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Job Matching, Risk Aversion, and Tenure*

by

Milton Harris and Yoram Weiss**

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J. L. Kellogg Graduate School of Management
Northwestern University
Evanston, Illinois 60201

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** Harris is at the Kellogg School of Management, Northwestern University. Weiss is at the Department of Economics, Tel-Aviv University. We are indebted to Truman Bewley, Bengt Holmstrom, Paul Milgrom, Rick Miller, Roger Myerson, and participants of the Uncertainty Workshop, Graduate School of Business, University of Chicago for very helpful input. Support from the National Science Foundation is gratefully acknowledged.

Job Matching, Risk Aversion, and Tenure

ABSTRACT

Two models are presented to explain the "tenure phenomenon." We use the term tenure phenomenon to describe the following five characteristics of an employment relation: (a) if a worker meets certain professional standards (the "tenure standard") he is thereafter observed to remain in his current career, (b) workers who have not met the tenure standard by a certain professional age (the "up-or-out age") seek alternative employment, (c) some workers drop out before the up-or-out age, (d) salaries, even among those with tenure, vary according to one's record, and (e) salaries are not reduced prior to termination, and, in some cases, termination occurs even though the worker would prefer to remain given his current record. To explain these characteristics, we consider two job matching models in which both employees and employers learn about a worker's productivity in the given task by observing his output over time. In the first model, both employers and workers are risk neutral. The first four characteristics of the employment relation described above are shown to hold in the model's solution. Further results on the tenure standard and up-or-out age are also derived. In the second model, workers are risk averse (but employers are still risk neutral). Characteristics (a), (b), (c), and (e) are shown to hold in this case while characteristic (d) does not hold.

JOB MATCHING, RISK AVERSION, AND TENURE

1. Introduction

This paper is an attempt to explain a phenomenon often observed, especially in the academic labor market, which we call the "tenure phenomenon" and associated tenure contracts. By the term tenure phenomenon, we simply mean a gradual sorting process (partially) described by the following four properties:

1. Tenure. Any scholar, when he has met certain professional standards (the "tenure standard"), is thereafter observed to stay at his current institution (or one of equivalent quality) until retirement, even if his output subsequently declines. This is known as achieving tenure.

2. Up-or-out. Those scholars who have not met the tenure standard by a certain professional age (the "up-or-out age") seek employment at another institution, usually of lower quality or reputation or of a non-academic nature. The up-or-out age is an upper bound on the waiting time for tenure.

3. Dropping Out. Some scholars (those with poor records) move to lower quality or non-academic institutions before the up-or-out age. This may be called dropping out early.

4. Getting Squeezed. Salaries, even among those scholars with tenure, vary according to one's research record or reputation. In particular, when one's research record is poor, his real wage

declines. This is known as getting squeezed.[1]

The tenure phenomenon is usually found in conjunction with a more-or-less explicit contract. This contract, in addition to specifying the up-or-out age and some description of the requirements for tenure, generally provides that the employer is allowed to terminate the relationship unilaterally before tenure is conferred. The adverb unilaterally is in some sense inessential to the contract because the worker can (almost) always be forced to leave by reducing his salary. Nevertheless, it seems to be the case that this alternative is not used, and workers expect not to be "squeezed" out by real salary reductions before the up-or-out age. That is, point 4 seems to apply mainly to scholars with tenure and tenure contracts exhibit the following feature:

5. The Boot. The employment of some scholars with poor research records is terminated at the employer's initiative while salaries are not reduced prior to this event. This is sometimes known as getting the boot.

The purpose of this paper is to provide an explanation of the above turnover pattern as an equilibrium response to imperfect information about worker productivity. Imperfect information results in a process of experimentation on the job which ends somewhat before the end of the worker's career. In applying this idea to the academic labor market, we incorporate

[1]It might be argued that this last point does not apply to state universities because in these institutions, salary is determined by rank. Thus individual scholars cannot receive salaries higher or lower than others of the same rank, regardless of their productivity. Yet most state university systems provide a multitude of grades within each rank, thus allowing considerable room for salary differentiation.

the following aspects (which may also be shared by other industries): large variability in individual performance, public information about each worker's record, and labor mobility which while prevalent is not costless.

The analysis proceeds in two stages; we first consider (in Section 2) the case in which workers are risk neutral and have access to a perfect capital market. In this pure job-matching framework, which is similar to Mortensen (1978), Jovanovic (1979), and Viscusi (1979), we show that the tenure characteristics (1) to (4) hold, but not (5).[2] In the second stage of the analysis (Section 3), we introduce risk aversion and assume that workers have no access to the capital market. In this framework, which is similar to Freeman (1977), Weiss (1981), and Harris and Holmstrom (1982), we show that characteristics (3) and (5) hold, that characteristics similar to (1) and (2) hold, but that (4) does not hold.

In both the risk neutral and risk averse models we retain the same informational structure. Scholars are heterogenous with respect to their ability to produce research. Everyone (including the worker himself) is assumed to be ignorant with regard to any given worker's productivity. Both the scholar and his employer learn about the scholar's productivity by observing his research output over time, and any information acquired is public. In this paper we assume that workers can be of two types with one type having higher expected output per period than the other. In each period a worker either produces a unit of output (a paper) or not. A Bayesian rule is used to update the estimate of the worker's expected output in each

[2]Viscusi (1979) provides a model similar to ours with a numerical example which exhibits a decreasing separation probability. Burdett (1978) shows that quit rates decline with job tenure in a search model with no learning. A more detailed comparison of our model with Mortensen (1978) and Jovanovic (1979) may be found in footnote 7.

period. Each scholar can also work in an alternative sector where his productivity is the same as any other scholar's, is fixed over time, and is known with certainty.

Under risk neutrality, each scholar currently in the research sector chooses whether to switch careers at each age so as to maximize his expected wealth. The situation is thus modeled as a mover-stayer or pure job-matching problem. Under risk neutrality there is no motivation to bind either the worker or the firm by an implicit or explicit contract. Workers with sufficiently favorable records will choose to remain with the firm for the rest of their career, irrespective of their future performance. Others will be forced by circumstances, i.e. their poor performance record, to leave. The firm does not provide insurance and is not bound to retain any worker whose expected contribution to the firm's profits is negative. Wages at each instant are equal to the perceived marginal productivity of the worker, where perceptions are determined by the observed cumulative output record, r. This results in an increasing sequence of "optimal stopping times", $t(r)$, $r = 0, 1, 2, \ldots$, where $t(r)$ is the longest a scholar with research record r would remain in the research sector. Moreover, we show that there is some finite r^* such that $t(r^* - 1) < A$ and $t(r) = A$ for all $r \geq r^*$, where A is retirement age. That is, scholars with research record $r^* - 1$ at age $t(r^* - 1)$ will switch to a non-research career while those with records of r^* or better will never switch. Thus r^* is the tenure standard and $t(r^* - 1)$ is the up-or-out age. Intuitively, once a scholar achieves a record of r^*, even if his record thereafter is poor, his perceived productivity does not decline as fast as his moving costs increase due to the finite lifetime. The other aspects of the tenure phenomenon (dropping out and getting squeezed) are also reflected in the equilibrium.

In the second part of the paper in which workers are risk averse and have no access to the capital market, the employment and salary strategies are derived from the maximization of the worker's expected utility given the expected profits of the firm (we retain the assumption that firms are risk neutral). The behavior of wages in this case reflects the desire of workers to smooth consumption over time and across different realizations of the output process. Wages do not always follow productivity, and, in particular, a worker who is retained by the firm will not suffer a reduction in wages just because his output record is not favorable [Freeman (1977), Weiss (1981), and Harris and Holmstrom (1982) also obtain this result but not in a job matching context]. As a result of this wage rigidity, a worker may choose to stay with the firm even though his expected productivity (as distinct from his wage) is below his productivity elsewhere. To mitigate such potential inefficiencies, the worker agrees, ex ante, to delegate the employment decision to the firm (subject to the constraint that the decision maximize the ex ante expected utility of the worker). For certain realizations of the output process, termination of employment will not be voluntary on the part of the worker, i.e., while it is optimal to agree ex ante to leave in certain situations in order to receive a higher current wage, leaving may not be ex-post-optimal for the worker once the situation is realized. It is in this sense that the model with risk averse workers exhibits characteristic (5), "the boot." A worker obtains tenure if for any future realization of the output sequence, the optimal strategy is to retain him. We again show that a tenure standard r^* exists, that is a worker who achieves r^* is assured of being retained thereafter. An up-or-out age T also exists in the sense that employment is never terminated later than that age. Contrary to the case of risk neutrality, however, some workers may achieve tenure without attaining r^* by age T. Finally, we consider some efficiency implications of the model with risk averse workers. We show that workers may stay in their current job even when it would be socially optimal to switch. The intuition for this has to do with risk aversion and is spelled out in Section 3.

Since our theoretical analysis yields somewhat different results depending upon the worker's risk preferences and his options outside the firm, one is naturally faced with the task of model selection. From a broader perspective, however, this is somewhat premature since we have neglected many potentially relevant aspects of the tenure arrangement. For instance the isssues of moral hazard (and work incentives), hierarchies (and other interactions across workers), non-marketability of the output (and non-profit considerations), and academic freedom are not analyzed in this paper [a provocative discussion is provided in Alchian (1977)]. We only maintain that experimentation on the job plays an important role in the tenure phenomenon, and this reflects itself in common features such as a tenure standard and up-or-out age which will also emerge in a more complete analysis.

2. A Model With Risk Neutral Workers

2.1. Model Description

The model is comprised of many "universities" (employers), called the "research sector," and many "scholars" (workers). There is also an exogenous "non-research" sector which absorbs all scholars who are forced out of a research career. The term "university" refers to a research institution. The teaching aspects of a job in a university are ignored. Further, a job in the non-research sector is assumed to involve no research. Scholars may not "time-share" between the two sectors.

A scholar in the research sector is assumed to produce either 0 or 1 "papers" at each age which the university can then market at a fixed price (which we normalize to 1). All scholars (including those in the non-research sector) are assumed to live a finite number of periods, denoted A. Scholars are homogeneous except that some have a high expected output per unit time q_H at any given age while others have a low expected output per unit of time q_L at any given age. Naturally, we assume $0 \leq q_L < q_H$. The output of a scholar of productivity $q \in \{q_L, q_H\}$ is assumed to follow a Poisson process with parameter q.

In a Poisson process, time (really time in process or "age" in this model) is viewed as continuous. At any age $a \in [0,A]$, the probability that a scholar of type q will have produced a cumulative output of $R_a = r$ papers is

$$Pr(R_a = r|q) = e^{-qa} (qa)^r / r! \qquad \text{for } r = 0,1,...$$

As mentioned above, q may be interpreted as the expected output per unit of time.

No one, including the scholar himself, knows the productivity of any given scholar. A priori, each scholar is believed by everyone to be low productivity with probability π independently of any other scholar ($0 < \pi < 1$). Beliefs about a given scholar of age $a > 0$ are assumed to be formed according to Bayes' rule, given the scholar's age, a, and his pattern of outputs at ages prior to a. It is easy to show that cumulative output r (for "record") is a sufficient statistic for the complete pattern of previous outputs [see, e.g., DeGroot, exercise 1, pages 183-184]. Thus the posterior probability that any given scholar of characteristics (a,r) [i.e., age a, record r] has low productivity, q_L, is

$$\pi(a,r) = Pr\ (q = q_L|\ R_a = r) = \frac{\pi e^{-q_L a}\ q_L^r}{\pi e^{-q_L a}\ q_L^r + (1-\pi) e^{-q_H a}\ q_H^r} \qquad (1)$$

for any $(a,r) \in C = [0,A] \times Z$ (Z is the set of natural numbers, 0,1,2,...). We will call C the set of feasible characteristics of scholars. Thus the expected productivity of a scholar $(a,r) \in C$ is given by

$$q(a,r) = q_L \pi(a,r) + q_H \left[1 - \pi(a,r)\right]. \tag{2}$$

From (1), it follows that π is strictly increasing in a for given r and strictly decreasing in r for given a. Thus from (2), q is strictly decreasing in a for given r and strictly increasing in r for given a.[3]

Salaries are assumed to be equal to perceived productivities, q(a, r).[4] At any age, a typical scholar must only decide whether to pursue a research career or a non-research career at that age. We assume that once a scholar has chosen a non-research career, he may not thereafter return to a research career.[5] Any scholar may, however, take a non-research job at any age (prior to retirement at age A). Non-research jobs are assumed to pay a guaranteed salary of W per year until retirement. Switching to a non-research career at any age $a > 0$ is assumed to involve a once-and-for-all moving cost K.[6] A

[3]These statements are correct only if $q_L > 0$. If $q_L = 0$, then $\pi(a, 0)$ and $q(a, 0)$ have the properties mentioned, but $\pi(a, r) \equiv 0$ and $q(a, r) \equiv q_H$ for any a and $r \geq 1$.

[4]In most job-matching models, this follows from a zero-profit condition. Since universities are not generally thought of as profit maximizing institutions, it would be more appropriate to model them as choosing a portfolio of scholars of various ages and records so as to maximize output subject to a salary budget constraint. This would result in salaries which are proportional to perceived productivities but would have no other significant effect on the results presented below.

[5]Yakowitz (1969, pp. 54-56) proves this for a discrete time version of our model in which the output process is Bernoulli and there are no moving costs in either direction. We conjecture that it can be proved for our model as well but have thus far been unable to do so.

[6]The term "moving" cost should not be taken too literally. The cost K is

rookie (scholar of age a=0) may choose a non-research career at no cost.

Scholars take the salaries offered in the research sector, $q(a,r)$, and the non-research salary W as given and make an expected wealth maximizing career choice at each age. All scholars are assumed to face the same, constant market real interest rate ρ per period. Let $V(a,r)$ be the expected wealth (present value of all future income) of a scholar (a,r) assuming that he makes an optimal career choice at each future age. Consider a very short time interval h and suppose that the scholar (a,r) decides to stay in the research sector during the period a to $a + h$. Then, to a close approximation, he will produce zero papers in that interval with probability $1-q(a,r)h$ and one paper with probability $q(a,r)h$ and more than one paper with probability zero. This statement becomes exact in the limit as h approaches zero.

It follows, therefore, that, to a close approximation, the expected wealth from staying in the research sector at (a,r) from a to $a+h$ is (for small h)

$$\Big[q(a,r)h + q(a,r)h\, V(a + h,\ r + 1) + \big(1 - q(a,r)h\big)\, V(a + h,r)\Big]/(1 + \rho h). \tag{3}$$

In this equation, $q(a,r)h$ is the salary over the period from a to $a + h$. It is assumed that the salary is paid at $a + h$ although as h gets small this is irrelevant. The expected (and actual) wealth from switching to the teaching sector at (a,r) is

meant to reflect all sorts of unreimbursed costs, monetary and psychological, occasioned by the switch in career. To the extent that direct moving expenses are reimbursed by the new employer in lieu of higher wages, these are included in K. Typically, there are very significant costs associated with such a switch which are not reimbursed, especially if the switch involves physical relocation of the family (loss of proximity to old friends, costs of acquiring information about the new community, costs of learning about the new job, etc.). Indeed it might reasonably be supposed that such costs increase with age. Letting K increase with a in the model would not affect the qualitative results but would tend to reduce the equilibrium up-or-out age.

$$v(a) = W\xi(a) - K \qquad \text{for } a > 0 \text{ and } v(0) = W\xi(0) \tag{4}$$

where $\xi(a)$ is the present value of a continuous flow of \$1 per period over the interval $[a,A]$ using interest rate ρ, i.e.

$$\xi(a) = (1/\rho)[1 - e^{-\rho(A-a)}] \tag{5}$$

Since the worker can choose to stay with the firm for an interval $(a,a+h)$ or to quit, the expected wealth of scholar (a,r) is either $v(a)$ as defined in (4) or the expression in (3), whichever is larger, i.e.

$$V(a,r) \tilde{=} \max\{v(a), [q(a,r)h + q(a,r)h\ V(a + h,\ r + 1) \\ + (1 - q(a,r)h)\ V(a + h,\ r)]/(1 + \rho h)\ \} \tag{6}$$

If $v(a)$ is the larger of the two expressions for small h, then $V(a,r) = v(a)$. If, however $v(a)$ is the smaller of the two, then multiplying by $1 + \rho h$ and subtracting $V(a,r)$ from both sides gives

$$\rho h V(a,r) \tilde{=} q(a,r)h + q(a,r)h\ V(a + h,\ r + 1) \\ + (1 - q(a,r)h)[V(a + h,r) - V(a,r)] - q(a,r)h\ V(a,r).$$

Now dividing by h and taking limits as h approaches zero yields

$$V(a,r) = (1/\rho)[q(a,r) + q(a,r)\ (V(a,r + 1) - V(a,r)) + \dot{V}(a,r)]$$

where dot over a function denotes its (partial) derivative with respect to age.

Therefore, we have

$$V(a,r) = \max\{v(a),\ (1/\rho)[q(a,r) + q(a,r)\ (V(a,\ r + 1) - V(a,r)) \\ + \dot{V}(a,r)]\} \tag{FE}$$

The above equation (FE) is the functional equation for the dynamic programming problem faced by any scholar. In order to define the problem fully, however, we must specify a boundary condition. The obvious boundary condition for this problem is that the present value of future income starting at retirement age A is zero for any record, i.e.

$$V(A,r) = 0 \qquad \text{for all } r. \tag{B}$$

The two equations (FE) and (B) define the value function V, if a solution of (FE) which satisfies (B) exists (a solution is constructed below).

Any scholar (a,r) will choose to exit from the research sector if and only if

$$\rho v(a) > q(a,r) + q(a,r)\,[V(a,r+1) - V(a,r)] + \dot{V}(a,r) \tag{7}$$

We define the exit set, E, as the set of characteristics which will result in exiting the research sector, i.e.

$$E = \{(a,r) \in C \mid (7) \text{ holds}\}.$$

Note that E is fully determined by, ρ, W and K.[7]

The analysis of the model consists of solving the optimal stopping problem defined by (FE) and (B) in order to characterize the exit set E. The solution will involve an increasing sequence of stopping times, $t(0) < t(1) <$

[7]To anyone familiar with the job matching literature, our model will look very similar to those of Mortensen (1978) and Jovanovic (1979). The main difference between our model and that of Jovanovic is that in the latter, workers are infinitely lived. The result is that the value of leaving is a constant which implies that a worker will leave at any time, no matter how long he has been in a particular match, if his current wage falls below his reservation wage. This is an event with positive probability at every time. Thus there is no "tenure phenomenon" in Jovanovic's model. Mortensen's (1978) model is similar to that of Jovanovic. There the dissolution of a match is caused by the discovery of a better one, an event which is unrelated to the observed output of the current match. Nonetheless, the model is similar to the present model in that it could produce an equilibrium with tenure if a finite horizon for the worker were introduced (search costs could play the role of moving costs in this model).

... such that (a,r) is in E if and only if $a > t(r)$. This analysis is carried out in Subsection 2.3. Before proceeding to the formal analysis it will be useful to present the main conclusion informally. This is done in the next subsection.

2.2. Informal Description of the Main Results

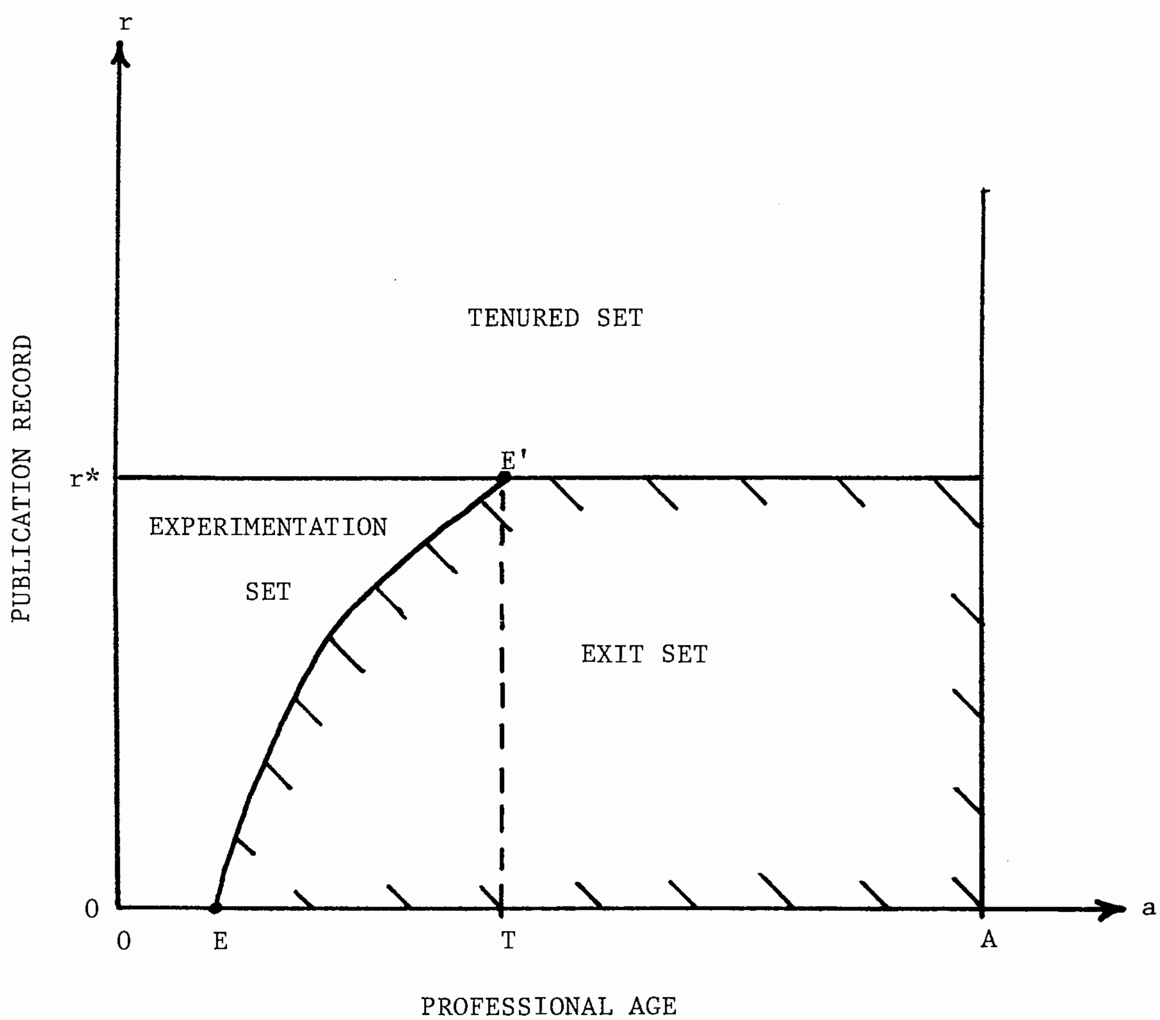
The nature of the solution to the problem stated in Section 1 can be described informally with the aid of Figure 1.[8] All the characteristics of the tenure phenomenon [except for (4)] derive from the shape of the boundary of the exit set defined in Section 1. This boundary is the curve EE' in Figure 1.

The boundary of the exit set must be a nondecreasing function of age. The reason is that the expected output per unit of time of a worker with a given number of papers is a decreasing function of age, i.e. $q(s,r) < q(t,r)$ for $s > t$. A worker who has attained a given record r earlier is more likely to be of high quality. Thus if a worker with r papers at some age chooses to leave, an older worker with the same record will not choose to stay.

The boundary of the exit set cannot touch the vertical lines through a=0 or a=A, i.e. E cannot be at a=0 nor can E' be at a=A. If one assumes that at time zero, the typical worker (with record r=0) is willing to enter the research sector, then due to positive mobility costs he will stay for a while even if no papers are forthcoming. Similarly, due to finite working life and positive costs of mobility, if the worker leaves, he will always do so strictly before retirement age. Indeed, at ages close to A, moving costs exceed the present value of the salary in the alternative sector.

[8]The figure is drawn as if r were a continuous variable for expositional purposes. A more accurate representation is given in Figure 2, below.

Figure 1: Properties of the Solution



Finally, there is a record r* sufficiently high such that the worker who attains it will never leave. The boundary of the exit set cannot cross the horizontal line through r*. This, again, is related to the finite horizon and the existence of moving costs. The rules for revising estimates of productivity, equations (1) and (2), and the finite horizon imply that the downward revision in the worker's estimated productivity is bounded, i.e. the worst possible future productivity estimate for a worker (a,r) is q(A,r). If this lower bound exceeds the wage in the alternative sector, W, minus the annual equivalent of moving costs, $K/\xi(a)$, the worker will clearly stay in the research sector until retirement.

The above considerations imply an exit set of the form drawn in Figure 1. It is clear that the mobility patterns associated with such an exit set will satisfy the properties which characterize the tenure phenomenon, i.e. properties 1-3 in the introduction. Workers with characteristics in the shaded region labeled "EXIT SET" in Figure 1 will not be observed in the research sector because when the boundary EE' is crossed, the worker switches to the alternative sector. Workers with records better than r* have tenure and will never be observed to leave the research sector, i.e. their probability of future exit is zero. Finally, workers with characteristics to the left of EE' and below r=r* may be viewed as experimenting on the job. Their probability of future exit is strictly between zero and unity. An interesting aspect of the gradual sorting process is that after some critical age (labeled T in Figure 1), experimentation, and thus mobility, will cease. All workers with record equal to or better than r* will choose to stay until retirement; everyone else will leave. This follows from the fact that realizations of the Poisson process are nondecreasing step functions in age-record space. The employment status of the worker can change only in the range in which the boundary of the exit set is strictly increasing, i.e. in the range [E,T] in Figure 1.

We turn now to a formal analysis. Readers not interested in the formalities can skip to Subsection 2.4 with no loss of continuity.

2.3. Formal Analysis

In what follows, we will be interested in scholars who remain in research until retirement, i.e. who achieve tenure. Such a scholar will follow a path in age-record space which does not pass through E. It will therefore be useful to have some notation for paths in general and paths which do not pass through E in particular. Accordingly, we make the following:

Definition. A path to a characteristic (a,r) is any feasible sample path of a Poisson process over the interval [0,a], i.e. any non-decreasing function $p\colon [0,a] \to Z$ such that $p(0) = 0$, $p(a) = r$ and p does not jump by more than one at any $t \in [0,a]$.

Let P(a,r) be the set of paths to (a,r) and $E'(a,r)$ be the set of paths to (a,r) which do not pass through E, i.e.

$$E'(a,r) = \{p \in P(a,r) \mid (t,p(t)) \notin E \text{ for all } t \in [0,a]\}.$$

Before proving the main results of this section, several lemmas are required. The proofs of these and all other lemmas are relegated to the Appendix.

Lemma 1. V(a,r) in (7) can be replaced by v(a), i.e., for any (a,r), (7) holds at (a,r,) if and only if

$$\rho v(a) > q(a,r) + q(a,r)\ [V(a,r + 1) - v(a)] + \dot{v}(a). \tag{8}$$

Lemma 2. If a scholar exits the research sector, then any scholar of the same age but inferior record will also exit, i.e., if $(a,r) \varepsilon E$ and $r' \leqslant r$, then $(a,r') \varepsilon E$.

Lemma 3. If there is some way of reaching (A,r) without exiting, then no one with a record of r or better will ever exit, i.e., suppose r is such that $E'(A,r) \neq \emptyset$. Then for any $s \geqslant r$ and any a, $(a,s) \notin E$.

Consider a scholar (a,r) where r is such that $E'(A,r) \neq \emptyset$. By Lemma 3, this scholar will remain in research until retirement. Consequently his salary will follow $q(t,s)$ for $t \geqslant a$, $s \geqslant r$. His current expectation about $q(t,s)$ is just his current expected product, $q(a,r)$. Therefore his current expected wealth is just the present value of $q(a,r)$ until retirement, namely $q(a,r)\xi(a)$. These statements are proved formally in Lemma 4.

Lemma 4. If $E'(A,r) \neq 0$, then for any a and $s \geqslant r$, $V(a,s) = q(a,s)\xi(a)$.

We are now ready to discuss the tenure properties of the model. We first define tenure as follows.

Definition. An age $T > 0$ and record r^* is called a <u>tenure characteristic</u> if

(i) No scholar ever exits the research sector after achieving a record of r^* or better.

(ii) If $r^* \geqslant 1$, $T < A$ and no scholar ever remains in the research sector after reaching age T with record $r^* - 1$ or worse. If $r^* = 0$, $T = A$.

(iii) If $r^* \geqslant 1$, there exists a path p to $(T, r^* -1)$ which does not pass through the exit set, and such that $p(a) = r^* -1$, for some $a < T$.

The age T is called the <u>up-or-out age</u> and the record r^* is called the <u>tenure</u>

standard. The first two conditions are self-explanatory. The third guarantees that not all scholars with records worse than r* exit before the up-or-out age.

We are now ready to prove the first main result of the paper.

Theorem 1. Suppose for some r, $E'(A, r) \neq \emptyset$. Let

$$r^* = \min \{r \varepsilon Z \mid E'(A, r) \neq \emptyset\},$$
$$X = \{a \ \varepsilon\ [0,A] \mid (a, r^* - 1) \notin E\} \text{ if } r^* \geq 1$$
$$= [0,A] \text{ if } r^* = 0,$$
$$T = \sup X.$$

Then (T, r^*) is the unique tenure characteristic and

$$X = [0, T) \text{ if } T < A,$$
$$X = [0, A] \text{ if } T = A.$$

Proof. We must first show $X \neq \emptyset$, for $r^* \geq 1$. Let $p \ \varepsilon\ E'(A, r^*)$. Then for some $\hat{a}$, $p(\hat{a}) = r^* - 1$. Therefore $(\hat{a}, r^* - 1) \notin E$, and $\hat{a} \ \varepsilon\ X$.

That (T, r^*) satisfies part (i) of the definition of a tenure characteristic is obvious from Lemma 3 and the definition of r^*.

To prove part (iii), suppose $r^* \geq 1$ and let $p \ \varepsilon\ E'(A, r^*)$. Also let $\hat{a}$ be such that $p(\hat{a}) = r^* - 1$. Then $\hat{a} \ \varepsilon\ X$ so $\hat{a} \leq T$. If $\hat{a} < T$, define p' by $p' \equiv p$ on $[0, \hat{a}]$, $p' \equiv r^* - 1$ on $[\hat{a}, T]$. If $\hat{a} = T$, then there is an $\bar{a} < \hat{a}$ such that $p(a) = r^* - 2$ on $[\bar{a}, \hat{a})$. Choose $\bar{a} < \delta < \hat{a}$ and define $p' \equiv p$ on $[0, \delta)$, $p' \equiv r^* - 1$ on $[\delta, T]$. Then p' is the required path.

Now suppose $r^* \geq 1$ and $T = A$. Then by the above construction, $E'(A, r^* - 1) \neq \emptyset$ contradicting the definition of r^*. Therefore $T < A$. Part (ii) now follows immediately from the definition of T and Lemma 2.

Next we show that $X = [0, T)$ if $T < A$. Clearly, by definition of T,

$X \subset [0, T)$. Let a be any element of $[0, T)$, and let $p \in E'(T, r^* - 1)$ as constructed above. Then $(a, p(a)) \notin E$ and $p(a) \leq r^* - 1$. Therefore by Lemma 2, $(a, r^* - 1) \notin E$. Hence $[0, T) \subset X$, and so $[0,T) = X$. If $T = A$, it is obvious that $X = [0, A]$.

Finally suppose (T', r') is a tenure characteristic. Then $r' \geq r^*$. Suppose $T' < T$. Let $p \in E'(T, r^* - 1)$. Then for $a \in (T', T]$, $(a, p(a)) \notin E$ which contradicts part (ii) of the definition since $r^* \leq r'$. Therefore $T' \geq T$. Now suppose $r' > r^*$. Then by Lemma 3, $T' = A$ violating part (ii) of the definition. Therefore $r' = r^*$. Finally, since $r' = r^*$, the argument that $T' \geq T$ can be reversed to yield $T \geq T'$. Thus $T' = T$.

Q.E.D.

In Theorem 1, we assumed that there is some record r which is sufficiently good that some path to (A, r) exists along which a scholar would not exit. We must now show that such an r exists. Before we can prove this, one additional assumption and a preliminary result which follows from the assumption are needed. The additional assumption is simply that scholars weakly prefer to start out in a research career, i.e. that (8) does not hold at $a = r = 0$. The preliminary result is

Lemma 5. (a) $q_H > W$ and

(b) $\lim_{h \to 0} V(h,0) = V(0,0)$, i.e., $V(\cdot,0)$ is continuous at 0.

We may now state (the proof is in the Appendix)

Lemma 6. For some r, $E'(A, r) \neq \emptyset$, i.e. there is a path to (A, r) which does not pass through E.

The second main result of the paper, namely that any solution of the model embodies the tenure phenomenon, follows trivially from Lemma 6.

Theorem 2. Define (T,r^*) by

$$r^* = \min \{r | E'(A, r) \neq \emptyset\}$$

$$T = \sup \{a \in [0,A] | (a,r^* - 1) \notin E\} \text{ if } r^* \geq 1$$

$$= A \text{ if } r^* = 0.$$

Then (T,r^*) is the unique equilibrium tenure characteristic.

Proof. By Lemma 6, $\{r | E'(A, r) \neq \emptyset\} \neq \emptyset$. Therefore r^* and T are well defined. The theorem now follows from Theorem 1. Q.E.D.

Suppose that $r^* \geq 1$ is an equilibrium tenure standard. Define

$$X(r) = \{a \in [0,A] | (a,r) \notin E\}$$

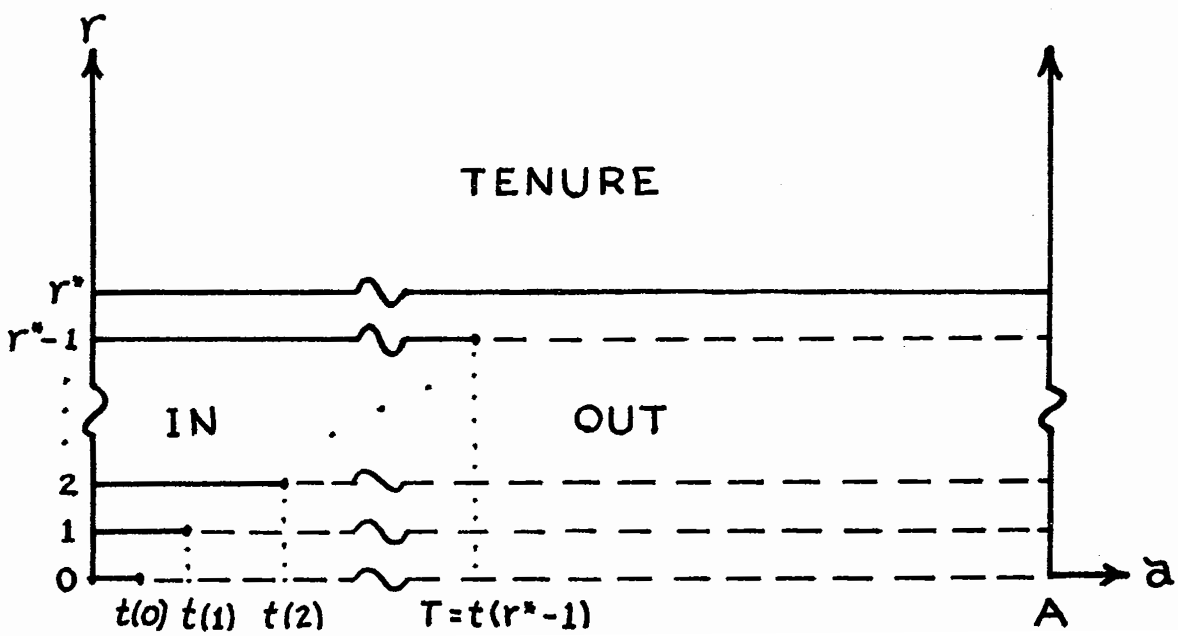
$$t(r) = \sup X(r) \quad \text{for } r = 0,\ldots, r^* - 1.$$

The following lemma will allow us to interpret $t(r)$ as an optimal stopping time.

Lemma 7. $t(r)$ is non-decreasing in r and $X(r) = [0, t(r))$.

We can now interpret $t(r)$ as an optimal stopping time in equilibrium. That is, if one has record r and is younger than $t(r)$ it is optimal to remain in research whereas if one reaches $t(r)$ with record r it is optimal to switch. Using Lemma 7, the equilibrium can be depicted as in Figure 2. In the figure, the horizontal broken lines collectively represent the exit set E. Scholars with characteristics on the solid horizontal lines below $r = r^*$ [i.e. on $X(r)$] are still in the research sector but have not achieved tenure. Any scholar which arrives at a point $(t(r),r)$ will exit so that no scholars with characteristics in the exit set (broken lines) will be observed in research. Scholars with characteristics on or above the line $r = r^*$ have

Figure 2: A Typical Equilibrium (risk neutral workers)



achieved tenure.

2.4. Computation and Comparative Dynamics

In this subsection we show how the equilibrium may, in principle, be calculated and derive some comparative dynamics results. One preliminary result is required.

Lemma 8. A scholar remains in research until retirement if and only if the present value of a constant income equal to his current wage until retirement exceeds the value of switching, i.e., $E'(A, r) \neq \emptyset$ if and only if $q(a, r)\xi(a) \geq v(a)$ for all a.

Using Theorem 2 and Lemmas 7 and 8, an equilibrium can be computed as follows. Define, r^* as the smallest $r \in Z$ such that $q(a, r)\xi(a) \geq v(a)$ for every a. We can now contruct V by backwards recursion on r starting with r^*. If $r^* = 0$, let $V(a, r) = q(a,r)\xi(a)$ for all (a, r). Otherwise let

$$H(a, r) = \rho v(a) - q(a, r) - q(a, r)[V(a, r + 1) - v(a)] - \dot{v}(a).$$

Now for $r \geq r^*$, let

$$V(a, r) = q(a, r)\xi(a).$$

By Lemma 8 and Theorem 2, r^* will be the equilibrium tenure standard. Define X(r) and t(r) as in Lemma 7. Then from Lemmas 1 and 7, t(r) may be calculated as the smallest root of H(a,r).

We may now solve the differential equation

$$\rho V(a, r) = q(a, r) + q(a, r)[V(a, r + 1) - V(a, r)] + \dot{V}(a, r) \tag{9}$$

with boundary condition

$$V[t(r),r] = v[t(r)] \tag{10}$$

recursively starting with $V(a, r^* - 1)$ and using $V(a,r^*) \equiv q(a,r^*)\,\xi(a)$.

It is now possible to derive some comparative dynamics results. It is easiest to deduce the behavior of the tenure standard, r^*. Using Lemma 8, we have that r^* is the smallest integer such that $q(a,r^*)\,\xi(a) \geqslant v(a)$ for every a. Dividing by $\xi(a)$, this relation is equivalent to

$$q(a,r^*) \geqslant W - K/\xi(a) \qquad \text{for all } a < A \tag{11}$$

(at $a = A$, $\xi(a) = 0$ and the original relationship is satisfied for any r). Any change which increases the right-hand-side of (11) will tend to increase r^* (although very small changes will have no effect on r^* because of the integer restriction). Therefore we have

Theorem 3. The tenure standard, r^*, will tend to increase with increases in the non-research wage, W, increases in retirement age, A, decreases in moving cost, K, and decreases in the real interest rate, ρ.

Proof. Note that $\xi(a)$ increases, for any a, with an increase in A or a decrease in ρ. The result follows from the remarks after equation (11).

Q.E.D.

Intuitively, any change which either makes non-research more atractive relative to research or reduces the annualized equivalent of moving costs ($K/\xi(a)$) will induce people with a record equal to the initial tenure standard to leave. Thus a better record is now required for it to be optimal to stay in research. This same reasoning indicates that any change which makes research more attractive, i.e. increases $q(a,r)$ for any (a,r), will have the opposite affect on r^*. Indeed we have

Theorem 4. The tenure standard, r^*, will tend to fall with increases in the productivity q_H or q_L, of either type of scholar, or with increases in the fraction of scholars who are highly productive, $1-\pi$.

Proof. It is easy to check that increases in q_H, q_L, and $1-\pi$ (decreases in π) shift $q(a,r)$ up for any (a,r). The result follows from (11). Q.E.D.

Comparative dynamics results on the up-or-out age, T, are more difficult to obtain. First recall that the up-or-out age $T = t(r^* - 1)$, i.e. T is the smallest root of $H(a,r^* - 1)$. We can write $H(a,r^* - 1)$ as

$$H(a,r^* - 1) = W\left[1 + q(a,r^* - 1)\,\xi(a)\right] - K\left[\rho + q(a,r^* - 1)\right]$$
$$-q(a,r^* - 1)[1 + q(a,r^*)\,\xi(a)].$$

Since it can be shown that $\dot{H}(T,r^* - 1) > 0$, we see that T varies inversely with W and directly with K provided that the changes in W or K are small enough not to affect r^*. Although it does not seem possible to say, in general, what happens to T when the changes in W or K also change r^*, some numerical examples suggest that increases in W will increase both r^* and T while increases in K have the opposite effect. To the extent that this holds generally, our model suggests that as non-research careers become more attractive, relative to research, a better record is required for tenure, but scholars remain in research longer without tenure.

3. Risk Aversion and Imperfect Capital Markets.

The analysis of Section 2 considered risk neutral workers with access to a perfect capital market. It was also assumed that there are no costs of mobility across firms in the research sector. We thus concluded that competition will force an equality between wages and the perceived productivity of the workers at any instant. There is thus no insurance aspect of tenure in that model--tenure is explained by the way information is revealed over time. Yet it would appear that the insurance features of a tenure contract are an important source of its appeal. Moreover the tenure phenomenon [as characterized by properties (1)-(4) listed in the Introduction] is rarely seen in practice without some associated, more or less explicit, tenure contract. Therefore, in this section, we investigate the extent to which worker risk aversion and lack of access to capital markets coupled with the model of information evolution and job matching of Section 2 can account both for the tenure phenomenon and insurance aspects of tenure contracts as embodied in property (5) of the Introduction. We show that if employees are risk averse and have no access to capital markets, some, but not all, features of the tenure phenomenon are retained and property (5) is satisfied by the equilibrium.

The development of wages and employment during the contract period can be derived from the principle that at each stage one cannot improve the expected utility of the worker given the expected profits of the firm. Consider a worker at age a and let X(a) be the present value of the expected net costs (i.e. wages minus output) to the firm over the remainder of the worker's life (i.e. from a to A). One may also think of X(a) as the net surplus (above average product) promised by the firm to the worker. Clearly the firm is indifferent among all rearrangements of wages and employment which keep X(a) fixed since the firm is assumed to be risk neutral and to have access to perfect capital markets. Define $V(x,a,r)$ as the maximal expected utility that a worker of age a can obtain given that $X(a) = x$ and given that his success record is r, where the maximization is over all feasible future wage and employment paths. In order to define the feasible set we note that the worker is free to move, at some cost, either to another firm in the research sector or to the alternative sector. Thus the choices of employment and wages are constrained by the requirement that expected utility must be at least as high as in these alternatives whenever the worker is to be retained. Since a competing firm in the research sector can always offer the worker $V(0,r,a)$ and just break even, the value of the contract cannot fall below that level by more than the mobility costs across firms (which we denote by c). Similarly it cannot fall below the value of the utility stream (net of mobility costs) in the alternative sector which we denote by $v(a)$.

To characterize variations in the contract we consider as control variables the current wage, w, the employment indicator, π, which takes on the value 1 if the worker is currently employed by a research firm and the value 0 otherwise, and the net cost of the worker to the firm next period, y. To simplify the exposition and to avoid some technical difficulties we shall discuss in this section the case in which employment and wages are subject to change only on a discrete set of equally spaced points in time. In each period a worker can produce either one (success) or zero papers (failure) according to a Bernoulli analogue of the Poisson process described in Section 2.[9] Wages are paid at the beginning of each period during which output is observed. Employment decisions are made after ouput is observed. Accordingly

[9]That is, the probabilty of producing one paper in any period is either q_H or q_L independently of what occurs in any other period.

we define π_0 and π_1 as the employment indicator contingent on not producing a paper (failure) and producing one (success) respectively. The worker is always guaranteed employment for a duration of one period. As in Section 2, once separated from the research sector, the worker cannot return. Since all firms are assumed to be identical and since information is public, the worker will not leave except to move to the alternative sector. Changes in the net cost of the worker to the firm reflect changes in the amount of insurance which occur due to the passage of time and the acquisition of new information. We denote by y_0 and y_1 the promised levels of net surplus next period contingent on a failure and a success respectively this period. Of course, for any given contract, y_0, y_1 are fully determined by the wage and employment commitments of the firm. Our purpose here is to characterize a selection of an optimal (or competitive) contract, and for this purpose we treat $w, \pi_0, \pi_1, y_0, y_1,$ as subject to choice, given a,r,x, thus allowing for potential revisions in the contract. Finally, let α and β denote the subjective and market discount factors facing the worker and the firm respectively. With the above definition we can state the basic recursive relation which defines $V(x,r,a)$ [where π and y denote (π_0,π_1) and (y_0,y_1) respectively].

$$V(x,r,a) = \max_{w,\pi,y} \{u(w) + \beta q(r,a)[V(y_1,r+1,a+1)\pi_1 + (1-\pi_1)v(a+1)] \qquad (12)$$
$$+ \beta\big(1-q(a,r)\big)[V(y_0,r,a+1)\pi_0 + (1-\pi_0)v(a+1)]\}$$

subject to

$$w \geq 0,\ \pi_i \in \{0,1\},\ y_i + q(a+1,r+i) \geq 0 \text{ if } \pi_i = 1,\ i=0,1 \qquad (13)$$

$$x = w - q(a,r) + \alpha[q(a,r)\pi_1 y_1 + \big(1-q(a,r)\big)\pi_0 y_0] \qquad (14)$$

$$V(y_i,r+i,a+1) \geq \mathrm{Max}[V(0,r+i,a+1)-c,\ v(a+1)],\ \text{if } \pi_i=1,\ i=0,1 \tag{15}$$

and the boundary conditions

$$x(A) = V(x,r,A) = 0 \text{ for all } x,r, \tag{16}$$

$$x(0) = 0, \qquad V(0,0,0) \geq v(0). \tag{17}$$

The constraints in (13) guarantee compactness and non-emptiness of the constraint set. Constraint (14) specifies all the potential revisions which are consistent with the same expected cost to the firm and which therefore do not affect the firm's position. Constraint (15) states that the new commitments to the worker must be sufficiently attractive to prevent quits unless the worker is to be discharged anyway ($\pi_i=0$). Comparing the formulations of the problem in (12) and (6) it is seen that we now allow for a separation of wages from productivity. To motivate such a separation we introduce a continuous, strictly increasing, concave utility function $u:R_+ \to R$ and make the strong assumption that the worker cannot borrow or lend outside the firm.[10]

In order to analyze the problem posed in (12)-(16), we must first establish some of the properties of the sequence of value functions $V(\cdot,\cdot,a)$ for $a=0,\ldots,A-1$. This is done in

Theorem 5. Assume u is continuous, strictly increasing, and concave and u'(0)

[10] Accordingly we need to modify the definition of v(a), given in (4). It should now be defined as

$$v(a) = u(W)\xi(a) - K \quad \text{for } a = 1,2,\ldots,A,$$

$$= u(W)\xi(a) \qquad \text{for } a = 0,$$

where $\xi(a) = \Sigma_{i=a}^{A} \beta^{i-a}$.

$= \infty$. Let $\{V(\cdot,\cdot,a) \mid a=0,\ldots,A-1\}$ be the sequence of functions defined by (12)-(16). Then for each (a,r), $V(\cdot,r,a)$ is continuous, strictly increasing, and concave in x for any x such that $x + q(a,r) \geqslant 0$. Moreover, for any x_0 such that $x_0 + q(a,r) > 0$, $V(\cdot,r,a)$ is differentiable with respect to x at x_0 and $V_x(x_0,r,a) = u'[\bar{w}(x_0,r,a)]$ where $(\bar{w},\bar{\pi},\bar{y})(x,r,a)$ solves the maximization problem in (12)-(15).

Proof. See Appendix.

We are now ready to state the first order conditions for the problem defined by (12)-(15). These can be written in the following form:

$$u'(w) = \lambda \text{ , where } \lambda \geqslant 0 \text{ is a Lagrange multiplier} \tag{18}$$

$$\text{if } \pi_i=1,\ V_x(y_i,r+i,a+1) \leqslant \lambda\alpha/\beta \quad \text{with equality if (15) is slack} \tag{19}$$

$$\pi_i = \begin{cases} 1 \text{ if } V(\hat{y}_i,r+i,a+1) - v(a+1) \geqslant \lambda\hat{y}_i\alpha/\beta \\ 0 \text{ otherwise} \end{cases} . \tag{20}$$

where $\hat{y}_i$ satisfies either (19) or (15) with equality,

along with the constraints (13)-(15).

Note that for any (x,r,a) and either value of i, if $\hat{y}_i(x,a,r) + q(a+1,r+i) > 0$ and $\bar{\pi}_i(x,r,a)=1$, then it follows from Theorem 5 and equation (19) that

$$u'[\bar{w}(x,r,a)] \geqslant (\beta/\alpha)V_x(\bar{y}_i(x,r,a),r+i,a+1) = (\beta/\alpha)u'[\bar{w}(\bar{y}_i,r+i,a+1)]$$

with equality if (15) is slack at $\bar{y}_i$. If, as is most commonly assumed, workers' subjective discount factor exceeds the market factor (i.e., $\beta \geqslant \alpha$), then, since u' is strictly decreasing, the above implies (for $\bar{\pi}_i=1$),

$\bar{w}(x,r,a) \leq \bar{w}[\bar{y}_i(x,r,a),r+i,a+1]$ for $i=0,1$. In what follows, we shall assume that $\beta \geq \alpha$. Therefore on the optimal path the wage is non-decreasing as long as the worker stays with the firm. If in fact $\alpha = \beta$, wage increases occur only to prevent the worker from leaving the firm [i.e. only when constraint (15) is effective] or to facilitate his leaving the research sector (i.e. $\pi_i = 0$).[11] Since the costs of mobility are assumed to be fixed, these contraints will be inffective at the end and beginning of the contract period.

Condition (20) states that a worker's employment can be terminated even though his expected utility upon staying one additional period with the firm is higher than in the alternative sector [i.e. (15) is not binding]. The reason for this is that by giving up the promised surplus for next period which he would have if he stayed (namely $\hat{y}_i$) he can have a higher level of consumption (w) currently. Indeed the extra consumption is equal to his expected net cost to the firm if he stayed, i.e. by agreeing to leave at the end of the period, the worker can capture his promised surplus as of the end of the period $(\hat{y}_i)$ in his current wage. This may induce him to agree ex ante to leave even if he would be better off staying given his promised surplus $(\hat{y}_i)$. In this case the worker's agreement to leave must be actively enforced by the firm (although, as will be seen in Lemma 9, this can be accomplished by taking away the surplus if he doesn't leave). This is the main difference from the discussion in Section 2 where, in the absence of insurance, all quits were voluntary. We now show that despite this important difference some of the tenure characteristics 1-3 defined in the introduction continue to hold. Due to downward rigidity of the wage along an optimal path, the fourth characteristic will not hold in this case.

[11]Similar results were obtained by Freeman (1977), Weiss (1981), and Harris and Holmstrom (1982) in models with no alternative sector.

Let us define a tenure standard r^* to be a critical value of r such that for all $r \geq r^*$ a worker who attains r at some age a is guaranteed to be employed by the firm throughout the remainder of his working life. This standard is defined so as to apply for any level of promised surplus which the worker may have when he attains r^*, i.e. for any history. It should be apparent though that the tenure decision depends also on the wage commitments that the worker has accumulated from the past as part of the insurance contract. These commitments may be different for workers of the same age and the same r because of differences in history. Note also that r^* is defined to satisfy only part (i) of the definition of a tenure characteristic given in Section 3. Indeed, in general, when workers are risk averse, there will not exist a single characteristic (T,r^*) which satisfies both parts (i) and (ii) of this definition.

To understand this last point and gain some additional insight into the wage process under risk aversion, consider two workers, "fast" and "slow". Although they are the same age a and have the same cumulative number of papers r, and therefore the same perceived productivity q, fast produced his r papers in his first r periods while slow produced his in his last r periods. Since fast had r successes in his first r periods, his wage increased rapidly to a level higher than slow's wage. Slow's wage, in fact, would still be well below this level even after achieving his r successes because these occurred over $a > r$ periods. Thus, because of the downward rigidity of fast's wage, even though by age a both have the same record and perceived productivity, fast has a higher current wage and promised surplus than does slow. This can result in it being optimal for fast to remain in research until retirement even if he never produces another paper and for slow to leave before retirement. In this case, if r^* is defined as in the previous paragraph, we see that although $r < r^*$ (since slow does not achieve tenure), there is no $T < A$ such that all workers with records worse than r^* at age T leave (since fast always has record $r < r^*$ and still achieves tenure). The point here is that a worker's current situation cannot be characterized simply by his current age and cumulative output record in this case.

Since we make a rather strong requirement from the tenure standard, one may wonder whether such a requirement is ever met in an optimal contract. We now show that this is indeed the case. We first provide an upper bound for the tenure standard then show that this upper bound is attained with positive probability. We shall make use of the following lemma:

Lemma 9. For all x,r,a, and for $i = 0,1$, $\pi_i = 0$ implies $v(a+1) > V(0,r+i,a+1)$.

Lemma 9 states that employment in the research firm will not be discontinued unless the worker can attain in the alternative sector a utility level which exceeds his expected utility within the firm in the absence of any surplus (i.e. when $x = 0$). This is quite obvious since otherwise the worker can gain from the continuation of his employment for at least one period without any loss to the firm. As mentioned above, Lemma 9 also implies that if it is ex ante optimal for a worker to leave, i.e., $\pi_i = 0$, then reducing the surplus, y_i, to zero also insures that leaving is ex post preferable. In other words, if a worker reneges on a promise to leave, the firm can react by reneging on its insurance commitment. In this case the worker will prefer to leave. We can use Lemma 9 to prove:

Theorem 6. For all x,r,a, $i = 0,1$, $\pi_i = 0$ implies $W \geqslant q(a+1,r+i)$ where W is the wage in the alternative sector.

Proof. From Lemma 9, we have

$$v(a+1) > V(0,r+i,a+1).$$

Since $V(0,r,a+1)$ is the value of an optimal policy it must exceed the value of any other feasible policy. Given the specified values for the state variables, it is clearly feasible to set $w = q(a+1,r+i)$ and $\pi_i = y_i = 0$, $i = 0,1$. Hence $V(0,r+i,a+1) \geqslant u[q(a+1,r+i)] + \beta v(a+2)$. Therefore

$$v(a+1) - \beta v(a+2) = u(W) - (1-\beta)K \geqslant u[q(a+1,r+i)] \text{ for } a = 0,\ldots,A-3$$

$$= u(W) - K \geqslant u[q(a+1,r+i)] \text{ for } a = A-2$$

and hence $W \geqslant q(a+1,r+i)$ for $a = 0,1,2,\ldots A-2$. Q.E.D.

It follows from Theorem 6 that employment will not be terminated unless it is perceived that the worker's productivity is higher in the alternative sector, otherwise one can always gain by postponing the decision one period. This consideration, which applies under risk neutrality, also holds when workers are risk averse even though wages in the research sector may be below current productivity due to the insurance premium implicit in the wage.

We can use the properties of the learning process to identify an upper bound for the tenure standard, r^*. Define $\hat{r}$ to be the lowest r such that $q(A-1,r) \geqslant W$. Then $r^* \leqslant \hat{r}$ To see this consider a worker who attains $r \geqslant \hat{r}$ at some time $a' \leqslant A - 1$. Since the worst possible realization is that he remains with r, and since q is monotone decreasing in a and monotone increasing in r, it is clear that $q(a,r) \geqslant W$ for all $a' \leqslant a \leqslant A - 1$ and all possible future values of r. Thus due to Theorem 6 the worker will be retained. To complete the argument we must also show that there is a positive probability of reaching $\hat{r}$ following the optimal contract. From the initial condition (17) there must exist paths on which productivity q exceeds W, otherwise the worker will never choose to work in the research sector. Due to the properties of the learning process [namely that $q(a+1,r+1) > q(a,r)$ for all (a,r)], there is a positive probability that a path which crosses W will not cross it again (as for instance, in the case of a complete run of successes). There is thus a positive probability of reaching $\hat{r}$ on the optimal path. Since $\hat{r} \geqslant r^*$ the above argument also shows that a tenure standard exists, i.e. there is a positive probability of reaching r^* on the optimal path.

We now turn to the other aspect of the tenure contract namely the up or out age. We define the up or out age T as the latest age a satisfying $\pi_i(x,r,a) = 0$ for some x,r,i. Thus for all $a > T$, $\pi_0(x,r,a) = \pi_1(x,r,a) = 1$ for all x,r, and the worker is assured tenure. For $a \leqslant T$ there is still some experimentation and the continuation of employment depends on performance. By the above definition $A-1$ is clearly a potential up or out age. However, if mobility costs are sufficiently high the up or out age will occur earlier than $A - 1$. To show this we shall provide an upper bound for the up or out age.

Recall that in Lemma 9 we proved that $\pi_i(x,r,a) = 0$ implies $v(a+1) > V(0,r+i,a+1)$. Since $V(0,r+1,a+1)$ is the value of an optimal policy, it must exceed the value of any other feasible policy and in particular the one in which the worker is always retained and is always paid his current perceived productivity q. This alternative policy provides no wage insurance but avoids all mobility costs. If mobility costs are non trivial we would expect them to dominate at the end of the contract period in which case $v(a) \leqslant \hat{V}(0,r,a)$ for all r, provided that a is "close" to A, where $\hat{V}(0,r,a)$ is the value of the permanent-employment, no-wage-insurance policy.

More precisely, assume that $v(A-1) < \hat{V}(0,0,A-1)$ or equivalently $u(W) - K < u[q(A-1,0)]$. Then even a worker with no successes will be retained in the last period. Define $\hat{a}$ as the highest a such that $v(a) \geq \hat{V}(0,0,a)$. Since we assume $v(A-1) < \hat{V}(0,0,A-1)$, there exists an $\hat{a}$ such that $\hat{a} < A-1$. It is easily seen that $\hat{a}$ is an upper bound for the up or out age. Suppose $a \geq \hat{a}$ but $\bar{\pi}_i(x,r,a) = 0$ for some x,r, and i. Then from Lemma 9

$$v(a+1) > V(0,r+i,a+1) \geq \hat{V}(0,r+i,a+1) \geq \hat{V}(0,0,a+1)$$

which contradicts the definition of $\hat{a}$.[12]

As with the upper bound for the tenure standard, the upper bound just derived for the up or out age also applies in the case of risk neutrality. The reason for the existence of an up or out age is the same in both cases. It is relatively expensive to get rid of a worker late in his career, and the gain from acquiring information by continuing the experimentation period is declining. Therefore, if one forces the worker to move out of the research sector it is optimal to do so sometime <u>strictly before</u> the end of his productive career.

Finally, with regard to efficiency, note that property 4 defined in the introduction (getting squeezed) does not apply when workers obtain wage insurance. As we have seen the optimal wage profile is non-decreasing, and thus tenured workers do not suffer a reduction in their real wages even if their performance justifies a downward revision in their perceived productivity.[13] As mentioned in the beginning of this section, this insurance

[12]The fact that $\hat{V}$ is increasing in r is used in the last inequality in the above chain. This follows from the fact that the distribution of q at age $t \geq a$ given $R_a = r$ stochastically dominates that of q at age t given $R_a = r' < r$.

can cause some ex post inefficiency. To see this note that a necessary condition for ex post social optimality is that the worker leave the research sector whenever the value of staying with zero expected cost to the firm (y_i = 0) is less than value of moving, i.e. whenever a+1 and r+i are such that

$$V(0,r+i,a+1) < v(a+1).$$

The first order condition for leaving, (20), can be written

$$V(y_i,r+i,a+1) - (\alpha\lambda/\beta)y_i < v(a+1).$$

But for $y_i \geq 0$, $\lambda = V_x(y_i,r+i,a+1)$ so by concavity of V and $\beta \geq \alpha$,

$$V(y_i,r+i,a+1) - (\alpha\lambda/\beta)y_i \geq V(0,r+i,a+1).$$

Thus, except for the case in which V is linear and $\alpha = \beta$, the first inequality does not, in general, imply the second, i.e. the worker may stay even when it is ex post socially inefficient to do so. The concavity of V in x is a result of the concavity of u which in turn implies both risk aversion and a desire for a smooth wage profile over time (if the worker has no access to the capital market). Each of these two aspects of worker preferences contribute to the inefficiency result. By agreeing to leave should productivity fall short of the worker's opportunity cost, the worker can capture y_i in his <u>current</u> wage, but this bonus payment may be worth less to him than the sequence of future wages it represents. This is because the worker would like to smooth his consumption over time. Moreover the worker can only capture the <u>expected</u> value of the firm's commitments to him, but he loses the insurance benefits he could have should the firm retain him.

4. Conclusions

We have presented two simple, job matching models in which the tenure phenomenon emerges as a property of the equilibrium allocations of the models. While the risk-neutral model exhibits all of the turnover aspects of tenure, it exhibits none of the contractual aspects. For this reason, a model with risk averse workers is analyzed. This model has similar (but not identical) turnover characteristics to the risk-neutral model and also exhibits the main contractual aspects of tenure. In particular, in the risk-averse case, firms provide wage insurance to the workers, and this in turn results in the phenomenon that not all separations are voluntary on the part of the worker.

Possibly the main drawback of the current model is that all aspects of moral hazard are omitted. It is therefore impossible to address issues such as the extent to which insurance aspects of tenure lead to excessive shirking by workers. This feature is an important one which should be addressed in future research in this area.

APPENDIX

Lemma 1. $V(a,r)$ in (7) can be replaced by $v(a)$, i.e., for any (a,r), (7) holds at $(a,r,)$ if and only if

$$\rho v(a) > q(a,r) + q(a,r)\,[V(a,r+1) - v(a)] + \dot{v}(a). \tag{8}$$

Proof. First suppose (7) holds at (a,r). Then, from (FE), $V(\cdot,r) \equiv v(\cdot)$ in a neighborhood of a. Therefore $\dot{V}(a,r) = \dot{v}(a)$ and (8) follows by substitution.

Now suppose (8) holds. Then (8) holds on a neighborhood of a so v satisfies (FE) on a neighborhood of a. This implies that $V(\cdot,r) \equiv v(\cdot)$ on this neighborhood, for suppose not. Then

$$\begin{aligned} \rho V(a,r) &= q(a,r) + q(a,r)\big[V(a,r+1) - V(a,r)\big] + \dot{V}(a,r) \\ &\geqslant \rho v(a) \text{ from (FE)} \\ &> q(a,r) + q(a,r)\,\big[V(a,r+1) - v(a)\big] + \dot{v}(a) \quad \text{from (8)} \end{aligned}$$

so

$$0 \geqslant q(a,r)\,\big[v(a) - V(a,r)\big] > \dot{v}(a) - \dot{V}(a,r)$$

or

$$\dot{v}(a) > \dot{V}(a,r).$$

But from (8)

$$\dot{v}(a) < \big[\rho + q(a,r)\big]\; v(a) - q(a,r)\,\big[1 + V(a,r+1)\big]$$

and from (FE)

$$\dot{V}(a,r) = \big[\rho + q(a,r)\big]V(a,r) - q(a,r)\big[1 + V(a,r+1)\big].$$

Therefore

$$v(a) > V(a,r)$$

which contradicts (FE). We conclude that $V(\cdot,r) \equiv v(\cdot)$ on this neighborhood of a and the result follows from substitution. Q.E.D

Lemma 2. If a scholar exits the research sector, then any scholar of the same age but inferior record will also exit, i.e., if $(a,r) \in E$ and $r' \leq r$, then $(a,r') \in E$.

Proof. By backwards induction on r. Clearly the lemma holds for $r' = r$. Suppose it holds for all $r \geq r' \geq s$ for some $s < r$. We will show it holds for $r' = s-1$. By Lemma 1, it suffices to verify that (8) holds at $(a,s-1)$. Now by the induction hypothesis and Lemma 1, (8) holds for all $r \geq r' \geq s$. Therefore $V(a,r) = v(a)$ for all $r \geq r' \geq s$ and

$$\rho v(a) > q(a,s) + q(a,s)\left[V(a,s+1) - v(a)\right] + \dot{v}(a)$$

or

$$\rho v(a) > q(a,s-1) + q(a,s-1)\left[V(a,s) - v(a)\right] + \dot{v}(a)$$

since $q(a,s-1) < q(a,s)$ and

$V(a,s) = V(a,s+1) = v(a)$.

This last inequality is just (8) at $(a,s-1)$. Q.E.D.

Lemma 3. If there is some way of reaching (A,r) without exiting, then no one with a record of r or better will ever exit, i.e., suppose r is such that $E'(A,r) \neq \emptyset$. Then for any $s \geq r$ and any a, $(a,s) \notin E$.

Proof. Suppose (a,s) is such that $s \geq r$ and $(a,s) \in E$. Now, for any $p \in P(A,r)$, $p(a) \leq r \leq s$ since p is non-decreasing. Therefore, by Lemma 2, $(a, p(a)) \in E$. Since this is true for any $p \in P(A,r)$, $E'(A,r) = \emptyset$ contrary to the assumption. Q.E.D.

Lemma 4. If $E'(A,r) \neq 0$, then for any a and $s \geq r$, $V(a,s) = q(a,s)\xi(a)$.

Proof. First we must establish a certain property of q. Using equations (1) and (2), it is easy to check that, for any (a,r),

$$\dot{q}(a,r) = -q(a,r)\left[q(a,r+1) - q(a,r)\right]. \tag{A1}$$

Since $\xi(A) = 0$, $q(a,s)\xi(a)$ satisfies (B). We need only show that it satisfies $s \geq r$

$$\rho V(a,s) = q(a,s) + q(a,s)\left[V(a,s+1) - V(a,s)\right] + \dot{V}(a,s).$$

But substituting $q(a,s)\xi(a)$ for $V(a,s)$ and $q(a,s+1)\xi(a)$ for $V(a,s+1)$ gives

$$\begin{aligned}\rho q(a,s)\xi(a) &= q(a,s) + q(a,s)\left[q(a,s+1) - q(a,s)\right]\xi(a) \\ &\quad + \left[\dot{q}(a,s)\xi(a) + q(a,s)\dot{\xi}(a)\right] \\ &= q(a,s)\left[1 + \dot{\xi}(a)\right] \quad \text{using (9)}, \\ &= q(a,s)\rho\xi(a) \quad \text{since } \dot{\xi}(a) = \rho\xi(a) - 1.\end{aligned}$$

Thus $q(a,s)\xi(a)$ does satisfy (FE). Q.E.D.

Lemma 5. (a) $q_H > W$ and

(b) $\lim_{h\to 0} V(h,0) = V(0,0)$, i.e., $V(\cdot,0)$ is continuous at 0.

Proof. Part (a) is obvious since otherwise the non-research wage would exceed the maximum possible research salary and no one would weakly prefer research at (0,0).

Now for $h > 0$, but small, the value of choosing research at (0, 0) is approximately

$$q(0,0)\,h + \left\{q(0,0)h\,V(h,1) + \left[1 - q(0,0)h\right]V(h,0)\right\}/(1 + \rho h).$$

The relation is exact in the limit as h approaches zero. Therefore since research is weakly preferred at (0,0), the limit of the above expression equals V(0,0). But that limit is $\lim_{h \to 0} V(h,0)$. Q.E.D.

Lemma 6. For some r, $E'(A, r) \neq \emptyset$, i.e. there is a path to (A, r) which does not pass through E.

Proof. Since $q_H > W$ and $\lim_{r \to \infty} q(A, r) = q_H$, for r sufficiently large, $q(A, r) > W$. Let $\bar{r}$ be any such r. Then $q(a, \bar{r}) > W$ for any $a \geq 0$. It follows that $(a, \bar{r}) \notin E$ for every $a > 0$. Now suppose $p \in E'(a, \bar{r})$ for some $a > 0$. Then define p' by $p' \equiv p$ on [0, a], $p' \equiv \bar{r}$ on [a, A]. In this case $p' \in E'(A, \bar{r})$ and we are done. Thus it suffices to show that for some $a > 0$, $E'(a, \bar{r}) \neq \emptyset$. Suppose that for every $a > 0$, $E'(a,\bar{r}) = \emptyset$. Then for any $a > 0$ there is a $0 < t < a$ such that $(t,0) \in E$, i.e. $v(t) = V(t,0)$. This implies (using Lemma 5) that

$$v(0) - K = \lim_{a \to 0} v(a) = \lim_{a \to 0} V(a,0) = V(0,0)$$

or since $K > 0$,

$$v(0) > v(0) - K = V(0, 0)$$

which is a contradiction. Q.E.D.

Lemma 7. t(r) is non-decreasing in r and $X(r) = [0, t(r))$.

Proof. Suppose $r' > r$ and $t(r') < t(r)$. By definition of t, there is $t(r') < a < t(r)$ such that $(a,r) \notin E$ but $(a,r') \in E$. Since $r' > r$, this contradicts Lemma 2. The remainder of the lemma is proved exactly as in the proof of Theorem 1. Q.E.D.

Lemma 8. A scholar remains in research until retirement if and only if the present value of a constant income equal to his current wage until retirement exceeds the value of switching, i.e., $E'(A, r) \neq \emptyset$ if and only if $q(a, r)\xi(a) \geq v(a)$ for all a.

Proof. If $E'(A, r) \neq \emptyset$, then by Lemma 4

$$V(a, r) = q(a, r)\xi(a).$$

From (FE), $V(a, r) \geq v(a)$ for all a, so $q(a, r)\xi(a) \geq v(a)$ for all a.

Now suppose $q(a,r)\xi(a) \geq v(a)$ for all a. Then this inequality holds for all $r' \geq r$. It is easy to check that $V(a, r') = q(a, r')\xi(a)$ for all $r' \geq r$ satisfies (FE) and (B) for all $r' \geq r$. Therefore, $(a, r) \notin E$ for all a. It can now be shown as in the proof of Lemma 6 that $E'(A, r) \neq \emptyset$. Q.E.D.

Theorem 5. Suppose u is continuous, strictly increasing, and concave and $u'(0) = \infty$. Let $\{V(\cdot,\cdot,a) | a = 0, \ldots, A-1\}$ be the sequence of functions defined by (12) - (16). Then for each (a,r), $V(\cdot, r, a)$ is continuous, strictly increasing and concave in x for any x such that $x + q(a,r) \geq 0$. Moreover, for any x_0 such that $x_0 + q(a,r) > 0$, $V(\cdot,r,a)$ is differentiable with respect to x at x_0 and $V_x(x_0,r,a) = u'[\bar{w}(x_0,r,a)]$ where $(\bar{w},\bar{\pi},\bar{y})$ solves the maximization problem in (12)-(15).

Before proving this theorem, we require the following

Lemma A1. Define the corresepondence

$$D(x,r,a) = \{(w,\pi,y) | (14) - (16) \text{ hold}\}$$

given $V(\cdot,\cdot,a+1)$. For any (a,r), if $V(\cdot,r,a+1)$ and $V(\cdot,r+1,a+1)$ are continuous in x, then D is continuous as a correspondence in x.

Proof. First note that $y_i \geqslant -q(a+1,r+i)$ for i=0,1 from (13). From (13) and (14), if $\pi_1 = 1$, then

$$y_1 \leqslant \left[x + q(a,r) + \beta\left(1-q(a,r)\right)\pi_0 q(a+1,r)\right]/\beta q(a,r)$$

If $\pi_1 = 0$, then y_1 is irrelevant and we may assume y_1 is bounded as above. Similar upper bounds can be derived from (13) and (14) for y_0 and w. Thus D(x,r,a) is bounded. By continuity in x of $V(\cdot,r+i,a+1)$ for i = 0,1, D(x,r,a) is also closed. Therefore D is compact valued. Continuity of D in x now follows from Theorem 2, Chapter 1 of Harris (1982). Q.E.D.

Proof of Theorem 5. From (12), the assumption that $x + q(A-1,r) \geqslant 0$, and the boundary condition (16),

$$V(x,r,A-1) = u\left(x+q(A-1,r)\right)$$

which is clearly continuous, increasing and concave in x. We proceed by backward induction. Suppose for some $a < A-1$, $V(\cdot,\cdot,a+1)$ has the required properties. Clearly, for any x,r such that $x+q(a,r) \geqslant 0$, $D(x,r,a) \neq \Phi$ since $w = x + q(a,r)$ and $\pi_1 = \pi_0 = y_1 = y_0 = 0$ is feasible. Therefore by continuity of u, continuity of $V(y_i,r+i,a+1)$ in y_i, Lemma A1 above, and Theorem 1, Chapter 1 of Harris (1982), V(x,r,a) is continuous in x.

Now suppose $x' > x$ and suppose (w,π,y) solves the maximization problem defined by (12) - (15) for x [a solution exists since u is continuous, $V(y_i,r+i,a+1)$ is continuous in y_i and D(x,r,a) is compact]. Let $w' = w+x'-x$. Clearly $(w',\pi,y) \varepsilon D(x',r,a)$ and $w' > w$. It follows that

$$V(x',r,a) \geqslant u(w') + \beta q(a,r)[V(y_1,r+1,a+1)\pi_1 + (1-\pi_1)v(a+1)]$$

$$+ \beta[1-q(a,r)][V(y_0,r,a+1)\pi_0 + (1-\pi_0)v(a+1)]$$

$$> V(x,r,a).$$

Thus $V(\cdot,r,a)$ is strictly increasing in x.

To show concavity, suppose (w,π,y) solve the maximization problem defined by (12) - (15) for x and (w',π',y') solve it for x', where $x' \neq x$. Let $\alpha \in (0,1)$,

$$(w_\alpha,\pi_\alpha,y_\alpha) = \alpha(w,\pi,y) + (1-\alpha)\,(w',\pi',y')$$

$$x_\alpha = \alpha x + (1-\alpha)x'.$$

First note that if $\pi_i \neq \pi_i'$ for some i, then one of π_i or π_i' can be assumed (without loss of generality) to be zero. In this case we can assume (without loss of generality) that $y_i = y_i'$ for that i. From this it follows easily that $(w_\alpha,\pi_\alpha,z_\alpha)$ is feasible for x_α since (14) is linear in x, w and either y_i or π_i (i=0,1) for fixed value of the other. Constraint (15) is satisfied by concavity of $V(y_i,r+i,a+1)$ in y_i. Therefore

$$V(x_\alpha,r,a) \geqslant u(w_\alpha) + \beta q\left[\pi_{1\alpha}V(y_{1\alpha},r+1,a+1) + (1-\pi_{1\alpha})v(a+1)\right]$$

$$+ \beta(1-q)\left[\pi_{0\alpha}V(y_{0\alpha},r,a+1) + (1-\pi_{0\alpha})v(a+1)\right] \qquad \text{(i)}$$

[where q = q(a,r)]. Concavity of u implies that

$$u(w_\alpha) \geqslant \alpha u(w) + (1-\alpha)u(w').$$

If $\pi_i = \pi_i'$, then $\pi_{i\alpha}V(y_{i\alpha},r+i,a+1) \geqslant \pi_i\alpha V(y_i,r+i,a+1) + \pi_i'(1-\alpha)V(y_i',r+i,a+1)$

for i=0,1 by concavity of V and $\pi_i = \pi_i'$. If $\pi_i \neq \pi_i'$, then

$$\pi_{i\alpha}V(y_{i\alpha},r+i,a+1) = \left(\alpha\pi_i + (1-\alpha)\pi_i'\right)V(y_{i\alpha},r+i,a+1)$$

$$= \alpha\pi_i V(y_i, r+i, a+1) + (1-\alpha)\pi_i' V(y_i', r+i, a+1)$$

since $y_i = y_i' = y_{i\alpha}$. Thus from (i) [with $q = q(a,r)$ and $v = v(a+1)$]

$$V(x_\alpha, r, a) \geq \alpha u(w) + (1-\alpha)u(w')$$

$$+ \beta q[\alpha\pi_1 V(y_1, r+1, a+1) + (1-\alpha)\pi_1' V(y_1', r+1, a+1)$$

$$+ \big(\alpha(1-\pi_1) + (1-\alpha)(1-\pi_1')\big)v\big] + \beta(1-q)\big[\alpha\pi_0 V(y_0, r, a+1)$$

$$+ (1-\alpha)\pi_0' V(y_0', r, a+1) + \big(\alpha(1-\pi_0) + (1-\alpha)(1-\pi_0')\big)v\big]$$

$$= \alpha V(x, r, a) + (1-\alpha)V(x', r, a).$$

and V is concave in x.

To show differentiability, first note that since $u'(0) = \infty$, $\bar{w}(x_0, r, a) > 0$. Consider the policy

$$y_i(x,r,a) = \bar{y}_i(x_0,r,a) \qquad \text{for any } x \text{ and } i = 0,1$$

$$\pi_i(x,r,a) = \bar{\pi}_i(x_0,r,a) \qquad \text{for any } x \text{ and } i = 0,1$$

$$w(x,r,a) = x - x_0 + \bar{w}(x_0,r,a) \qquad \text{for any } x.$$

For x in some neighborhood of x_0, this is a feasible policy since $\bar{w}(x_0,r,a) > 0$, $(\bar{w},\bar{\pi},\bar{y})$ is feasible for x_0, V is continuous in x, and

$$w-q + \beta\big[\pi_1 y_1 q + \pi_0 y_0 (1-q)\big] = x - x_0 + w - q + \beta\,[\bar{\pi}_1 \bar{y}_1 q + \bar{\pi}_0 \bar{y}_0 (1-q)] = x$$

by feasibility of $(\bar{w},\bar{\pi},\bar{y})$. Define

$$\hat{V}(x,r,a) = u\big(w(x,r,a)\big) + \beta q\big[V\big(y_1(x,r,a), r+1, a+1\big)\pi_1(x,r,a)$$

$$+ \big(1-\pi_1(x,r,a)\big)v(a+1)\big] + \beta(1-q)\big[V\big(y_0(x,r,a), r, a+1\big)\pi_0(x,r,a)$$

$$+\left(1-\pi_0(x,r,a)\right)v(a+1)\big].$$

Since (w,π,y) is feasible for x in a neighborhood of x_0, $\hat{V} \leqslant V$ on this neighborhood. Moreover

$$(w,\pi,y) = (\bar{w},\bar{\pi},\bar{y}) \text{ at } x = x_0$$

so $\hat{V}(x_0,r,a) = V(x_0 r,a)$. Finally, $\hat{V}$ is differentiable with respect to x at x_0 and

$$\hat{V}_x(x_0,r,a) = u'\left(w(x_0,r,a)\right) = u'\left(\bar{w}(x_0,r,a)\right)$$

since everything multiplied by β in the expression for $\hat{V}$ is independent of x. Therefore, by Lemma 3, Chapter 1 of Harris (1982), V is differentiable with respect to x at x_0 and

$$V_x(x_0,r,a) = \hat{V}_x(x_0,r,a) = u'\left(\bar{w}(x_0,r,a)\right). \qquad \text{Q.E.D.}$$

Lemma 9. For all x,r,a, and for i = 0,1, $\pi_i = 0$ implies $v(a+1) > V(0,r+i,a+1)$.

Proof. From the first order conditions, $\pi_i = 0$ implies

$$v(a+1) > V(y_i,r+i,a+1) - \lambda y_i \alpha/\beta.$$

Therefore, due to constraint (15) it must be the case that $y_i > 0$ and since V is strictly increasing in x, $V(y_i,r+i,a+1) > V(0,r+i,a+1)$. Therefore the only effective constraint in (15) arises from the second branch and we can rewrite (15) simply as $V(y_i,r+1,a+1) \geqslant v(a+1)$. There are two cases to consider:

Case 1. Constraint (15) is not binding, that is $V(y_i,r+i,a+1) > v(a+1)$. From the first order conditions $V_x(y_i,r+i,a+1) = \lambda\alpha/\beta$. Due to the concavity of the value function in x (Theorem 5),

$$V(y_i, r+i, a+1) - V_x(y_i, r+i, a+1)y_i \geq V(0, r+i, a+1).$$

We therefore conclude $v(a+1) > V(0,r+i,a+1)$.

Case 2. The constraint (15) is binding. In this case we must have

$$v(a+1) = V(y_i, r+i, a+1).$$

Since $y_i > 0$ and V is strictly increasing in x, we can again conclude that $v(a+1) > V(0,r+i,a+1)$. Q.E.D.

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