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The Effect of Liquidity Constraints on Consumption:  
Cross-Sectional Analysis

by

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ABSTRACT

The life cycle-permanent income hypothesis is tested on cross-section data. The test procedure does not require that the alternative hypothesis of liquidity constraints be explicitly formulated. Measurement errors are allowed to be correlated with any of the variables in the equation to be estimated. The basic idea is to compare the Tobit and OLS estimates of a reduced-form equation for consumption and carry out a Hausman-type specification test. The test statistic overwhelmingly rejects the life cycle-permanent income hypothesis. The estimated effect of liquidity constraints is to reduce aggregate consumption about 10% below the desired level.

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## 1. Introduction

The basic postulate of the life cycle-permanent income hypothesis is that households behave as if they maximize a dynamic utility function subject only to the lifetime budget constraint without being constrained by imperfect capital markets. This postulate, if true, casts serious doubts on the effectiveness of macroeconomic stabilization policies such as temporary tax cuts. If, on the other hand, households are subject to borrowing constraints (or, to use James Tobin's terminology, liquidity constrained), short-run stabilization policies will have some influence on aggregate demand.<sup>1</sup>

Because of the forward-looking nature of the life cycle-permanent income hypothesis, convincing empirical testing of the postulate is impossible unless the hypothesis is coupled with a sensible assumption about expectations formation. Recently, Hall(1978), Sargent(1978), Flavin(1981), and Hayashi(forthcoming) have tested the life cycle-permanent income hypothesis on U.S. aggregate time-series data under the assumption that expectations are rational. Their test results are mixed, mainly because of the low power of time-series tests.

Subsequently, Hall and Mishkin(forthcoming) turned to panel data to find that food consumption is more sensitive to current disposable income than is predicted by the hypothesis. This work is followed by Bernanke(1981) who examined expenditure on automobiles using a different data set. He found no evidence against the life cycle-permanent income-rational expectations hypothesis.

The basic testing strategy common to the above-mentioned work is to look at the relationship between changes in consumption and current disposable income. The life cycle-permanent income-rational expectations hypothesis predicts no correlation between the two; a statistically significant correlation suggests that households are liquidity constrained. It would be highly desirable to extend this analysis to total consumption (as opposed to food consumption or durable goods expenditure), but unfortunately no panel data exist in this country for total consumption for more than one period. Cross-section data on total consumption do exist in this country, but one needs a different line of approach to test the hypothesis on such data.

A natural approach would be to derive two consumption functions -- one from the life cycle-permanent income hypothesis (i.e., the household's intertemporal optimization without borrowing constraints) and the other from the alternative hypothesis of liquidity constraints (i.e., intertemporal optimization with borrowing constraints) -- and see which consumption function fits the data better. There are at least two problems with this approach. First, we have the familiar problem that we, as econometricians, cannot observe the household's expectations about future income, so that any variable that helps predict future income can show up in the consumption function, which makes it very difficult to distinguish one consumption function from the other. Second, neither the life cycle-permanent income hypothesis nor the

alternative hypothesis of liquidity constraints delivers an explicit formulation for the level of consumption. Even under the assumption that the dynamic utility function is time-separable with constant degree of relative risk aversion, no closed-form solution for optimal consumption rule has been derived when future labor income is stochastic. Moreover, the life cycle-permanent income hypothesis is not very specific about how the family structure should be incorporated in the consumption function. The problem becomes even less tractable if the additional constraint of imperfect capital markets is imposed. For example, work by Levhari, Mirman, and Zilcha(1980) shows that the optimal consumption rule under uncertainty with borrowing constraints is quite complicated.

This paper is an attempt to test the life cycle-permanent income hypothesis on a single-time cross-section data set compiled by the Board of Governors of the Federal Reserve System in the early 1960s. The basic idea is to select by some a priori criterion a subset of households in the sample who are not likely to be liquidity constrained. Consumption by such households must largely be the result of intertemporal optimization without borrowing constraints. A very general reduced-form equation for consumption is estimated for such households by the Tobit procedure to account for the selectivity bias. The same equation is estimated by OLS (ordinary least squares) on the entire sample. If the two estimates of the same reduced-form equation for consumption are different, one would conclude that some of the households in the sample are liquidity constrained. A

statistical test of this can be carried out using a Hausman(1978) type specification test. The test procedure is valid even if measurement errors are correlated with the variables in the reduced-form equation. Since the Tobit estimate of the reduced-form equation is consistent even if some of the households in the population are subject to borrowing constraints, we can use it to predict desired consumption, namely the level of consumption dictated by the life cycle-permanent income hypothesis.

The plan of the paper is as follows. Section 2 discusses some theoretical issues concerning the formulation of the life cycle consumption function, and presents a reduced-form equation for consumption which simply is a regression of consumption on the variables available in our cress-section data. Section 3 explains how the Tobit procedure can be applied to consistently estimate the reduced-form equation in the presence of liquidity constrained households in the sample. Section 4 is a brief description of the data. In section 5, parameter estimates by OLS and by Tobit are presented and the Hausman test is carried out. We then calculate desired consumption predicted by the Tobit estimate of the reduced-form equation and compare it to actual consumption. Also in section 5, some diagnostic tests of the normality and heteroskedasticity assumptions which are used to justify the Tobit procedure are also undertaken. Section 6 contains concluding remarks.

## 2. Formulation of the "Life Cycle" Consumption Function

In this paper we do not attempt to construct "permanent income" or "lifetime resources" from the available data when we formulate the optimal consumption rule for the household's intertemporal optimization without borrowing constraints. Since this non-theoretical approach is somewhat unconventional, we devote this section to justify it. One of the most popular versions of the life cycle-permanent income hypothesis is to write the optimal consumption rule for a household as

$$(2.1) \quad c^* = \alpha (A + H),$$

where  $c^*$  is the household's optimal consumption,  $A$  is assets,  $\alpha$  is the propensity to consume out of total wealth (lifetime resources)  $A+H$ . This propensity would depend on the age of the household head. Human wealth  $H$  is defined as the present discounted value of current and expected future after-tax labor income. This consumption function can be derived from the standard deterministic intertemporal utility maximization problem (without borrowing constraints) with time-additive preference where the instantaneous utility function is of the form  $c^\gamma / \gamma$ , ( $\gamma < 1$ ). Permanent income is usually defined as the interest rate times lifetime resources  $A+H$ .

There are several theoretical and practical problems associated with the formulation (2.1), especially when we do not have longitudinal data on consumption.<sup>2</sup> First, if the family size affects instantaneous utility, the propensity to consume out of lifetime resources will depend on the future family size planned by the household. Such information is not usually

available.

Second, neither human wealth nor permanent income is observable. Since they depend on expectations about future income, any variables that help predict future income will show up with significant coefficients if neither permanent income nor human wealth is included in the consumption function. One way to get around this is to explicitly specify the stochastic process for after-tax labor income and find a closed-form representation of  $H$  as a distributed lag function of current and past labor income.<sup>3</sup> A practical problem with this is that we need longitudinal information on after-tax labor income extending for more than a few years back in order to get a realistic distributed lag representation of human wealth. A theoretical problem is the fact that income tax is a nonlinear function of the household's income. Since non-labor income is a part of the household's income, the stochastic process for after-tax labor income is affected by the amount of assets held by the household. It follows from this that human wealth will depend on assets in a nonlinear fashion as well as on current and past labor income.

Third, the derivation of the consumption function (2.1) from optimization assumes that the household has no subjective uncertainty about future after-tax labor income. This is clearly unrealistic, especially for the young. If the household faces a stochastic stream of after-tax labor income, we can no longer obtain a convenient closed-form solution like (2.1). In fact, it seems that no operational definition of permanent income or human

wealth is possible except for the tautological one that permanent income is something that is proportional to the optimal consumption. Another source of complication arises when risky assets whose rates of return are stochastic are present. It is true that, as Hakansson(1970) and Merton(1971) have shown, one can still obtain a closed-form solution for the optimal consumption rule like (2.1) if the stochastic rates of return are independently distributed over time. This, however, does not carry over to the case where future after-tax labor income is uncertain or stochastic.

The foregoing argument seems to suggest that any attempt to explicitly formulate the optimal consumption rule as a function of the variables that are typically available in cross-section data is bound to be misspecified. For this reason we choose to take a non-theoretical approach which can be briefly stated as follows. Let  $x$  be a vector of the variables that are available in our cross-section data and let  $c^*$  be the optimal consumption which is the solution to the household's (possibly stochastic) intertemporal optimization problem without borrowing constraints. Suppose we have a random sample of  $(c^*, x)$  from a common distribution and write the least squares projection of  $c^*$  on  $x$  as  $x'a$ . Thus  $c^*$  can be written as

$$(2.2) \quad c^* = x'a + e,$$

where  $e$  is uncorrelated with any elements of  $x$ . This error term  $e$  summarizes the household-specific component of the optimal consumption  $c^*$ . For example, if the household is more risk averse than the average household with the same value of  $x$ , the



error term will tend to be negative. We can think of (2.2) as a reduced-form representation of the optimal consumption for the household's intertemporal optimization without borrowing constraints. Our approach is similar in spirit to Sim's(1980) vector autoregressive modelling on time-series data. One advantage of our non-theoretical approach is that we do not have to commit ourselves to any particular version of the life cycle-permanent income hypothesis.

### 3. Methodology

In this paper we make a clear distinction between desired consumption  $c^*$  and actual consumption  $c$ . Desired consumption comes from the household's intertemporal optimization (or the life cycle-permanent income hypothesis) where the lifetime budget constraint is the only relevant constraint. At the end of the previous section we have presented the "reduced-form" equation for consumption (2.2) which relates the vector of observable variables  $x$  to desired consumption  $c^*$ . Actual consumption, however, may not be the same as desired consumption, because the household may not be able to borrow as much as it wants to finance current consumption. If borrowing constraints are binding, actual consumption will be lower than desired consumption. Otherwise, desired consumption will be equal to actual consumption. We will say that households are liquidity constrained or subject to borrowing constraints if their actual consumption is less than desired consumption. Households who are not liquidity constrained will be called the life cycle households. In spite of recent attempts by several authors (see e.g., Livari, Mirman and Zilcha[1980]), deriving an operational, closed-form optimal consumption rule under uncertainty with borrowing constraints remains an elusive subject. No attempt is made in this paper to formulate or estimate an optimal consumption rule for the households whose intertemporal optimization is constrained by borrowing constraints. Our basic strategy here is to try to estimate the reduced-form equation (2.2) and compare the value of consumption predicted by it with

actual consumption.

In this paper actual consumption is calculated as disposable income minus saving (i.e., net changes in assets). Since disposable income (and possibly saving) are measured with error, measured consumption CON differs from actual consumption by measurement error  $u$ :

$$(3.1) \quad \text{CON} = c + u .$$

Since the measurement error  $u$  consists of measurement error for disposable income and (possibly) for saving, it may be correlated with the vector  $x$  of observable variables. The least squares projection of  $u$  on  $x$  (which will include labor income and assets as its elements) is written as:

$$(3.2) \quad u = x'd + v ,$$

where  $v$  is, by construction, uncorrelated with any element of  $x$ .

If the life cycle hypothesis is true, we have

$$(3.3) \quad c = c^* .$$

Combining (2.2),(3.1)-(3.3) gives

$$(3.4) \quad \text{CON} = x'b + (e + v),$$

where  $b = a + d$  and the error term  $e+v$  is uncorrelated with  $x$  by construction. This equation, too, will be called the reduced-form equation for consumption. No attempts will be made in this paper to identify  $a$  and  $d$  separately. It will turn out in section 5 that  $b$  is the parameter we should be interested in. If no households in the population are liquidity constrained, then (3.4) applies to all households in the sample. Provided that the error term is identically and independently distributed across households, an asymptotically efficient estimator of  $b$  is

obtained by ordinary least squares (OLS) and an asymptotically efficient estimator of  $\text{Var}(e+v)$  is the sum of squared residuals divided by the sample size. If, on the other hand, some of the households in the population are liquidity constrained, then the reduced-form equation (3.4) does not apply to such households with the same parameter values and the OLS estimate of  $b$  will be biased and inconsistent.

Is there any way to consistently estimate the reduced-form equation (3.4) even if some of the households in the population are liquidity constrained? Clearly, identification of  $b$  rests on whether or not one can observe consumption by the life cycle households. In the context of single-time cross-section, some a priori criterion has to be utilized to identify at least some of the households that are not liquidity constrained. One such criterion popular in the literature (Kowalewski and Smith[1979] and Bernanke[1981]) is the level of liquid assets or the ratio of it to consumption. The idea, of course, is that a household with ample liquid assets relative to consumption will have no difficulty executing the optimal consumption rule dictated by the life cycle-permanent income hypothesis. Another plausible criterion is the saving rate. It would be unlikely that the household's saving rate is positive and yet its desired consumption exceeds actual consumption. However, consumption is not the only item a household has to finance; debts of various kinds must be paid off on schedule. Following Tobin and Dolde(1971) and Kowalewski and Smith(1979), we will call payments on mortgages and noninstallment debts the contractural saving.

The crucial identifying assumption in this paper is that the household is not liquidity constrained if the ratio of measured consumption to disposable income minus contractual saving plus .5 times the amount of liquid assets is less than .85. In other words we assume

$$(3.5) \quad c^* + u = \text{CON} \quad \text{if} \quad \text{CON} < U = .85*(\text{YD} - \text{CT} + .5*\text{LIQ}),$$

where YD = measured disposable income, CT = contractual saving, and LIQ = liquid assets. (A more precise definition of these variables will be given in the next section.) We choose this particular threshold value U because it is undoubtedly lower than any reasonable estimate of the amount that the household can spend for current consumption; if consumption is less than that very conservative estimate U, we can conclude that the household is not subject to borrowing constraints.<sup>4</sup> Now, since c is never greater than c\*, condition (3.5) implies:

$$(3.6) \quad \text{CON} < U \quad \text{if and only if} \quad c^* + u < U.$$

Now define the following limited dependent variable:

$$(3.7) \quad y = \begin{cases} \text{CON} & \text{if } \text{CON} < U. \\ U & \text{otherwise.} \end{cases}$$

Then (3.6) and (3.7) imply that

$$(3.8) \quad y = \begin{cases} x'b + e + v & \text{if } x'b + e + v < U, \\ U & \text{otherwise,} \end{cases}$$

since  $c^* + u = x'b + e + v$  by (3.1) and (3.4).

The model (3.8) is the one considered by Tobin(1958) and Amemiya(1973), and the parameters of the model can be estimated by maximum likelihood procedure (Tobit) under the assumption that

(1) the expectation of  $e+v$  conditional on  $x$  and  $U$  is zero, (2) the distribution of  $e+v$  conditional on  $x$  and  $U$  is normal and homoskedastic. In our empirical analysis the vector  $x$  will consist not only of the variables available in our cross section data set (age, family size, assets, labor income, and  $U$ ) but also their squared terms. Thus we can expect that assumption (1) above is (at least approximately) satisfied. Since assumption (2) is something that cannot be justified on a priori basis, we will carry out some Lagrange multiplier tests of homoskedasticity and normality at the end of section 5. If there is any clear violation of assumption (1), the Lagrange multiplier tests will be able to detect it.

The intuitive idea for using Tobit runs like this: Since we are confident that the households with ample liquid assets or with high saving ratio are not liquidity constrained, we would like to use their consumption data to estimate the reduced-form equation for consumption. But since we suspect that at least some of those households who do not have ample liquid assets or whose saving ratio is low are liquidity constrained, we do not use their consumption data except for the fact that their consumption is high relative to their liquid assets or disposable income.

Thus two different estimators of the reduced-form equation for consumption can be obtained. The OLS estimator is efficient under the null hypothesis that all households in the population share the same reduced-form equation (3.4) with common parameter values. The Tobit estimator is consistent (and asymptotically

normal) even if some of the households are liquidity constrained. The test procedure that immediately comes to mind is Hausman's (1978) specification test which is to compare the efficient OLS estimates and the consistent but inefficient Tobit estimates. It should be noted that for testing purposes we can allow the possibility that measurement error has nonzero mean and/or is correlated with any elements of the vector of the right hand side variables  $x$ . It should also be noted that a perfect split of the sample into liquidity constrained households and non-liquidity constrained households by the subsample selection rule  $CON < U$  are not needed here. All that is necessary for the consistent estimation of the reduced-form equation (3.4) by Tobit is that the subsample selection rule  $CON < U$  does not pick up liquidity constrained households; there may well be life cycle households that do not satisfy  $CON < U$ . For example, young life cycle households whose desired consumption exceeds current disposable income would not satisfy  $CON < U$ .

It is true, however, that because of measurement errors there can be a non-zero probability that liquidity constrained households get in the subsample under the sample selection rule  $CON < U$ . To illustrate this, suppose, somewhat unrealistically, that the reduced-form equation for the liquidity constrained is

$$(3.9) \quad c = YD^* - CT,$$

where  $YD^*$  is the true value of disposable income. If measurement error  $u$  consists entirely of measurement error for current disposable income so that  $u = YD - YD^*$  where  $YD$  is measured disposable income, then (3.9) implies that measured consumption

CON is equal to measured disposable income and the probability of mis-selection is zero. If, on the other hand, measurement error for saving is nonzero so that  $u$  consists of measurement errors for disposable income and for saving, then (3.9) implies  $CON = YD - CT + s$ , where  $s$  is the measurement error for saving. If  $s$  is normally distributed, the probability that some liquidity constrained households satisfy  $CON < U$  is not zero, so that the Tobit procedure will end up estimating a mixture of (3.4) and (3.9).

This problem of mis-selection does not appear to be a serious one for the following reasons. First, since the unique feature of the data set we will use in the subsequent analysis is its exhaustive coverage of various kinds of assets, the variance of measurement error for saving is likely to be small relative to that for disposable income. Second, even though it may not literally be a consistent estimate of (3.4), the Tobit estimate will be a very close approximation. If the true distribution of the error term does not have long tails like a normal distribution, the probability of liquidity constrained households ending up in the subsample may well be zero, in view of the high value of saving ratio (15%) used for the threshold value  $U$ , and the normality assumption will still be a good approximation. Even if the error term does have long tails, the probability of mis-selection will be negligibly small. Third, it should be noted that the Tobit estimate is consistent and asymptotically normal under the null hypothesis that there are no liquidity constrained households in the population. Thus the Hausman



specification test is still valid. The Hausman test will be carried out in section 5 where estimation results are reported.

#### 4. The Data

The cross-section data for the calculations reported in this paper came from the 1963/64 Survey of Financial Characteristics of Consumers conducted by the Board of Governors of the Federal Reserve System. A complete description of the survey is in Projector and Weiss(1966). The survey collected detailed information for income, the value of various categories of assets as well as for socio-economic characteristics of the households for two years 1962 and 1963. The quality of data is believed to be very good relative to other available data sets.<sup>5</sup>

The variables used in the analysis are as follows.

YD63 = 1963 disposable income excluding capital gains, after estimated federal income and payroll taxes,<sup>6</sup>

W63 = after-tax labor income, defined as before-tax labor income multiplied by  $(1-TR)$  minus estimated social security contributions, where TR is the ratio of federal income tax to before-tax household income,

ASSET = total market value of financial and physical assets (including the actuarial value of life insurance, pensions, annuities, royalties, real estates, and automobiles), at the beginning of 1963,

SAVING = saving during 1963, defined as net changes in assets (including automobiles and houses) after the exclusion of capital gains,

CON = measured consumption during 1963, defined as  
 $YD63 - SAVING,$

LIQ = amount of net liquid assets, defined as demand deposits, plus saving accounts, plus bonds, plus common stocks, minus loans secured by stocks and bonds, minus installment and noninstallment debts,

HOUSE = market value of houses at the beginning of 1963,

CT = contractual saving during 1963 in installment and mortgage debts,

$U = .85*(YD63 - CT + .5*LIQ)$ , the threshold value for creating the limited dependent variable (see [3.7] in section 3),

AGE = age of the household head as of December 1962,

FSZ = family size.

The following households are excluded from the initial sample of 2164 households. (1) households with missing data for the relevant variables (360 cases), (2) the self-employed and farmers (433 cases), (3) households whose 1963 disposable income is less than \$1,000 (79 cases), (4) households whose assets are greater than or equal to one million dollars (39 cases), and (5) households with negative consumption (43 cases). This reduced the sample size to 1210 observations. The self-employed and farmers are eliminated as their income is least accurately reported and is likely to be understated. In the subsequent analysis, we will deflate the equation by disposable income to avoid heteroskedasticity. The reason for excluding low- and high-income households is to avoid extreme values when a heteroskedasticity correction is made.

It became apparent in a preliminary analysis that the consumption behavior by the retired is very much different from the rest of the households in the sample. This may be seen from Table 1 where the sample means of consumption and other variables are reported for four age brackets. The relationship between consumption and disposable income for the households whose heads are 65 or over looks very different. This is probably because the stochastic process for labor income changes after the age of retirement. For this reason households whose heads are 65 or over are also excluded from the sample. This reduced the sample size by 168 from 1210 to 1042. The sample mean, standard deviation, and skewness of the variables listed above for the sample of 1042 observations are reported in Table 2.

## 5. Results

In the subsequent analysis, the vector  $x$  in the reduced-form equation (3.4) or (3.8) consists of the following twenty-one variables: ASSET, ASSET\*(AGE-45), ASSET\*((AGE-45)\*\*2), ASSET\*FSZ, W63, W63\*(AGE-45), W63\*((AGE-45)\*\*2), W63\*FSZ, U, U\*(AGE-45), U\*((AGE-45)\*\*2), U\*FSZ, A\*\*2, (A\*\*2)\*(AGE-45), (A\*\*2)\*((AGE-45)\*\*2), (A\*\*2)\*FSZ, U\*\*2, (U\*\*2)\*(AGE-45), (U\*\*2)\*((AGE-45)\*\*2), (U\*\*2)\*FSZ and HOUSE. To avoid misspecification, no a priori (linear or nonlinear) constraints are imposed on this reduced-form equation for consumption. The discussion in section 2 implies that we have no believable restrictions to be imposed on the equation. To account for possible differences in the consumption behavior by low- and high-income households, squared terms in ASSETS and U are included in the equation. Neither the constant nor the square of W63 is included because in a preliminary regression analysis they did not pick up significant coefficients.<sup>7</sup> For the same reason labor income in 1962 was not included in the equation; the correlation coefficient between labor income in 1963 and labor income in 1962 was more than .95. The reason for including HOUSE is to treat homeowners and non-homeowners symmetrically; the calculated consumption CON does not include service flows from houses which will be represented by the HOUSE variable in the equation with a negative coefficient. We include U and U squared in order to make the assumption that the error term  $e+v$  is orthogonal to U plausible.

Not surprisingly, inspection of the residuals from a preliminary regression analysis revealed considerable

heteroskedasticity across households of different income sizes. Since the Tobit estimation to be carried out shortly will assume that the error term  $e+v$  is identically distributed across households, a heteroskedasticity correction is necessary. To this end, disposable income YD63 is used to deflate the equations (3.4) and (3.8). In other words the reduced-form equation we actually estimate is a projection of  $CON/YD63$  on  $x/YD63$ . Of course, there is no guarantee that this deflation by YD63 completely removes heteroskedasticity in the error term  $e+v$ . Later in this section we will carry out a Lagrange multiplier test for heteroskedasticity. The parameter estimates obtained from applying OLS to the deflated equation are reported in Table 3. It should be kept in mind that the coefficient  $b$  in (3.4) is the sum of  $a$  in (2.2) and  $d$  in (3.2). The fact that the squared terms have significant coefficients would imply that the proportionality assumption common in the usual formulation of the life cycle-permanent income hypothesis is unwarranted.

Of the whole sample of 1042 households, 445 households satisfied the criterion that  $CON < U$ . Table 5 displays the sample mean and standard deviation of the variables for the subsamples of 445 and 597 observations. Although the subsample selection rule  $CON < U$  does not necessarily favor high-income households since it is based on the ratio of  $CON$  to  $YD-CT+.5*LIQ$ , it ended up selecting relatively rich households into the subsample of 445 observations. As would be expected, the average age is considerably higher for the households with  $CON < U$ .

The model (3.8) (after the deflation by YD63) is estimated by maximum likelihood under the assumption that the error term is normal and homoskedastic, and results are reported in Table 4. As in the OLS estimates, the HOUSE coefficient picked up the wrong sign. One possible reason for this is the fact that LIQ and HOUSE are the two major component of ASSET. Risk averse households tend to hold a larger portion of ASSET in the form of liquid assets and would consume less than households with the same value of  $x$  who are less risk averse. The positive HOUSE coefficient can be interpreted as representing this negative effect of LIQ on consumption. The reason that the average of  $x'b$  over the subsample of 445 households is considerably higher than the average of measured consumption on the same subsample (as is reported in Table 5) is, of course, that the subsample selection rule (3.5) tends to select those high-saving households whose individual specific component  $e$  is negative or whose measured income overstates the true income.

The two sets of estimates -- OLS and Tobit -- appear to be different from each other. As Hausman(1978) has shown, the right distance between the two sets of estimates is given by the difference in the variance matrices for the two estimates, as the efficient estimator of  $b$ ,  $b_{OLS}$ , is asymptotically uncorrelated with the difference  $b_{TOBIT} - b_{OLS}$ , under the null hypothesis that equation (3.4) applies to all households in the population with the same parameter value. This fact can also be directly verified by looking at the Taylor expansion of the estimators around the true value of  $b$ . Table 6 presents the difference

$b_{\text{TOBIT}} - b_{\text{OLS}}$  along with asymptotic standard errors which are obtained by taking the square roots of  $(1/N \text{ times})$  the diagonal elements of  $V_{\text{TOBIT}} - V_{\text{OLS}}$  evaluated at  $b_{\text{TOBIT}}$ , where  $N$  is the sample size and  $V_{\text{TOBIT}}$  and  $V_{\text{OLS}}$  are the asymptotic variance matrices of  $b_{\text{TOBIT}}$  and  $b_{\text{OLS}}$ , respectively. It is quite clear that the two parameter estimates are very different from each other in a quantitative and statistical sense. As Hausman(1978) has shown, the Wald-type statistic:

$$(b_{\text{TOBIT}} - b_{\text{OLS}})' (V_{\text{TOBIT}} - V_{\text{OLS}})^{-1} (b_{\text{TOBIT}} - b_{\text{OLS}})$$

is asymptotically distributed as chi-squared with 21 degrees of freedom under the null hypothesis of no misspecification. In the present case the statistic is 624.2, which emphatically rejects the null hypothesis that all households in the population share the same reduced-form equation for consumption.<sup>8</sup>

A less formal but probably more interesting way to evaluate the importance of liquidity constraints is to compare the sample mean of predicted desired consumption  $x' b_{\text{TOBIT}}$  to the sample mean of measured consumption on the entire sample of 1042 observations. It can be easily shown from equations (2.2), (3.1)-(3.4) that the population mean of  $c^* - c$  can be consistently estimated by the sample mean of  $x b_{\text{TOBIT}} - \text{CON}$ , if the Tobit estimate is a consistent estimate of  $b$  and the sample mean of  $x$  converges in probability. This is why our interest has been centered around the consistent estimation of  $b$ . The weighted mean of  $x' b_{\text{TOBIT}}$  is .967 and the weighted mean of measured consumption is .876. The effect of liquidity constraints is to reduce consumption to about 9.4% below the desired level, on the



average. From the viewpoint of macroeconomic stabilization policies, a more relevant measure is the unweighted mean of consumption. The unweighted mean of measured consumption is \$7,079 which is about 11% below the unweighted mean of predicted desired consumption  $x_{b\_TOBIT}$  of \$7,935. Thus, the quantitative importance of liquidity constraints does not seem as large as the difference between the Tobit and the OLS estimates of the reduced-form equation for consumption might suggest.

Table 7 carries out a similar comparison by the age of the household head. As would be expected, the effect of borrowing constraints is most evident for young households. Not only the discrepancy between predicted desired consumption and measured consumption is largest for the young, but also their predicted desired consumption exceeds their disposable income. For only 15% (46 cases out of 298) of the households whose heads are under 35 measured consumption is greater than the predicted desired consumption  $x'_{b\_TOBIT}$ .

We conclude this section by carrying out some Lagrange multiplier tests for non-normality and heteroskedasticity. Following Lee(1981) we assume that the error term  $w = e+v$  (after the deflation by YD63) is a member of the general Pearson family of distributions whose density function can be written as

$$(4.1) \quad f(w) = \exp \left[ \int_0^w \frac{c_3 - z}{c_0 - c_3 z + c_4 z^2} dz \right] / \int_{-\infty}^{\infty} \exp \left[ \int_0^z \frac{c_3 - t}{c_0 - c_3 t + c_4 t^2} dt \right] dz .$$

The variance under this general Pearson distribution is  $c_0/(1-3c_4)$ . There are several different ways to incorporate heteroskedasticity into this distribution. We assume that the variance is a linear function of ASSET and YD63 so that  $c_0$  is written as

$$(4.2) \quad c_0 = \sigma^2 + c_1*ASSET + c_2*YD63.$$

The normality assumption is that  $c_1 = c_2 = 0$ , and the homoskedasticity assumption is that  $c_3 = c_4 = 0$ . Our null hypothesis, therefore, is that  $c_1 = c_2 = c_3 = c_4 = 0$ . The Lagrange multiplier test is based on the fact that the score vector under the null hypothesis has mean zero and its variance is the elements of the information matrix that correspond to the parameters constrained by the null hypothesis. Its attractive feature is that we do not have to compute the maximum likelihood estimates under the alternative hypothesis. The reader is referred to Engel(forthcoming) for an excellent exposition of the Lagrange multiplier principle. The Lagrange multiplier statistics are calculated for the following alternative hypotheses:

Heteroskedasticity ( $c_1 = 0, c_2 = 0; c_3 = c_4 = 0$ ): LM1 = 3.2,

Non-normality ( $c_1 = c_2 = 0; c_3 = 0, c_4 = 0$ ): LM2 = 12.9,

Heteroskedasticity and Non-normality ( $c_i = 0, i = 1,2,3,4$ ):

LM3 = 16.1.

To calculate the above statistics, consistent estimates of the relevant information matrix are necessary; we used the formula given by Lee(1981) to obtain such estimates. Under the null hypothesis, LM1 and LM2 are distributed asymptotically chi-

squared with two degrees of freedom and LM3 is distributed asymptotically chi-squared with four degrees of freedom. Thus the normality assumption can be rejected at a significance level of .5% while the homoskedasticity assumption can be easily accepted.

## 6. Conclusion

The basic message of this paper can be summarized as follows. The sample was divided into high- and low-saving households. The coefficients in the reduced-form equation for consumption (i.e., the regression of consumption on the variables available in our cross-section data) for the high-saving households appeared to be quite different from those for the rest, even after we removed the selectivity bias due to the fact that the sample selection was based on the dependent variable. When the reduced-form equation for the high-saving households was extrapolated to the low-saving households, it overpredicted their actual consumption. Our interpretation of this finding was that the low-saving households were unable to consume as much as they want due to borrowing constraints. This is admittedly not the only interpretation, but is the one that seems to be most natural.

One might want to comment on this by saying that the high- and low-saving households are simply two different types of consumers with respect to their preferences. Our response to this is two-fold. First, the error term in our reduced-form equation for consumption does include the individual differences in preferences that cannot be captured by the right hand side variables. The error term for the high-saving households tend to be negative. This is precisely the selectivity bias that can be removed by the Tobit procedure under the assumption that the error term is normal. As it turned out, the normality assumption was rejected, but it seems likely that basically the same

conclusion will hold under a more suitable distributional assumption.<sup>9</sup> Second, if it is in fact the case that two household groups differ with respect to their preferences, one would like to explain why they are different; in particular, one would have to explain why the saving rate is the relevant criterion in dividing households into two different types of consumers.

Can one say something about the effect of a temporary tax cut on aggregate consumption based on the results obtained in this paper? This paper has shown that the effect of liquidity constraints is to reduce average consumption about 10% below the desired level. What the paper could not show is what fraction of the sample is liquidity constrained. If every household in the population is liquidity constrained so that the 10% discrepancy is spread over the entire population of households, then the direct effect of the temporary tax cut will be to increase consumption on an almost dollar-for-dollar basis. (This, of course, abstracts from the macroeconomic interaction of aggregate consumption and other variables.) If, however, only a small fraction of the households are deeply liquidity constrained accounting for the 10% discrepancy and if the rest of the households are not liquidity constrained at all, then the effect of the temporary tax cut will be small.

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### Footnotes

1. See Tobin(1980) for his latest account of liquidity constraints and their implication to macroeconomic stabilization policies. In this paper we use the words "liquidity constraints" and "borrowing constraints" interchangeably. We will not use the word "quantity constraints", because it is usually used to describe the situation where labor supply is exogenously given to the household. This paper assumes that labor supply is given, i.e., the household is a "income taker". Although this is a standard assumption in the literature on consumption function, it would be preferable to treat both consumption and labor supply as choice variables. Unfortunately, our data have no information on labor supply or wage rate.
2. If longitudinal data on total consumption were available, we would operate on the Euler equation (the first order condition for intertemporal optimality), as Hansen and Singleton (forthcoming) did using aggregate time series data.
3. See Hansen and Sargent(forthcoming) for more details on this approach.
4. The reason that LIQ has a coefficient of .5 in (3.5) is that we wanted to guard against the possibility that some of the liquid assets reported in our data is not readily cashable. Since the distribution of liquid assets in our data is very skewed (see Table 2), our particular choice of the LIQ coefficient did not considerably affect the composition of households satisfying  $CON < U$ . The reason that  $YD-CT+.5*LIQ$  is further multiplied by .85 is to reduce the probability that measured consumption by liquidity constrained households satisfies  $CON < U$  due to measurement error for saving. This point is further discussed on pp.14-16 of the text.
5. I also looked at a University of Michigan Survey Research Center panel study entitled Consumer Durables and Installment Debts, 1967-1970, which has longitudinal data on saving and income. It turned out that calculated consumption (defined as income minus saving) was negative for more than two cases out of ten.
6. The data set contains no information about taxes. Federal income tax was calculated by following the instructions in a handbook named Your Federal Income Tax (1964 edition, U.S. Internal Revenue Service publication No.17). The tax deductability of mortgage payments was incorporated in the calculation. Other taxes were ignored. Property tax could be a substantial omission, but this will be picked up by the variable HOUSE in the reduced-form equation.



7. The number of variables that can be put on the right hand side of the equation was dictated by the computational feasibility of the Tobit procedure; when the number of the right hand side variables was greater than twenty-five, the Hessian matrix of the log of likelihood function ceased to be negative definite.

8. The Hausman statistic is 1129.5 when the hypothesis is that both the coefficients in the equation and the variance of the error term  $e+v$  are the same.

9. It is in principle possible to carry out the Tobit procedure with the general Pearson family of distributions. The expression for the Hessian matrix of the log of likelihood function became too complicated to be computationally feasible with a limited computing budget.

TABLE 1: Sample Statistics for the Four Age Groups

| variable | age < 35 | 35 ≤ age < 50 | 50 ≤ age < 65 | 65 < age |
|----------|----------|---------------|---------------|----------|
| CON      | \$5041   | \$7518        | \$8363        | \$5354   |
| YD63     | \$5889   | \$8767        | \$9878        | \$6069   |
| W63      | \$5798   | \$8384        | \$8768        | \$2506   |
| ASSET    | \$5903   | \$22733       | \$55363       | \$67626  |
| LIQ      | \$14     | \$6273        | \$27223       | \$45981  |
| HOUSE    | \$6025   | \$16545       | \$19623       | \$15117  |
| CT       | \$617    | \$933         | \$633         | \$167    |
| FSZ      | 3.67     | 4.30          | 2.86          | 2.14     |
| AGE      | 28.3     | 42.1          | 56.5          | 71.9     |
| #cases   | 298      | 412           | 332           | 168      |

TABLE 2: Sample Statistics

Sample Size = 1042.

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| variable name | mean      | standard deviation | skewness |
|---------------|-----------|--------------------|----------|
| CON           | \$7079.0  | 7642.6             | 4.846    |
| YD63          | \$8297.7  | 7210.7             | 3.718    |
| W63           | \$7766.4  | 6513.0             | 3.461    |
| ASSET         | \$28316.3 | 83500.8            | 6.771    |
| LIQ           | \$11157.9 | 53543.7            | 7.628    |
| HOUSE         | \$14517.9 | 32604.2            | 15.560   |
| U             | \$11160.2 | 27032.6            | 7.031    |
| CT            | \$747.0   | 1344.0             | 7.250    |
| FSZ           | 3.66      | 1.95               | 1.152    |
| AGE           | 42.7      | 11.7               | .015     |

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TABLE 3: OLS Estimates

|  | 1   | AGE-45  | (AGE-45)**2                                   | FSZ   |
|--|---|---|---|---|
| ASSET                                  | .0934<br>(.0196)                              | -.00141<br>(.000641)                          | .0 <sup>4</sup> 665<br>(.0 <sup>4</sup> 569)  | -.0246<br>(.00449)                            |
| W63                                    | .881<br>(.0637)                               | .00786<br>(.00283)                            | -.0 <sup>3</sup> 769<br>(.0 <sup>3</sup> 196) | -.0567<br>(.0147)                             |
| U                                      | -.303<br>(.0784)                              | -.0114<br>(.00365)                            | .00124<br>(.0 <sup>3</sup> 253)               | .126<br>(.0196)                               |
| ASSET**2                               | -.0 <sup>6</sup> 807<br>(.0 <sup>7</sup> 793) | -.0 <sup>7</sup> 105<br>(.0 <sup>8</sup> 258) | .0 <sup>8</sup> 200<br>(.0 <sup>9</sup> 244)  | .0 <sup>6</sup> 147<br>(.0 <sup>7</sup> 228)  |
| U**2                                   | .0 <sup>5</sup> 621<br>(.0 <sup>6</sup> 842)  | .0 <sup>8</sup> 675<br>(.0 <sup>7</sup> 759)  | -.0 <sup>7</sup> 210<br>(.0 <sup>8</sup> 373) | -.0 <sup>5</sup> 131<br>(.0 <sup>6</sup> 207) |
| HOUSE                                  | .0167<br>(.0108)                              |   |   |   |
| estimate of Var(e+v) = .150<br>(.0169) |   |   |   |   |

mean of dependent variable (CON/YD) = .876, sample size = 1042.

Note: Numbers in parentheses are standard errors. The point estimate of the coefficient of ASSET, for example, is .0934 which is the (1,1) element of the above matrix. The point estimate of the coefficient of (U\*\*2)\*FSZ is -.00000131.

TABLE 4: Tobit Estimates

|          | 1   | AGE-45  | (AGE-45)**2                                   | FSZ   |
|----------|---|---|---|---|
| ASSET    | .0101<br>(.0268)                              | .00180<br>(.000792)                           | -.0 <sup>4</sup> 499<br>(.0 <sup>4</sup> 703) | -.0186<br>(.00599)                            |
| W63      | .900<br>(.0743)                               | -.00611<br>(.00342)                           | -.0 <sup>3</sup> 188<br>(.0 <sup>3</sup> 229) | -.0257<br>(.0172)                             |
| U        | -.159<br>(.0952)                              | .00270<br>(.00428)                            | .0 <sup>3</sup> 500<br>(.0 <sup>3</sup> 294)  | .0915<br>(.0238)                              |
| ASSET**2 | -.0 <sup>6</sup> 742<br>(.0 <sup>6</sup> 164) | -.0 <sup>7</sup> 257<br>(.0 <sup>8</sup> 882) | .0 <sup>8</sup> 384<br>(.0 <sup>9</sup> 796)  | .0 <sup>6</sup> 203<br>(.0 <sup>7</sup> 390)  |
| U**2     | .0 <sup>5</sup> 705<br>(.0 <sup>5</sup> 141)  | .0 <sup>7</sup> 378<br>(.0 <sup>7</sup> 921)  | -.0 <sup>7</sup> 242<br>(.0 <sup>8</sup> 618) | -.0 <sup>5</sup> 176<br>(.0 <sup>6</sup> 330) |
| HOUSE    | .0399<br>(.0152)                              |   |   |   |

estimate of  $\text{Var}(e+v) = .121$   
(.0105)

Log of likelihood function = -443.04, sample size = 1042.

Note: Numbers in parentheses are standard errors.

TABLE 5: Sample Statistics of the Two Subsamples

| variable name | 445 observations<br>(CON < U) |                    | 597 observations<br>(CON $\geq$ U) |                    |
|---------------|-------------------------------|--------------------|------------------------------------|--------------------|
|               | mean                          | standard deviation | mean                               | standard deviation |
| CON           | \$7980.0                      | 8875.3             | \$6407.4                           | 6502.3             |
| YD63          | \$10847.2                     | 9354.1             | \$6397.2                           | 4142.3             |
| W63           | \$9860.8                      | 8340.2             | \$6205.2                           | 4068.9             |
| ASSET         | \$53034.4                     | 119475.3           | \$9891.6                           | 27351.8            |
| LIQ           | \$26122.1                     | 79318.7            | \$3.7                              | 5362.5             |
| HOUSE         | \$20929.2                     | 45458.4            | \$9737.4                           | 16239.1            |
| U             | \$19683.4                     | 39488.1            | \$4807.1                           | 4480.7             |
| CT            | \$751.3                       | 1656.9             | \$743.7                            | 1053.4             |
| FSZ           | 3.28                          | 1.65               | 3.95                               | 2.10               |
| AGE           | 46.2                          | 11.1               | 40.1                               | 11.5               |

Weighted and Unweighted mean of  $x' b_{\text{TOBIT}}$ :

|                          |       |        |
|--------------------------|-------|--------|
| for the 445 observations | .927, | \$9966 |
| for the 597 observations | .997, | \$6421 |
| for the whole sample     | .967, | \$7935 |

Weighted and Unweighted mean of measured consumption CON:

|                          |       |        |
|--------------------------|-------|--------|
| for the 445 observations | .724, | \$7980 |
| for the 597 observations | .989, | \$6407 |
| for the whole sample     | .876, | \$7079 |

TABLE 6: Difference between Tobit and OLS Estimates  
and Associated Standard Errors

|   | 1  | AGE-45  | (AGE-45)**2                                   | FSZ   |
|---|--|---|---|---|
| ASSET                                     | -.0832<br>(.0190)                            | .00321<br>(.000549)                           | -.0 <sup>3</sup> 116<br>(.0 <sup>4</sup> 488) | .00604<br>(.00448)                            |
| W63                                       | .0190<br>(.0448)                             | -.0140<br>(.00234)                            | .0 <sup>3</sup> 580<br>(.0 <sup>3</sup> 148)  | .0310<br>(.0110)                              |
| U   | .143<br>(.0627)                              | .0141<br>(.00281)                             | -.0 <sup>3</sup> 738<br>(.0 <sup>3</sup> 189) | -.0347<br>(.0163)                             |
| ASSET**2                                  | .0 <sup>7</sup> 647<br>(.0 <sup>6</sup> 148) | -.0 <sup>7</sup> 362<br>(.0 <sup>8</sup> 854) | .0 <sup>8</sup> 184<br>(.0 <sup>9</sup> 766)  | .0 <sup>7</sup> 560<br>(.0 <sup>7</sup> 333)  |
| U**2                                      | .0 <sup>6</sup> 847<br>(.0 <sup>5</sup> 120) | .0 <sup>7</sup> 311<br>(.0 <sup>7</sup> 624)  | -.0 <sup>8</sup> 321<br>(.0 <sup>8</sup> 520) | -.0 <sup>6</sup> 450<br>(.0 <sup>6</sup> 274) |
| HOUSE                                     | .0232<br>(.00778)                            |   |   |   |
| estimate of Var(e+v) = -.0283<br>(.00667) |  |   |   |   |

Hausman statistic for the hypothesis that the coefficients in the reduced-form equation are the same = 624.2.

Hausman statistic for the hypothesis that both the coefficients and the variance of the error term are the same = 1129.5

TABLE 7: Comparison of the Averages for Measured  
and Predicted Desired Consumptions for the Three Age Groups

|   | age < 35 | 35 ≤ age < 50 | 50 ≤ age < 65 |
|---|----------|---------------|---------------|
| measured consumption<br>(weighted)            | .876     | .851          | .907          |
| predicted desired<br>consumption (weighted)   | 1.083    | .924          | .916          |
| measured consumption<br>(unweighted)          | \$5041   | \$7518        | \$8363        |
| predicted desired<br>consumption (unweighted) | \$6440   | \$8195        | \$8956        |
| disposable income YD63                        | \$5889   | \$8767        | \$9878        |
| #cases where CON < U                          | 78       | 172           | 195           |
| #cases where CON < x'b                        | 46       | 109           | 83            |
| #cases  | 298      | 412           | 332           |

Note: Predicted desired consumption x'b is evaluated at the Tobit estimate.