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MANAGERIAL INCENTIVES, INVESTMENT AND
AGGREGATE IMPLICATIONS

PART 1: SCALE EFFECTS

by

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Abstract: We explore a managerial model of investment behavior in which an incentive problem arises because one input factor (managerial effort) is not publicly observed. We show that an optimal incentive contract leads to investment levels which are below first-best in low states and that in the aggregate this phenomenon can account for greater cyclical variability in production and investment. From the perspective of incentive scheme design, a novel feature of the model is that screening takes place over two variables (investment and output) rather than one as is customary.

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1. Introduction

The idea that optimal risk sharing can sometimes interfere with productive efficiency is a key insight in the economics of uncertainty. This possibility may arise when agents are differentially informed about either an action undertaken by one of the parties, or some exogenous characteristic or event. In the insurance literature, the former phenomenon is known as moral hazard and the latter as adverse selection. Recently, adverse selection considerations have been advanced as contributing to the magnitude of aggregate fluctuations by affecting efficient wage-employment bargains.¹ In this paper, and its sequel, we extend the analysis of efficient risk sharing under asymmetric information to a study of the aggregate implications of incentive compatible managerial compensation schemes.² The current analysis

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emphasizes how adverse selection and moral hazard affect the sensitivity of desired and actual input levels to exogenous, imperfectly verifiable changes in the firm's operating environment. The sequel paper investigates how such considerations affect the portfolio of aggregate investment projects undertaken in competitive economies.

We examine a situation in which risk neutral shareholders wish to compensate risk averse managers at the lowest possible cost; a particular example of a principal and agent relationship which has been investigated by, among others, Ross [1973], Wilson [1968], Shavell [1979], Harris and Raviv [1979] and Holmstrom [1979]. Our point of departure from these earlier works is to permit the manager's payment to depend not only on the observed outcome (profit) and whatever exogenous information may be relevant, but also on observed values of the manager's decision variables, such as the level of investment or other factor inputs. We show that it is generally desirable to use information about both inputs and outputs to reward the manager. Our main result is that such considerations will result in ex post inefficient production decisions; factor inputs will be distorted downwards from levels which would prevail if managers were rewarded a function of observed profits alone, or a fortiori, if managers were risk neutral or if all information were common. Under some conditions, this phenomenon can account for greater cyclical variability in aggregate production and investment. Thus the model gives rise to a new channel by which asymmetric information (in this case between managers and shareholders) in the presence of risk aversion can amplify the effects of exogenous shocks on aggregate variables.

The model focuses on possible confusion between events outside of managerial control and the level of managerial effort which both affect the firm's observed profitability. It is assumed that the manager can observe a

signal about the prospective return to both investment and effort and must choose the appropriate value of these inputs conditional upon this signal. Neither the value of this signal, nor the level of effort is directly observable by the shareowners, although the level of investment is common knowledge. Profits are assumed to be a non-stochastic function of investment and managerial effort given the realized value of the signal.

In section 2 we consider the optimal compensation scheme between a risk averse manager and a risk neutral owner when the signal can take only two values, high and low. The manager and the owner act so as to maximize expected utility. The owner's utility is assumed to depend only on wealth, and the manager's on both wealth and effort. We interpret effort as either having a monetary equivalent or as an expenditure. It is straightforward to show that complete insurance is non-optimal; the manager would always report that he had observed a low signal and reduce effort in a good state so that profits in both states would be equal. To make the contract incentive compatible requires that the manager be made sufficiently better off when he reports a high signal. To achieve this, we show that both the level of investment and effort undertaken in the good state is equal to their first best levels (i.e., when everything is observable to everyone), but both the level of investment and managerial effort is less in the bad state than it would be under complete information. The intuition behind this result is that a reduction in investment beneath its first best level in the bad state has only a second order effect on profits, but imposes a first order cost on the manager if he reports a low signal when he, in fact, observes a high state. In this way the difference in his compensation between the high and low state can be made smaller, while still maintaining incentive compatibility. Thus, asymmetric information and managerial risk aversion amplifies the variability

in both observed profits and investment for this firm.

In Section 3.1 we show how these considerations can result in greater cyclical variability in aggregate investment and profits. As in Section 2, we assume that each manager's observed signal can take on one of two values. Aggregate uncertainty is introduced by positing that the fraction of managers who observe a high signal is itself a random variable, which, for simplicity, is also assumed to take on one of two possible values. An important assumption of this section is that managers do not know the aggregate shock at the time investment and effort must be chosen. However, this information becomes public when all firms report profits. Such aggregate information is shown to be valuable for each individual manager's compensation schedule. Specifically, we show that when a manager reports a low signal, his compensation is higher when the aggregate state is bad than when the aggregate state is good. The idea that the manager's compensation depends on his relative performance is an implication of earlier work (cf. Holmstrom [1979, 1982], Shavell [1979]).

In Section 3.2 we discuss briefly the same aggregate model, but under the assumption that managers do observe the aggregate shock before determining input. Our main point here is that now fluctuations in aggregate output and investment may be either higher or lower than in a Walrasian world. Thus, uncertainty about aggregate variables at the time of decision making is relevant for explaining how asymmetric information can amplify the effects of exogenous shocks.

The final section contains a comparison with the wage-employment model of Grossman and Hart [1981] with some alternative interpretations of results.

2. Analysis of a Single Firm

2.1 Model

We will study a firm, which is run by a risk-averse manager and is owned by a risk-neutral syndicate. The manager's utility function over wealth is $u(\cdot)$; $u' > 0$, $u'' < 0$. The manager's task is to control the firm's two inputs. One input, labeled I , is publicly observed, and the other, labeled e , is privately observed by the manager (but not the owners). For concreteness we will refer to I as investment and to e as the manager's effort, although other interpretations are possible. For instance, I could stand for labor input (either by the manager or other workers) in terms of man hours, if that is publicly observed.

The firm's output is stochastic. Let α be the random productivity parameter. We will assume α can take only two values: "low" denoted α_1 and "high" denoted α_2 . The firm's revenue in state $\alpha = \alpha_i$ is given by

$$(2.1) \quad z = f_i(I, e), \quad i=1,2.$$

We will assume that the manager's level of effort carries a monetary cost, which is independent of his income level. This assumption is borrowed from Grossman and Hart [1981] and, as they observe, can be rationalized by viewing the cost of effort as an opportunity cost for alternative income (rather than as a value for leisure). It is convenient then to measure investment and effort levels in terms of their cost. In (2.1) the requisite transformation from physical units to cost units has already been made. Since revenue functions are indexed by i in a general way, the possibility that cost functions also depend on i is subsumed in (2.1).

Interpreting I and e as monetary costs yields the profit function

$$(2.2) \quad \Pi_i(I, e) = f_i(I, e) - I - e, \quad i=1,2.$$

Regarding f_i we will make the following assumptions:

Assumption 1: For $i = 1, 2$,

- a. f_i is strictly increasing, twice continuously differentiable and strictly convex.
- b. $\partial^2 f_i / \partial I \partial e > 0$, i.e., I and e are complements.
- c. $\partial f_i / \partial I < 1$, $\partial f_i / \partial e < 1$ for large enough I and e .

These are standard assumptions; part c merely assures that input levels will be finite.

The effect of α is described by:

Assumption 2: For all (I, e) ,

- a. $f_2 > f_1$,
- b. $\partial f_2 / \partial I > \partial f_1 / \partial I$,
- c. $\partial f_2 / \partial e > \partial f_1 / \partial e$.

Both total revenue and marginal revenue of each input factor is assumed higher when α is high.

The firm's product, net of investment costs, will be denoted y_i in state i ; i.e.,

$$(2.3) \quad y_i = f_i(I, e) - I.$$

The effort level required to produce y with an investment level I in state i is denoted $e_i(I, y)$ and defined through the relation:

$$(2.4) \quad y = f_i(I, e_i(I, y)) - I.$$

The profit function (value added) in terms of I and y is denoted $\pi_i(I,y)$ and is defined by

$$(2.5) \quad \begin{aligned} \pi_i(I,y) &= f_i(I,e_i(I,y)) - I - e_i(I,y) \\ &= y - e_i(I,y). \end{aligned}$$

We will find it convenient to have a shorthand notation for the production decision pair and will often write $x = (I,y)$. Thus, $e_i(x)$ and $\pi_i(x)$ refer to $e_i(I,y)$ and $\pi_i(I,y)$ respectively.

Assumption 1a implies that $e_i(\cdot)$ is strictly convex and $\pi_i(\cdot)$ is strictly concave. For future reference we note some other implications as well. From Assumptions 1a and 2a follows that

$$(2.6) \quad \Delta(x) \equiv \pi_2(x) - \pi_1(x) = e_1(x) - e_2(x) > 0, \text{ for all } x.$$

From the definition of $e_i(\cdot)$ and (2.4), we get

$$(2.7) \quad \frac{\partial e_i(x)}{\partial y} = \frac{1}{\partial f_i(I,e_i(x))/\partial e}, \text{ and}$$

$$(2.8) \quad \frac{\partial e_i(x)}{\partial I} = \frac{1 - \partial f_i(I,e_i(x))/\partial I}{\partial f_i(I,e(x))/\partial e}.$$

Using Assumption 1a and 2c in combination with (2.6) and (2.7) we have

$$(2.9) \quad \frac{\partial e_2(x)}{\partial y} < \frac{\partial e_1(x)}{\partial y} \text{ and } \frac{\partial \pi_2(x)}{\partial y} > \frac{\partial \pi_1(x)}{\partial y}.$$

A central feature of the model is that information is asymmetric. The

manager has superior information relative to the owners on two accounts. He can observe the choice of e and he can observe the realization of α before production decisions are made. The owners can observe neither e nor α , only I and y .

The manager's informational expertise rationalizes his presence in the firm. Since he knows factor productivities while owners do not, he should be delegated some authority in making production decisions. Yet, there is a problem with his expertise. His preferences for production decisions do not coincide with the owners'. He will wish to substitute investment for effort in an attempt to reduce effort cost, which he has to carry himself because effort is not publicly observed.³

Owners deal with this incentive problem by designing a contract, which pays the manager a reward as a function of the observable variables y and I . Alternatively, the contract specifies how much the manager should pay the owners as a function of y and I . The latter view will be taken here. Let $s(I,y)$ be the contingent payment schedule from the manager to the owners. Since α can take on only two values, the manager will use at most two pairs (I_1, y_1) (when $\alpha = \alpha_1$) and (I_2, y_2) (when $\alpha = \alpha_2$) given any $s(\cdot)$ schedule. Therefore, a contract, denoted by δ , can be written as a pair of 3-tuples.

$$\delta = \{(I_1, y_1, s_1), (I_2, y_2, s_2)\},$$

where $s_1 = s(I_1, y_1)$ and $s_2 = s(I_2, y_2)$. As is well-known by now, we will require that δ is designed so that the manager chooses (I_1, y_1, s_1) when $\alpha = \alpha_1$, i.e., that δ is incentive compatible.

At the time of contracting it is assumed that the owners and the manager share the same information about α and therefore hold the same beliefs about

the probability that α will be high. Let this probability be ϕ . The optimal contracting problem which chooses an ex ante efficient contract, subject to incentive constraints is given by

$$(2.10) \quad \underset{\delta}{\text{Max}} U = (1 - \phi)u(\pi_1(x_1) - s_1) + \phi u(\pi_2(x_2) - s_2),$$

subject to

$$(i) \quad \pi_1(x_1) - s_1 \geq \pi_1(x_2) - s_2,$$

$$(ii) \quad \pi_2(x_2) - s_2 \geq \pi_2(x_1) - s_1,$$

$$(iii) \quad (1 - \phi)s_1 + \phi s_2 = 0.$$

Constraints (i) and (ii) are the incentive compatibility constraints; we must make sure the manager wishes to choose (I_i, y_i, s_i) when he observes $\alpha = \alpha_i$. Constraint (iii) is a zero-profit constraint for the owners. We will call a solution to (2.10) second-best and denote it $\delta^a = (s_1^a, s_2^a, x_1^a, x_2^a)$.

2.2 First-Best

Before studying the second-best solution it is useful to see what a first best solution looks like. This solution solves (2.10) without imposing the incentive compatibility constraints (i) and (ii). Let

$$(2.11) \quad c_i = \pi_i(x_i) - s_i, \quad i=1,2,$$

be the manager's level of consumption. Then first-best solution,

$$\delta^* = (s_1^*, s_2^*, x_1^*, x_2^*),$$

is characterized by:

$$(2.12) \quad c_1^* = c_2^*,$$

$$(2.13) \quad \frac{\partial e_i(x_i^*)}{\partial I} = 0, \quad i=1,2,$$

$$(2.14) \quad \frac{\partial e_i(x_i^*)}{\partial y} = 1, \quad i=1,2,$$

$$(2.15) \quad (1 - \phi)s_1^* + \phi s_2^* = 0.$$

By Assumption 2, $\pi_2^* > \pi_1^*$ and hence $s_2^* > s_1^*$ by (2.12). Also, by Assumption 1, $I_2^* > I_1^*, y_2^* > y_1^*$ and $e_2^* > e_1^*$. It is worth emphasizing that the value of ϕ does not affect the production decisions x_1^* and x_2^* , but of course the level of profits and consumption. This will not be the case for the second-best solution.

2.3 Second-Best

We proceed to characterize the second-best contract δ^a .

Lemma 2.1: Under Assumptions 1 and 2, $c_2^a > c_1^a$.

Proof: We know $\pi_2(x) > \pi_1(x)$ for all x , from (2.6). This together with (2.10)(ii) implies the sequence:

$$c_2^a = \pi_2(x_2^a) - x_2^a > \pi_2(x_1^a) - s_1^a > \pi_1(x_1^a) - s_1^a = c_1^a \quad \text{Q.E.D.}$$

This proposition shows that the second-best solution, at least in terms of the manager's consumption is worse than first-best for which (2.12) holds. As we will see shortly, the productive decisions will also be distorted in search of a balance between optimal risk-sharing and efficient

production.

Lemma 2.2: Under Assumptions 1 and 2, constraint (2.10)(ii) will be binding at an optimum.

Proof: Suppose (2.10)(ii) is not binding. Consider a perturbation ds_1 , ds_2 such that $ds_1 < 0$ and $(1 - \phi)ds_1 + \phi ds_2 = 0$. Such a perturbation leaves (2.10)(iii) intact, satisfies for small enough values ds_1 constraint (2.10)(ii) by the contrapositive assumption and furthermore relaxes (2.10)(i). The perturbation is therefore feasible. The effect on the objective function is $dU = (1 - \phi)(u'(c_1^a) - u'(c_2^a))ds_1 > 0$ by Lemma 2.1. Q.E.D.

Lemma 2.2 merely reflects the fact that the gap between c_2 and c_1 should be minimized subject to incentive compatibility constraints. This requires c_2 to be as low as permitted (and c_1 as high as permitted) by (2.10)(ii).

Lemma 2.3: Under Assumptions 1 and 2, constraint (2.10)(i) will not be binding at an optimum. Hence, $x_2^a = x_2^*$. Furthermore, $\Delta(x_2^*) > \Delta(x_1^*) > \Delta(x_1^a)$.

Proof: We note first that $\Delta(x_2^a) > \Delta(x_1^a)$ implies that (2.10)(i) is not binding. This follows from the sequence:

$$c_2^a - \Delta(x_2^a) = c_1^a + \Delta(x_1^a) - \Delta(x_2^a) < c_1^a.$$

The equality follows from the fact that (2.10)(ii) is binding.

Let \hat{x}_1, \hat{x}_2 be the production decisions in an optimal solution to the relaxed program (2.10)(ii)-(iii). If we can show that $\Delta(\hat{x}_2) > \Delta(\hat{x}_1)$, the argument above shows that \hat{x}_1, \hat{x}_2 will be optimal in the full program (2.10)(ii)-(iii) as well; that is, $\hat{x}_1 = x_1^a$ and $\hat{x}_2 = x_2^a$.

Clearly, $\hat{x}_2 = x_2^*$. Because x_2^* and x_1^* are unique maximizers of $\pi_2(\cdot)$ and $\pi_1(\cdot)$, respectively

$$\pi_2(x_2^*) - \pi_2(x_1^*) > 0 > \pi_1(x_2^*) - \pi_1(x_1^*).$$

This implies $\Delta(\hat{x}_2) = \Delta(x_2^*) > \Delta(x_1^*)$. We claim $\Delta(x_1^*) > \Delta(\hat{x}_1)$. Suppose to the contrary that $\Delta(x_1^*) < \Delta(\hat{x}_1)$. We will show that then \hat{x}_1 cannot be optimal in (2.10)(ii)-(iii) because $x_1 = x_1^*$ would be a better choice.

Let s_1' and s_2' be such that (2.10)(ii) and (iii) hold as equalities when the production decisions are (x_1^*, x_2^*) . Because (x_1^*, x_2^*) are productively efficient, the expected level of consumption $\phi(\pi_2(x_2^*) - s_2') + (1 - \phi)(\pi_1(x_1^*) - s_1')$ is greater than when (\hat{x}_1, x_2^*) is implemented (in a way which satisfies (2.10)(ii)-(iii) as equalities). Also, since $c_2 = c_1 - \Delta(x_1)$ when (2.10)(ii) is binding, the variance in consumption is lower with (x_1^*, x_2^*) than with (\hat{x}_1, x_2^*) if $\Delta(\hat{x}_1) > \Delta(x_1^*) > 0$. Consequently, the objective function in (2.10) must be higher when (x_1^*, x_2^*) is implemented than when (\hat{x}_1, x_2^*) is implemented so \hat{x}_1 cannot be optimal.

This proves $\Delta(\hat{x}_1) < \Delta(x_1^*)$. The fact that we will have a strict inequality follows from differentiability by a routine envelope argument, since there are strict risk-sharing gains from choosing x_1 so that

$$\Delta(x_1) < \Delta(x_1^*).$$

Q.E.D.

We combine the results above in the following proposition.

Proposition 2.1: Under Assumptions 1 and 2, a second-best solution satisfies:

$$(a) \quad c_2^a > c_1^a, \quad x_2^a > x_1^a,$$

(b) $x_2^a = x_2^*$,

(c) $\Delta(x_1^a) < \Delta(x_1^*) < \Delta(x_2^*)$,

(d) (2.10)(i) holds as a strict inequality,

(e) (2.10)(ii)-(iii) hold as equalities.

Proof: We have proved everything except $s_2^a > s_1^a$. From (e) we can solve for s_1^a and s_2^a . This gives:

$$s_1 = -\phi(\pi_2(x_2^*) - \pi_2(x_1^a)),$$

(2.16)

$$s_2^a = (1 - \phi)(\pi_2(x_2^*) - \pi_2(x_1^a)).$$

Since x_2^* maximizes $\pi_2(\cdot)$ uniquely,

$$\pi_2(x_2^*) - \pi_2(x_1^a) > 0 \text{ and so } s_2^a > 0 > s_1^a. \quad \text{Q.E.D.}$$

Notice that (a) shows that there will be coinsurance at the optimum. In models which screen on a one-dimensional variable (c) would directly imply $x_1^a < x_1^*$. A novel feature of our model is that we screen on two variables, y and I , and as a consequence, proving $x_1^a < x_1^*$ will require some more work and an additional assumption. We turn next to this issue.

The key to locating the optimal choice of x_1 is the following simple observation:

Lemma 2.4: For some constant m , $x_1 = x_1^a$ solves the program:

$$(2.17) \quad \text{Min } \pi_2(x_1) \text{ subject to } \pi_1(x_1) = m.$$

Proof: If the statement were false, we could move x_1 along the curve $\pi_1(x_1) = m$ and decrease the value of $\pi_2(x_1)$ while keeping s_1 and s_2 unchanged. This would not change c_2 or c_1 . It would maintain (2.10)(iii) as an equality and relax (2.10)(ii). But then an improvement could be made by changing s_1 and s_2 so that (2.10)(ii) is restored as an equality; a contradiction. Q.E.D.

From (2.17) follows that x_1 must lie on what may be called the contract curve of $\pi_2(\cdot)$ and $\pi_1(\cdot)$. In terms of marginal rates of substitution we must have:

$$(2.18) \quad \frac{\partial \pi_1(x_1^a)/\partial I}{\partial \pi_1(x_1^a)/\partial y} = \frac{\partial \pi_2(x_1^a)/\partial I}{\partial \pi_2(x_1^a)/\partial y},$$

since we know $x_1^a \neq x_1^*$. Note that by (2.9), (2.18) implies that both $\partial \pi_1(x_1^a)/\partial y$ and $\partial \pi_2(x_1^a)/\partial y$ have to be non-zero. In contrast, we could have both numerators in (2.18) be zero. This in fact is a case of special interest and we will return to it shortly.

From (2.18) and (2.9) follows that either

$$(2.19) \quad \frac{\partial \pi_1(x_1^a)}{\partial I} = \frac{\partial \pi_2(x_1^a)}{\partial I} = 0, \text{ or}$$

$$(2.20) \quad \frac{\partial \pi_1(x_1^a)/\partial y}{\partial \pi_2(x_1^a)/\partial y} = \frac{\partial \pi_1(x_1^a)/\partial I}{\partial \pi_2(x_1^a)/\partial I}.$$

Applying (2.7) and (2.8) this can be written as:

$$(2.21) \quad \frac{\partial f_1(I_1^a, e_1(x_1^a))/\partial e - 1}{\partial f_2(I_1^a, e_2(x_1^a))/\partial e - 1} = \frac{\partial f_1(I_1^a, e_1(x_1^a))/\partial I - 1}{\partial f_2(I_1^a, e_2(x_1^a))/\partial I - 1} .$$

Before using these equations we will need some results on the structure of the indifference curves of $\pi_2(\cdot)$ and $\pi_1(\cdot)$. Define,

$$(2.22) \quad \begin{aligned} I_i(y) &= \operatorname{argmax}_I \pi_i(I, y) , \\ y_i(I) &= \operatorname{argmax}_y \pi_i(I, y) , \quad i=1,2. \end{aligned}$$

These represent profit maximizing choices of I given y and conversely. Notice that

$$(2.23) \quad \frac{\partial e_i(I_i(y), y)}{\partial I} = 0, \text{ and}$$

$$(2.24) \quad \frac{\partial e_i(I, y_i(I))}{\partial y} = 1.$$

Thus, $I_i(y)$ and $y_i(I)$ are determined so that the effort level is chosen efficiently (given y or I). Since $e_i(\cdot)$ is strictly convex, $I_i(\cdot)$ and $y_i(\cdot)$ are well-defined functions (not correspondences) and there is only one fixed point of (2.22), namely (I_i^*, y_i^*) .

Lemma 2.5: Under Assumption 1, $I_i(y)$ is non-decreasing.

Proof: Straightforward by Assumption 1b and the concavity of $\pi_i(\cdot)$.

Q.E.D.

Next we will show that $y_i(I)$ is above or below $I_i^{-1}(I)$ (the inverse function of $I_i(\cdot)$) depending on whether I is smaller or greater than I_i^* . Since the range of $I_i(\cdot)$ need not cover all investment levels, we introduce

the convention that $I_i^{-1}(I) = \infty$ if $I > \sup_{y_i} I_i(y_i)$ and $I_i^{-1}(I) = -\infty$ if $I < \inf_{y_i} I_i(y_i)$.

Lemma 2.6: Under Assumption 1, $I (\succ) I_i^*$ implies $y_i(I) (\succ) I_i^{-1}(I)$.

Proof: If I is outside the range of $I_i(y)$, the statement follows from the earlier convention and Lemma 2.5. So let I be within the range of $I_i(y)$.

Assume $I > I_i^*$. Lemma 2.5 implies $I_i^{-1}(I) > y_i^*$. The gradient at $(I, I_i^{-1}(I))$ is $(0, \beta)$, since I is optimal given $I_i^{-1}(I)$. Concavity of $\pi_i(\cdot)$ implies that $\pi_i(\cdot)$ increases when going from $(I, I_i^{-1}(I))$ towards the optimum (I_i^*, y_i^*) . Consequently, since $I_i^{-1}(I) > y_i^*$, we must have $\beta < 0$; ($\beta = 0$ would mean we are at (I_i^*, y_i^*)). By concavity of $\pi_i(\cdot)$, the optimal y given $I, y_i(I)$, must therefore be such that $y_i(I) < I_i^{-1}(I)$.

The case $I < I_i^*$ is proved analogously.

Q.E.D.

Lemma 2.7: Under Assumption 1, $y_i(I)$ is strictly increasing for $I < I_i^*$.

Proof: Let $\varepsilon_i(I)$ be defined by the relationship $y_i(I) = f_i(I, \varepsilon_i(I)) - I$. From (2.24) follows that $\partial f_i(I, \varepsilon_i(I)) / \partial \varepsilon = 1$. We have therefore:

$$y_i'(I) = \frac{\partial f_i(I, \varepsilon_i(I))}{\partial I} - 1 + \frac{\partial \varepsilon_i(I)}{\partial I}.$$

Complementarity of I and e (Assumption 1b) implies that $\partial \varepsilon_i(I) / \partial I \geq 0$. It also implies that $\partial f_i(I, \varepsilon_i(I)) / \partial I - 1 > 0$, or else a gradient process would converge to an optimum below I in contradiction with $I < I_i^*$. Thus,

$y_i'(I) > 0$ as claimed.

Q.E.D.

The lemmas above tell us about the structure of the indifference curves of $\pi_i(\cdot)$. The results are useful to summarize in a diagram.

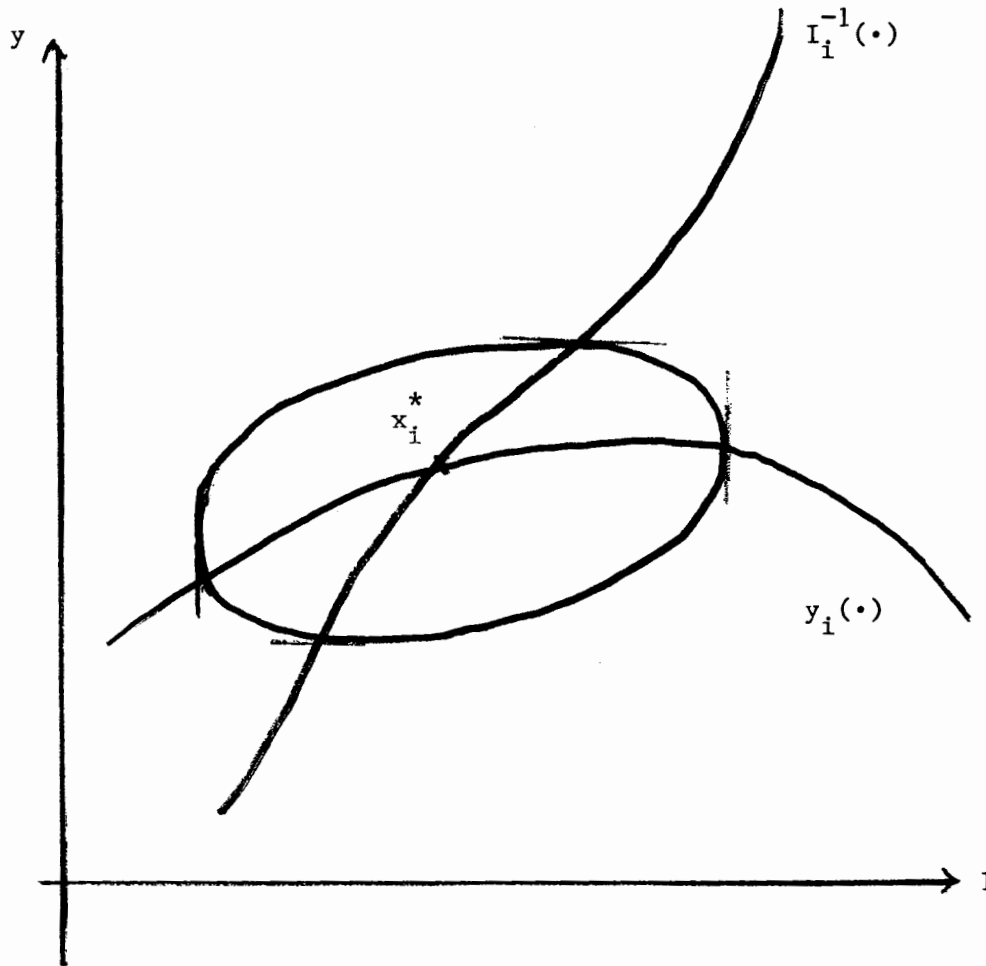


Figure 1

The fact that $I_i^{-1}(\cdot)$ slopes upward (it could be vertical) was established in Lemma 2.5. Lemma 2.6 told us that $y_i(\cdot)$ is below $I_i^{-1}(\cdot)$ for $I > I_i^*$ and above it for $I < I_i^*$, while Lemma 2.7 showed that $y_i(\cdot)$ is strictly increasing for $I < I_i^*$. In general, $y_i(\cdot)$ will not be increasing everywhere (for instance for the technology in (2.28) below). This underscores that I and y are not in a symmetric position in the problem.

Our objective now is to show that $y_1 < y_1(I_1)$ and $I_1 < I_1(y_1)$, implying that we must be in the southwest quadrant of the cross in Figure 1, and hence that $x_1 < x_1^*$. These results will be direct consequences of Lemma 2.4.

Lemma 2.8: Under Assumptions 1 and 2, a second-best solution satisfies $y_1^a < y_1(I_1^a)$.

Proof: We noted already in conjunction with (2.21) that $y_1^a = y_1(I_1^a)$ is not possible. Make the contrapositive assumption that $y_1^a > y_1(I_1^a)$. Let $y_1^i < y_1(I_1^a) < y_1^a$ be such that

$$(2.25) \quad \pi_1(I_1^a, y_1^i) = \pi_1(I_1^a, y_1^a) .$$

From (2.9) follows that

$$(2.26) \quad \pi_2(I_1^a, y_1^i) < \pi_2(I_1^a, y_1^a) .$$

Equations (2.25) and (2.26) contradict Lemma 2.4.

Q.E.D.

For the final step $I_1^a < I_1(y_1^a)$, we will need an additional assumption.

Assumption 3: For all $x = (I, y)$,

$$(2.27) \quad \frac{\partial f_2(I, e_2(x))}{\partial I} > \frac{\partial f_1(I, e_1(x))}{\partial I} .$$

This assumption states that the marginal product of capital is higher in the high state for all output levels y and investment levels I . Obviously, since $e_2(x) < e_1(x)$, Assumption 3 is stronger than the earlier assumed 2b which had marginal product higher in the high state for all input levels I and e .

Two examples of technologies for which (2.27) holds are the additively separable technology:

$$(2.28) \quad f_i(I, e) = h_i(I) + g_i(e), \quad i=1,2,$$

and the multiplicatively separable technology:⁴

$$(2.29) \quad f_i(I, e) = h(I)g_i(e).$$

For the additive technology Assumption 2a is equivalent to Assumption 3 and for the multiplicative technology (2.27) holds as an equality since $g_2(e_2(x)) = g_1(e_1(x))$ for all x .

Note that in (2.29) $h(\cdot)$ is not indexed. If $h(\cdot)$ were indexed, (2.27) would not hold in general. An example is the following:

$$(2.30) \quad f_i(I, e) = (h(I) + \alpha_i)g(e), \quad \alpha_2 > \alpha_1 > 0.$$

Now, $g(e_2(x)) < g(e_1(x))$ for all x and so the reverse of (2.7) holds. For this technology it will be the case that $I_1^a > I(y_1^a)$ and we are unable to conclude that investment will be below I_i^* .

Lemma 2.9: Under Assumptions 1-3, $I_1^a < I_1(y_1^a)$ for the second-best solution.

Proof: By Assumption 2, and (2.6),

$$(2.31) \quad \frac{\partial f_2(I_1^a, e_2(x_1^a))}{\partial e} - 1 > \frac{\partial f_2(I_1^a, e_1(x_1^a))}{\partial e} - 1 > \frac{\partial f_1(I_1^a, e_1(x_1^a))}{\partial e} - 1 > 0.$$

The last step is a restatement of Lemma 2.8.

We know that either (2.19) or (2.21) holds. If (2.19) holds, then $I_1^a = I_1(y_1^a)$. If (2.21) holds, then it implies together with (2.21) that:

$$(2.32) \quad 0 < \frac{\partial f_1(I_1^a, e_1(x_1^a))/\partial I - 1}{\partial f_2(I_1^a, e_2(x_1^a))/\partial I - 1} < 1.$$

If $\partial f_1/\partial I - 1 < 0$, so is $\partial f_2/\partial I - 1$ or else the first inequality in (2.32) could not hold. From the second inequality in (2.32) follows that

$\partial f_1/\partial I - 1 > \partial f_2/\partial I - 1$, contradicting Assumption 3. Consequently, $\partial f_1(I_1^a, e_1(x_1^a))/\partial I - 1 > 0$, or $I_1^a < I_1(y_1^a)$ by concavity of $\pi_1(\cdot)$. Q.E.D.

Putting Lemmas 2.6-2.9 together yields our basic result that resources are underemployed in the low state:

Proposition 2.2: Under Assumptions 1-3, second-best has $I_1^a < I_1^*, y_1^a < y_1^*$ and $e_1^a < e_1^*$.

Proof: With reference to Figure 1, we have already proved in the earlier lemmas that $y_1^a < y_1^*$, $I_1^a < I_1^*$. The strict inequality $I_1^a < I_1^*$ follows unless $I_1^{-1}(\cdot)$ is horizontal. However, the only case for which $I_1^{-1}(\cdot)$ is horizontal is the separable one defined in (2.28). But then $\partial f_1/\partial I - 1 > 0$, since equality cannot hold by Assumption 2b. Hence $I_1^a < I_1^*$ also in that case.

Finally, $e_1^a < e_1^*$ follows by the facts that: $I_1^a < I_1^*$, e and I are complementary and $y_1^a < y_1(I_1^a)$. Q.E.D.

2.4 Value of Screening on Input I

As mentioned, a novel feature of our model is that we may screen both on inputs and outputs. It is natural to ask whether this yields an improvement over contracts which screen on output alone. The answer depends on the

technology.

If contracts are only contingent on output y , the manager is free to set I at his preferred level. This level is $I_1(y_1^a)$ when $\alpha = \alpha_1$ and $I_2^* = I_2^*(y_2^*)$ when $\alpha_2 = \alpha_2$. Therefore, screening on investment will not be valuable if and only if the second-best solution characterized above is such that

$I_1^a = I_1(y_1^a)$. From the definition of $I_1(\cdot)$ (see (2.22)), $I_1^a = I_1(y_1^a)$ will obtain if and only if (2.19) is true, which in turn holds if (2.27) holds as an equality. We conclude therefore:

Proposition 3: If (2.27) holds as an equality for all x , then additional screening on I is valueless. If (2.27) holds as a strict inequality for all x , then additional screening on I has strictly positive value.

Note that since (2.19) obtains when screening is valueless, both the high state manager and the low state manager agrees on what investment is optimal for any given y . This explains why nothing can be achieved by constraining the choice of I .

The multiplicative technology defined in (2.29) provides an example where screening on investment is valueless, because as we observed earlier, (2.27) is an equality in that case.

Whenever screening on y alone is as good as screening on I and y , the model reduces to the standard single-variable case.⁵ One may check that the single-crossing property (the cross-partial condition), which is essential in the single-variable case, is implied by our assumptions. Since I_1 will be chosen so that $I_1 = I_1(y_1)$, we have (using (2.22)):

$$(2.23) \quad \frac{\partial \pi_i(I_1(y_1), y_1)}{\partial y} = \frac{\partial \pi_i(I_1(y_1), y_1)}{\partial y} + \frac{\partial \pi_i(I_1(y_1), y_1)}{\partial I} \cdot \frac{\partial I_1(y_1)}{\partial y} = \frac{\partial \pi_i(I_1(y_1), y_1)}{\partial y}.$$

The fact that $\partial\pi_2(I_1(y_1), y_1)/\partial y > \partial\pi_1(I_1(y_1), y_1)/\partial y$ is a straightforward consequence of our assumptions.

Of course, in a more general sense, our model will always reduce to an equivalent single-variable problem, because (2.20) (or (2.19)) will yield a relationship between I and y . Above this relationship was $I = I_1(y)$. The first equality in (2.33) will always hold, but whether the single-crossing property will be true or not in the general case we do not know.

Finally, it is worth noting that while additional screening on I may have no benefits, it will never be optimal to delete y from the contract. This is so, because by (2.9), $y_1(I) < y_2(I)$ for all I and so preferences for y given I do not depend on whether $\alpha = \alpha_1$ or α_2 .

2.5 Comparative Statics

Given the technology, two parameters affect the choice of x_1 : the probability of a high state (ϕ) and the manager's risk aversion. These effects can be signed.

From Proposition 2.1 we know that (2.10)(ii) and (iii) will be binding. We can solve for s_1 and s_2 from these equations. The result is given in (2.16). Substituting (2.16) into the objective function of (2.10) and taking derivatives yields the following first-order conditions for y_1^a and I_1^a :

$$(2.34) \quad \frac{dU}{dy} = -(1 - \phi) \left[\frac{\phi \partial\pi_2(x_1^a)}{\partial y} \frac{u'(c_1^a) - u'(c_2^a)}{u'(c_1^a)} - \frac{\partial\pi_1(x_1^a)}{\partial y} \right] = 0.$$

$$(2.35) \quad \frac{dU}{dI} = -(1 - \phi) \left[\frac{\phi \partial\pi_2(x_1^a)}{\partial I} \frac{u'(c_1^a) - u'(c_2^a)}{u'(c_1^a)} - \frac{\partial\pi_1(x_1^a)}{\partial I} \right] = 0.$$

When ϕ is changed, c_1^a and c_2^a will also change, but so that $c_2^a = c_1^a + \Delta(x_1^a)$ (as (2.10) (ii) is an equality). If $u(\cdot)$ exhibits non-decreasing absolute risk-aversion, $[u'(c_1^a) - u'(c_2^a)]/u'(c_1^a)$ will increase in ϕ , because c_2^a and c_1^a

will both increase (in order for (2.10) (iii) to hold). Now, $\partial\pi_2(x_1^a)/\partial y > 0$ by Lemmas 2.8 and 2.9. Also, $c_1^a < c_2^a$ by Proposition 2.1 and so

$u'(c_1^a) - u'(c_2^a) > 0$. Consequently, differentiating (2.34) totally with respect to ϕ implies $dy_1^a/d\phi < 0$. By Lemma 2.9 and Assumption 3, $\partial\pi_2(x_1^a)/\partial I \geq 0$. If equality prevails then Lemma 2.5 and $dy_1^a/d\phi < 0$ imply $dI_1^a/d\phi < 0$, while if a strict inequality holds, the same conclusion follows from differentiating (2.35) totally with respect to ϕ . Thus, we have shown:

Proposition 2.4: An increase in the probability of a high state (ϕ) will decrease y_1^a and I_1^a if the manager's utility function displays non-decreasing absolute risk-aversion.

The result is intuitive and reflects the trade-off between productive efficiency and risk-sharing. The more likely the low state is, the more we care about productive efficiency as long as risk-aversion does not increase too much due to the decrease in the level of expected income.

Based on the same trade-off we can also conclude:

Proposition 5: If the manager becomes more risk-averse (in the sense of a concavification of u) y_1^a and I_1^a will decrease.

Proof: If we replace $u(\cdot)$ by $\hat{u}(\cdot) = v(u(\cdot))$ where v is concave and increasing, $\hat{u}(\cdot)$ is a more risk-averse utility function than $u(\cdot)$. It is straightforward to check that then

$$\frac{\hat{u}(c_2) - \hat{u}(c_1)}{\hat{u}(c_1)} > \frac{u(c_2) - u(c_1)}{u(c_1)}.$$

The conclusion follows from (2.34) and Lemma 2.5 as before.

Q.E.D.

3. Aggregate Analysis

If there is a large number of firms of the kind described above with productive shocks that are independent, then the aggregate implications from asymmetric information are merely that a lower overall level of investment and production will obtain relative to the full-information world. Next we want to study the effects of asymmetric information when the productivity shocks of the firm are dependent; that is, when there is some aggregate uncertainty. Our specific objective is to establish that under some circumstances asymmetric information will magnify fluctuations in output and investment.

The simplest way to introduce aggregate uncertainty into the model is to let ϕ represent the fraction of firms which have a high α . Thus, conditional on ϕ , the probability that a firm has a high α is ϕ . We will assume that ϕ can take only two possible values, $\phi_2 > \phi_1$.

Let the ex ante probability that $\phi = \phi_2$ be p . Let $p_{ji} = \Pr\{\phi = \phi_j, \alpha = \alpha_i\}$ be the joint probability distribution of ϕ and α . We have, of course,

$$p_{11} = (1 - p)(1 - \phi_1) , \quad p_{12} = (1 - p)\phi_1 ,$$

$$p_{21} = p(1 - \phi_2) , \quad p_{22} = p\phi_2 .$$

The marginal distribution of α is given by $\bar{\phi} = \Pr\{\alpha = \alpha_2\} = p_{12} + p_{22}$ and $1 - \bar{\phi} = \Pr\{\alpha = \alpha_1\} = p_{11} + p_{21}$. We denote the conditional distribution by $l_{ji} = \Pr\{\phi = \phi_j | \alpha = \alpha_i\}$. Since $\phi_2 > \phi_1$,

$$(3.1) \quad \frac{p_{22}}{p_{12}} > \frac{p_{21}}{p_{11}} , \text{ and}$$

$$(3.2) \quad l_{22} > l_{21} , \quad l_{11} > l_{12} .$$

As before, we will assume that a contract is designed ex ante, when the manager and the owners hold the same probabilistic beliefs given by the matrix (p_{ji}) above. We will also assume, as is natural, that aggregate shocks are publicly observed and that contracts therefore can be indexed by the realization of ϕ . To what extent they are indexed depends on whether ϕ is observed before or after x is chosen. If it can be observed before, then both the production decision x and the payments s from the manager to the owners can be indexed by ϕ . In this case the analysis of the previous section will apply rather directly as we will discuss shortly. Perhaps the more interesting case is when ϕ cannot be observed at the time x is chosen, but before rewards are paid. Then the analysis of optimal contracts is more complicated because of the risk the indexing of s may introduce into a contract. We proceed to look at that case first.

3.1 Manager Uninformed About ϕ

Since ϕ is observed after x is chosen but before s is paid, while α is observed as before, a contract δ is now a pair of 4-tuples:

$$\delta = \{(I_1, y_1, s_{11}, s_{21}), (I_2, y_2, s_{12}, s_{22})\}.$$

Here, $x_i = (I_i, y_i)$ is the manager's choice if $\alpha = \alpha_i$ and s_{ji} is his payment to the owners if $\phi = \phi_j$ and $\alpha = \alpha_i$.

An optimal contract now solves the following program:

$$(3.3) \quad \begin{aligned} \text{Max}_{\delta} & p_{11} u(\pi_1(x_1) - s_{11}) + p_{21} u(\pi_1(x_1) - s_{21}) + \\ & p_{12} u(\pi_2(x_2) - s_{12}) + p_{22} u(\pi_2(x_2) - s_{22}), \end{aligned}$$

subject to

- (i)
$$p_{11}u(\pi_1(x_1) - s_{11}) + p_{21}u(\pi_1(x_1) - s_{21})$$

$$\geq p_{11}u(\pi_1(x_2) - s_{12}) + p_{21}u(\pi_1(x_2) - s_{22}),$$
- (ii)
$$p_{12}u(\pi_2(x_2) - s_{12}) + p_{22}u(\pi_2(x_2) - s_{22})$$

$$\geq p_{12}u(\pi_2(x_1) - s_{11}) + p_{22}u(\pi_2(x_1) - s_{21}),$$
- (iii)
$$p_{11}s_{11} + p_{21}s_{21} + p_{12}s_{12} + p_{22}s_{22} = 0,$$

where x_i and $\pi_i(\cdot)$ are as before.

The formulation above deserves brief comment. Constraints (i) and (ii), requiring incentive compatibility, are based on the view that the ex ante policy of truth-telling is optimal for the manager. Of course, this is equivalent to requiring that conditional on α , the manager is better off telling the truth. These constraints would read:

- (i')
$$\ell_{11}u(\pi_1(x_1) - s_{11}) + \ell_{21}u(\pi_1(x_1) - s_{21})$$

$$\geq \ell_{11}u(\pi_1(x_2) - s_{12}) + \ell_{21}u(\pi_1(x_2) - s_{22}),$$
- (ii')
$$\ell_{12}u(\pi_2(x_2) - s_{12}) + \ell_{22}u(\pi_2(x_2) - s_{22})$$

$$\geq \ell_{12}u(\pi_2(x_1) - s_{11}) + \ell_{22}u(\pi_2(x_1) - s_{21}).$$

If we multiply (i') by $(1 - \phi)$ and (ii') by $\bar{\phi}$ we get back to (i) and (ii).

Constraint (3.3) (iii) assumes that owners are risk neutral. This may appear to be a bad assumption now that we deal with aggregate risks as well as idiosyncratic risks. We retain it for technical simplicity with the following

rationalization. If owners were modeled as risk averse with respect to aggregate shocks, then the manager could absorb some of that risk and would certainly do so in the second-best solution. However, the manager's risk absorption capacity is negligible relative to the risk sharing a stock market (i.e., the ownership syndicate) can offer. Therefore, by assuming that owners are risk neutral, we are merely capturing the fact that they are so relative to the manager.

The optimal solution to (3.3) will be denoted $x_i^b, s_{ji}^b (i, j=1, 2)$. The consumption of the manager in state $\alpha = \alpha_i$, $\phi = \phi_j$ is:

$$(3.4) \quad c_{ji} = \pi_i(x_i) - s_{ji} = y_i - e_i(I_i, y_i) - s_{ji}.$$

As in the previous section we will show that (3.3) (ii) is binding but (3.3) (i) is not at an optimum. The argument is slightly more elaborate now because it is not evident that the manager's expected utility from consumption is higher when $\alpha = \alpha_2$ than when $\alpha = \alpha_1$, as the conditional probabilities on ϕ change with α .

Lemma 3.1: Under Assumptions 1 and 2, constraint (ii) in (3.3) is binding at an optimum.

Proof: Let λ_1 be the multiplier for (i), λ_2 for (ii) and μ for (iii). Naturally, $\mu > 0$.

The first-order conditions for s_{11} and s_{21} are (dropping the superscript for simplicity):

$$(3.5) \quad -\lambda_2 \frac{p_{12}}{p_{11}} u'(c_{11} + \Delta(x_1)) + (1 + \lambda_1)u'(c_{11}) - \mu = 0,$$

$$(3.6) \quad -\lambda_2 \frac{p_{22}}{p_{21}} u'(c_{21} + \Delta(x_1)) + (1 + \lambda_1)u'(c_{21}) - \mu = 0.$$

Suppose $\lambda_2 = 0$. Then (3.5) and (3.6) imply $c_{11} = c_{21} \equiv c_1$. The first-order condition for s_{12} is:

$$(3.7) \quad (1 + \lambda_2)u'(c_{12}) - \lambda_1 \frac{p_{11}}{p_{12}} u'(c_{12} - \Delta(x_2)) - \mu = 0.$$

Combined with (3.5) this gives (since $\lambda_1 > 0$, $\lambda_2 = 0$):

$$u'(c_{12}) = \mu + \lambda_1 \frac{p_{11}}{p_{12}} u'(c_{12} - \Delta(x_2)) > \mu > \frac{\mu}{(1 + \lambda_1)} = u'(c_1).$$

Consequently, $c_{12} < c_1$. Similarly, the first-order condition for s_{22} is:

$$(3.8) \quad (1 + \lambda_2)u'(c_{22}) - \lambda_1 \frac{p_{21}}{p_{22}} u'(c_{22} - \Delta(x_2)) - \mu = 0,$$

which together with (3.5), gives $c_{22} < c_1$.

The assumption $\lambda_2 = 0$ has led to the conclusion that c_{12} and c_{22} are both below $c_1 = c_{11} = c_{21}$. Since $\Delta(x) > 0$, this contradicts (3.3) (ii). Hence, $\lambda_2 > 0$ and the claim follows. Q.E.D.

We will go on to show that at an optimum (3.3) (i) will not be binding. Again, the strategy is to solve for an optimum without (3.3) (i) and check that the solution to the relaxed program satisfies (3.3) (i).

Denote by \hat{x}_i, \hat{s}_{ji} ($i, j = 1, 2$) the solution to (3.3) with constraint (i) dropped.

Lemma 3.2: Under Assumptions 1 and 2, $\hat{x}_2 = x_2^*, \hat{s}_{12} = \hat{s}_{22} = \hat{s}_2$ and $\Delta(\hat{x}_2) = \Delta(x_2^*) > \Delta(x_1^*) > \Delta(x_1)$.

Proof: When (i) is not imposed, the optimal choice of x_2 , s_{12} , s_{22} will necessarily maximize $p_{12}u(c_{12}) + p_{22}u(c_{22})$ subject to (ii) and (iii); (here $\hat{x}_1, \hat{s}_{11}, \hat{s}_{21}$ are kept fixed). The first claim follows immediately.

For the second part $\Delta(x_2^*) > \Delta(x_1^*)$ was proved before. We claim $\Delta(x_1^*) > \Delta(\hat{x}_1)$.

Suppose not. Then we can choose $x_1 = x_1^*$, $s_{11} = \pi_1(x_1^*) - \hat{c}_{11} > \pi_1(\hat{x}_1) - \hat{c}_{11}$ and $s_{21} = \pi_1(x_1^*) - \hat{c}_{21} > \pi_1(\hat{x}_1) - \hat{c}_{21}$, because $\hat{x}_1 \neq x_1^*$. It follows if $\Delta(\hat{x}_1) > \Delta(x_1^*)$ that this choice will satisfy (ii), leave (iii) with a surplus and leave the objective unchanged. But this contradicts the fact that we are at an optimum at $\hat{x}_1, \hat{s}_{11}, \hat{s}_{21}$. Q.E.D.

Lemma 3.3: Under Assumptions 1 and 2, constraint (i) in (3.3) is not binding at an optimum.

Proof: Throughout, keep x_1 and x_2 at the optimal levels \hat{x}_1 and $\hat{x}_2 = x_2^*$ in the relaxed program (3.3) (ii)-(iii). We show first that (\hat{x}_1, \hat{x}_2) can be implemented incentive compatibly without indexing payments on ϕ and that (i) is not binding in that case. Next we show that any improvement obtained in the relaxed program (3.3) (ii)-(iii) by indexing payments on ϕ , maintains (i) as a strict inequality. This proves the claim.

Choose s_1' and s_2' so that (ii) and (iii) hold as equalities. Then,

$$(3.9) \quad \begin{aligned} \pi_1(x_1) - s_1' &= \pi_2(x_2) - s_2' - \Delta(x_1) \\ &> \pi_2(\hat{x}_2) - s_2' - \Delta(\hat{x}_2) = \pi_1(\hat{x}_2) - s_2'. \end{aligned}$$

The first equality is (ii) while the inequality follows from Lemma 3.2.

Inequality (3.9) shows that if we do not index on ϕ , then (ii) and (iii) imply

that (i) is not binding.

Now, consider the optimal payments $\hat{s}_2, \hat{s}_{11}, \hat{s}_{21}$ (\hat{s}_2 is not indexed by ϕ according to Lemma 3.2). We claim $s_2 \geq s_2'$. Suppose not, i.e., $s_2' > \hat{s}_2$. Then, by (iii),

$$\begin{aligned}
 (3.10) \quad & pu(\pi_2(\hat{x}_2) - s_2') + (1-p)u(\pi_1(\hat{x}_1) - s_1') \\
 & > pu(\pi_2(\hat{x}_2) - \hat{s}_2) + (1-p)u(\pi_1(\hat{x}_1) - s_1'') \\
 & > pu(\pi_2(\hat{x}_2) - \hat{s}_2) + (1-p)[\lambda_{11}u(\pi_1(\hat{x}_1) - \hat{s}_{11}) \\
 & \quad + \lambda_{21}u(\pi_1(x_1) - s_{21})] \\
 & = p_{11}u(\pi_1(\hat{x}_1) - \hat{s}_{11}) + p_{21}u(\pi_1(\hat{x}_1) - \hat{s}_{21}) \\
 & \quad + (p_{12} + p_{22})u(\pi_2(\hat{x}_2) - \hat{s}_2).
 \end{aligned}$$

The first and second inequalities follow from the concavity of u , the facts that $\pi_2(\hat{x}_2) - s_2' > \pi_1(\hat{x}_1) - s_{11}'$ and $\hat{s}_1 < s_2', \hat{s}_1 < s_1'$ and (iii). From (3.10) we see that the objective function is lower for $(\hat{s}_2, \hat{s}_{11}, \hat{s}_{21})$ than for (s_1', s_2') , contradicting the optimality of the former. Consequently $\hat{s}_2 \geq s_2'$.

When moving from (s_1', s_2') to $\hat{s}_{11}, \hat{s}_{21}, \hat{s}_2$ the component of the objective function relating to $\alpha = \alpha_2$ does not go up $\hat{s}_2 \geq s_2'$. Hence, the component of the objective function relating to $\alpha = \alpha_1$ cannot go down, which means the left side in (i) cannot go down. On the other hand, the right side of (i) does not go up as $\hat{s}_2 \geq s_2'$. In sum, (i) is maintained as a strict inequality when payments are $(\hat{s}_{11}, \hat{s}_{21}, \hat{s}_2)$. Therefore, solving (3.3) (ii)-(iii) yields a

solution to (3.3) (i)-(iii) as well.

Q.E.D.

We can now apply the results from the previous sections to prove:

Proposition 3.1: Under Assumptions 1-3, a solution to the second-best program (3.3) has $I_2^b = I_2^*$, $y_2^b = y_2^*$, $I_1^b < I_1^*$, $y_1^b < y_1^*$.

Proof: Just as in the simpler model of the previous section, it is clear from (3.3) that a necessary condition for x_1 to be optimal is that it solves (2.17). Since Proposition 2.2 was based on (2.17) alone, it follows that

$$I_1^b < I_1^*, y_1^b < y_1^*.$$

The claim $I_2^b = I_2^*$, $y_2^b = y_2^*$ was established in Lemmas 3.2 and 3.3. Q.E.D.

An immediate consequence of the preceding proposition is our main conclusion:

Proposition 3.2: Under Assumptions 1-3, the aggregate levels of production and investment are lower and fluctuate more when there is incomplete information than when there is full information.

Proof: The aggregate level of investment when $\phi = \phi_i$ is $\bar{I}_i^b = \phi_i I_2^* + (1 - \phi_i) I_1^b$. Therefore, $\bar{I}_2^b - \bar{I}_1^b = (\phi_2 - \phi_1)(I_2^* - I_1^b)$. In the full-information case this difference is $\bar{I}_2^* - \bar{I}_1^* = (\phi_2 - \phi_1)(I_2^* - I_1^*)$. Since $I_1^b < I_1^* < I_2^*$ and $\phi_2 > \phi_1$ the conclusion follows. Q.E.D.

In the model of the earlier section the means for improving risk-sharing without violating incentive compatibility was to decrease x_1 . When the aggregate levels of investment and output (that is, ϕ) in the economy can be used as signals about individual firms' productivities, a second instrument for controlling incentives is made available: indexation of payments on ϕ . We show next that such indexation will be used in the spirit of relative

performance evaluation: if the manager claims that the firm is in a low productivity state and it turns out that the aggregate level is high, he will receive less income than if the aggregate level is low ("confirming" his claim):

Proposition 3.3: Assume the manager's utility function displays decreasing or constant absolute risk-aversion. Then $c_{11}^b > c_{21}^b$, i.e., $s_{11}^b < s_{21}^b$).

Proof: Let λ_2 be the multiplier for (3.3) (ii) and μ the multiplier for (3.3) (iii). It follows from first-order conditions for s_{11}^b and s_{21}^b that (dropping subscripts for notational simplicity):

$$(3.12) \quad -\lambda_2 \frac{p_{12}}{p_{11}} u'(c_{11} + \Delta(x_1)) + u'(c_{11}) = \mu > 0,$$

$$(3.13) \quad -\lambda_2 \frac{p_{22}}{p_{21}} u'(c_{21} + \Delta(x_1)) + u'(c_{21}) = \mu > 0.$$

By (3.1), (3.12) implies:

$$(3.14) \quad -\lambda_2 \frac{p_{22}}{p_{21}} u'(c_{11} + \Delta(x_1)) + u'(c_{11}) < \mu.$$

From the inequality in (3.13):

$$\lambda_2 \frac{p_{22}}{p_{21}} < \frac{u'(c_{21})}{u'(c_{21} + \Delta(x_1))}.$$

Thus, since $u'' < 0$,

$$(3.15) \quad -\lambda_2 \frac{p_{22}}{p_{21}} u''(c_{21} + \Delta(x_1)) + u''(c_{21})$$

$$\begin{aligned}
 &< -\frac{u'(c_{21})}{u'(c_{21} + \Delta(x_1))} u''(c_{21} + \Delta(x_1)) + u''(c_{11}) \\
 &= \left[\frac{-u''(c_{21} + \Delta(x_1))}{u'(c_{21} + \Delta(x_1))} - \frac{-u''(c_{21})}{u'(c_{21})} \right] u'(c_{21}) \leq 0.
 \end{aligned}$$

The last inequality follows by non-decreasing absolute risk-aversion and the fact that $\Delta(x_1) > 0$.

From (3.13)-(3.15) it follows that $c_{11} > c_{21}$. Q.E.D.

The intuition behind this result seems clear. If $s_{11} < s_{21}$, then $E[s_{j1} | \alpha = \alpha_1] < E[s_{ji} | \alpha = \alpha_2]$ since $\lambda_{11} > \lambda_{12}$. The expected payment to the owners is higher in the high aggregate state than in the low aggregate state when x_1 is chosen. Since they are the same if we do not index on ϕ , it appears that choosing $s_{11} < s_{21}$ is a new means for discouraging the high state manager from posing as a low state manager.

The matter is somewhat more complicated, however. Even if $\lambda_{11} = \lambda_{12}$ (i.e., $\phi_1 = \phi_2$) so that indexation would serve no purpose along the lines described above, it may pay to use randomized payment schemes (see Maskin [1981]). Whether s_{11} is greater or less than s_{21} in such schemes depends on the shape of the utility function. Thus, the combined effect is in general ambiguous. However, if absolute risk-aversion is non-decreasing, then Maskin [1981] has shown that randomized schemes will not be used (if $\lambda_{11} = \lambda_{12}$) and so the remaining effect is the one argued above.

Proposition 3.3 also shows that indexation has positive value in accordance with the general results of Holmstrom [1979] and Shavell [1979]. Indexation relieves (partly) the burden of policing incentives by lowering I_1 . We may therefore expect that I_1 is higher when we can index payments on ϕ than when we cannot. A partial confirmation of this is the following result:

Proposition 3.4: Under the assumption that the manager's utility function displays constant absolute risk-aversion, the investment level in the low productivity state ($\alpha = \alpha_1$) is higher when one can index on the aggregate state (ϕ) than when one cannot.

Proof: Let I_1^a be the low state investment level in an optimal solution to problem (2.10) when $\phi = \bar{\phi}$ and let I_1^b as before be the low state investment level in an optimal solution to (3.3). What is claimed in the proposition is that $I_1^b > I_1^a$.

Equation (2.35) determines I_1^a . A first-order condition for I_1^b can be derived as follows. Consider payment perturbations $ds_{11} = ds_{21} = ds_1$ and ds_2 and an investment perturbation dI_1 in (3.3) such that (3.3) (ii)-(iii) remain binding. Straightforward calculations (eliminating ds_2) yield the condition:

$$(3.16) \quad \frac{dU}{dI_1} = (1 - \bar{\phi}) \left\{ - \frac{ds_1}{dI_1} [E(u' | \alpha_1) - u'(c_2)] + \frac{\partial \pi_1(x_1)}{\partial I_1} E(u' | \alpha_1) \right\}.$$

Constant absolute risk-aversion implies that (3.3) (ii) can be written (since $s_{12} = s_{22} = s_2$) as:

$$(3.17) \quad \bar{\phi} u'(\pi_2(x_2) - s_2) = p_{12} u'(\pi_2(x_1) - s_{11}) + p_{22} u'(\pi_2(x_1) - s_{21}).$$

Differentiating (3.3) (ii) and using (3.17) we get:

$$(3.18) \quad ds_2 = ds_1 - \frac{\partial \pi_2(x_1)}{\partial I_2} dI_1.$$

Differentiating (3.3) (iii) we get ds_2 in terms of ds_1 which after substitution into (3.18) yields:

$$(3.19) \quad ds_1 = \phi \frac{\partial \pi_2(x_1)}{\partial I_1} dI_1.$$

Finally, substituting (3.19) into (3.16) gives the condition for I_1^b :

$$(3.20) \quad \frac{dU}{dI_1} = -(1 - \bar{\phi}) \left[\bar{\phi} \frac{\partial \pi_2(x_1)}{\partial I_1} \left[1 - \frac{u'(c_2)}{E(u' | \alpha_1)} \right] - \frac{\partial \pi_1(x_1)}{\partial I_1} \right] = 0.$$

Notice, that (3.20) reduces to (2.35) if $s_{11} = s_{21}$.

Now, if $I_1 = I_1^a$ in (3.20), we must have improved risk-sharing (since (3.3) yields a higher utility than (2.10)). Thus, $u'(c_2^b) > u'(c_2^a)$ and

$E(u'(c_1^b) | \alpha_1) < u'(c_1^a)$. Consequently, the inner bracketed term in (3.20) is smaller than the corresponding terms in (2.35) and so $dU/dI_1 > 0$ at $I_1 = I_1^a$.

Therefore, $I_1^b > I_1^a$.

Q.E.D.

3.2 Manager Informed About ϕ

We will now briefly discuss the case where the manager makes decisions after observing both α and ϕ . Since owners eventually observe ϕ as well, the situation is analytically equivalent to one where both parties observe the aggregate state before x has to be determined. However, the contract is still assumed to be made before any information is released (either about α or ϕ).

A contract consists now of four 3-tuples:

$$\delta = \{(I_{11}, y_{11}, s_{11}), (I_{12}, y_{12}, s_{12}), (I_{21}, y_{21}, s_{21}), (I_{22}, y_{22}, s_{22})\}$$

The first index is for the aggregate state, the second is for the firm state. If $\phi = \phi_1$, then the manager can choose between the first two 3-tuples; if $\phi = \phi_2$ he can choose between the latter two.

An optimal contract, denoted δ^C , solves the program:

$$(3.21) \quad \text{Max}_{\delta, k_1, k_2} (1-p)[\phi_1 u(\pi_2(x_{12}) - s_{12}) + (1-\phi_1)u(\pi_1(x_{11}) - s_{11})] + \\ p[\phi_2 u(\pi_2(x_{22}) - s_{22}) + (1-\phi_2)u(\pi_1(x_{21}) - s_{21})],$$

subject to

- (i) $\pi_1(x_{j1}) - s_{j1} \geq \pi_1(x_{j2}) - s_{j2}, \quad j=1,2$
- (ii) $\pi_2(x_{j2}) - s_{j2} \geq \pi_2(x_{j1}) - s_{j1}, \quad j=1,2,$
- (iii) $\phi_j s_{j2} + (1-\phi_j)s_{j1} = k_j, \quad j= 1,2,$
- (iv) $pk_2 + (1-p)k_1 = 0.$

We have written the zero-profit constraint as two constraints (iii) and (iv) to make it evident that for the optimal choices of k_1 and k_2 , (3.21) separates into two independent problems of the same structure as (2.10), except that in (3.21) expected payments to the owners are k_j rather than zero. For constant absolute risk-aversion this difference has no bearing on the choice of production decisions.

We will restrict attention to the constant absolute risk aversion case, since our purpose is to show by way of examples that results concerning aggregate fluctuations are generally ambiguous when the manager can observe ϕ before choosing input levels.

Since (3.21) is essentially two separate programs, it follows from Section 2 that optimal investment levels are $I_{12}^c = I_{22}^c = I_2^*$ and

$I_{21}^c < I_{11}^c < I_1^*$ (as $\phi_2 > \phi_1$; cf. Proposition 2.4). Aggregate investment in state $\phi = \phi_j$ will be $\bar{I}_j^c \equiv \phi_j I_2^* + (1 - \phi_j) I_j^c$. Both for the high and the low aggregate state $\bar{I}_j^c < \bar{I}_j^*$, i.e., aggregate levels in the second-best are below first-best levels. We can write the fluctuation as

$$(3.22) \quad \bar{I}_2^c - \bar{I}_1^c = (\phi_2 - \phi_1)(I_2^* - I_1^*) + (1 - \phi_1)(I_1^* - I_{11}^c) - (1 - \phi_2)(I_1 - I_{21}^c).$$

Expression (3.22) shows that $\bar{I}_2^c - \bar{I}_1^c < \bar{I}_2^* - \bar{I}_1^*$, if and only if the difference between the last two terms is negative. Either case is possible as can be seen by changing the technology. Consider an additively separable technology (2.28) for which $I_1^* > I_{11}^c > I_{21}^c$. Now change $h_1(I)$ so that it becomes flat beyond I_{11}^c (with the kink appropriately smoothed to preserve our assumption of differentiability). This will not change I_2^* , I_{11}^c , I_{21}^c but will bring I_1^* as close as we wish to I_{11}^c . Consequently, we can make the middle term in (3.22) arbitrarily small and hence $\bar{I}_2^c - \bar{I}_1^c < \bar{I}_2^* - \bar{I}_1^*$. In other words, aggregate levels may fluctuate less in the second-best than in the first-best if the manager can observe ϕ . This is not possible when the manager cannot observe ϕ , as we showed in Proposition 3.2.

The reverse may also obtain. In the example discussed above, change $h_1(I)$ so that instead $h_1(I)$ becomes arbitrarily steep below I_{11}^c . That way we can bring I_{21}^c as close as we wish to I_{11}^c . Since $1 - \phi_2 < 1 - \phi_1$, the difference between the last two terms in (3.22) can be made positive and hence $\bar{I}_2^c - \bar{I}_1^c > \bar{I}_2^* - \bar{I}_1^*$.

We conclude that the aggregate level of investment may fluctuate either more or less when the manager first observes ϕ than when he does not. The same is true for output. We can further show that the optimum in (3.21) can either be higher or lower than in (3.3). In other words, the value of having

the manager better informed about the future (in this case ϕ) can be either positive or negative. This observation conforms with earlier findings in the principal-agent literature that a more informed agent may or may not be worth more to the principal (see, e.g., Green and Stokey [1980]). In the present context the two opposing forces are easily identified. On the benefit side (3.21) has more instruments for controlling the manager, because x can be indexed by ϕ . On the cost side the constraints in (3.21) are tighter. All feasible points in (3.21) (with $I_{11} = I_{21}$ and $y_{11} = y_{21}$, of course) are also feasible in (3.3).

4. Conclusion

In this paper, we have shown that asymmetric information between owners and managers will generally distort input levels downwards from that level where marginal revenue product equals factor cost. In some settings this can amplify the magnitude of fluctuations in aggregate investment levels. As in recent models of equilibrium business cycles, we have shown that the magnitude of such fluctuations depends on the availability of aggregate information at the time allocation decisions must be made.

In several aspects, our analysis is similar to the wage-employment model in Grossman and Hart [1981]. They consider a situation in which workers cannot directly observe the marginal product of labor while firms can. exovided firms are risk averse and profits unobservable, they show that efficient risk sharing may necessitate a reduction in labor input relative to a Walrasian (spot) labor market, or a first-best allocation in which all information is public. However, the assumption that firms are risk averse appears unattractive. If uncertainties in production are firm specific, then a stock market ought to be able to spread the risk so that owners will act as if they are (almost) risk neutral with respect to these idiosyncratic risks.

On the other hand, if risk is common to all firms, then aggregate information will reveal publicly the state of labor productivity and hence not be a source of incentive problems. In either case, and presumably even if there is a mixture of both types of risk, incentive problems should disappear. The assumption that profits are unobservable also appears dubious, especially for publicly traded companies with active share markets.

Our explicit model of managerial control with an unobservable factor input makes the tradeoff between efficient risk bearing and productive efficiency more plausible. This formulation completely separates the function of screening on observable factor inputs from the problem of risk sharing with suppliers of factor inputs, which is the driving force of Grossman and Hart. If we interpret I , the observable factor, as labor, then the substantive conclusions of the model would be robust to risk aversion on behalf of labor. In this case, labor and shareowners could make a separate insurance agreement because they may have common information, and the incentive problem between the manager and owner would be unaffected.

Our analysis of the principal-agent problem when the principal's reward depends on both inputs and outputs is of some interest in contexts beyond those considered here. For example, our model could be interpreted as an optimal taxation model of the type first proposed by Mirrlees (1971). If labor hours were observable, as well as income, but not ability or effect, then social welfare could be improved by making taxes depend on both hours worked and income. For the case of two types of individuals, an optimal tax schedule would force low ability individuals to work less than they would otherwise desire in the absence of redistributive taxes.

Footnotes

¹See Azariadis [1981], Chari [1981], Green and Kahn [1982] and Grossman and Hart [1981] for partial equilibrium analyses. A general equilibrium model is found in Grossman, Hart and Maskin [1982]. Hart [1982] provides an excellent overview of this literature.

²In independent work Hart [1982] and Maskin [1982] discuss managerial models similar to ours.

³This channel of incongruity in investment preferences is different from that in Wilson [1968] and Ross [1973]. They rely on differences in risk-preferences induced by optimal risk-sharing. Since we assume there is no investment uncertainty (α is revealed to the manager) there would be no difference in preferences without the introduction of unobservable effort.

⁴This technology is the one considered in Hart [1982].

⁵This is the fact Hart [1982] exploits.

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