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EXPECTATIONS, PLANS AND REALIZATIONS:

IN THEORY AND PRACTICE*

by

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INTRODUCTION

To the extent that economic behavior is purposeful and takes place over time, expectations and plans clearly play a major role.[1] Yet there has been considerable difficulty in formulating models of producer and consumer behavior that incorporate these elements in meaningful ways and that are also simple enough to permit empirical application. Hicks (1946) found a solution to the problem of formulating a dynamic theory of the firm under certainty by dating all variables and applying a static theory to the thus expanded set.[2] But such a solution fails to reveal the dynamic structure of decisions and constraints, and it does not explicitly deal with uncertainty, the costs of information, or the costs of formulating plans and decisions. In principle, theorists know how to set the problem up as a dynamic programming problem under uncertainty, in which the conditional distributions of future unknown exogenous variables are estimated by using all available information up to the current period (Nerlove, 1972).[3] While costly information is more difficult to incorporate since its value is usually not known until it is acquired, presumably a suitable Bayesian framework can be devised for incorporating this element.

In this paper, I attempt to peek into the "black box" of the firm and to explore some very simple models of expectation formation and planning, using data on a group of French and German manufacturing firms who report over time on both expectations and their subsequent realization. Although much simpler than the "correct" approach described in the preceeding paragraph, these models work well, provide a straight-forward, less rigorous, yet practical framework for the empirical analysis of plans and expectations, and are a useful point of departure for future research.

The plan of the paper is as follows: First, I discuss a number of different models of expectation and plan formation, the French and German survey data and problems connected with their analysis; next I present the results of a series of empirical investigations, dealing with the relation of prior plans and expectations to subsequent realizations, with the formation and revision of production plans, price expectations, and expectations of future demand, and with some simple models of expectations and plan fulfillment. In conclusion, a summary of the results and the questions they raise for future research is presented. A technical appendix contains a more detailed discussion of the principal methodological tool.

PART I: BACKGROUND

1. Models of Expectation Formation and Planning

The Hicksian model of dynamic planning under certainty is the basis for a more empirically relevant framework for the analysis of plans and expectations, which is developed in the work of Modigliani and Cohen (1961) and which is the starting point for the investigations reported here.[4] The Hicksian assumption of certainty means that information about the future value of a variable is single-valued and costless. We continue to regard expectations and plans as single-valued but to recognize that the economic agent knows that they may turn out to be wrong. As of a particular date information about the future can be acquired only at a cost, albeit a cost which decreases for a particular future date as that date draws near. Planning and decision making are themselves costly activities. Therefore only what is necessary to plan will be planned, only decisions which cannot be postponed will be made, and only the information about the future necessary to those plans and decisions and only to the accuracy warranted by the cost of error will be gathered. Plans will not always be fulfilled, single-valued expectations will often turn out to be wrong, and both will be continually revised.

Extrapolative Expectations

Use of weighted averages of past readily observable data, or linear extrapolations, has negligible cost and yields a single-valued expectation, which is perhaps why such extrapolations have been so popular with econometricians.[5] The class of extrapolative expectations includes adaptive expectations and expectations based on forecasts from time-series models (including quasi-rational expectations) and some formulations of rational expectations.

Let X_t be the realized value of a variable in period t (in the data used below, this is reported on a survey at the end of period t). Let ${}_{t+1}X^*_t$ be the value projected for period t+1 in period t. The general form of extrapolative expectations is

$$(1) \qquad {}_{t+1}X^*_t = \sum_{j=0}^{\infty} w_j X_{t-j} \quad ,$$

where the w_j are some fixed weights, which may, for example, have been chosen to yield a forecast, X^*, with certain statistical properties.[6] The form (1) can be extended in an obvious manner to periods farther in the future than the next and to forecasts involving the past values of several variables. Adaptive expectations result when the weights decline geometrically,

$$w_j = \beta(1-\beta)^j, \; 0<\beta<1 \quad ,$$

in which case

$$(2) \qquad {}_{t+1}X^*_t - {}_tX^*_{t-1} = \beta[X_t - {}_tX^*_{t-1}] \quad .$$

In this form, the model is sometimes known as the error-learning model.

Below, I use both the form (2) and a less restrictive form

(3) $$ {}_{t+1}X^{*}_{t} = \beta_1 X_t + \beta_2 \; {}_{t}X^{*}_{t-1} \; . $$

It is well known that in the case of any covariance-stationary time series the minimum mean-square error (MMSE) forecast can always be written as a linear function of past one-step-ahead forecast errors

(4) $$ {}_{t+k}X^{*}_{t} = \sum_{j=0}^{\infty} b_{kj}[X_{t+1-j} - {}_{t+1-j}X^{*}_{t-j}] \; , \; k>0, $$

where

$$ \sum_{j=0}^{\infty} | b_{kj} |^2 < \infty \; . $$

In general, MMSE, forecasts for covariance-stationary time series have greater stability of long-term expectations than of short-term, although this conclusion may be modified by adding trends, introducing non-stationarities in the manner of Box-Jenkins, or adding seasonal components. This property of MMSE forecasts may account, in part, for the commonly observed regressivity of expectations.[7]

Rational Expectations

In the early 1960's, Mills (1962) introduced the notion of "implicit expectations" and Muth (1961) of "rational expectations." The idea of the former is to employ future values as proxies for anticipations of them on the grounds that, on the whole, economic agents forecast successfully and errors are small. Rational expectations are based on a broadly similar hypothesis but one which, especially in recent years, has taken different specific forms. The underlying idea is simply that economic agents behave purposefully in collecting and using information just as they do in other activities. In this general form the hypothesis is a compelling one, but in practice this idea is often translated into the requirement that expectations are, in the model at hand, formed in a way that is stochastically consistent with the behavior of the realized values of the variables in question. This is clearly a much stronger hypothesis, which one can reasonably dispute. Clearly, our models of behavior are imperfect and, however attractive consistency may be, it would be unreasonable to insist that expectations and behavior are necessarily generated by the same stochastic approximation, with every restriction pertaining to the one incorporated in the other.[8]

A more attractive but weaker form of the rational expectations hypothesis is simply that there is no pattern of systematic error.[9] Purposeful economic agents have incentives to eliminate such errors up to a point justified by the costs of obtaining the information necessary to do so. (See Feige and Pearce, 1976.) The most readily available and least costly information about the future value of a variable is its past values. Moreover, in the absence of structural change, the final form of an econometric model leads under fairly general conditions to univariate relations between the current value of a variable and its own past values.[10] Thus one possible approach to modeling expectation formation, consistent with the rational expectations hypothesis, would be to generate expectations as MMSE forecasts (also conditional expectations in the mathematical sense) from empirical time-series models for the variables to be forecast by the economic agents whose behavior we are studying. Elsewhere (Nerlove, Grether, Carvalho, 1979), I and others have called such expectations quasi-rational. They are not fully efficient because they neglect some of the restrictions implied by the behavioral model; but, of course, those restrictions may not be correct because the model itself is only an approximation. In the absence of structural change, quasi-rational expectations satisfy the minimal requirement of rational expectations: they are unbiased forecasts of the realized future values. Moreover, there will be no systematic components in the forecast errors.

Inference about Expectation and Plan Formation

In the form in which they are usually applied, all of the foregoing approaches to the modeling of expectations and plans have one common feature: a model of expectation formation is embedded in a behavioral model. We can infer how expectations are formed and how they influence plans and behavior only indirectly by observing realized outcomes and the values of variables representing external forces impinging on the firm. This means that errors in the formulation of a behavioral model affect conclusions regarding expectation formation and plans.[11] For this reason it is important to obtain independent evidence on expectations and plans and to devise models which are able to explain variations in such variables directly.

In recent years, a number of studies have been made using aggregates of expectational data obtained from surveys. Turnovsky (1970), Carlson (1977) and Jacobs and Jones (1980) have analyzed aggregates from a survey of general price-level expectations. Similar studies of aggregates from surveys have been made by Knoebl (1974), deMenil and Bhalla (1975) and Carlson and Parkin (1975). deLeeuw and McKelvey (1981) analyze information collected in the year-end surveys of business plant and equipment by the U.S. Bureau of

Economic Analysis on the expected prices of capital goods purchased and the prices of goods and services sold. There are few studies which make use of the micro-data themselves to study reported expectations and plans directly and to test various models of the formation and revision process.[12]

While the use of aggregates derived from surveys is an important first step in the analysis of expectations and plans, such analysis should be supplemented by studies based on the microdata themselves for several reasons: First, the micro-data should be consistent with hypotheses regarding the behavior of the aggregates; for example, expectational aggregates could provide unbiased forecasts of realized aggregates, as asserted by the theory of rational expectations, yet forecasts of individual agents could be systematically and persistently biased. Second, some factors affecting deviations between expectations or plans and subsequent realizations may affect all individuals simultaneously yet vary from period to period and some factors may affect individuals; only through analysis of the micro-data can we disentangle those two groups of effects. Finally, individual variation in variables related to expectations, plans and realizations may be reduced or obscured in aggregated data.

2. The French and German Business Tests

For many years and in a large number of countries, data have been collected from individual firms on expectations, plans, appraisals, and past realizations for a variety of variables (Strigel, 1977). The oldest and most famous of these surveys is that done by the Ifo-Institut, Munich, every month since November 1949 for the Federal Republic of Germany. Other surveys, such as the quadrimestrial survey conducted by the Service de la Conjoncture of the

Institut National de la Statistique et des Etudes Economiques (INSEE), beginning in 1951 for France, are administered less frequently but often contain data on a greater number of variables. These business tests, as they are called, offer a unique source of data for the analysis of how expectations and plans are formed and revised at the level of the individual firm. The empirical investigations reported in this paper are based on monthly data, in the case of Germany, for approximately 4500 individual establishments over the period January 1977 through December 1978.[13] For France, the data are for approximately 1600 firms for the period June 1974 through June 1978, collected in March, June and November.[14]

Almost all the data are categorical, indeed most are trichotomous.[15] The categorical data can be classified into three groups:

(i) Variables that reflect plans or expectations (<u>ex ante</u> data).

(ii) Variables referring to realizations (<u>ex post</u> data).

(iii) Variables indicating appraisals.

Responses are in the form: Increase (+), no change (=), or decrease (-); or greater than normal (+), normal (=), or less than normal (-); or too large (+), about right (=), or too small (-).[16] In most previous work, so-called balances have been used to aggregate the categorical responses obtained from business-test surveys. In this procedure, the number or percent of respondents reporting a "-" is subtracted from the number or percent reporting a "+". As noted by Carlson and Parkin (1975), the aggregate balances neglect the information afforded by the no-change, "=", category. The balances are typically used as time-series data, for example, as leading indicators, or in analyses relating one series to another. It is easy, however, to construct an example for two categorical variables which are in fact independent but for which the balances show a perfect positive correlation over time (Koenig, Nerlove, Oudiz, 1981a). This situation may arise because changing common environmental factors influence all individuals over time. As such, the problem is analogous to spurious correlation among time series, but the use of micro-data may permit the true nature of the relationship (in the example, independence) to be established, even though it may not be possible to specify or observe the common environmental factors responsible for the spurious relationship among the aggregate variables. The methods used here to analyse the micro-data from the Ifo and INSEE surveys do not aggregate categories and do treat the categories symmetrically.

The data available from the German (Ifo) and the French (INSEE) surveys are summarized in Table 1.

Table 1: German (Ifo) and French (INSEE) Business-Test Variables

Variable	Plans or Expectations	Realizations	Appraisals
Business Conditions (Ifo)	G^*	G	--
Demand (INSEE)	D+	D	D^a
Production	Q^*	Q	--
Inventories of Finished Products	--	L (INSEE)	L^a
Backlog of Orders	--	S (Ifo)	S^a
Domestic Selling Prices	P^*	P	--
Incoming New Orders (Ifo)	--	A	--

The variable designations employed are German and French mnemonics; thus, G is "Geschäftslage," etc. Data are also available for exports and, for the French surveys, on stocks of materials and semifinished products, foreign demand, delivery delays, labor force and hours, expectations of the general level of industrial production, general price level and general wage level. Continuous data on capacity utilization and on weeks of production represented by inventories of finished products and order backlogs are available in certain months from the Ifo surveys and from all INSEE surveys. Continuous data on the wage rate for the firm are avilable from the French survey.[17]

3. Some Problems Encountered in the Analysis of Categorical Data

The difficulties encountered in the analysis of categorical data may be illustrated by the problems of defining a change over time and a forecast error (surprise or failure to fulfill a plan). For example, firms report whether they plan to increase production, decrease it, or maintain it at its present level. Subsequently, they report in the same categories what has happened to the level of production. In the absence of any order among the categories, each individual prediction would be merely right or wrong: either the firm correctly anticipated that it would increase production or it did not. There is no sense in which one prediction would be closer to what subsequently happened than another. When the categories are ordered, as in the case of the business-test responses, a greater, but still limited, possibility exists for such comparisons; for example, when an increase occurs, a prediction of no change is better than a prediction of a decrease although both are wrong.

In what follows, I deal only with consecutive pairs of surveys at three-month intervals for the German data and at three-, four-, or or five-month intervals for the French data. Consequently, the pre-subscript indicating the period for which a forecast or plan is made may be dropped.

Let X^*_{-1} represent a reported expectation or plan on a prior survey and let X be the corresponding realization reported on a subsequent survey. In Koenig, Nerlove, Oudiz (1981a), we define a surprise, or failure to fulfill a plan, as a new variable, EX, with categories given by the table.[18]

		X^*_{-1}: +	+	−
	+	=	+	+
EX = X	=	−	=	+
	−	−	−	=

Figure 1

For the most part, the responses already reflect changes rather than levels. In some instances, however, one may wish to examine changes in the direction of change, realized or anticipated. This may be done by defining a new variable, ΔX, by means of table similar to that used in the definition of EX

		X_{-1} +	X_{-1} =	X_{-1} -
$\Delta X = X$	+	=	+	+
	=	-	=	-
	-	-	-	=

Figure 2

where X_{-1} refers to the realization reported on a prior survey.

In Part II, the first empirical investigation reported deals with the question of how well firms forecast future changes or to what extent plans are fulfilled. In particular we are concerned with the temporal stability of the cross-sectional conditional distributions of realizations given prior expectations/plans. In contrast to variables which are quantitative, relations between predictions and outcomes cannot be numerically expressed and distances between forecasts and outcomes cannot be defined in the usual way. Criteria such as mean-absolute error or mean-square error cannot be used to assess the goodness-of-fit of forecasts of aggregates or of individual data or to compare forecasts made at one time with those made at another. As noted above, forecasts are either right or wrong or, at best, more or less wrong.

Forecasting individual observations are what Hildenbrand, et al. (1977) call "forecasting events." In the case of quantitative variables, it is possible to form aggregates of individuals. Thus, for example, one might aggregate individual firms' expectations of future sales and compare these with subsequently realized industry sales. Similar aggregation procedures may be applied to categorical data and, indeed, may be highly desirable because of the "all or nothing" character of the individual forecasts of events. Let X^f be a forecast of a variable, the realization of which in a subsequent period we denote by X. Let X^f and X have the same categories. One possibility would be to analyse forecast errors defined as EX is defined in Figure 1, replacing X^*_{-1} by X^f; this converts individual forecast errors into a third categorical variable. An alternative is to consider the properties of a 2-way table relating forecasts and realizations by counts of firms falling in each combination of categories. Replacing "+", "=", "-", in subscripts by 1,2,3, respectively, this table is

		X^f +	X^f =	X^f −	
X	+	N_{11}	N_{12}	N_{13}	$N_{1.}$
X	=	N_{21}	N_{22}	N_{23}	$N_{2.}$
X	−	N_{31}	N_{32}	N_{33}	$N_{3.}$
		$N_{.1}$	$N_{.2}$	$N_{.3}$	

Figure 3

where $N_{i.}$ and $N_{.j}$ are the marginal row and column totals. The marginal distributions and related balances of forecast and realization may be compared over time to assess, for example, systematic bias; but it should be understood that, like all aggregates, such variables may conceal rather than reveal what is actually going on.

Perfect forecasting in the context of categorical variables may be defined as a situation in which every firm correctly forecasts the direction of change.

Clearly it is possible, in the context of quantitative variables, for perfect categorical forecasts to be very far off the mark, and, conversely, for imperfect categorical forecasts to be close quantitatively. Comparison of the marginals may also be misleading as illustrated by the example in Figure 4.

		X^f +	X^f =	X^f −	
X	+	0	25	25	50
X	=	25	50	25	100
X	−	25	25	0	50
		50	100	50	

Figure 4

In the table of Figure 4, the margins fit perfectly yet the forecasts are quite bad.[19]

Even if expectations or plans do not forecast subsequent realizations perfectly, in the sense defined, there is always a close relation between the two. Because realizations are not independent of earlier expectations or plans the conditional distribution of X depends on X^*_{-1}. Let the conditional distribution of X given X^*_{-1} be $P(X \mid X^*_{-1})$ and let $P(X)$ and $P(X^*_{-1})$ be the respective marginal distributions. Then knowledge of the conditional distribution would enable us to deduce the marginal distribution of X exactly from the marginal distribution of X^*_{-1} by means of the identity

$$(1) \qquad P(X) = \Sigma\, P(X \mid X^*_{-1})\, P(X^*_{-1})$$

where the summation is taken over the categories of X^*_{-1}.

In a true forecasting context, the current conditional is not known. If the conditional distribution does not vary very much from one survey date to the next, an estimate, such as

$$(2) \qquad \hat{P}(X \mid X^*_{-1}) = P(X_{-1} \mid X^*_{-2})$$

may be used in (1) to estimate the marginal distribution of X from that of X^*_{-1}. We may imagine an implicit random variable X^f_c for which

$$\Sigma\hat{P}(X \mid X^*_{-1})\ P(X^*_{-1})$$

is the marginal distribution.[20] Call this implicit random variable the <u>conditional forecast</u> of X. It is conditional on data from two previous surveys.

The possibility of constructing conditional forecasts provides an additional reason for studying the time stability of the conditional distributions $P(X \mid X^*_{-1})$, although this question is of intrinsic interest.

In formulating models of expectation/plan formation and revision as well as in the analysis of the temporal stability of conditional distributions of realizations given prior expectation/plans, it is highly useful to have a parametric formulation of the joint multivariate distribution of three or more categorical variables and the associated conditional distributions. The parametric formulation used in the empirical investigations reported below is the log-linear probability model. (See the appendix.) The two most important properties of the model for my purposes here are: (1) The conditional probabilities associated with a joint distribution characterized by the log-linear probability model are also log-linear in form but involve a reduced set of parameters. (2) The model decomposes the logarithm of the probability, joint or conditional, into "effects" analogous to the usual analysis-of-variance (ANOVA) effects. In this formulation the log probability is represented as the sum of an over-all mean (which is chosen so that the probabilities sum to one), main effects for each categorical variable, bivariate interaction effects for pairs of categorical variables, and so on.

In this formulation, component probabilities may be defined corresponding to each effect configuration; and, in this way, it is also possible to define measures of partial bivariate association, for example, based on the Goodman-Kruskal Gamma coefficient, normally defined only for the two variable case.[21]

Models which contain all possible effect configurations are called <u>saturated</u>. In general, one would want to restrict attention to models containing fewer configurations, for example, only main-effect and bivariate-effect configurations. Such restrictions may be justified on grounds of parsimonious representation, ease of interpretation, and considerations related to estimability.

One way to test for time stability is to formulate a log-linear probability model for the conditional distribution

$$P(X \mid X^*_{-1}, T)$$

where T is a dichotomous variable representing two survey dates and the variables X and X^*_{-1} represent realization and prior expectations or plans on either of the two survey dates. An appropriate test for stability is then whether either the bivariate interaction configuration T x X or the trivariate interaction configurative $TxXxX^*_{-1}$ is significantly different from zero.[22]

Methods of analysis based on the log-linear probability model do not, in general, permit structural estimation in the traditional econometric sense as developed for the analysis of continuous data.[23] Conditional probability models may, however, be formulated. These models contain relatively few parameters and correspond under some circumstances to reduced-form equations from a structural model. Estimation of conditional probability models permits inferences about direction and strength of associations, although, as is typically the case in the analysis of cross-section data, inferences about the direction of causality are hazardous, since it is hardly ever appropriate to assume that *post hoc* implies *propter hoc*. Nonetheless, throughout the remainder of this paper, I do generally assume that timing determines direction of causality, i.e., that prior events, plans and expectations reported by the firm influence current expectations and plans and not *vice versa*. Causality is assumed to run in the direction past to present.

PART II: EMPIRICAL INVESTIGATIONS

Koenig, Nerlove, and Oudiz (1979,1981a,b,c) report a variety of results derived from the French and German business-test data related to expectations, plans and realizations. In this section I selectively review and summarize the main findings and report a few additional complementary analyses.

1. Prior Expectations and Plans in Relation to Realizations[24]

How well do firms' expectations or plans predict subsequent realizations? To what extent is the relation between expectations or plans and subsequent realizations a stable one, irrespective of how well the former predict the latter? These are the first questions one asks about the French and German business-test data. I argue in Part I that comparing marginal distributions of firms' forecasts (expectations or plans) with the marginal distributions of realizations could be a rather misleading indicator of the quality of the forecasts. Inspection of the contingency tables between X^*_{-1} and X, however, suggests that the marginals do provide a fairly accurate summary of the extent to which the observations are concentrated along the diagonal. The balances derived from the marginals, while helpful in revealing broad tendencies, conceal considerable information. In particular, the apparent stability over time in the marginal distributions for the German data conceals considerable individual variations in response over time.

In Koenig, Nerlove, Oudiz (1981c), we present marginal distributions and balances for both the French and German data for prices, demand, and production for realizations, expectations or plans, and conditional forecasts. Tests for the presence of significant bivariate and trivariate interaction configurations involving the variable T, the realization, and the expectation or plan are also presented for each triple of consecutive survey dates. Here I illustrate these results with two examples: production plans and realizations for the German data and demand expectations and realizations for the French data. The marginal distributions and balances are presented in Tables 2-3 and χ^2 -tests of stability in Tables 4-5.

The first thing to note is that there is very little variation in the

Table 2: German Data. Marginal Distributions and Balances. Realizations, Firms' Forecasts and Conditional Forecasts. Production. $\bar{Q}$ and Q*. (Percent)

Period and Variable	Marginal Distribution			Balance
	+	=	-	
April 1977 - July 1977				
$\bar{Q}$	15	53	31	-16
Q^{*}_{-1}	11	77	12	-1
Q^{f}_{c}	21	53	26	-5
July 1977 - October 1977				
$\bar{Q}$	17	53	30	-13
Q^{*}_{-1}	7	79	14	-7
Q^{f}_{c}	16	53	31	-15
October 1977 - January 1978				
$\bar{Q}$	16	49	35	-19
Q^{*}_{-1}	10	77	13	-3
Q^{f}_{c}	15	51	34	-19
January 1978 - April 1978				
$\bar{Q}$	18	53	29	-11
Q^{*}_{-1}	5	76	19	-14
Q^{f}_{c}	18	50	32	-14
April 1978 - July 1978				
$\bar{Q}$	18	53	29	-11
Q^{*}_{-1}	9	77	14	-5
Q^{f}_{c}	18	53	29	-11
July 1978 - October 1978				
$\bar{Q}$	23	53	24	-1
Q^{*}_{-1}	8	78	14	-6
Q^{f}_{c}	20	53	27	-7

Table 3: French Data. Marginal Distributions and Balances. Realizations, Firms' Forecasts and Conditional Forecasts. Demand. D and D*. (Percent)

Period and Variable	Marginal Distribution			Balance
	+	=	-	
November 1974 - March 1975				
D	10	32	58	-48
D^*_{-1}	8	44	48	-40
D^f_c	10	36	54	-44
March 1975 - June 1975				
D	11	33	56	-45
D^*_{-1}	14	53	33	-19
D^f_c	14	34	52	-38
June 1975 - November 1975				
D	18	42	40	-22
D^*_{-1}	12	52	36	-24
D^f_c	10	33	57	-47
November 1975 - March 1976				
D	30	43	27	3
D^*_{-1}	20	55	25	-5
D^f_c	21	43	36	-15
March 1976 - June 1976				
D	39	41	20	19
D^*_{-1}	28	57	15	13
D^f_c	33	44	23	10
June 1976 - November 1976				
D	30	46	24	6
D^*_{-1}	23	62	15	8
D^f_c	38	41	21	17

Table 3 (continued):

Period and Variable	Marginal Distribution +	Marginal Distribution =	Marginal Distribution −	Balance
November 1976 - March 1977				
D	22	49	29	-7
D^*_{-1}	10	57	33	-23
D^f_c	25	44	31	-6
March 1977 - June 1977				
D	20	47	33	-13
D^*_{-1}	20	61	19	1
D^f_c	26	50	24	2
June 1977 - November 1977				
D	18	45	37	-19
D^*_{-1}	13	59	27	-14
D^f_c	18	46	36	-18
November 1977 - March 1978				
D	18	47	35	-17
D^*_{-1}	14	59	27	-13
D^f_c	18	45	37	-19
March 1978 - June 1978				
D	23	49	28	-5
D^*_{-1}	21	60	19	2
D^f_c	20	48	32	-12

Table 4: German Data. Chi-square Values and Associated Probabilities for Tests of Significance of Certain Interactions in the Model $\{\bar{Q}|Q^*_{-1}, T\}$.

Period and Number of Observations	Bivariate Interaction $T \times \bar{Q}$ χ^2	Prob.	Trivariate Interaction $T \times \bar{Q} \times Q^*$ χ^2	Prob.
Jan. '77, Apr. '77, July '77, 2476	19.6	0.000	2.28	0.684
Apr. '77, July '77, Oct. '77 2293	0.560	0.756	3.84	0.428
July '77, Oct. '77, Jan. '78 2218	1.39	0.499	1.56	0.815
Oct. 77, Jan '78, Apr. '78 2497	5.07	0.079	2.29	0.683
Jan. '78, Apr. '78, July '78 2650	0.440	0.802	1.89	0.755
Apr. '78, July '78, Oct. '78 2640	1.83	0.401	3.28	0.512

Table 5: French Data. Chi-square Values and Associated Probabilities for Tests of Significance of Certain Interactions in the Model $\{D|D^*_{-1}, T\}$.

Period and Number of Observations	Bivariate Interaction T x D χ^2	Bivariate Interaction T x D Prob.	Trivariate Interaction T x D x D^*_{-1} χ^2	Trivariate Interaction T x D x D^*_{-1} Prob.
June '74, Nov. '74, Mar. '75 835	2.94	0.230	16.5	0.002
Nov. '74, Mar. '75, June '75 926	6.25	0.043	6.29	0.178
Mar. '75, June '75, Nov. '75 1160	57.9	0.000	4.67	0.323
June '75, Nov. '75, Mar. '76 1220	21.3	0.000	4.22	0.377
Nov. '75, Mar. '76, June '76 1153	7.00	0.030	9.66	0.047
Mar. '76, June '76, Nov. '76 1109	4.98	0.083	3.59	0.464
June '76, Nov. '76, Mar. '77 1000	1.06	0.588	3.53	0.473
Nov. '76, Mar. '77, June '77 1003	20.1	0.000	4.66	0.324
Mar. '77, June '77, Nov. '77 1223	0.157	0.924	4.44	0.350
June '77, Nov. '77, Mar. '78 1215	0.447	0.800	3.69	0.450
Nov. '77, Mar. '78, June '78 1355	5.59	0.061	9.22	0.056

marginal distributions of $\bar{Q}$, Q^{*}_{-1} and Q^{f}_{c} for the German data and substantial variation in D, D^{*}_{-1} and D^{f}_{c} for the French data. This is characteristic of the other variables examined, little variation for the German data, much variation for the French data, irrespective of concentration in the no-change categories. Despite concentration in the no-change category for prices and production and in the minus category for incoming orders, the balances for the German data exhibit a similar lack of variability. While the French responses tend to be concentrated in the no-change category for prices and production, there is considerable variability for demand. This variability is also exhibited by the balances.

The second thing to note is that the strong bias towards the no-change category in the production plans or demand expectations as compared with their respective realizations is almost completely corrected by the conditional forecasts. In the case of the German firms the balances of both realizations and plans are consistently negative, but the balances of the plans generally tend to underestimate the absolute values of the balances of the realizations; that is, German firms underestimate both increases and decreases, but they tend to report a planned decrease in production proportionately less frequently. This too is corrected by the conditional forecasts. While the general tendency to forecast no change in demand by French firms is corrected by the conditional forecasts, the balances are often further off for the conditional forecasts than for the firms' own projections. This is a consequence of a more stable conditional distribution $P(\bar{Q} \mid Q^{*}_{-1})$ for the German firms than the conditional distribution $P(D \mid D^{*}_{-1})$ for the French firms.

The third thing to note appears in the stability tests reported in Table 4 for the German firms and in Table 5 for the French firms. In only one case is the bivariate interaction configuration T x $\bar{Q}$ significantly different from zero for the German firms and in no case is the trivariate interaction configuration significantly different from zero; thus, in all but one case, we can accept the null hypothesis of no change in consecutive pairs of surveys of $P(\bar{Q} \mid Q^*_{-1})$. In the one case we cannot accept the null hypothesis, the change appears to be confined to the main effect.[25] For the French firms, on the other hand, the bivariate effect configurations TxD are significantly different from zero at the 10% level in seven out of eleven cases, while the trivariate effect configurations $TxDxD^*_{-1}$ are significant in only three of the eleven cases at the same level. This suggests that the conditional distribution of realizations given prior expectations/plans is more unstable for the French than for the German firms but that, in both cases, the instability is concentrated in the main effect parameters of the distribution. Since these parameters reflect factors affecting all firms simultaneously, it would not be unreasonable to conclude that economy-wide variations are causing changes in the conditional distributions. The same finding of relative stability for the German firms and changes which are confined to main effect parameters for the most part reappears in the data for business conditions, prices and production.

The comparative instability of the French conditional distributions of realizations given prior expectations or plans is perhaps directly attributable to the greater changes taking place in the French economy over the period for which I have data. Clearly, the more variation, the more difficult it will be to forecast demand or prices or other relevant variables and the less likely it will be for firms to adhere to production plans. The more variation in other relevant variables, the less stable will be the relation between prior forecasts and subsequent realizations.

2. Results on Formation of Production Plans[26]

In this section I report results from a model of the formation of production plans. This model relates the conditional probability of reporting a planned increase, no change, or a decrease in production to recent past changes in production, appraisal of inventory levels (for firms carrying inventories of finished products) and expectations of future demand or business conditions. In the notation introduced above, the model estimated from the data is $\{Q^* \mid \bar{A} \text{ or } D\ ,\ L^a,\ G^* \text{ or } D^*\ \}$.[27] We would expect to find that the probability that a firm will report a planned increase in production will be higher if it has recently experienced an increase in demand or incoming orders, if it reports inventories too low, or if it expects an improvement in business conditions or demand for the product. Conversely, the probability of reporting a planned decrease in production should be higher if the opposite is true. Thus, we would expect a positive association between Q^* and $\bar{A}$ or D and between Q^* and G^* or D^*. Depending on the order of the responses for L^a, we expect a positive association (too low, about right, too high) for the German firms or a negative association (above average, average, below average) for the French firms.

The principle tool in the statistical analyses reported here is the log-linear probability model and the component gamma coefficient, γ, derived from that model (see Kawasaki, 1979), based on the familiar bivariate Goodman-Kruskal measure of association for pairs of ordered categorical variables.[28]

Tables 6 and 7 report results for the German data and Tables 8 and 9 for the French data. The first of each pair of tables shows the component gamma, its associated t-statistic, and the configuration χ^2 for each bivariate interaction between Q^* and one of the conditioning variables in the model $\{Q^* \mid \bar{A} \text{ or } D,\ L^a,\ G^* \text{ or } D^*\}$. The second of each pair presents the complete set

Table 6: German Data. Conditional Model $\{Q^* | \bar{A}, L^a, G^*\}$. Bivariate Component Gamma, t-ratio, and Configuration Chi-square.

Period. Number of Observations. Item.	Bivariate Interaction		
	$Q^* \times \bar{A}$	$Q^* \times L^a$	$Q^* \times G^*$
April 1977 (1842)			
γ	0.509	0.543	0.875
t	(7.79)	(5.65)	(25.6)
χ^2	79.2	65.8	340.
July 1977 (1897)			
γ	0.545	0.443	0.909
t	(7.30)	(4.05)	(34.1)
χ^2	66.1	30.2	333.
October 1977 (2026)			
γ	0.362	0.558	0.897
t	(5.12)	(5.93)	(27.7)
χ^2	44.0	42.1	370.
January 1978 (1881)			
γ	0.406	0.501	0.933
t	(6.01)	(4.51)	(43.7)
χ^2	52.8	55.1	371.
April 1978 (2054)			
γ	0.523	0.300	0.889
t	(8.83)	(2.39)	(26.0)
χ^2	95.1	45.9	338.
July 1978 (2011)			
γ	0.416	0.497*	0.922
t	(4.34)	(6.42)	(37.5)
χ^2	59.3	28.1	403.
October 1978 (2018)			
γ	0.530	0.322	0.880
t	(8.77)	(2.71)	(23.4)
χ^2	70.1	26.3	350.

*Partially estimated configuration. 1 degree of freedom.

Table 7: German Data. Estimates of the Conditional Model $\{Q^* | \bar{A}, L^a, G^*\}$ for October 1977 and October 1978. (t-ratios in parenthesis below estimates.)

Period. Number of Observations. Item.		Variable at Level 1	Level 2	Level 3
October 1977 (2026)				
Main Effect		-0.992	1.657	-0.665
Q^*		(6.39)	(15.6)	(4.07)
Bivariate Interaction		0.445	-0.103	-0.342
$Q^* \times \bar{A}$		(3.82)	(0.748)	(2.74)
		-0.099	0.224	-0.125
		(1.32)	(2.76)	(1.75)
		-0.346	-0.122	0.467
		(3.12)	(1.09)	(5.12)
γ	0.362			
	(5.12)			
Bivariate Interaction		0.713	0.077	-0.791
$Q^* \times L^a$		(3.77)	(0.602)	(4.45)
		-0.084	-0.001	0.085
		(0.645)	(0.002)	(0.820)
		-0.529	-0.077	0.706
		(3.03)	(0.644)	(5.20)
γ	0.558			
	(5.93)			
Bivariate Interaction		1.616	-0.286	-1.330
$Q^* \times G^*$		(9.22)	(2.05)	(5.59)
		-0.205	0.410	-0.205
		(1.43)	(4.39)	(1.46)
		-1.411	-0.124	1.536
		(6.49)	(0.993)	(9.64)
γ	0.897			
	(29.7)			

Table 7 (continued):

Period. Number of Observations. Item.		Variable at Level 1	Level 2	Level 3
October 1978 (2018)				
Main Effect		-1.051	1.599	-0.548
Q^*		(5.76)	(14.5)	(3.68)
Bivariate Interaction		0.558	0.174	-0.731
$Q^* \times \bar{A}$		(5.07)	(1.46)	(5.09)
		-0.037	0.088	-0.051
		(0.502)	(1.17)	(0.623)
		-0.521	-0.262	0.783
		(4.64)	(2.36)	(7.65)
γ	0.530			
	(8.77)			
Bivariate Interaction		0.436	-0.183	-0.254
$Q^* \times L^a$		(2.62)	(1.65)	(1.85)
		-0.074	0.190	-0.116
		(0.589)	(2.49)	(1.32)
		-0.363	-0.007	0.370
		(1.73)	(0.062)	(2.85)
γ	0.322			
	(2.71)			
Bivariate Interaction		1.646	-0.600	-1.046
$Q^* \times G^*$		(8.81)	(4.98)	(6.06)
		-0.135	0.460	-0.325
		(0.752)	(4.54)	(2.55)
		-1.510	0.140	1.371
		(4.68)	(0.824)	(7.49)
γ	0.880			
	(23.4)			

Table 8: French Data. Conditional Model $\{Q^* | D, L^a, D^*\}$. Bivariate Component Gamma and t-Ratio. Configuration Chi-square.

Period. Number of Observations. Item.	Bivariate Interaction		
	$Q^* \times D$	$Q^* \times L^a$	$Q^* \times D^*$
November 1974 (779)			
γ	0.535	-0.412	0.877
t	(5.96)	(3.44)	(20.7)
χ^2	41.3	17.3	250.
March 1975 (832)			
γ	0.596	-0.227	0.932
t	(6.03)	(1.56)	(41.1)
χ^2	35.6	13.5	287.
June 1975 (884)			
γ	0.196	-0.403	0.893
t	(1.72)	(3.40)	(30.7)
χ^2	29.0	30.2	342.
November 1975 (920)			
γ	0.401	-0.377	0.907
t	(4.72)	(3.72)	(37.0)
χ^2	31.2	17.2	375.
March 1976 (906)			
γ	0.503	-0.162	0.946
t	(6.52)	(1.08)	(47.5)
χ^2	43.4	2.73	351.
June 1976 (886)			
γ	0.380	-0.162	0.909
t	(4.31)	(1.07)	(31.0)
χ^2	24.0	9.78	334.
November 1976 (894)			
γ	0.365	-0.299	0.990*
t	(3.92)	(2.43)	(303.9)
χ^2	29.1	9.01	291.

*Partially estimated configuration. 1 degree of freedom.

Table 8 (continued):

Period. Number of Observations. Item.	Bivariate Interaction		
	$Q^* \times D$	$Q^* \times L^a$	$Q^* \times D^*$
March 1977 (956)			
γ	0.258	-0.417	0.927
t	(2.68)	(3.40)	(40.1)
χ^2	26.0	30.8	329.
June 1977 (972)			
γ	0.362	-0.175	0.918
t	(4.21)	(1.26)	(40.5)
χ^2	26.0	12.1	403.
November 1977 (1085)			
γ	0.465	-0.310	0.912
t	(6.42)	(2.71)	(39.0)
χ^2	35.2	22.4	388.
March 1978 (1039)			
γ	0.364	-0.173	0.957
t	(4.03)	(1.19)	(63.9)
χ^2	21.8	13.7	402.
June 1978 (1077)			
γ	0.351	-0.166	0.926
t	(4.19)	(1.16)	(41.9)
χ^2	16.8	7.16	423.

Table 9: French Data. Estimates of the Conditional Model $\{Q^* | D, L^a, D^*\}$ for March 1976 and March 1978. (t-ratios in parenthesis below estimates.)

Period. Number of Observations. Item.		Variable at Level 1	Level 2	Level 3
March 1976 (906)				
Main Effect		0.178	1.010	-1.188
Q^*		(1.25)	(8.18)	(5.75)
Bivariate Interaction		0.709	-0.070	-0.639
$Q^* \times D$		(5.98)	(0.72)	(3.91)
		-0.204	0.119	0.085
		(1.80)	(1.29)	(0.54)
		-0.504	-0.049	0.554
		(3.99)	(0.52)	(3.77)
γ	0.503			
	(6.52)			
Bivariate Interaction		-0.216	0.019	0.198
$Q^* \times L^a$		(1.55)	(0.16)	(1.09)
		0.040	0.014	-0.054
		(0.36)	(0.14)	(0.34)
		0.177	-0.033	-0.144
		(0.96)	(0.19)	(0.53)
γ	-0.162			
	(1.08)			
Bivariate Interaction		1.844	-0.222	-1.622
$Q^* \times D^*$		(9.59)	(1.24)	(4.95)
		-0.296	0.622	-0.326
		(2.11)	(5.30)	(1.63)
		-1.548	-0.400	1.948
		(7.89)	(2.83)	(9.44)
γ	0.946			
	(47.5)			

Table 9 (continued):

Period. Number of Observations. Item.		Variable at Level 1	Level 2	Level 3
March 1978 (1039)				
Main Effect		-0.313	1.066	-0.754
Q^*		(1.88)	(8.92)	(4.29)
Bivariate Interaction		0.506	-0.173	-0.333
$Q^* \times D$		(3.90)	(1.77)	(2.12)
		-0.077	0.128	-0.051
		(0.74)	(1.70)	(0.43)
		-0.429	0.045	0.385
		(3.63)	(0.55)	(3.10)
γ	0.364			
	(4.03)			
Bivariate Interaction		-0.264	0.068	0.196
$Q^* \times L^a$		(1.90)	(0.56)	(1.01)
		-0.024	0.216	-0.192
		(0.25)	(2.44)	(1.32)
		0.289	-0.284	-0.005
		(1.94)	(2.08)	(0.02)
γ	-0.173			
	(1.19)			
Bivariate Interaction		2.080	-0.292	-1.788
$Q^* \times D^*$		(10.63)	(1.84)	(6.53)
		-0.300	0.483	-0.183
		(1.81)	(4.29)	(1.08)
		-1.780	-0.191	1.971
		(6.53)	(1.20)	(10.04)
γ	0.957			
	(63.9)			

of estimates for the conditional probability model for two of the available survey dates: October 1977 and October 1978 for the German firms and March 1976 and March 1978 for the French firms. The χ^2 -statistic with four degrees of freedom would be appropriate for testing the null hypothesis that the entire configuration is absent.[29]

In Tables 6 and 8, we see that the directions of association are all as expected and highly significant for the most part. Note that the sign of γ for the bivariate association between Q^* and L^a is positive for the German data and negative for the French data, which is exactly as it should be, since the first category for L^a in the German survey corresponds to the response "too small" while that for the French survey corresponds to the response "above normal." The relationship between Q^* and G^* or D^* is clearly the strongest and most significant of the three conditional bivariate interactions of the model, with component gamma coefficients of about 0.9 and χ^2 -values of about 300. The association between Q^* and $\bar{A}$ or D is somewhat weaker but still quite significantly positive. The relationship is more unstable over time for the French firms than for the German firms. The relationship between Q^* and L^a is always significant and relatively stable over time for the German firms, but the relationship between the two variables is frequently not significant and quite unstable over time for the French firms.

Similiar results are obtained for firms which do not carry inventories of finished products using the model $\{Q^* \mid \bar{A} \text{ or } D, S^a, G^* \text{ or } D^*\}$.

Tables 7 and 9 exhibit for two survey dates the actual parameter estimates (given a particular choice of basis for the log-linear probability model, in this case the deviation-contrast basis) for the main and bivariate interaction configurations in the models $\{Q^* \mid \bar{A}, L^a, G^*\}$ and $\{Q^* \mid D, L^a, D^*\}$. Examination of these tables (as well as similar results for other survey dates, not presented here) reveals a similar stability and significance of the relation between Q^* and G^* or D^* in comparison with the relation between Q^* and $\bar{A}$ or D and between Q^* and L^a. Moreover, the parameters for the bivariate interactions $Q^* \times G^*$ and $Q^* \times D^*$ are numerically very similiar, suggesting that the German business-test variable G^*, expected business conditions, is in fact a measure of expected changes in future demand.[30]

In order to see whether these models of the formation of production plans, which make good economic sense, in fact do better than naive models, I estimated several naive models for the same data, identical to those estimated in the next section for price expectations and expectations of future demand (extrapolative, adaptive, and error-learning). While it is clear that these models all fit the data well, their parameters appear to be much less stable than the models with more economic content.[31] A strong and stable serial correlation between Q^* and Q^*_{-1} does emerge from these estimates (stronger and more stable for the German than for the French data). To see whether such serial correlation might dominate the economic variables, I estimated conditional models

$$\{Q^* \mid \bar{A}, L^a, G^*, Q^*_{-1}\}$$

and

$$\{Q^* \mid D, L^a, D^*, Q^*_{-1}\}$$

for firms with inventories. Although the significance and strength of the economic variables were somewhat reduced, the relationship between Q^* and Q^*_{-1} by no means dominated.[32]

3. Estimates of Models of the Formation of Expectations of Prices and Future Demand[33]

This section examines some simple specific models, adapted here to the analysis of categorical data: Extrapolative expectations, adaptive expectations, and error-learning models.

Extrapolative Expectations: Two-Period Case

In the model of purely extrapolative expectations, currently reported expectations, depend upon immediately preceeding realizations:

$$P\left(X^* \mid X, X_{-1}\right),$$

where X may be prices (P), demand (D), or incoming orders $\left(\bar{A}\right.$, in which case $\left.X^* = G^*\right)$. Estimation of this model requires complete data on each firm included in the sample for a five month period in the German case and for the preceding survey in the French case. Although, in general, extrapolative expectations can refer to any weighted sum of past values, I call the two-period case extrapolative here. Two forms of naive model are special cases: Current expectations reflect only what has happened in the immediately preceding period, and current expectations reflect what has happened one and two periods previously in such a way that current changes are expected to be augmented or decremented by the change in the rate of change between last period and two periods ago.[34]

Adaptive Expectations

In its original early formulation, the adaptive expectations model related the change in expected normal prices to the difference between last period's realized price and last period's expectation (Nerlove, 1956, 1958). The variables X^* and X already represent changes in levels, but one can consider changes in X^* in relation to surprises as defined above. Below I call this the "error-learning" model of expectation formation. In the notation used in this paper, I would write the error-learning model as $\{\Delta X^* \mid EX\}$. Clearly this model represents a special restricted case of the adaptive expectations model that places no quantitative restrictions on the relations between X^*_{-1} and the previous expectations and realizations, X^*_{-1} and X, respectively. I view the more general form as more appropriate in the case of qualitative data since this form allows the strength of the association between current expectations and immediately preceding ones to differ from the strength of the association of the former with realizations. Thus, if one writes, as for continuously measured variables,

$$X^* = \beta X + (1-\beta)\, X^*_{-1} \ ,$$

it suggests that, the stronger the association between X^* and X relative to the association between X^* and X^*_{-1} , the larger the coefficient of expectation, β. The model is thus $\{X^* \mid X^*_{-1}, X\}$ with no further restriction.

An Error-Learning Model of Expectation Formation

As we noted in Koenig, Nerlove, Oudiz (1981a), there is a strong relationship between P^*_{-1} and $\bar{P}$ or P in both the French and German data. This same relation exists between G^*_{-1} and $\bar{A}$ and D^*_{-1} and D . Although the relationship is a strong one, it is not, however, as we showed above, stable for the French data. Because the form of the adaptive expectations model proposed above introduces no restrictions on the relation among the three variables, it is possible that the relation between X and X^*_{-1} so dominates that it is impossible to find a stable or well-defined relation between either X^* and X or X^* and X^*_{-1} when both X and X^*_{-1} are included in the model.[35] Thus, I also estimate a restrictive form of the adaptive expectations model, which I call the error-learning model. In the notation of Part I, it is

$$\{\Delta X^* \mid EX\},$$

where X^* stands for P^* or G^* and X for D or $\bar{A}$. In the next two sections I also examine joint models of production plans and price expectations and models of expectation or plan fulfillment.

Empirical Results

The results of fitting the three conditional probability models, extrapolative, adaptive, and error-learning, to data on prices and demand for German and French firms are given in summary form in Tables 10-15.[36] The price data for the French firms have been recategorized so that "-" now corresponds to "≤ 0 % ," "=" now corresponds to "0% < and ≤5%," and "+" now corresponds to

Table 10: German Data. Prices and Business Conditions. Extrapolative Expectations. Conditional Models: $\{P^*|\bar{P}, \bar{P}_{-1}\}$ and $\{G^*|\bar{A}, \bar{A}_{-1}\}$. Bivariate Component Gamma and t-Ratio. Configuration Chi-square.

Period. Number of Observations. Item.	Bivariate Interaction			
	$P^* \times \bar{P}$	$P^* \times \bar{P}_{-1}$	$G^* \times \bar{A}$	$G^* \times \bar{A}_{-1}$
July 1977 (2914, 1913)				
γ	0.812	0.186	0.528	0.262
t	(19.3)	(2.29)	(10.6)	(5.03)
χ^2	260.	39.8	198.	49.4
October 1977 (2759, 1759)				
γ	0.599	0.314	0.524	0.243
t	(7.89)	(3.24)	(11.00)	(3.95)
χ^2	124.	28.6	218.	34.0
January 1978 (3176, 1964)				
γ	0.801	-0.075	0.415	0.385
t	(13.7)	(0.710)	(8.58)	(7.92)
χ^2	155.	11.9	157.	93.6
April 1978 (3072, 1922)				
γ	0.724	0.237	0.558	0.253
t	(14.6)	(2.13)	(12.1)	(4.45)
χ^2	127.	39.0	216.	36.1
July 1978 (3174, 2063)				
γ	0.854	0.041	0.510	0.261
t	(19.7)	(0.384)	(11.2)	(5.47)
χ^2	199.	20.4	191.	38.1
October 1978 (3183, 2054)				
γ	0.837	0.264	0.537	0.165
t	(16.8)	(2.15)	(13.7)	(3.19)
χ^2	234.	13.5	181.	40.8

Table 11: French Data. Prices and Demand. Extrapolative Expectations: $\{P^* | P, P_{-1}\}$ and $\{D^* | D, D_{-1}\}$. Bivariate Component Gamma and t-Ratio. Configuration Chi-square.

Period. Number of Observations. Item.	Bivariate Interaction			
	$P^* \times P$	$P^* \times P_{-1}$	$D^* \times D$	$D^* \times D_{-1}$
November 1974 (843, 1014)				
γ	0.191	0.034	0.644	0.204
t	(4.04)	(0.695)	(12.38)	(1.77)
χ^2	77.6	31.7	169.	17.4
March 1975 (827, 1080)				
γ	0.043	0.149	0.486	0.081
t	(0.64)	(2.71)	(7.62)	(1.13)
χ^2	92.1	172.	88.6	2.1
June 1975 (979, 1313)				
γ	0.228	0.347	0.573	0.096
t	(3.38)	(7.76)	(10.98)	(1.36)
χ^2	59.0	98.2	156.	10.1
November 1975 (1094, 1425)				
γ	0.107	0.069	0.462	-0.075
t	(1.86)	(1.04)	(10.01)	(1.16)
χ^2	90.7	44.3	123.	4.4
March 1976 (1122, 1364)				
γ	-0.108	0.182	0.298	0.198
t	(1.96)	(3.24)	(5.68)	(3.19)
χ^2	50.8	89.4	89.3	9.6
June 1976 (1034, 1278)				
γ	0.144	0.129	0.466	0.023
t	(2.75)	(2.77)	(9.45)	(0.39)
χ^2	91.9	54.8	113.	5.6

Table 11 (continued):

Period. Number of Observations. Item.	Bivariate Interaction			
	$P^* \times P$	$P^* \times P_{-1}$	$D^* \times D$	$D^* \times D_{-1}$
November 1976 (958, 1277)				
γ	-0.094	0.191	0.435	0.126
t	(2.00)	(3.78)	(7.48)	(1.84)
χ^2	33.3	39.4	129.	15.2
March 1977 (941, 1132)				
γ	-0.279	-0.049	0.337	0.271
t	(3.94)	(0.88)	(5.63)	(4.51)
χ^2	96.2	18.5	84.0	21.0
June 1977 (1151, 1377)				
γ	-0.272	0.021	0.431	0.0003
t	(2.66)	(0.31)	(8.23)	(0.005)
χ^2	142.	39.1	95.6	1.00
November 1977 (1048, 1383)				
γ	-0.037	0.053	0.450	0.117
t	(0.51)	(0.74)	(8.99)	(1.99)
χ^2	39.3	39.6	130.	24.6
March 1978 (1193, 1556)				
γ	-0.253	0.164	0.172	0.261
t	(4.48)	(2.48)	(3.01)	(5.03)
χ^2	123.	77.5	74.5	5.07
June 1978 (1161, 1541)				
γ	-0.062	0.125	0.374	0.042
t	(0.92)	(2.39)	(7.18)	(0.68)
χ^2	94.7	57.3	88.4	30.2

Table 12: German Data. Prices and Business Conditions. Adaptive Expectations. Conditional Models: $\{P^*|P^*_{-1}, \bar{P}\}$ and $\{G^*|G^*_{-1}, \bar{A}\}$. Bivariate Component Gamma and t-Ratio. Configuration Chi-square.

Period. Number of Observations. Item.	Bivariate Interaction			
	$P^* \times P^*_{-1}$	$P^* \times \bar{P}$	$G^* \times G^*_{-1}$	$G^* \times \bar{A}$
April 1977 (3456, 2581)				
γ	0.530	0.692	0.786	0.465
t	(5.57)	(12.7)	(24.0)	(11.1)
χ^2	60.9	116.	384.	239.
July 1977 (3278, 2633)				
γ	0.474	0.685	0.673	0.416
t	(4.53)	(11.1)	(16.8)	(9.05)
χ^2	68.8	277.	385.	200.
October 1977 (3206, 2405)				
γ	0.675	0.562	0.741	0.423
t	(8.61)	(7.03)	(19.3)	(9.05)
χ^2	80.6	173.	314.	220.
January 1978 (3575, 2652)				
γ	0.591	0.764	0.729	0.343
t	(5.82)	(11.22)	(18.13)	(7.50)
χ^2	57.5	203.	374.	169.
April 1978 (3597, 2802)				
γ	0.576	0.663	0.721	0.453
t	(7.27)	(11.55)	(19.11)	(10.89)
χ^2	97.7	160.	426.	227.
July 1978 (3656, 2864)				
γ	0.536	0.773	0.713	0.432
t	(5.24)	(12.71)	(18.18)	(10.32)
χ^2	88.6	143.	369.	197.
October 1978 (3586, 2748)				
γ	0.711	0.760	0.614	0.472
t	(7.76)	(12.50)	(13.62)	(12.55)
χ^2	72.8	213.	325.	199.

Table 13: French Data. Prices and Demand. Adaptive Expectations. Conditional Models: $\{P^*|P^*_{-1}, P\}$ and $\{D^*|D^*_{-1}, D\}$. Bivariate Component Gamma and t-Ratio. Configuration Chi-square.

Period. Number of Observations. Item.	Bivariate Interaction $P^* \times P^*_{-1}$	$P^* \times P$	$D^* \times D^*_{-1}$	$D^* \times D$
November 1974 (728, 939)				
γ	0.132	0.165	0.434	0.612
t	(2.44)	(3.18)	(4.23)	(10.7)
χ^2	30.7	55.1	37.9	140.
March 1975 (764, 1004)				
γ	0.218	-0.010	0.419	0.378
t	(3.57)	(0.140)	(4.92)	(4.92)
χ^2	26.0	67.7	42.5	52.7
June 1975 (911, 1268)				
γ	0.214	0.124	0.556	0.447
t	(3.49)	(1.61)	(10.4)	(7.46)
χ^2	67.4	37.2	123.	96.2
November 1975 (1018, 1386)				
γ	0.411	-0.105	0.325	0.381
t	(8.39)	(1.58)	(5.34)	(7.57)
χ^2	121.	65.7	64.9	85.9
March 1976 (1046, 1326)				
γ	0.278	-0.217	0.304	0.247
t	(5.13)	(3.57)	(5.05)	(4.56)
χ^2	71.2	50.0	48.6	70.5
June 1976 (976, 1231)				
γ	0.263	0.032	0.540	0.309
t	(4.74)	(0.515)	(9.22)	(5.11)
χ^2	82.7	61.5	128.	58.7

Table 13 (continued):

Period. Number of Observations. Item.	Bivariate Interaction			
	$P^* \times P^*_{-1}$	$P^* \times P$	$D^* \times D^*_{-1}$	$D^* \times D$
November 1976 (913, 1247)				
γ	0.188	-0.139	0.275	0.433
t	(3.59)	(2.75)	(3.53)	(7.12)
χ^2	54.4	28.5	46.1	119.
March 1977 (818, 1097)				
γ	0.355	-0.463	0.292	0.335
t	(5.68)	(6.21)	(3.74)	(5.54)
χ^2	51.5	88.0	26.7	82.7
June 1977 (1087, 1325)				
γ	0.268	-0.387	0.499	0.290
t	(3.78)	(3.96)	(8.59)	(4.92)
χ^2	61.5	80.4	108.	50.4
November 1977 (1013, 1345)				
γ	0.214	-0.112	0.300	0.396
t	(3.21)	(1.42)	(4.67)	(7.33)
χ^2	47.9	28.5	96.1	106.
March 1978 (1039, 1498)				
γ	0.162	-0.274	0.389	0.116
t	(2.65)	(4.40)	(6.76)	(1.89)
χ^2	61.1	93.9	74.1	69.5
June 1978 (1063, 1473)				
γ	0.258	-0.202	0.468	0.298
t	(4.54)	(2.80)	(8.93)	(5.44)
χ^2	53.4	60.1	88.5	80.1

Table 14: German Data. Prices and Business Conditions. Error-Learning Model: $\{\Delta P^* | \overline{EP}\}$ and $\{\Delta G^* | \overline{EA}\}$. Bivariate Component Gamma and t-Ratio. Configuration Chi-square.

Period. Number of Observations. Item.	Bivariate Interaction: $\Delta P^* \times \overline{EP}$	Bivariate Interaction: $\Delta G^* \times \overline{EA}$
April 1977 (3456, 2581)		
γ	0.729*	0.727
t	(30.1)	(27.3)
χ^2	321.	347.
July 1977 (3278, 2633)		
γ	0.845	0.687
t	(23.8)	(22.3)
χ^2	610.	341.
October 1977 (3206, 2405)		
γ	0.808	0.673
t	(19.06)	(21.74)
χ^2	394.	331.
January 1978 (3575, 2652)		
γ	0.845	0.620
t	(19.25)	(18.81)
χ^2	196.	290.
April 1978 (3597, 2802)		
γ	0.859	0.708
t	(27.94)	(26.02)
χ^2	213.	329.
July 1978 (3656, 2864)		
γ	0.923	0.661
t	(35.90)	(20.59)
χ^2	580.	302.
October 1978 (3586, 2748)		
γ	0.890	0.669
t	(30.46)	(23.59)
χ^2	491.	281.

*Partially estimated configuration. 1 degree of freedom.

Table 15: French Data. Prices and Demand. Error-Learning Model: $\{\Delta P^* | EP\}$ and $\{\Delta D^* | ED\}$. Bivariate Component Gamma and t-Ratio. Configuration Chi-square.

Period. Number of Observations. Item.	Bivariate Interaction	
	ΔP^* x EP	ΔD^* x ED
November 1974 (728, 939)		
γ	0.770	0.788
t	(18.0)	(20.2)
χ^2	142.	258.
March 1975 (764, 1004)		
γ	0.630	0.696
t	(10.14)	(14.5)
χ^2	138.	165.
June 1975 (911, 1268)		
γ	0.635	0.781
t	(9.17)	(20.2)
χ^2	200.	254.
November 1975 (1018, 1386)		
γ	0.686	0.693
t	(12.87)	(19.6)
χ^2	120.	234.
March 1976 (1046, 1326)		
γ	0.468	0.663
t	(8.34)	(18.1)
χ^2	89.7	186.
June 1976 (976, 1231)		
γ	0.553	0.602
t	(8.38)	(13.6)
χ^2	88.1	146.

Table 15 (continued):

Period. Number of Observations. Item.	Bivariate Interaction	
	ΔP* x EP	ΔD* x ED
November 1976 (913, 1247)		
γ	0.392	0.725
t	(6.35)	(16.3)
χ^2	58.5	242.
March 1977 (818, 1097)		
γ	0.645	0.702
t	(11.99)	(17.8)
χ^2	111.	213.
June 1977 (1087, 1325)		
γ	0.738	0.672
t	(15.35)	(17.1)
χ^2	170.	177.
November 1977 (1013, 1345)		
γ	0.534	0.727
t	(8.70)	(21.4)
χ^2	83.3	267.
March 1978 (1039, 1498)		
γ	0.473	0.577
t	(9.25)	(13.5)
χ^2	67.9	169.
June 1978 (1063, 1473)		
γ	0.579	0.746
t	(10.63)	(23.8)
χ^2	132.	249.

"5% <." This recategorization makes use of the quantitative information on price realizations and price expectations/plans supplied in the French surveys and eliminates the concentration in the "+", category for both variables, caused by general inflation.

Table 10 and 11 present results for extrapolative models for price expectations/plans and demand expectations. For German firms there is a strong positive and highly significant relation between the anticipated change in price or in demand and the immediately preceding realization but the relation between P^* and $\bar{P}_{-1}$ is much weaker, more unstable, and frequently insignificant. Although the relation between G^* and $\bar{A}_{-1}$ is weaker than between G^* and $\bar{A}$, it is stable and always significant. The results for the German firms contrast sharply with those for the French firms. For the latter, there is a very unstable, often negative relation between P^* and P. While the relation between P^* and P_{-1} is positive, except for March 1977, it is unstable and usually insignificant. On the other hand, the relation between D^* and D is always positive and significant for the French firms.

The results for the unrestricted adaptive expectations model are presented in Tables 12 and 13. The relation between $\bar{P}$ and between G^* and $\bar{A}$ is only very slightly weakened for German firms by introducing P^*_{-1} and G^*_{-1} respectively; on the other hand, the "serial correlation (P^* and P^*_{-1} or G^* and G^*_{-1}) is strong and stable. In the case of French firms price expectation/plans, the interaction $P^* \times P^*_{-1}$ dominates the relationship, the association between P^* and P is usually insignificant and frequently negative when significant. This finding contrasts sharply with the results reported in Table 11. On the other hand, the results for demand are very similar to those found for the German firms.

The relative stability and strength of the relation between P^* and P^*_{-1} and between D^* and D^*_{-1}, together with the persistence of positive association between anticipations and immediately past realizations for both demand and prices in the case of the German data and for demand in the case of the French data suggest that the adaptive expectations model is to be preferred to the extrapolative. However, the negative association between P^* and P_{-1} for the French data suggests that the adaptive expectations may be explosive. When price data, which have not been recategorized, are used, this result is not found: The association between P^* and P is positive, usually significant and usually greater than the association between P^* and P^*_{-1}. It is difficult to find an explanation for why the elimination of a high concentration in the "+,+" category should cause the association to change sign in an unreasonable way.

Additional restrictions in a model sometimes aid in achieving more reasonable and stable results. This is the case here. Results for the error-learning models are presented in Tables 14 and 15. The associations between ΔP^* and $E\bar{P}$ and between ΔG^* and $E\bar{A}$ are strongly positive, highly significant, and very stable over time for the German data. The same result is obtained for the association between ΔP^* and EP and between ΔD^* and ED for the French data, although the price results are less stable over time for the French than for the German data.

I conclude that the error-learning model of the formation of price and demand expectations is well supported by the data and better than either the simple extrapolative model or the unrestricted adaptive-expectations model. It is the best explanation of anticipation of both prices and demand I have found so far.

4. Estimates of a Joint Model of Price Expectations and Production Plans

As Koenig, Nerlove, and Oudiz (1981a) note, price expectations may be more in the nature of plans than of forecasts. To the extent that this is true there should be a relation between the two processes of expectation/plan revision. Not only should revisions in expectations/plans for prices and production be related in the current period, but, if revisions of each are related to past surprises (or non-fulfillment) of the same variable, we would expect cross-effects as well; i.e., we would expect changes in production plans to be related to surprises in price expectations and changes in price expectations to be related to lack of fulfillment in production plans. A simple test of this hypothesis involves fitting the model

$$\{\Delta P^*, \Delta Q^* \mid E\bar{P}, E\bar{Q}\}$$

to the German data and the model

$$\{\Delta P^*, \Delta Q^* \mid EP, EQ\}$$

to the French data. The results are summarized in Tables 16-17. Of course, ΔP^* is strongly positively associated with $E\bar{P}$ or EP, and ΔQ^* is strongly positively associated with $E\bar{Q}$ or EQ . That much could have been expected from previous results. Moreover, there is a weak but generally significant positive association between ΔP^* and ΔQ^* for the German data. But in both cases a remarkable result occurs: The association between ΔP^* and $E\bar{Q}$ is negative and usually significant for the German data and the association between ΔP^* and EQ is erratic and almost always insignificant for the French data; there is no significant association between ΔQ^* and $E\bar{P}$ or EP for either the German or French data.

Table 16: German Data. Price Expectations and Production Plans. Joint Error-Learning Model: $\{\Delta P^*, \Delta Q^* | E\bar{P}, E\bar{Q}\}$. Bivariate Component Gamma and t-Ratio. Configuration Chi-square.

Period. Number of Observations. Item.	$\Delta P^* \times \Delta Q^*$	$\Delta P^* \times E\bar{P}$	$\Delta P^* \times E\bar{Q}$	$\Delta Q^* \times E\bar{P}$	$\Delta Q^* \times E\bar{Q}$
April 1977 (3177)					
γ	0.197	0.735*	-0.038	-0.030	0.719
t	(3.59)	(29.3)	(0.792)	(0.487)	(25.36)
χ^2	22.8	297.	3.10	10.7	436.
July 1977 (2936)					
γ	0.381	0.835	-0.264	-0.104	0.700
t	(5.65)	(21.31)	(4.11)	(1.25)	(22.35)
χ^2	60.7	489.	16.7	13.6	421.
October 1977 (2802)					
γ	0.398	0.794	-0.209	-0.099	0.793
t	(5.43)	(16.78)	(3.10)	(1.08)	(28.31)
χ^2	60.2	313.	22.5	9.58	436.
January 1978 (3016)					
γ	0.369	0.847	-0.339	-0.061	0.740
t	(6.90)	(19.30)	(7.29)	(0.940)	(25.20)
χ^2	77.8	163.	52.9	27.0	356.

*Partially estimated configuration. 1 degree of freedom.

Table 16 (continued):

Period. Number of Observations. Item.	$\Delta P^* \times \Delta Q^*$	$\Delta P^* \times E\bar{P}$	$\Delta P^* \times E\bar{Q}$	$\Delta Q^* \times E\bar{P}$	$\Delta Q^* \times E\bar{Q}$
April 1978 (3154)					
γ	0.380	0.867	-0.327	-0.082	0.811
t	(7.36)	(29.03)	(7.21)	(1.19)	(34.92)
χ^2	61.5	179.	48.4	14.1	515.
July 1978 (3176)					
γ	0.285	0.924	-0.140	-0.056	0.789
t	(4.04)	(35.75)	(2.24)	(0.674)	(27.43)
χ^2	63.3	463.	13.9	41.4	402.
October 1978 (3118)					
γ	0.367	0.881	-0.182	0.078	0.786
t	(4.91)	(26.51)	(2.87)	(0.920)	(29.3)
χ^2	41.2	402.	22.4	6.98	452.

Table 17: French Data. Price Expectations and Production Plans. Joint Error-Learning Model: $\{\Delta P^*, \Delta Q^* | EP, EQ\}$. Bivariate Component Gamma and t-Ratio. Configuration Chi-square.

Period. Number of Observations. Item.	$\Delta P^* \times \Delta Q^*$	$\Delta P^* \times EP$	$\Delta P^* \times EQ$	$\Delta Q^* \times EP$	$\Delta Q^* \times EQ$
November 1974 (640)					
γ	0.290	0.786	-0.099	-0.050	0.749
t	(2.78)	(17.71)	(0.939)	(0.412)	(13.23)
χ^2	18.1	128.	4.49	4.5	101.
March 1975 (677)					
γ	0.110	0.652	0.106	-0.230	0.744
t	(1.11)	(9.93)	(1.01)	(2.24)	(14.15)
χ^2	5.90	124.	4.37	7.88	121.
June 1975 (824)					
γ	0.028	0.668	0.051	0.052	0.780
t	(0.294)	(9.12)	(0.505)	(0.440)	(17.6)
χ^2	5.10	179.	1.87	3.40	119.
November 1975 (900)					
γ	0.225	0.700	-0.140	-0.003	0.736
t	(2.75)	(12.40)	(1.56)	(0.004)	(16.79)
χ^2	8.2	108.	8.20	1.4	174.

Table 17 (continued):

Period. Number of Observations. Item.	$\Delta P^* \times \Delta Q^*$	$\Delta P^* \times EP$	$\Delta P^* \times EQ$	$\Delta Q^* \times EP$	$\Delta Q^* \times EQ$
March 1976 (937)					
γ	0.151	0.445	-0.067	-0.074	0.770
t	(1.97)	(7.26)	(0.88)	(0.860)	(17.4)
χ^2	9.54	77.0	1.50	3.30	166.
June 1976 (878)					
γ	0.032	0.545	0.058	-0.166	0.708
t	(0.373)	(7.87)	(0.689)	(1.76)	(14.4)
χ^2	2.30	76.4	0.698	6.56	144.
November 1976 (807)					
γ	0.038	0.360	0.183	0.057	0.658
t	(0.457)	(5.38)	(2.37)	(0.606)	(11.2)
χ^2	8.90	41.8	7.51	5.69	112.
March 1977 (722)					
γ	0.046	0.668	0.106	0.096	0.707
t	(0.529)	(11.7)	(1.21)	(0.946)	(13.0)
χ^2	3.22	94.8	4.55	1.37	114.

Table 17 (continued):

Period. Number of Observations. Item.	$\Delta P^* \times \Delta Q^*$	$\Delta P^* \times EP$	$\Delta P^* \times EQ$	$\Delta Q^* \times EP$	$\Delta Q^* \times EQ$
June 1977 (974)					
γ	-0.002	0.743	-0.011	0.146	0.644
t	(0.022)	(14.7)	(0.138)	(1.40)	(11.6)
χ^2	0.696	155.	0.277	4.84	98.0
November 1977 (910)					
γ	-0.073	0.522	0.152	0.111	0.692
t	(1.01)	(8.03)	(2.07)	(1.23)	(14.9)
χ^2	3.09	72.6	10.5	1.99	130.
March 1978 (943)					
γ	0.115	0.478	-0.004	0.022	0.660
t	(1.69)	(8.95)	(0.064)	(0.288)	(13.6)
χ^2	5.50	59.9	2.00	0.728	179.
June 1978 (958)					
γ	0.061	0.575	-0.047	0.032	0.741
t	(0.800)	(10.0)	(0.633)	(0.342)	(17.0)
χ^2	2.68	116.	3.95	0.759	160.

Clearly common factors influencing revisions in production plans and price expectations could account for the positive association between ΔP^* and ΔQ^* for the German firms. These factors may not operate as powerfully for French firms. I established that an error-learning model fits well for price expectations above; that it also fits well for production plans is not remarkable although the production planning model gives a more economically appealing explanation. The important point is that, to the extent that one can describe revisions of price expectations and of production plans by error-learning models, one can do so nearly independently of each other. The lack of independence arises from the negative association between ΔP^* and $\bar{EQ}$ for the German data, which is all the more unexpected since one would anticipate a greater upward revision in price plans/expectations given a positive surprise in production on the hypothesis that shifts in demand drive the system. If on the other hand, supply bottlenecks are largely responsible for short-falls in production and if German firms, but not French, respond by revising prices upward to protect inventory levels, the observed results would follow.

5. Estimates of a Model of Expectation or Plan Fulfillment

The theoretical discusson of Part 1 suggests that a firm's plans and expectations will be revised when unexpected events occur. A simple version of this theory is that when demand increases, perhaps unexpectedly, prices and producton plans are revised upward. The theory of the imperfectly competitive firm does not, of course, predict what will happen to prices and output given an upward shift in the demand function that a firm faces. If demand is higly elastic or if firms are reluctant to raise prices once announced, one can be reasonably confident that the quantity supplied will increase. The response of prices remains conjectural, but the more competitive are firms, the greater the likelihood of some increase in prices when demand increases.

In the previous section, I argued that the results for the joint model of revisions of price expectations/plans and production plans were consistent with the hypothesis that production surprises, i.e., nonfulfillment of production plans, are largely the result of supply bottlenecks but that French and German firms respond differently when revising price expectations/plans. As a partial test of this hypothesis, I present estimates of a model relating surprises in price expectations/plans and surprises in production plans to a measure of surprise in demand expectations ($E\bar{A}$ for the German data and ED for the French data). Tables 18-19 report the results.

Surprises in demand are strongly and significantly associated with failures to fulfill production plans for the German data; surprises in demand are less strongly, but always positively and significantly, associated with surprises in price expectations or with failure to fulfill price plans. For the French data, on the other hand, one observes only the strong positive and significant association between surprises in demand and failure to fulfill production plans. The association between price expectations/plans and surprises in demand is weak, erratic and usually insignificant.

These results suggest that unexpected changes in demand are indeed important for both French and German firms, but French firms revise their price expectations/plans largely in response to previous errors in fulfillment of these same expectatins or plans, independently of an unexpected deviations of production changes from planned changes. Unexpected changes in demand are associated with these last-mentioned deviations but not with deviations of realized price changes from expected or planned price changes.

Table 18: German Data. Prices and Production. Fulfillment Models: $\{E\bar{P}|E\bar{A}\}$ and $\{E\bar{Q}|E\bar{A}\}$. Bivariate Component Gamma and t-Ratio. Configuration Chi-square.

Period. Number of Observations. Item.	Bivariate Interaction	
	$E\bar{P} \times E\bar{A}$	$E\bar{Q} \times E\bar{A}$
April 1977 (2543, 2487)		
γ	0.229	0.751
t	(5.54)	(32.9)
χ^2	31.2	530.
July 1977 (2601, 2543)		
γ	0.161	0.677
t	(2.87)	(25.0)
χ^2	9.37	421.
October 1977 (2401, 2304)		
γ	0.350	0.710
t	(5.93)	(26.36)
χ^2	38.1	419.
January 1978 (2614, 2509)		
γ	0.196	0.718
t	(4.41)	(28.83)
χ^2	21.2	523.
April 1978 (2757, 2674)		
γ	0.170	0.678
t	(3.91)	(20.95)
χ^2	15.4	475.
July 1978 (2834, 2733)		
γ	0.115	0.672
t	(2.36)	(25.82)
χ^2	5.83	492.
October 1978 (2726, 2579)		
γ	0.191	0.697
t	(3.58)	(28.37)
χ^2	13.1	521.

Table 19: French Data. Prices and Production. Fulfillment Models: {EP|ED} and {EQ|ED}. Bivariate Component Gamma and t-Ratio. Configuration Chi-square.

Period. Number of Observations. Item.	Bivariate Interaction	
	EP x ED	EQ x ED
November 1974 (687, 939)		
γ	0.052	0.764
t	(0.618)	(18.3)
χ^2	4.36	238.
March 1975 (755, 985)		
γ	0.036	0.738
t	(0.438)	(16.3)
χ^2	2.26	251.
June 1975 (943, 1240)		
γ	-0.033	0.824
t	(0.317)	(26.2)
χ^2	11.6	395.
November 1975 (1033, 1348)		
γ	0.086	0.878
t	(1.24)	(40.9)
χ^2	13.5	502.
March 1976 (1057, 1318)		
γ	0.076	0.813
t	(1.21)	(29.0)
χ^2	7.30	406.
June 1976 (973, 1217)		
γ	0.159	0.840
t	(2.25)	(30.8)
χ^2	10.7	338.

Table 19 (continued):

Period. Number of Observations. Item.	Bivariate Interaction	
	EP x ED	EQ x ED
November 1976 (974, 1231)		
γ	0.122	0.827
t	(1.87)	(29.1)
χ^2	5.11	366.
March 1977 (810, 1076)		
γ	0.063	0.846
t	(0.85)	(27.8)
χ^2	2.2	330.
June 1977 (1072, 1307)		
γ	0.216	0.812
t	(2.91)	(26.9)
χ^2	15.3	393.
November 1977 (1095, 1317)		
γ	-0.044	0.862
t	(0.642)	(34.2)
χ^2	2.29	409.
March 1978 (1095, 1497)		
γ	-0.062	0.743
t	(1.01)	(23.2)
χ^2	8.20	424.
June 1978 (1106, 1438)		
γ	0.027	0.808
t	(0.394)	(29.0)
χ^2	1.55	426.

Clearly the groups of French and German firms are not homogeneous. If it were possible to divide firms into subgroups, it is likely that somewhat different and possibly sharper conclusions would emerge for the different subgroups.

SUMMARY OF RESULTS AND DIRECTIONS FOR FURTHER RESEARCH

I have stressed the importance of analyzing individual data on expectations, plans, and realizations for the study of how such expectations and plans are formed and the extent to which they are fulfilled. The conclusions that can be drawn from the analyses in this paper are limited by the categorical nature of the data and the short time periods for which the data are available. Throughout there are important differences between the results for French firms and for German firms. The reason for such differences is an important topic for further research.

In the first series of reported empirical investigations, firms' expectations or plans are much more concentrated in the no-change category than are the realizations that they forecast. This means that firms understate the proportion of realizations in the increase and decrease categories, but the net effect of such underestimation differs markedly between French and German firms. For prices, German firms consistently overestimate the balance between increases and decreases; they consistently underestimate, in absolute terms, the balance between increases and decreases for demand (measured by incoming new orders) and production. For French firms, on the other hand, there is no such consistent bias; moreover, the firms' forecasts are much better, on the whole, than those of the German firms.

The conditional distributions of realizations given prior expectations or plans estimated from past data may be used to correct the firms' forecasts. These distributions are quite stable over time for German firms and the corrected forecasts are quite close to marginal distributions of the realizations. But the conditional distributions for the French firms are unstable over time and the corrected forecasts are sometimes worse than the firms' own forecasts. The nature of the instability suggests that economy-wide changes affecting all firms simultaneously may be responsible. Finding such variables and measuring their influence is a topic for further investigation.

In the second reported investigation of a simple conditional probability model for production plans, I find what one would expect to find. Firms that report a recent increase in incoming orders or demand, report that they regard their inventory levels as too low or their backlog of orders as too high, and also report that they expect demand to increase (or business conditions, so called, for their products to improve), are much more likely than firms that report the opposite to report that they plan to increase production. The importance of this finding is not so much to verify the obvious, but, first, to give us some confidence in the data for analyzing less trivial hypotheses, and, second, to yield quantitative measures of the effects of different variables. With respect to the latter, the most quantitatively important influence on production plans are expectations of future demand or business conditions; moreover, the magnitudes of the parameters attached to these variables in the conditional distributions for the French and German firms are so similar as to suggest that the two variables, based on responses to two quite different questions, measure roughly the same thing. Lagged production plans, while significant, do not reduce the significance of the economic variables perceptibly. Mechanical models of production plan formation, such as the error-learning model, fit well, but not better than models with more economic content.

The third series of empirical investigations deals with the formation of price and demand expectations. Of the models considered, the error-learning model, which relates changes in expectations to surprises in the same variable, gives the best and most parsimonious explanation of the data. The model, however, has relatively little economic content; in continuous form, it has been widely used in studies of supply, investment behavior, and the like.

The fourth investigation looks at a joint error-learning model of price expectations and production plans. As noted the univariate error-learning model for production plans has little economic content but fits the data well. Consequently it may be viewed as a description of the process. When a joint model of the two processes is estimated, the remarkable conclusion emerges that they are almost independent of each other for both the sample of French firms and the sample of German firms, although some negative association between changes in price expectations/plans and deviation of changes in actual production from planned changes for German firms is found. This is consistent with the hypothesis that supply bottlenecks are largely responsible for short-falls in production. But this hypothesis is contradicted by the additional finding that unexpected changes in demand are strongly associated with deviations of production from planned levels. An important topic for further research is whether the same finding will emerge when more economically meaningful models, especially for production planning, are related to one another.

The final series of investigations deals with the fulfillment of plans and expectations. The simple hypothesis tested is whether deviations between prior expectation of demand and realizations in the current period affect the deviations between price expectations and production plans from their respective realizations. They do, except for French firms' price expectations. That surprises affect fulfillment of production plans for both French and German firms but fulfillment of price expectations only for German firms, suggests that German industry may be more competitive on the whole than French industry. This, however, is obviously a topic requiring much further research.

FOOTNOTES

1 Many people have attempted to replace the word "expectations" to avoid confusion with the same word used to mean the mathematical expectation. This nicety seems doomed to failure by usage; in this paper I use the terms "expectations" and "anticipations" interchangeably.

2 Hicks himself was clearly not happy with this solution; see Hicks (1977, pp. vi-vii).

3 In Nerlove, Grether, Carvalho (1979, Chapter 14), I and others have applied a simplified version of this method to analyze the behavior of U.S. cattle producers.

4 The analysis of Modigliani and Cohen has been applied, inter alia, to the study of investment decisions by Eisner (1978) and McKelvey (1980).

5 Mincer (1969) has considered this class of expectation-formation model in great detail.

6 For example, the minimum mean-square error (MMSE) forecasts of a time series with rational spectral density can be expressed this way. These are also the conditional expectations (mathematical) of the future value given the infinite past up to the time the forecast is made.

7 Other explanations are possible: Regressivity will be observed, for example, if the forecast is an unbiased estimate of the future value; the latter can be represented by the forecast plus an error uncorrelated with it.

8 See McCallum (1980).

9 Frydman and Schankerman have argued in an unpublished paper that the unbiasedness test is not an appropriate test of the rationality of forecasts because nonrational expectations, based on an incomplete or erroneous set of exogenous variables, may also be unbiased. Unbiasedness is thus a necessary

but not a sufficient condition for rationality in this weak form.

[10] See Zellner and Palm (1974) and Wallis (1977). An intermediate form expresses each variable in terms of its own past and the past of other variables entering the model. It is in this form that Nerlove, Grether and Carvalho (1979) use the result in formulating quasi-rational expectations for prices in their study of the U.S. beef cattle industry.

[11] See Jacobs and Jones (1980, p. 272). Mincer (1969) argues that it is also necessary to observe forecasts for several successive future periods at a given time.

[12] Elsewhere, in collaboration with Koenig and Oudiz (1979, 1981a, 1981b, 1981c), I have studied production plans and their formation, models of price-expectation formation and the forecasting properties of expectations and plans using microdata from the Ifo and INSEE business test surveys. The present paper grows out of this work. See also Theil (1966, pp. 417-24), Eisner (1978), Koenig (1980), and McKelvey (1980).

[13] Although the surveys are conducted at the establishment rather than the firm level, the information collected refers to that level and the questionnaire is filled out by a respondent at the establishment level. Some initial experiments in which multiple-product firms were eliminated, suggest that the establishment responses may be treated as independent observations.

[14] For the French firms, responses are obtained separately by product line. In the case of multiple-product firms, only the information for the principal product was used, because we could not consider responses for different product lines to be independent. Since it is possible to match the responses to the business-test survey with surveys of investment plans and expenditures and financial circumstances, which reflect decision plans, and expectations at the level of the firm rather than the product line, it will be necessary

to devise methods of analysis aggregating product lines or treating them as nonindependent observations in order to link the various bodies of data.

15 In the case of prices, the French survey asks for a percentage increase or decrease, actual or expected. We have made use of this information only to re-categorize the responses in order to mitigate the problem of general inflation. Besides the difficulties involved in combining categorical and non-categorical data in the same analysis, we have observed that when asked for quantitative data, particularly with reference to expectations, firms tend to respond categorically. Hence, it may be preferable to use the data in categorical form even when the response is quantitative. The categorical character of quantitative responses may be due to the uncertainty attached to a single-valued response.

16 The order of responses for the appraisal of inventory levels is reversed in the German and French survey instruments: + corresponds to "too small" in the German data and "above normal" in the French data. We have left the order as is, so that the signs of measures of association between inventory appraisals and other variables are reversed between the two bodies of data in the results reported in Part II.

17 In the German survey reported realizations describe monthly changes of variables, whereas reported expectations or plans are for a three or a six month period (three for prices and production, six for business conditions, a variable which can be interpreted as demand for the product). This difference in the unit period of observation creates problems in the analysis of the relation between expectations or plans and realizations. The approach used here aggregates the data on realizations for three consecutive months into a new variable having a unit period of three months. A discrepancy in the unit period for business conditions remains.

The French data are collected every three, four or five months but the unit periods to which the expectations or plans refer are always identical to the periods for which the realizations are reported on the subsequent survey.

The method of aggregating the German data is as follows: Let X_t be a realization reported on the survey taken at date t; it refers to the change over the preceeding month. Let $\bar{X}_t$ be the aggregate referring to the preceding three months. The method of aggregation is as follows:

If the sign of X_{t-i}, i=0,1,2 is identical, set $\bar{X}_t$ equal to that sign.

If the responses differ in sign for each survey, the firm is deleted.

If the sign of X_{t-1}, i=0,1,2 is equal for two periods and the sign for the third period is not opposite in sign, set $\bar{X}_t$ equal to the sign of the categories reporting changes; otherwise the firm is deleted.

18 In principle, a five-category variable could have been defined by making the categories in the far southwest and far northeast corners separate categories. Thus, we might have a "++" category for the combination X^*_{-1} = "-" and X = "+" and a "--" category for X^*_{-1} = "+" and X = "-".

19 The Goodman-Kruskal Gamma coefficient for the table is -0.2, indicating a weak negative association between forecast and outcome. (See Goodman and Kruskal 1979.) Kawasaki (1979) has extended the Goodman-Kruskal gamma to a measure of partial bivariate association in a multivariate context, Kawasaki's extension is extensively employed below.

20 A rigorous definition of the random variable X^f_c, for which $\Sigma\hat{P}(X \mid X^*_{-1})P(X^*_{-1})$ is the marginal distribution, may be given although I will not attempt one here. Since the marginal distributions may be misleading, a more interesting possibility is to define a random variable

X_c^f with the same marginal distribution but also a complete joint distribution with the realization X. In the 3x3 case this may be done as follows: For each firm n we observe X(n); according to whether the firm reports an increase = category 1, no change = category 2, or a decrease = category 3, we place that firm in a set F_1, F_2, or F_3. Using $\hat{P}(X\ X_{-1}^*)$ we may find for each firm having a particular response $X_{-1}^*(n)$ on the previous survey a vector of probabilities $(p_1(n), p_2(n), p_3(n))'$ that it will experience a realization in category 1,2, or 3. The joint distribution is estimated as

X_c^f

		+	=	-
	+	$\sum_{n\epsilon F_1} p_1(n)$	$\sum_{n\epsilon F_1} p_2(n)$	$\sum_{n\epsilon F_1} p_3(n)$
X	=	$\sum_{n\epsilon F_2} p_p(n)$	$\sum_{n\epsilon F_2} p_2(n)$	$\sum_{n\epsilon F_2} p_3(n)$
	-	$\sum_{n\epsilon F_3} p_1(n)$	$\sum_{n\epsilon F_3} p_2(n)$	$\sum_{n\epsilon F_3} p_3(n)$

A rigorous definition of this random variable X_c^f may be given, but I will not attempt it here. I owe this suggestion to Seiichi Kawasaki.

21 See footnote 18.

22 In an earlier paper (Koenig, Nerlove, Oudiz, 1981a), we estimated the joint distributions on pairs of consecutive survey dates and tested whether the main effect parameters and whether the bivariate interaction effect parameters were the same for the two pairs. Estimation of the conditional model $\{X\ \ X_{-1}^*, T\}$ and the test indicated is not only simpler but no

assumptions need be made about interactions between expectations reported on two survey dates or between realizations reported on two survey dates.

23 However, recent work of Vuong (1981) shows how suitable restrictions can lead to structural models within the log-linear model framework.

24 Results reported in this section are drawn from a more extensive discussion in a joint paper with H. Koenig and G. Oudiz presented at the 15th CIRET Conference, September 30, 1981, Athens, Greece.

25 Note that this does not mean that we would also find stability of $(P(\bar{Q} \mid Q^*_{-1})$ if we took more widely separated pairs of surveys.

26 This section reports revised analyses similar to those reported in Koenig, Nerlove, Oudiz (1979). The main differences are (1) the greater number of periods, and (2) the use of temporally aggregated incoming new orders as a measure of demand in the previous period (see footnote 17).

27 A similar set of results is available for firms that do not carry inventories of finished products, based on the model

$$\{Q^* \mid \bar{A} \text{ or } D, S^a, G^* \text{ or } D^*\}.$$

Since the results are broadly similar, I do not report them here.

28 The component gamma coefficient is based on a decomposition of the joint probability of several ordered categorical variables by the configurations in a log-linear probability model. If one considers a model involving only main effects and bivariate interaction effects, the component gamma for a pair of variables is obtained from the table of exponential bivariate effect parameters for that pair, normalized to sum to one, by the usual formula. In the conditional probability models, estimates of which are reported here, I limit interactions between Q^* and the conditioning variables to no higher order than bivariate, although the interactions among conditioning variables are unconstrained.

29 The maximum-likelihood estimates for the parameters of a particular configuration in a hierarchical log-linear probability model do not exist if the marginal table corresponding to that configuration contains one or more zero cells which are not a priori zero. Note, however, that it is not sufficient for estimability for all the cells of the corresponding marginal table to be filled.

In the case of the German data for July 1978 a zero occurred in the marginal table for $Q^* \times L^a$; consequently, all four parameters of the bivariate interaction configuration for Q^* and L^a could not be estimated. The basis vectors corresponding to all but the first parameter, $\beta_{12}(1,1)$, were arbitrarily suppressed. Thus the χ^2-value given is associated with only one degree of freedom. The same problem occurred for the French data for November 1976 in the marginal table for $Q^* \times D^*$.

30 It is perhaps particularly important not to attribute any causal significance to this relationship since the same, latent, variables may cause both a high probability of a planned increase in production and an expectation of increased demand for the product.

31 The error-learning model for changes in production fits the best and is more stable, particularly for the French data, than the others. However, these results are not inconsistent with those from the models with more economic content.

32 Space limitation precludes even a summary of the numerical results here. I hope to include them in a subsequent paper.

33 The results for prices presented here draw on Koenig, Nerlove, Oudiz (1981a). Those for demand or business conditions are wholly new. Estimation procedures are different.

34 This formation differs somewhat from the usual one in that it refers to rates

of change rather than levels and changes in rates of change rather than past changes.

35 This is the counterpart of collinearity.

36 The same set of models were fit to production plans and realizations in order to verify that the models with economic content presented above were more stable. Although the mechanical models fit well, the results were not as stable or as intuitively appealing as those from the economic model presented in the previous section.

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Technical Appendix to "Expectations, Plans and Realizations in Theory and Practice"

1. Characteristics of the Business-Test Data

Categorical data encountered in typical economic contexts frequently present a number of peculiarities not encountered in other applications. Some of these in this paper are as follows.

First, there are not only many cells which are empty (cell count equal zero) but there is also considerable clumping, particularly in the no-change or "=" categories. Such clumping and the presence of a large number of zero cell counts for category combinations, which may be implausible but not of *a priori* zero probability, cause particular difficulty in the analysis of business-test data as compared with other types of categorical survey or biomedical data.

A second characteristic of the business-test data, which is of considerable importance in their analysis, is that the categories are *ordered*, that is, the variables are ordinal. In many other types of surveys or other contexts in which categorical data are collected, there is no particular order to the data. The respondent is male or female; the member of a particular occupational class; agrees, disagrees or has no opinion with respect to a certain statement; etc. In contrast, expecting demand to increase is more than expecting it to remain the same, which, in turn, is more than expecting it to decrease. For such variables it is meaningful to consider measures of ordinal bivariate association, e.g., does the statement that inventories are too low tend to be associated with a statement that planned production is to be increased and *vice versa*? If so we would say there is a positive association between inventory appraisals (too low, "+", etc.) and planned production (increase, "+", etc.).

A third characteristic of categorical data encountered in economic contexts is that in the analysis of such data we are often interested in distinguishing between exogenous or explanatory variables and jointly dependent categorical variables, that is, our models may often be formulated in terms of conditional probabilities. In the business-test this characteristic may be illustrated by an hypothesis with respect to the conditional probability of reporting planned production in a certain category, given responses with respect to order backlogs, expected future demand, and inventory appraisals. An example in which the distinction between jointly dependent and explanatory variables is still more explicit is our earlier analysis (Nerlove and Press, 1973, 1976, 1980) of the adoption of several modern agricultural practices by Filipino farmers. The primary purpose of the investigation was to assess the relative importance of factors associated with the adoption of high-yielding rice varieties (HYV) in the crop year 1967-68. In his original investigation, Mangahas (1970) treated the agricultural practices as purely explanatory variables and did not attempt to explain the joint occurrence of the simultaneous adoption of a number of modern agricultural practices. In the analyses presented earlier, and also reported in Nerlove and Press (1980), we attempt to treat the complex of modern agricultural practices, including the adoption of HYV, simultaneously. The joint possibilities of simultaneous adoption or non-adoption were treated as conditional upon such variables as the age, educational attainment and tenancy status of the farmer, the area of the farm and the type of irrigation facilities available, if any, and whether the farmer was a cooperator with the local Experiment Station. Some of these variables, such as area of farm, age and schooling, are continuous, whereas the others are categorical.

2. General Discussion of the Log-Linear Probability Model

Methodological problems of the analysis of cross-classified data are the subject of considerable current interest (Reynolds, 1977; Fienberg, 1977; Upton, 1978). I and collaborators have dealt elsewhere at some length with a particular methodological approach especially useful in the analysis of categorical data of the kind we encounter in the German and French business-tests (Nerlove and Press, 1976, 1978; Koenig and Nerlove, 1979; Koenig, Nerlove, and Oudiz 1979a and 1979b). The approach is based on a parameterization of the probabilities characterizing large multi-dimensional contingency tables, which was developed by Birch (1963), Mosteller (1968), Bishop (1969), Haberman (1974b), and Goodman (1978). Good expository accounts are contained in Everitt (1977, pp. 80-107), Fienberg (1977), Haberman (1974 and 1978-79), Payne (1977), and Plackett (1974). More abstract accounts are contained in Collombier (1980), Haberman (1974b), Kawasaki (1979), Link (1982), Nerlove and Press (1978), and Vuong (1981). The approach also lends itself to a suitable generalization of measures of ordinal bivariate association in analyses involving more than two categorical variables.

Briefly the parameterization of the log-linear probability model may be described as follows: Let $\mathcal{A} = \{A_1, \ldots, A_q\}$ be a set of categorical random variables, which may take on, respectively, $I_1, \ldots, I_q$ possible values. If we have a sample of N observations on the q categorical random variables, we might arrange these in an $I_1 x I_2 x \ldots x I_q$ table of counts corresponding to a similar arrangement of the probabilities

(1) $p_{i_1, \ldots, i_q}, i_1 = 1, \ldots, I_1, \; i_2 = 1, \ldots, I_2, \ldots, i_q = 1, \ldots, I_q$.

Alternatively, order the logarithms of the

$$Q = \prod_{k=1}^{q} I_k$$

probabilities (1) into a Qxl vector by some principle, e.g., lexicographically,

(2) $$\log p = \begin{bmatrix} \log p_{1 \ldots 1} \\ \vdots \\ \log p_{I_1 \ldots I_q} \end{bmatrix}$$

The vector log p may be thought of as a point in R^Q. Let M be a linear manifold in R^Q of dimension m, $0 < m \leq Q$. The class of models for which the Qxl vector U_0 consists entirely of ones is in M and

(3) $$\log p \in M \text{ such that } \langle p, U_0 \rangle = 1,$$

where p is the vector of probabilities corresponding to log p and $\langle x,y \rangle$ denotes the inner product of x and y, is defined as the class of log-linear probability models. Since M is a linear manifold in R^Q, there exist m independent vectors, not necessarily orthogonal, which span M, one of which may be U_0 as defined above. Because log p is contained in M, it may be represented in terms of the basis vectors $U_1, \ldots, U_{m-1}$ and $U_m = U_0$.

There are clearly many possible choices of a basis for M. One of the most interesting and useful of these is the choice that, in the case $M = R^Q$, allows us to represent the logarithms of the probabilities in a traditional analysis-of-variance format

$$\log p_{i_1, \ldots, i_q} = \mu + \alpha_1(i_1) + \ldots + \alpha_q(i_q) + \beta_{12}(i_1, i_2) + \ldots + \beta_{q-1,q}(i_{q-1}, i_q) + \ldots + \omega_{1, \ldots, q}(i_1, \ldots, i_q), \tag{4}$$

where $\alpha_1(\ ), \ldots, \omega_{1, \ldots, q}(\ )$ satisfy the usual constraints:

$$\begin{aligned} &\alpha_1(.) = \alpha_2(.) = \ldots = \alpha_q(.) = 0, \\ &\beta_{12}(i_1, .) = 0,\ \beta_{12}(., i_2) = 0,\ \ldots, \beta_{q-1,q}(., i_q) = 0, \\ &\omega_{1, \ldots, q}(i_1, \ldots, i_{q-1}, .) = 0,\ \ldots, \omega_{1, \ldots, q}(., i_2, \ldots, i_q) = 0. \end{aligned} \tag{5}$$

The dot used in place of an index denotes summation over that index. The parameters $\alpha_1(i_1), \ldots, \omega_{1, \ldots, q}(i_1, \ldots, i_q)$ have the usual analysis-of-variance interpretation: μ denotes an overall effect; $\alpha_1(i_1)$ denotes an effect due to A_1 (at "level" i_1); $\beta_{12}(i_1, i_2)$ denotes a bivariate interaction effect between A_1 and A_2 (at "levels" i_1 and i_2, respectively); and $\omega_{1, \ldots, q}(i_1, \ldots, i_q)$ denotes a q-order interaction among $A_1, \ldots, A_q$ (at "levels" $i_1, \ldots, i_q$, respectively). Because of the constraints imposed, this characterization is frequently called the deviation-contrast basis. It is the one used in this paper. In general, it makes no use of any order among the categories of a categorical variable (e.g., "+" is greater than "=" is greater than "-").

In terms of the vector log p defined in (2) and the vector of parameters

$$B = (\alpha_1(1), \ldots, \omega(I_1, \ldots, I_q))',$$

a design matrix A, $Qx \{I_1 + \ldots + I_1 I_2 + \ldots + Q\}$, may be defined such that

(6) $$\log p = AB.$$

The number of independent elements of B, however, is only equal to Q, which is the number of probabilities. To express log p in terms of the basis vectors, arranged in a Q x Q matrix

$$U = [U_0, \ldots, U_{Q-1}],$$

a matrix L of rank Q is defined such that

(7) $$\Theta = LB$$

is a Q x 1 vector, whose elements are a subset of the elements of B such that together with the restrictions (5) they are sufficient to determine all of the parameters. In terms of Θ

(8) $$\log p = U\Theta$$

Any basis which allows decomposition into main and interaction effects can legitimately be called an analysis-of-variance parameterization, but even within this class there are many different choices (Bock, 1975, pp. 239-43). An alternative to the deviation-contrast basis is based on assigning scores to ordinal categories, and is useful in interpreting directions and other characteristics of association among ordinal variables. (See Goodman, 1979).

Kawasaki (1979, Chapter 2) shows how the basis for a general multivariate log-linear probability model may be generated from so-called elementary bases for univariate models by direct (Kronecker) product operations, provided the main and interaction effects are reordered in a certain way. For example, in the case of the univariate dichotomy, the elementary basis consists of the columns of the matrix

$$U_A = U_B = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

The basis for the bivariate dichotomy consists of the columns of

$$U^{*} = U_A \otimes U_B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \text{ reordered to } U = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

which yields the representation for $\log p = (\log p_{11}, \log p_{12}, \log p_{21}, \log p_{22})'$ as

$$\log p = U\Theta = U \begin{bmatrix} \mu \\ \alpha_1(1) \\ \alpha_2(1) \\ \beta_{12}(1,1) \end{bmatrix},$$

where μ corresponds to the overall effect, α_1 to the main effect for the first variable, α_2 to the main effect for the second variable, and β_{12} to the bivariate interaction effect. The values of the parameters for other combinations of indices are recovered from the restrictions (5).[2]

The bivariate trichotomous case is only slightly more difficult. For the deviation-contrast basis, the elementary basis for a single trichotomous variable consists of the columns of

$$U_A = U_B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & -1 \end{bmatrix}$$

Thus, for the bivariate trichotomy we have $U^* = U_A \times U_B$ and

$$U = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & -1 & -1 & -1 & -1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & -1 & -1 & 0 & 0 & -1 & -1 \\ 1 & -1 & -1 & 1 & 0 & -1 & 0 & -1 & 0 \\ 1 & -1 & -1 & 0 & 1 & 0 & -1 & 0 & -1 \\ 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

where $\log p = U\Theta$ and Θ is the vector

$$\Theta = \begin{bmatrix} \mu \\ \alpha_1(1) \\ \alpha_1(2) \\ \alpha_2(1) \\ \alpha_2(2) \\ \beta_{12}(1,1) \\ \beta_{12}(1,2) \\ \beta_{12}(2,1) \\ \beta_{12}(2,2) \end{bmatrix}$$

U is the appropriately reordered version of U^*.[3]

The remaining parameters $\alpha_1(3)$, $\alpha_2(3)$, $\beta_{12}(1,3)$, $\beta_{12}(2,3)$, $\beta_{12}(3,1)$, $\beta_{12}(3,2)$ and $\beta_{12}(3,3)$ are recovered as before, from the restrictions (5).

In general, the elementary basis for a single q-category variable is

$$U = \begin{bmatrix} \underline{1}_q & I_{q-1} \\ 1 & -\underline{1}_q' \end{bmatrix} ,$$

where $\underline{1}_q$ is a column vector consisting entirely of ones and I_{q-1}, is an identity matrix of order q-1. Taking the Kronecker product of any number of these yields a matrix of basis vectors for the corresponding multivariate case; the order of the vectors does not correspond to the order of the effects parameters in Θ , but re-ordering them appropriately is not difficult.[4]

In general, we have the following result for the q-way case:

$$(9) \quad \log p_{i_1, \ldots, i_q} = \sum_{j_1, \ldots, j_q} \{ \prod_{k=1}^{q} u_k (i_k, j_k) \, \tau (j_1, \ldots j_q)\},$$

where k=1, ... ,q are the q categorical variables, i_k takes on the values 1, ... ,I_k and j_k takes on the values 0,1,... ,I_k-1 for the kth categorical variable, $\tau(j_1, \ldots, j_q)$ represents an effect of order corresponding to the number of non-zero indices (thus, $\tau(j_1, 0, \ldots, 0)$, $j_1 = 1, \ldots, I_1$, is the first main effect), and where

$$\begin{aligned} u_k(i_k, j_k) &= 1, \text{ if } j_k = 0 \text{ or } i_k = j_k , \\ &= -1, \text{ if } i_k = I_k , \\ &= 0, \text{ otherwise.} \end{aligned}$$

$\tau(j_1, \ldots, j_q)$ with indices in a different order clearly represents the conventional ordering of the effects, Equation (9) is a generalization of

Theorem 1 of Nerlove and Press (1976, p. 13).

Haberman (1974a) suggests an alternative basis derived from scores constructed from orthogonal polynomials. (Plackett, 1974, Chapter 8, also mentions scoring for ordinal variables, but does not develop the idea.)

In current terminology, the collection of parameters characterizing a main or interaction effect is called a configuration; thus the general q-variate case has, potentially, main effects or univariate configurations, bivariate interaction configurations, and so on up to the q-way interaction configuration. A log-linear probability model which contains all possible configurations is called saturated. In general, saturated models are not very interesting since such models place no restrictions on, and therefore, simply represent an alternative description of, the probabilities underlying the contingency table. Moreover, if there are any sampling zeros at all, it will not be possible to estimate the parameters of a saturated model. Instead, one normally wishes to consider models involving only a subset of many possible configurations; the most important class of such models is called hierarchical.

A model is hierarchical if the inclusion of any interaction configuration implies the inclusion of all lower-order interaction configurations involving only the variables in the higher order configuration. Equivalently, exclusion of any configuration implies exclusion of all higher-order configurations that include all of the variables included in the lower-order configuration. Hierarchical models are generally more plausible in the log-linear probability context than nonhierarchical models (Payne, 1977). The rationale is most easily explained in the context of a 2-way table. In this case, there are only five models in the hierarchical class. Let A and B be the two categorical variables;

in an obvious notation, some of the models are

Saturated model: A, B, AB;

Independence model: A, B (only main effects).

A model which is not hierarchical is

One main effect eliminated: A, AB.

As shown elsewhere (e.g., Koenig, Nerlove, Oudiz, 1979a, 1979b), the main effects essentially reflect proportional variations in the probabilities for each variable across categories; leaving out a main effect would place the burden of explaining such a proportional variation affecting all categories of a particular variable on the interaction(s) of that variable with another or others. This is clearly unreasonable. A similar argument may be given for higher-order interactions in relation to lower-order interactions.

Hierarchical models have some special properties related to estimation as well. In the paper, I generally make use of the class of models involving only main and bivariate interaction effects, that is, the simplest hierarchical model which allows dependence among the variables.

3. Measures of Partial Bivariate Association

One of the most important topics in the analysis of categorical data is the measurement of association among ordinal variables, especially partial association, controlling for the influence of additional variables when more than two variables are considered at the same time. This is analogous to the estimation of regression coefficients or partial correlations in multiple regression analysis, if attention is restricted to conditional log-linear probability models. As shown in Nerlove and Press (1976) and elsewhere, one can always interpret joint probabilities in terms of a series of conditional probabilities, and vice versa. Thus, the analogy may be carried over to situations in which joint dependence among several categorical variables, some of which may be ordinal, is of interest. Besides Haberman (1974a), extensive discussions of measures of bivariate association between two ordinal categorical variables are contained in Davis (1967), Wilson (1974), Hildebrand, Laing and Rosenthal (1977), Reynolds (1977, Chapter 3), Upton (1978, pp. 34-8), and Goodman (1979). An important and frequently used measure is the Goodman-Kruskal gamma coefficient, developed in a series of four papers, reprinted as Goodman and Kruskal (1979). This measure has been generalized by Kawasaki (1979, Chapter 6; 1980), in the context of multivariate log-linear probability models, to a so-called component gamma coefficient, γ , which is a measure of partial bivariate association based on the bivariate-interaction parameter estimates from a joint or a conditional log-linear probability model. Davis (1967) extends the Goodman-Kruskal coefficient to the multivariate case in a manner based directly on the observed contingency table and without reference to the log-linear model representation of the contingency table probabilities. Haberman (1974a) suggests that his proposed scoring technique also yields useful measures of bivariate association in the multivariate case. This idea has been developed in detail for log-linear probability models in unpublished papers by Quang Vuong (1979a, 1979b).

The Goodman-Kruskal gamma coefficient is defined for 2-way tables; to generalize it to the multivariate case in order to measure partial bivariate association, Kawasaki (1979, 1980) shows that the joint probability for any log-linear model can be written as a constant times the product of probabilities associated with each configuration, i.e., each main effect, each bivarite interaction effect, etc. Thus, for example, neglecting trivariate and higher order interactions, a gamma coefficient defined on a particular bivariate component probability configuration represents the "pure" partial bivariate association "cleansed" of main effects and other bivariate interactions. Kawasaki (1979) calls such a measure of partial association a component gamma coefficient. Asymptotic variances of the component gamma coefficient may be obtained from the values of, and variance-covariance matrices for, the underlying parameters of the configurations.

To illustrate Kawasaki's decomposition consider the case of two ordinal variables A and B, indexed by i_A and i_B, respectively. The restrictions

$$\alpha_A(.) = 0,\ \alpha_B(.) = 0,\ \beta_{AB}(.,i_B) = 0,\ \beta_{AB}(i_A,.) = 0,$$

are assumed to hold. One may write

$$(1)\quad p_{i_A i_B} = C \frac{\exp[\alpha_A(i_A)]}{\sum_{j_A} \exp[\alpha_A(j_A)]} \cdot \frac{\exp[\alpha_B(i_B)]}{\sum_{j_B} \exp[\alpha_B(j_B)]} \cdot \frac{\exp[\beta_{AB}(i_A,i_B)]}{\sum_{j_A,j_B} \exp[\beta_{AB}(j_A,j_B)]},$$

where

$$C = \frac{\sum_{j_A} \exp[\alpha_A(j_A)] \cdot \sum_{j_B} \exp[\alpha_B(j_B)] \cdot \sum_{j_A,j_B} \exp[\beta_{AB}(j_A,j_B)]}{\sum_{j_A,j_B} \exp[\alpha_A(j_A) + \alpha_B(j_B) + \beta_{AB}(j_A,j_B)]}$$

is a constant when all the parameter values are given. Each of the remaining three terms can be thought of as a "probability" associated with a particular configuration, each depending on the relevant index or indices, positive, and summing to one over those indices. Think of it as the multiplicative contribution of that particular configuration to the over-all joint probability when the contributions of all the other configurations are isolated in like manner. In contrast, the marginal probability represents the gross contribution of a variable, or variables, operating through all main and interaction effects.

Denoting each of the component probabilities, as they are called by Kawasaki (1979, p. 154; 1980), by a subscript referring to the effect configuration with which the probability is associated, we rewrite (1) as

$$(2) \qquad p_{i_A i_B} = C\, p_1(i_A)\, p_2(i_B)\, p_{12}(i_A, i_B).$$

In general, the q-variate log-linear probability model may be decomposed in a similar manner

$$(3) \qquad p_{i_1, \ldots, i_q} = C\, p_1(i_1)\, p_2(i_2) \cdot \ldots \cdot p_{1 \ldots q}(i_1, \ldots, i_q).$$

As noted above, in most economic contexts it is rarely possible to include higher-order than bivariate interaction configurations. When, however, trivariate and higher-order interactions are included in the model, definition and measurement of partial association becomes difficult. Perhaps the simplest way to proceed is to regard the measure of bivariate association between two variables, say A and B, as a function of the level of a third variable C, or of a third and a fourth variable, etc.

The component probabilities have an associated contingency table of the dimensionality of the configuration; thus, the Goodman-Kruskal gamma can be computed for that table associated with a bivariate configuration just as it can for an ordinary two-way table. Moreover, if the parameters of the configuration have been estimated by maximum-likelihood, one has an asymptotic variance-covariance matrix for those parameters; since the component probabilities are functions of the parameters associated with the configuration, the Goodman-Kruskal gamma is too; thus, an asymptotic standard error may be calculated by the delta method (Kawasaki, 1979, pp. 161-63).

Note that the component gamma, γ , is not equal to the Goodman-Kruskal gamma even in the case of a two-way table (except in the 2x2 case), because the latter depends on main effects as well as the bivariate interaction, whereas γ depends only on the bivariate interaction. I find this a desireable feature of the component gamma in an economic context: Economic variables affecting all individuals observed will have the effect of concentrating certain responses in specific categories, different at one time than at another; an ordinary gamma would be affected by such variations, yet the underlying association between the two variables might well be unaffected; thus one might prefer a component gamma for ascertaining stability or instability of association over time.

As suggested by equations (1)-(3), the parameters of an interaction configuration may be associated with a "component probability"; thus the bivariate interaction configuration corresponds to a two-way contingency table. For example, in the 3x3 case, one can arrange the bivariate interaction parameters, of whichh only the four in the upper-left corner cells need be estimated, in a 3x3 table:

	+	=	-
+	$\beta(1,1)$	$\beta(1,2)$	$\beta(1,3)$
=	$\beta(2,1)$	$\beta(2,2)$	$\beta(2,3)$
-	$\beta(3,1)$	$\beta(3,2)$	$\beta(3,3)$

I have used category 1 to indicate "+", 2 to indicate "=", and 3 to indicate "-". It is clear that, if one set $\beta(1,2) = \beta(2,1) = \beta(2,2) = 0$, the table would have $\beta(1,1) = \beta(3,3) = -\beta(1,3) = -\beta(3,1)$ and would indicate strong positive or negative association. A large positive value of $\beta(2,2)$ indicates concentration in the no-change categories; whereas a negative value indicates its absence. The presence or the absence of significant concentration in the no-change categories is particularly important in econometric applications. Large positive or negative values for the off-diagonal elements $\beta(1,2)$ and $\beta(2,1)$, in relation to the diagonal, reflect variation in the degree of concentration of one variable with respect to changes in the other and tend, given the corner values and the center, to lower the degree of bivariate association, positive or negative.

Using a priori information and/or assumptions related to the ordinal character of the data may become important when some of the bivariate configurations are not estimable due to a very large number of sampling zeros. This point is considered in more detail in Section 5 below.

4. Conditional Log-Linear Probability Models

The conditional probabilities in the log-linear model are also log-linear of simple form, but the marginal probabilities are more complex except in the case of independence. It is this result which makes the log-linear probability representation useful if conditional probabilities have a natural and intuitive interpretation. They are less useful when the marginal probabilities are of particular interest or significance. I suggest in the Filipino farmer example, described in Section 1, that conditional probabilities are indeed of special interest in econometric applications. Moreover, often some of the conditioning variables are continuous, as they were in that example.

The subject of log-linear conditional probability models may be usefully introduced by the following result proved in Nerlove and Press (1976, pp. 19-21). Let the set of random variables $\mathcal{A}$, described at the beginning of Section 2 above, be partition into two nonoverlapping subsets $\mathcal{A}_1$ and $\mathcal{A}_2$ such that $\mathcal{A}_1 \cup \mathcal{A}_2 = \mathcal{A}$. If $\eta = \{i_1, \ldots, i_q\}$, is defined to be the set of indices associated with the q variables in $\mathcal{A}$, the set of indices may be partioned accordingly into two parts η_1 and η_2 such that $\eta_1 \cup \eta_2 = \eta$. Let

Θ_{η_1} = the sum of all main effects and interaction effects with indices in η_1;

$\Theta_{\eta_1 \eta_2}$ = the sum of interaction effects having at least one index in η_1 and one index in η_2.

Then, the conditional probabilities associated with the log-linear probability model are also log-linear probabilities, but involve a reduced set of main and interaction effects. In particular, the conditional probability of the random variables in $\mathcal{A}_1$ given those in $\mathcal{A}_2$ involves only the main effects pertaining to $\mathcal{A}_1$ themselves and between those random variables and the random variables in $\mathcal{A}_2$, but not the main effects pertaining to $\mathcal{A}_2$ or any interactions involving only random variables in $\mathcal{A}_2$. More formally,

$$\text{Prob}\left[A_1 = a_1, \ldots, A_k = a_k \;\middle|\; A_{k+1} = a_{k+1}, \ldots, A_q = a_q\right]$$

$$(1) \quad = p_{i_1, \ldots, i_k \mid i_{k+1}, \ldots, i_q} = \frac{\exp\left(\Theta_{\eta_1} + \Theta_{\eta_1 \eta_2}\right)}{\sum_{\eta_1} \exp\left(\Theta_{\eta_1} + \Theta_{\eta_1 \eta_2}\right)} .$$

Although it is not apparent from (1), the fact that the conditional probabilities must sum to one means that these probabilities also depend implicitly on the main and interaction effects for the random variables in $\mathcal{a}_2$. (See Kawasaki, 1979, pp. 61-6).[5] In particular, this dependence has implications for the estimation of joint and conditional log-linear probability models.

In Nerlove and Press (1976, pp. 29-32), we prove a special case (limited to discrete random variables) of a general result proved by Gourieroux and Monfort (1979); for any joint probability distribution, which is strictly positive over the set (discrete or continuous) on which it is defined, the univariate conditional probabilities taken together uniquely determine the joint probability distribution from which they arose. This result may tempt one to estimate conditional probabilities rather than joint probabilities, particularly if some of the interaction configurations among conditioning variables are nonestimable, a subject discussed in the next section. Unfortunately, because these probabilities depend implicitly on the nonestimable parameters, one cannot avoid the problem in this manner.[6]

Note also that, if main effects and/or interaction effects are functions of some continuous or discrete variables, these functional forms are exactly preserved in the conditional probabilities. In this case, however, maximum-likelihood methods based on fitting certain marginal tables cannot be used; instead one must resort to other iterative procedures of the sort employed by Nerlove and Press (1973 and 1976).

A fundamental result concerning conditional probability models is related to estimation (Kawasaki, 1979, pp. 71-3). Essentially, it is that maximum-likelihood estimates of the parameters of a conditional probability model, subject to any constraints, are exactly the same as those derived from a joint probability model which contains among its parameters also all those in the conditional model that involve the conditioning parameters subject to the same constraints. The result holds ony when all variables are categorical and when all interactions among the conditioning variables are included in the joint model. The result is difficult to prove in an elegant fashion and I do not attempt a proof here. Kawasaki's (1979) proof has been put more rigorously and elegantly in recent unpublished work by Quang Vuong (1979a and 1979b).

5. Estimation and Partial Estimation

The theory underlying maximum-likelihood estimation of the parameters in the log-linear probability model was developed for the three-way case by Birch (1963). Bishop, Fienberg and Holland (1975, Chapter 3) give an exhaustive discussion for hierarchical models. The maximum-likelihood estimates are functions of the marginal frequencies corresponding to the highest-order configurations included in the model. (I neglect here the possibility that some parameters may depend upon continuous explanatory variables.) Birch (1963) and Bishop, et al. (1975) show that a number of different sampling schemes lead to the same maximum-likelihood estimates.

Provided attention is restricted to the class of models which are hierarchical, it can be shown that the marginal tables corresponding to the highest-order interaction configuration in the model are sufficient statistics for the interaction configuations (Bishop, et al. 1975, pp. 64-73). The cell frequencies are unbiased estimates of the expected frequencies of the model so that the marginal tables corresponding to the configurations included in the model are fit exactly. Thus, it is not necessary to compute estimates of the actual interaction parameters since the expected frequencies of the model can be computed by the iterative proportional fitting algorithm introduced by Deming and Stephan (1940). Birch (1963) showed this for the three-way case but assumed positive cell counts in the full table, which is too restrictive in the case of unsaturated models.

In his review of Bishop, et al. (1975), Haberman (1976) complains that the restriction to hierarchical models is too severe, that the parameters and estimates of their asymptotic variances and covariances are useful to compute, and that proceeding directly to estimated frequencies for calculation of the values of the likelihood function and tests of goodness-of-fit of one model against another is misleading. Moreover, proceeding directly to maximum-likelihood estimates of the expected frequencies leads Bishop, et al. (1975, p. 218) into error in stating rules for the calculation of appropriate degrees of freedom in various Chi-square tests (Haberman, 1976, p. 820). The reason for this difficulty is closely related to the fact that Bishop, et al., rarely compute parameter estimates. Indeed, most analyses using log-linear probability models in fields other than econometrics usually specify which configurations are to be included and test the model against various alternatives directly. If a marginal table for a given configuration contains a sampling zero, then the maximum-likelihood estimates of the corresponding cell frequency will be exactly zero; in this case, the full table will also contain some zero estimates. It follows that the maximum-likelihood estimates of the parameters of the log-linear model in question, which assumes strictly positive probabilities, do not exist. It may, however, be possible to estimate some of the parameters of the configuration and the parameters of other configurations for which the corresponding marginals have no sampling zeros if the model is suitably restricted.

The fundamental result on estimability is given by Haberman (1974b, Theorem 2.2, p. 37), who states a necessary and sufficient condition for the existence of maximum-likelihood estimates for any log-linear probability model, not merely those which are hierarchical, in relation to the observation vector (location of zeros in the full table). Haberman's condition does not make use of the fact that the model under consideration belongs to the hierarchical class, if indeed it does. For hierarchical models the following condition is true:

> If a maximum-likelihood estimate exists for a hierarchical log-linear probability model, then any marginal table corresponding to a configuration contained in the model has no empty cells.[7]

Note that this does not mean that a table which has all the sufficient marginal tables filled is estimable. It is perfectly possible for the appropriate margins to be filled without a maximum-likelihood estimate existing for the parameters of the model.

Kawasaki (1979, pp.84-121) has developed an algorithm to determine the estimability of any given model for any given observation vector. Unfortunately, at least as it has been possible to program Kawasaki's algorithm, this computation is often more time consuming than simply estimating the model (Link, 1980). I am tempted to recommend that the minimally sufficient set of marginal tables be examined. If a zero appears, one knows the model is not estimable; if no zero appears, try to estimate the model. However, since convergence is occasionally slow and because, in any case, the criteria by which convergence is decided is, to some extent, arbitrary, I hesitate to recommend this procedure. The question of deciding estimability remains unresolved at this time.

The suggestion is sometimes made that, to make a nonestimable model estimable, one simply add a small number to sufficiently many empty cells in the full table. I would expect the parameter estimates of those configurations which would have been nonestimable to be quite sensitive to exactly what small numbers were added, particularly so, if the corresponding marginal table contained a zero. Not only is this true, but, as Kawasaki (1979, pp. 111-18) shows, all of the estimates and their standard errors are highly sensitive. Thus, the problem of nonestimability cannot be resolved in this way.

When there are one or more empty cells in a marginal table corresponding to a configuration, then a log-linear probability model in the hierarchical class of models is not estimable. However, the model may become estimable if we suppress some of the parameters of the configuration and the corresponding basis vectors. For example, consider a model involving four trichotomous variables, A,B,C, and D, and suppose the marginal table (AB) contains a zero. Then a hierarchical model containing all main effects and all bivariate interaction effects but no higher-order effects is not estimable. A hierarchical model omitting the bivariate configuration AB but containing main effects for both A and B is estimable (except for a pathological case neglected here). The interaction configuration AB is determined by four parameters and four associated basis vectors in any analysis-of-variance representation; it is possible that if one deleted one or more of these parameters and their associated basis vectors the resulting model containing a partial bivariate interaction configuration would become estimable. Indeed, this frequently occurs; thus, the question arises as to which parameters and associated vectors to suppress to make optimal use of the data.

It may be thought that, if, the suppression of one parameter renders the model estimable, then it would not matter which of the four parameters was included for the value of the maximized likelihood function. Unfortunately, as Kawasaki (1979, p. 139) shows, the value of the maximized likelihood function is affected significantly. Thus, in principle, one could find that suppression yielding the largest value of the maximized likelihood function and choose that as the estimate of the model containing the partial configuration. The implications of such a procedure in an econometric context are, mind-boggling, since many marginal tables are likely to contain at least one zero. It is one thing to search over four parameters (characterizing a bivariate configuration among two trichotomous variables); it is quite another to search over the 16 possibilities for two nonestimable interaction configurations, or 64 for three. And what if some marginal tables have more than one zero? Obviously, the option of searching is not a viable one in situations in which many, many zero cells are encountered, as is the case in the econometric applications considered in this paper. In the results reported in the text, in those cases in which a bivariate configuration was not fully estimable, only the parameter $\beta(1,1)$ was retained, the remaining three were suppressed.

A final point to be made here is one alluded to in the preceding section. One may think that, if a nonestimable configuration were to be found among the conditioning variables in a conditional log-linear probability model, there would be no cause for concern since such interactions do not appear explicitly in the conditional model. Unfortunately, this conjecture is false since the parameters of these configurations enter the conditional probabilities implicitly. Thus one might as well estimate a joint probability model containing all of the configurations among conditional and conditioning variables that one needs to define the conditional probability model in question. Given a joint model in the hierarchical class with no nonestimable configurations, the efficient Deming-Stephan algorithm may be employed. However, if some configurations are only partially estimable, a modification of this algorithm must be used, which is much less efficient computationally. An alternative calculation using a form of the Newton-Raphson algorithm may be preferred, in which case direct calculation of the parameters of the conditional, rather than the joint, probability model may be considered. In any case, such calculation is necessary if any of the conditioning variables are continuous, as in the Filipino farmer example.

Footnotes for Appendix

1 The material in this appendix is drawn partly from M. Nerlove and S.J. Press, Invited General Methodology Lecture, 1980 Meeting of the American Statistical Association, Houston, Texas, August 11, 1980.

2 The reordering necessary is that the columns of the U matrix appear in order of the effects parameters rather than the indices attached to the probabilities in lexicographic order. This principle requires us to interchange the second and third column of $U_A \otimes U_B$ in the 2x2 case. See Kawasaki (1979, pp. 37-56).

The Kronecker product of the elementary basis matrices gives the basis appropriate to the parameters in the following order: Let τ be a vector with elements

$$\tau(j_1, \ldots, j_q),$$

$j_k = 0, \ldots, I_k-1$, such that $\tau(j_1, \ldots, j_q)$ is an effect parameter of the order indicated by the number of nonzero arguments, at the "level" given by the values of those arguments; thus, for example,

$$\tau(1,0, \ldots, 0) = \alpha_1(1)$$
$$\tau(1,2, \ldots, 0) = \beta_{12}(1,2),$$

etc. Then, $\log p = U^{*}\tau$, but the elements of τ are arranged lexicographically, which does not place together all main-effect parameters, all bivariate interaction effect parameters, etc.

3 In this case the Kronecker product $U_A \otimes U_B$ gives the vectors corresponding to the parameters in the following order:

$$(\mu,\ \alpha_2(1),\ \alpha_2(2),\ \alpha_1(1),\ \beta_{12}(1,1),\ \beta_{12}(1,2),\ \alpha_1(2),\ \beta_{12}(2,1),\ \beta_{12}(2,2))'.$$

to obtain U from U^* requires interchanging the second and fourth columns, the third and seventh, etc.

4 See footnote 2.

5 This relationship depends on including all interaction configurations, not assumed zero, in both the conditional and joint models. Thus, if three-way and higher order interactions are assumed absent in the conditional model they are also assumed absent in the joint model and vice versa.

6 This should be intuitive by analogy to multiple regression: To obtain estimates of the regression coefficients, the variance-covariance matrix of the independent variables must be estimable. (See also footnote 5 above.)

7 Furthermore, the MLE exists if and only if there exists an observation vector such that it has positive frequencies in every cell that agree with the given observation vector on the margins corresponding to included configurations.

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