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INCENTIVES IN GOVERNMENT CONTRACTS

by

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I. Introduction

A government's economic activities can be broadly grouped into two major functions. First, the government directs and regulates the economic process on the basis of its legal power and competence. Second, it participates in the market place as an economic agent, producing and consuming a variety of commodities and services.

This paper will be concerned with the case of a government monopsony. Economic theory has provided a variety of models for the market outcome in monopoly situations. However, it is commonly accepted that the government should not be allowed to exert its dominant market and bargaining power to effect a most favorable outcome for itself, but that government contracts should satisfy certain fairness and equity conditions. As a consequence, most countries have developed a detailed legal framework for the structure of government contracts.

In the case, where the supply side is comprised of several competitors, a bidding scheme might be utilized to determine an "appropriate" price. But in numerous important cases there will be only one domestic supplier, leading to a situation of bilateral monopoly.

The implementation of an equitable outcome is straightforward, if the government has precise knowledge about all the relevant aspects of the project and its associated cost. The problem becomes significantly more complex in the case of uncertainty. Development contracts for national defense are a common example of this condition.

In contracting for the development of a jet fighter, all the desired technical characteristics (weight, speed, capacity etc.) can be specified in advance. However, total research and development expenditures up to the actual production stage may be seen as a random variable with large dispersion. For these high risk projects most NATO countries have adopted procedures of the following nature: At a preliminary stage they will attempt to get an idea of the involved costs. If the project is undertaken, the government will conduct an audit of the contracting firm after the project is completed. The firm will then be reimbursed for all costs incurred, provided that an upper cost bound, specified at the end of the preliminary stage, has not been violated. In addition, the firm receives a compensation payment which is a function of the actual cost level.

These practices have caused two major problems. First, in the attempt to optimally allocate the funds of a fixed budget, the government must conduct cost - benefit studies under conditions of uncertainty. Therefore the one would like to rely on the firm's "best" cost estimate, since in general the firm will have superior knowledge about the necessary inputs for the successful completion of the project. However, the firm might be induced to distort its private information in order to favorably impact the decision of whether or not the project is undertaken at all.

Second, an audit of the firm will enable the government to verify what inputs have actually been used, but in general it will not permit it to judge how efficiently these resources were utilized. Any payment schedule that attempts to positively correlate the compensation payment with the actual cost level of the project (an idea that reflects a fair rate of return condition) will induce a moral hazard problem, since the firm would punish itself for efficient practice.

The objective of this paper is to give a first characterization of the class of contracts (compensation functions) that simultaneously address these two problems.

In section II the general model is introduced; results are stated and shown in Section III. Finally, in Section IV, our approach will be discussed in light of some related work.

II. The Model

The realization of a complex project can be decomposed into a number of planning and production stages. We will distinguish between a negotiation and an implementation phase.

In the negotiation phase the government specifies all the relevant characteristics of the project and asks the firm for its "best" estimate of the minimal cost for the realization of the project. If the estimate is below a certain critical value, the project is undertaken and the implementation phase starts. Finally, at the end of the implementation phase, the firm will be audited so that the incurred costs can be verified. The compensation payment then is a function of the cost estimate and the actual cost level.

Typically, the firm itself will be uncertain about the actual cost level during the negotiation phase. We will model this uncertainty by a probability measure that the firm holds once it is informed about the technical aspects of the project. Let [a, b] denote a real interval such that both parties agree that total cost will fall into this interval with certainty. The firm's beliefs are described by a cumulative distribution function of the form:

$$F: \ [a,b] \to [0,1], \ F(a) = 0, \ F(b) = 1, \ F \text{ monotone increasing}$$

Therefore $0 \leq r \leq F(x_2) - F(x_1) \leq 1$ means that minimal cost will be a number between x_1 and x_2 with probability r. To design a contract that elicits the best cost estimate truthfully we need to specify what the firm perceives the critical value to be.

One could imagine that the government, in the attempt to optimally allocate its budget, collects cost estimates for all projects under consideration and then solves a knapsack problem in order to determine what projects to undertake. Therefore the firm faces a critical value, denoted by A which represents the highest cost estimate that it could submit and still be awarded the project. The critical value A itself is a random variable in the negotiation phase.

Define $\tilde{G}(E) = P[A \leq E]$: probability that the announced cost estimate E exceeds the critical value.

For convenience we can set: $\tilde{G}(a) = 0$, $0 \leq \tilde{G}(E) < 1$ for $E \varepsilon [a,b)$, $\tilde{G}$ monotone increasing

We will make the simplifying assumption that $\tilde{G}$ is of common knowledge in the negotiation phase.

The firm then will be asked to report expected minimal cost, i.e. the number

$$E = \int_{[a,b]} x \, dF(x)$$

when F represents the firm's distribution function. If x is the actual (ex post verified) cost, the payment to the firm is given by

$$P = x + H(E, x)$$

Where

H: [a,b] x [a,b] $\to \mathbb{R}$ denotes the compensation function.

Figure I

Project Specification

Cost estimate: E

$E > A$ | $E \leq A$

Stop | Implementation

Actual Cost: x

Compensation: H(E,x)

We will restrict attention to compensation functions that are continuous in the x variable. Denote by $U: \mathbb{R} \to \mathbb{R}$ the firm's von Neuman - Morgenstern utility function such that $U' > 0$ and $U'' \leq 0$.

If the firm behaves "properly", its expected utility under the contract $H(\cdot, \cdot)$ is given by:

$$\Gamma_F(E) \equiv \int_{[a,b]} U\left(H(E,x) + w\right) G(E)\, dF(x) + \left(1 - G(E)\right) U(w)$$

where w denotes initial wealth and $G(E) \equiv 1 - \tilde{G}(E)$ Without loss of generality we can rescale U such that $U(w) = 0$.

The principal goal of this paper is to characterize the class of all compensation functions that satisfy the following two conditions:

(a) It is the firm's best interest to report the minimal cost estimate E truthfully.

(b) Independently of the announced value E the firm seeks to minimize cost in the implementation phase.

Requirement b) is assumed to be satisfied, if for all values of $E, H(E, \ )$ is a monotone decreasing function, i.e.

(1) $$H(E, x_1) \geq H(E, x_2) \text{ for all } E \in [a,b], b \geq x_2 \geq x_1 \geq a.$$

Define $\Lambda(E) = \{F | F$ is a probability distribution function on $[a,b]$ such that $\int_{[a,b]} x \, dF(x) = E\}$

Postulate a) can be formalized through:

(2) $$\Gamma_F(E) \geq \Gamma_F(\tilde{E}) \text{ for all } \tilde{E} \in [a,b], \ F \in \Lambda(E)$$

A complication arises from the fact that the firm can always choose to produce in a cost-efficient way. In other words the firm could "build in" cost facotors that will shift its cost distribution function to the right, thereby giving less probability weight to lower values. Such behavior should never be in the contracting firm's interest. Postulate a) then takes the form:

(3) $\Gamma_F(E) \geq \Gamma_{\bar{F}}(\tilde{E})$ for all $\tilde{E} \in [a,b]$ and all $\bar{F}$ with $F(x) \geq \bar{F}(x)$ for all $x \in [a,b]$

In this formulation, building in cost factors means that the firm can always create a new "true" distribution function that is dominated by the original one in the sense of first order stochastic dominance.

III. Results

We have argued that postulates (a) and (b) are met, if H(,) is chosen such that conditions (1) and (3) hold. The following result yields a useful reduction:

Proposition 1: If a compensation function satisfies (1) and (2) then it also satisfies (3).

Proof: Suppose the contrapositive for some $\tilde{E} \in [a,b]$ and $\bar{F} \leq F$ so that

$$\Gamma_{\bar{F}}(\tilde{E}) - \Gamma_F(E) = \varepsilon > 0$$

Then there exist continuous distribution functions

F^* and $\bar{F}^*$ such that $\bar{F}^* \geq F^*$,

and $\Gamma_{\bar{F}^*}(\tilde{E}) - \Gamma_{F^*}(E) > \frac{\varepsilon}{2}$

Condition (2) implies $\Gamma_{F*}(E) \geq \Gamma_{F*}(\tilde{E})$. Therefore $\Gamma_{\bar{F}*}(\tilde{E}) - \Gamma_{F*}(\tilde{E}) > \frac{\varepsilon}{2}$.
Integrating by parts we obtain:

$$\Gamma_{\bar{F}*}(\tilde{E}) - \Gamma_{F*}(\tilde{E}) = G(\tilde{E}) \left[U\left(H(\tilde{E},x) + \omega\right) (\bar{F}* - F*)(x) \, I_a^b \right.$$

$$\left. - \int_{[a,b]} (\bar{F}* - F*)(x) \quad dV(x) \right]$$

$$\text{where} \qquad V(x) \equiv U(H(\tilde{E},x) + \omega).$$

The first term in the brackett is zero while the value of the integral is nonpositive since $\bar{F}* \geq F*$ and $V(x)$ is a monotone decreasing function by (1). Hereby we have established a contradiction.

A compensation function is said to be proper, if it satisfies conditions (1) and (2), i.e. postulates a) and b).

The following proposition shows how the class of proper compensation functions varies with the firm's risk preferences.

<u>Proposition 2</u>: H(,) is a proper compensation function for a risk neutral firm if and only if

$\bar{H}:[a,b] \times [a,b] \to \mathbb{R}$ defined by

$\bar{H}(E, x) \equiv U^{-1}\left(H(E, x)\right) - w$ is a proper compensation function for a firm with risk preferences described by the utility function U.

Proof: $\int_{[a,b]} G(E)U\left(\bar{H}(E,x) + w\right) dF(x) \geq \int_{[a,b]} G(\tilde{E})U\left(\bar{H}(\tilde{E},x) + w\right) dF(x)$

$$\text{iff} \quad \int_{[a,b]} G(E)H(E,x)\, dF(x) \geq \int_{[a,b]} G(\tilde{E})H(\tilde{E},x)\, dF(x)$$

Furtheremore $\bar{H}(E,x)$ is monotone decreasing in x for all $E \in [a,b]$ if and only if $H(E,x)$ is.

Our first concern is to give a complete characterization of the class of proper contracts. Intuitively, it is apparent that for any two values $E, \tilde{E} \in [a,b]$ the corresponding functions $H(E,\cdot)$ and $H(\tilde{E},\cdot)$ have to be carefully "balanced" so as to ensure the incentive compatibility of the contract. The following theorem yields a simple pointwise condition that is necessary and sufficient for equation (2).

Theorem 1: $H(\cdot,\cdot)$ satisfies condition (2) if and only if for all

$x_1, x_2 \in [a,b]$ with $\beta x_1 + (1-\beta)x_2 = E$, $0 \leq \beta \leq 1$:

$$\beta\Delta(E,\tilde{E},x_1) + (1-\beta)\Delta(E,\tilde{E},x_2) \geq 0 \text{ for all } \tilde{E} \in [a,b] \tag{4}$$

$$\text{where } \Delta(E,\tilde{E},x) \equiv G(E)\, H(E,x) - G(\tilde{E})\, H(\tilde{E},x)$$

Proof: Necessity is easily shown by assuming that (4) didn't hold for some combination of $(x_1, x_2, E, \tilde{E})$.

Then we can construct the distribution function

$$F(x) = \begin{cases} 0 & \text{for } x < x_1 \\ \beta & \text{for } x_1 \leq x \leq x_2 \\ 1 & \text{for } x > x_2 \end{cases}$$

so that $\int x dF(x) = E$ and

$$\int_{[a,b]} \Delta(E,\tilde{E},x)\, dF(x) = \beta\Delta(E,\tilde{E},x_1) + (1-\beta)\, \Delta(E,\tilde{E},x_2) < 0$$

This is in contradiction to condition (2).

Let $F \varepsilon \Lambda(E)$ and $\varepsilon > 0$ be arbitrarily small. Then there exist numbers $(x_1 \ldots x_n)$, $(p_1 \ldots p_n)$ Such that

$$a \leqslant x_i \leqslant x_{i+1} \leqslant b, \qquad p_i \geqslant 0, \quad \sum_{i=1}^{n} p_i = 1, \quad \sum_{i=1}^{n} p_i\, x_i = E$$

and

$$\left| \sum_{i=1}^{n} \Delta(E,\tilde{E},x_i) p_i - \int_{[a,b]} \Delta(E,\tilde{E},x) dF(x) \right| < \varepsilon$$

Take

$$x_k < E < x_{k+1} \text{ and } 0 < \alpha_{ij} < 1 \quad \text{such that } \alpha_{ij}\, x_i + (1 - \alpha_{ij})\, x_j = E$$

$$\text{for} \quad i \in I \equiv \{1 \ldots k\},\ j \in J \equiv \{k+1, \ldots, n\}.$$

Then there exist $k \cdot (n-k)$ nonnegative numbers $\{\gamma_{ij}\}_{i\varepsilon I, j\varepsilon J}$ such that

$$p_i = \sum_{j\varepsilon J} \gamma_{ij} \alpha_{ij} \text{ and } p_j = \sum_{i\varepsilon I} \gamma_{ij}\, (1-\alpha_{ij})$$

By hypothesis: $\alpha_{ij}\Delta(E,\tilde{E},x_i) + (1 - \alpha_{ij})\, \Delta(E,\tilde{E},x_j) \geqslant 0$

Multiplying the above inequality with γ_{ij} and summing up over I and J we obtain:

$$\sum_{i=1}^{n} \Delta(E,\tilde{E},x_i)\ p_i \geq 0$$

Since ε can be chosen arbitrarity small the conclusion follows.

The above theorem immediately gives rise to the following.

Proposition 2: If H is a proper compensation function, then for any two points $E,\tilde{E} \in [a,b]$ with $E < \tilde{E}$:

$$\Delta(E, \tilde{E},x) \geq 0 \text{ for } x \leq E$$

$$\Delta(E, \tilde{E}, x) \leq 0 \text{ for } x \geq \tilde{E}$$

Proof: Suppose that $\Delta(E,\tilde{E},x_1) < 0$ for $x_1 < \tilde{E}$. Condition (4) in theorem two then implies that $\Delta(E,\tilde{E},x_2) > 0$ for $x_2 > \tilde{E}$.

This in turn implies that $\Delta(E,\tilde{E},z) < 0$ for $E < z < \tilde{E}$, since $\beta\Delta(E,\tilde{E},x_2) + (1-\beta)\Delta(E,\tilde{E},z) \leq 0$ when $\beta\ x_2 + (1-\beta)z = \tilde{E}$. However $\Delta(E,\tilde{E},x,) < 0$ and $\Delta(E,\tilde{E},z) < 0$ is in contradiction to condition (4) which estabilishes the claim.

Proposition two can be interpreted as follows: Let $E < \bar{E}$ and $G(E) = G(\bar{E})$. Compared to $\bar{E}$, the low cost estimate E induces a higher payment at a low cost value x. At a high cost value x the opposite is the case.

These results indicate that the class of proper contracts is quite broad and therefore a social planner will have significant choice possibilities. As mentioned in the introduction it is often times argued that a "fair" contract should positively correlate the compensation payment with the actual cost size. This is impossible under a proper contract, however one might ask whether at least the expected payoff can be made on increasing function of the expected cost.

To formalize this idea, let F and $\bar{F}$ be two distribution functions such that $F \varepsilon \Lambda(E)$ and $\bar{F} \varepsilon \Lambda(\bar{E})$. A contract is said to satisfy positive correlation (in an expected value sense), if

$$E < \bar{E} \text{ implies } \Gamma_F(E) < \Gamma_{\bar{F}}(\bar{E})$$

Proposition 3: There does not exist a proper contract that satisfies positive correlation.

Proof: Suppose that $\bar{F} \geqslant F$ and $\bar{F} > F$ on some subinterval so that $\bar{E} > E$. Positive correlation requires that $\Gamma_F(E) < \Gamma_{\bar{F}}(\bar{E})$. On the other hand the contract is proper and therefore satisfies (3) as demonstrated in proposition one. Hence $\Gamma_F(E) \geqslant \Gamma_{\bar{F}}(\bar{E})$ which shows the claim.

Finally, it may be instructive to look at a special class of proper compensation functions, namely those that are linear in the x - variable.

Denote by $v\colon [a,b] \to \mathbb{R}_+$ and $w\colon [a,b] \to \mathbb{R}_+$ two functions that satisfy the following inequalities:

For all $E, \tilde{E} \in (a,b)$, such that $E < \tilde{E}$,

$$(6) \quad E\left[G(E)w(E) - G(\tilde{E})w(\tilde{E})\right] \leq v(E)G(E) - v(\tilde{E})G(\tilde{E}) \leq \tilde{E}\left[G(E)w(E) - G(\tilde{E})w(\tilde{E})\right]$$

Then $H(E, x) = v(E) - w(E)x$ is a proper compensation function. To see this, we compute:

$$\int_{[a,b]} \Delta(E,\tilde{E},x)\, dF(x) = G(E)\left[v(E) - w(E)E\right] - G(\tilde{E})\left[v(\tilde{E}) - w(\tilde{E})E\right]$$

This expression is nonnegative for all $\tilde{E} \in [a,b]$ exactly when condition (6) holds.

An example of a pair of functions that satisfy condition (6) is:

$$v(E) = \begin{cases} \dfrac{b^2 - E^2}{2G(E)} & \text{for } a \leq E < b \\ 0 & \text{for } E = b \end{cases}$$

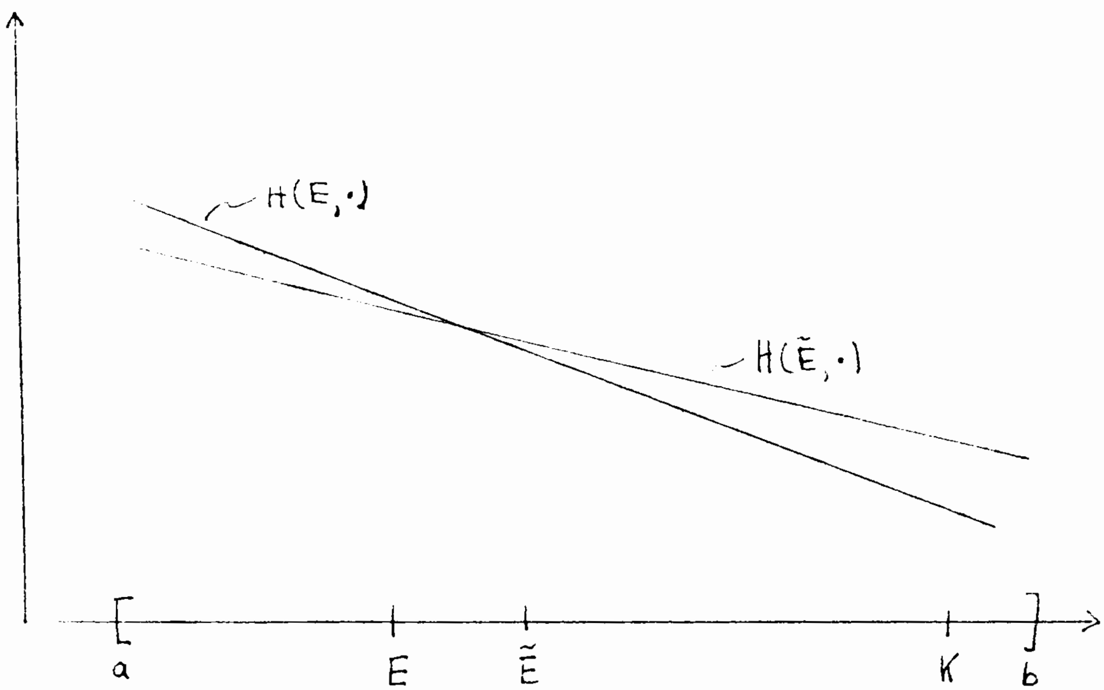
and

$$w(E) = \begin{cases} \dfrac{b - E}{G(E)} & \text{for } a \leq E < b \\ 0 & \text{for } E = b \end{cases}$$

For the special case

$$G(E) = \begin{cases} 1 & \text{for } E \in [a,K] \\ \dfrac{1}{b-k}(b-E) & \text{for } E \in (K,b] \quad a < K < b \end{cases}$$

figure 2 give an illustration of a linear compensation function.



Under such a contract the straight lines turn around clockwise for decreasing E intersecting in between the respective expected values; furthermore G(E)[v(E) - w(E)E] is monotone decreasing in E. These properties have to be hold in view of the established results.

IV Discussion

The problem under consideration falls in the broad range of the agency literature. One branch of this literature, which was originated by the new Soviet incentive model, investigated the design of payment schedules for situations similar to the one described in our model. Weitzmann (1976), Bonin (1976), Thomson (1979) and Holmstrom (1981) discuss compensation schemes that are closely related to the Soviet bonus system. In all of these models the agent (firm) reports some statistic of a privately known probability distribution. The payment to the agent then is a function of this report and the actual outcome which the principal (government) gets to observe.

A possible translation of the reformed Soviet bonus scheme to our model would be:

$$\bar{H}(E, X) = B + u(b - x) + \begin{cases} u \cdot v\ (E - x) & \text{for } E \leq x \\ u \cdot w\ (x - E) & \text{for } x \leq E \end{cases}$$

$$1 \geq u \geq 0,\ 1 > w \geq 0,\ B \geq 0$$

It is straightforward to verify that such a scheme does not satisfy postulate a). Weitzmann (1976) and Thomson (1979) have shown that by a proper choice of the parameters (u, v, w) the firm can be induced to truthfully report the q - quantile of the distribution, i.e. it is in the firm's best interest to report that number $\tilde{E}$ for which

$$q = \int_{[a,\tilde{E}]} dF(x)$$

in order to maximize $\int_{[a,b]} \bar{H}(E,x)\, dF(x)$ with respect to E

Thomson (1979) furthermore, established that the class of contracts that exhibit this property is very narrow.

In a public decision making context the government will be faced with a large number of projects, the cost outcomes of which are stoctastically almost independent. Since the actual costs are unknown in the decision making period the planner will have to rely on estimates. For this purpose eliciting the expected value appears to be more informative than the q - quantile.

In our model formulation the firm is indifferent between any two cost levels as long as they result in the same monetary payoff. This specification abstracts from possible perquisities that the firm may derive in the course of the project. A standard example of such side benefits is technological knowledge gained through research in connection with the project. In analogy to the existing principal - agent models, where effort aversion by the agent is explicitly taken into consideration, one therefore might introduce a benefit funciton that reflects the monetary vlaue of those perquisites.

A crucial specification of our model is that the government knows the firm's risk preferences and the probability distribution function G. It can be shown that there does not exist a compensation function that is proper for a "broad" class of utility and distribution functions. In other words a compensation function that is proper for given U and G would not necessarily exhibit this property for slightly perturbed U and G.

In finding optimal contracts a next step might be to incorporate a utility function for the government. Such a function maps ordered pairs of environments (distribution functions) and economic outcomes (costs and compensation payments) into utility index values. The task then is to find a contract that maximizes this utility function over the class of all proper compensation schemes.

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