	----	----	----	2.336
						(-4.010)
γ_{13}	----	----	----	----	----	-1.242
						(1.520)
LL^{o}[2]	-102.60	-102.60	-615.5	-614.8	-1230.0	-276.3
LL^{*}[3]	-52.95	-59.33	-322.4	360.0	-690.6	-160.09
ρ^2	0.4838	0.4217	0.4762	0.4151	0.4390	0.4176
%Bus	86.39	85.69	86.40	85.70	86.00	89.40
%CP[4]	86.81	86.60	86.4	85.7	86.04	69.72
E_{bus}	-0.026	-0.009	-0.026	-0.009	-0.0160	-0.010
E_{taxi}	-1.307	-0.490	-1.271	-0.467	-0.841	-0.720
E_{total}	-0.196	-0.076	-0.195	-0.074	-0.131	-0.085
Cases(N)	148	148	888	888	1776	142

1 All statistics reported are the means of the daily models estimated for the six day period.

2 log-likelihood at zero

3 log-likelihood at convergence

4 percent correctly predicted

5 estimated by pooling morning and evening commutes

6 estimated after deleting the six cases choosing alternatives 7, 8, 10 and 12 (see Table 1).

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