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THE EFFECT OF SYNDICATION ON THE CORE OF MARKETS WITH TRANSFERABLE UTILITY AND CONTINUUM OF TRADERS

by

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ABSTRACT

The core of market games with continuum of players and syndicates is studied using the thoery of the value of non-atomic games. In general syndication tends to enlarge the core of a market game and the unorganized players are exploited by the additional elements of the core. Conditions under which the unorganized players are not exploited are presented. These results for market games are used to prove parallel results for markets with transferable utility. In these markets the conditions under which the unorganized traders are not exploited, are weaker than the conditions needed for markets without utility transfers.

1. Introduction

This paper deals with the effect of syndication on the core of transferable utility markets (monetary markets) with continuum of traders. Each transferable utility market with continuum of traders is associated with a non-atomic game in which the worth of a coalition of traders is the maximal cumulative utility they can get together by reallocating their initial resources. The core of the market is the core of this game, namely the set of all payoff vectors (i.e finitely additive measures on the space of the traders) that give each coalition not less than its worth. A payoff vector fails to be in the core if it is blocked by some coalition; i.e. if some coalition is paid less than its worth. The game associated with the market yields another payoff vector; the Shapley value of the game. By introducing money into the market, as a linear factor of the utility functions the monetary competitive equilibrium (m.c.e.) is defined. With each m.c.e. a payoff vector is associated, that pays each coalition its utility-wise value which is the same as its monetary value. It is shown in [A-S] that under certain differentiablity requirements, the core of the market contains only one element which coincides with the Shapley value and the (unique) m.c.e. payoff.

By a market game we mean a non-atomic game which is super additive and homogeneous of degree one. The family of market games contains all the games associated with the monetary markets.

The introduction of syndicates into either a monetary market or a market game prevents certain coalitions from blocking payoff vectors. A blocking coalition can not contain a proper subset of a syndicate. The generation of syndicates tends therefore to enlarge the core. The question we study is, how the unorganized players (i.e. player who are not members of the syndicates) are paid by the additional elements in the core. This problem is discussed, for market games, in the first part of the paper. The techniques used are those developed by Aumann and Shapley in [A-S] for studying the value of non-atomic games. In the second part of the paper the results for market games are implemented to monetary market. It is shown that in general the unorganized players do not benefit by syndication. No coalition of unorganized players gets more from the additional elements in the core then it gets in the old core, but such coalitions may get less than that. In terms of the monetary market, any element in the core of the market with the syndicates pays coalitions of unorganized traders not more than they get in the m.c.e. payoff.

The exploitation of small traders (those who are not atoms) in non-trasnferable utility (n.t.u.) markets with some large traders (atoms) was proved by Shitovitz who extensively studied these markets in [S]. In these markets the exploitation is revealed in the fact that for each allocation in the core there is a vector of efficiency prices under which the monetary value of the bundle of a small trader is not greater than the value of his initial bundle. Although small traders are monetarily exploited they can still have higher utility of their core bundles than of their initial bundles as was pointed out by

Aumman [A]. This anomaly disappears of course in the transferable utility markets.

The exploitation of the unorganized player can be avoided under certain symmetry conditions imposed on the set of syndicates. We prove such results both for market games and monetary markets. The results for monentary markets are analogous to the results of Shitoviz for non-transferable utility markets but achieved under weaker conditions. One of the differences is that for n.t.u. markets these results are true only for syndicates in which all the traders are of the same type (atoms) as was shown by Champsaur and Laroque [C-L], while in monetary markets this limitation is not needed.

The existence of a big enough sector of small traders of the same type as the syndicates sector is another condition under which the unorganized traders are not exploited. The condition given in Theorem 6.3 is weaker then the condition given by Gabszewicz and Mertens for n.t.u. markets [G-M].

The paper has two parts. The first one (sections 2-4) deals with market games and the second one (sections 5-9) deals with monetary markets. Section 2 describes the model of market games and some preliminary results. The main results for market games are stated in Section 3, and proved in section 4. In section 5 preliminaries for monetary markets are given. The main results for these markets are stated in section 6. In section 7 we show how syndicates can be atomized without effecting the market. We prove the main results in section 8 and discuss them in section 9.

Part I

The Core of Market Games with Syndicates

2. The Model of Market Games with Syndicates

Let $(I, \mathcal{C})$ be a measure space isomorphic to $([0,1],\mathcal{B})$ where $\mathcal{B}$ is the σ-field of Borel subset of $[0,1]$. An elements of $\mathcal{C}$ is called a coalition. A game is a real valued function v, defined on $\mathcal{C}$ such that $v(\emptyset) = 0$. A non decreasing sequence of sets in $\mathcal{C}$, $\Omega = \{\emptyset = S_o \subseteq S_1 \subseteq \cdots \subseteq S_n = I\}$ is called a chain. The variation of v over a chain Ω, $\|v\|_\Omega$ is defined by:

$$\|v\|_\Omega = \sum_{i=1}^{n} |v(S_i) - v(S_{i-1})|.$$

The variation $\|v\|$ of v, is defined by

$$\|v\| = \operatorname*{Sup}_{\Omega} \|v\|_\Omega$$

where the sup is taken over all chains Ω . The linear space of all games with bounded variation is BV. The function $\|\cdot\|$ is a norm on BV. The linear subspace of BV which contains all the finitely additive set functions is denoted by FA. The subspace of FA which contains all the non-atomic measures on $(I,\mathcal{C})$ is NA. Denote by $\mathcal{G}$ the set of automorphisms of $(I,\mathcal{C})$. For each $\theta \in \mathcal{G}$ define $\theta^* v$ by $(\theta^* v)(S) = v(\theta S)$ for each S in $\mathcal{C}$. A set of games Q is called symetric if for each $\theta \in \mathcal{G}$, $\theta^* Q \subseteq Q$. Let Q be a symetric linear subspace of BV. A game v is monotonic if $S \subseteq T$ implies $v(S) \leq v(T)$. A value on Q is a mapping $\phi: Q \to FA$ such that:

(2.1) ϕ is linear.

(2.2) ϕ is symetric; i.e. for each $\theta \varepsilon \mathcal{G}$, $\phi\theta^* = \theta^*\phi$.

(2.3) ϕ is positive; i.e. for each montonic game v, ϕv is monotonic.

(2.4) ϕ is efficient i.e.; for each game v, $(\phi v)(I) = v(I)$.

The space pNA is the closed (in the variation norm on BV) linear space generated by games of the form $p \circ \mu$ where p is a polynomial of n variables and $\mu = (\mu_1,\ldots,\mu_n)$ is a vector of n measures in NA. We denote by pNA' the space of all games that are the limit of polynomials in measures uniformly on $\mathcal{C}$; i.e. $v \varepsilon$pNA' if there is a sequence of vectors of NA-meausres $(\mu^n)_{n=1}^{\infty}$ and a sequence of polynomials $(p_n)_{n=1}^{\infty}$ such that $\sup_{S\varepsilon} |P_n(\mu^n(S)) - v(S)| \underset{n\to\infty}{\to} 0$. The space pNA' is a linear space which contains pNA, since convergence in the variation norm implies uniform convergence. A family of coalitions is called a diagonal neighborhood if there is a positive integer k and a vector of NA-measures $\mu = (\mu_1,\ldots,\mu_k)$ such that the set $\mu(\mathcal{D}) = \{\mu(S) \mid S \varepsilon \mathcal{D}\}$ contains a neighborhood of the diagnoal $[0,\mu(I)]$ in R^k. The linear subspace of BV, which contains all the games which vanish on some diagonal neighborhood is denoted by DIAG. The closed linear space generated by pNA DIAG is called pNAD. There is a unique continuous value on pNAD which we shall denote by ϕ. (The existance of this value is proved in proposition 43.13 of [A-S], the uniquness is proved in [N]). Let us denote by F the set of Borel measurable functions on I with values in [0,1]. By proposition 22.16 in [A-S] there is a unique correspondence $v \to v^*$ which assigns to each v in pNA' a function v^* defined on F such that,

$$(2.5)\quad (\alpha v + \beta w)^* = \alpha v^* + \beta v^*$$

$$(2.6)\quad (vw)^* = v^* w^*$$

(2.7) $\mu^*(f) = \int f d\mu$

(2.8) $v \to v^*$ is continuous in the uniform convergence

where α and β are scalars, v and w game in pNA' and $\mu \in$ NA. The function v^* has also the property

$$v^*(\chi_S) = v(S)$$

for each S in $\mathcal{C}$, where χ_S is the characteristic function of S. v^* is called a the <u>extension</u> of v.

From properties (2.5) and (2.6) it follows that for games of the form $p \circ \mu$ where p is a polynomial and μ a vector of measures in NA, $(p \circ \mu)^* = p \circ \mu^*$. The continuity of $v \to v^*$ implies that for any $v \in$ pNA', v^* is the uniform limit of functions of the form $p \circ \mu^*$. A game v in pNA' is called a <u>market game</u> if it is superadditive and homegeneous of degree 1.

Superadditivity means,

$$v(S \cup T) \geq v(S) + v(T)$$

for any disjoint sets T and S.

Homogeneity of degree 1 means,

$$v^*(\alpha\chi_S) = \alpha v(S)$$

for each α in [0,1] and each S. It follows that for a game which is homegeneous of degree 1

$$v^*(\alpha f) = \alpha v^*(f)$$

for any α in [0,1] and f in F. The <u>core</u> of a game v is the set C(v) which contains all the finitely additive set functions ν, such that for each S

$$\nu(S) \geq v(S)$$

and

$$\nu(I) = v(I).$$

The following lemma is proposition 44.28 in [A-S].

Lemma 2.1 Let v be a market game in pNAD pNA'. Then the core of v contains one element which is ϕv. Moreover for each S and $0 < t < 1$

$$(\phi v)(S) = \lim_{\tau \to 0} \frac{v^*(t\chi_I + \tau\chi_S) - v^*(t\chi_I)}{\tau} \quad .$$

A syndicate structure is a set of disjoint coalitions, called syndicates. For a given syndicate structure $\omega = \{A_1, A_2, \ldots\}$ let us denote $I_1 = \bigcup_i A_i$ and $I_0 = I \setminus I_1$. We shall refer to the players of I_0 as the unorganized players. An ω-coalition is a set in the σ-field $\mathcal{C}_\omega$ defined by

$$\mathcal{C}_\omega = \{S | S \supseteq A_i \text{ or } S \cap A_i = \emptyset \text{ for each } i\}$$

The ω-core of v, $C_\omega(v)$ containes all the finitely additive set functions ν, such that for each ω-coalition S,

$$\nu(S) \geq v(S)$$

and

$$\nu(I) = v(I).$$

Clearly $C(v) \subseteq C_\omega(v)$. We shall identify any two members of $C_\omega(v)$ which agree on all the ω-coalitions.

Lemma 2.2 Let v be a game in pNA' and let ω be a syndicate structure. Then each member of $C_\omega(v)$ is a σ-additive measure which has no atoms in I_0.

Proof. Proposition 44.27 in [A-S] claims that elements of the core of a game in pNA' are σ-additive measures with no atoms. The same proof with minor changes is applicable to our lemma.

Since for a given ν in $C_\omega(v)$ we are interested only in the values that ν assigns to ω-coalitions, we can assume that ν has no atoms in each of the syndicates and thus by lemma 2.2. assume that $\nu \in NA$.

3. Statement of the main results for market games.

Let $\omega = \{A_1, A_2 \ldots\}$ be a syndicate structure. For a market game v, in $pNAD \cap pNA'$ the core C(v) contains only one element namely ϕv. But in general the ω-core $C_\omega(v)$ may contain other elements (i.e. measures which differs from ϕv on some ω-coalition). We claim in the first theorem that for the unorganized players, ϕv is the best element in $C_\omega(v)$.

Theorem 3.1 Let v be a market game in $pNAD \cap pNA'$. Then for each ν in $C_\omega(v)$ and for each coalition S of unorganized players

$$\nu(S) \leq \phi v(S).$$

Certain symmetry conditions on the syndicate structure guarantee that the unorganized players are not hurt by elements of $C_\omega(v)$ with respect to ϕv. Let ξ be non-atomic vector measure. A coalition S is said to be of ξ-type for the game v, if for any $S_1, S_2 \subseteq S$ for which $\xi(S_1) = \xi(S_2)$ and any T disjoint to S_1 and S_2, $v(T \cup S_1) = v(T \cup S_2)$.

Theorem 3.2 Let v be a market game in $pNAD \cap pNA'$ and let ξ be a non-atomic vector measure such that $I_1 = \bigcup_i A_i$ is of ξ-type. If there exists $0 < \alpha < 1$ and a subset of syndicates $\{A_{i_j}\}$, such that $\xi(\bigcup_j A_{i_j}) = \alpha \xi(I_1)$ then each ν in $C_\omega(v)$ coincides with ϕv on each $S \subseteq I_0$, on $\bigcup_j A_{i_j}$ and on $I_1 \setminus \bigcup_j A_{i_j}$.

From this theorem we deduce:

Corollary 3.3 Let v be a market game in pNAD $\cap$ pNA' and let ξ be a non-atomic scalar measure such that I_1 is of ξ-type. If there is i such that $0 < \xi(A_i) < \xi(I_1)$ then $C_\omega(v) = \{\phi v\}$.

4. Proofs

We prove theorem 3.1 using the following lemmas 4.1 and 4.2.

Lemma 4.1 Let v be a game in pNA' and $\nu \in C_\omega(v)$. If g is a function which vanishes on I_1 and $B \leq I_1$ is an ω-coalition then

$$\nu^*(\chi_B + g) \geq v^*(\chi_B + g).$$

Proof: Let g and B be as in the lemma. For a given $\varepsilon > 0$ chose a polynomial in measures $p\circ\mu$ such that for each $f \in F$

$$(4.1) \qquad |v^*(f) - p(\mu^*(f))| < \varepsilon.$$

For the vector measure $\lambda = (\mu,\nu)$ as a non-atomic vector measure defined on I_o, there exists by Lyapunov's theorem a coalition $S \leq I_o$ such that

$$\int_{I_o} g d\nu = \nu(S)$$

$$\int_{I_o} g d\mu = \mu(S)$$

and since g vanishes outside I_o,

$$\nu^*(g) = \int_I g d\nu = \nu(S)$$

$$\mu^*(g) = \int_I g d\mu = \mu(S).$$

Since $B \cup S$ is an ω-coalition and $B \cap S = \emptyset$ we conclude by (4.1) that

$$\nu^*(\chi_B + g) = \nu(B \cup S)) \geqslant v(B \cup S) \geqslant p(\mu(B \cup S)) - \varepsilon = p(\mu^*(\chi_B + g)) - \varepsilon \geqslant$$

$$\geqslant v^*(\chi_B + g) - 2\varepsilon$$

Since this inequality holds for any $\varepsilon > 0$ the lemma is proved.

Q.E.D.

<u>Lemma 4.2</u> For a market game v is pNAD pNA' there exists the limit

$$\lim_{\tau \to 0^-} \frac{v^*(\chi_I + \tau\chi_S) - v^*(\chi_I)}{\tau}$$

for each S, and it is equal to $(\phi v)(S)$.

<u>Proof</u> The proof is a variant of the proof of lemma 27.2 in [A-S] and we omit it.

<u>Proof of Theorem 3.1</u> Let ν be in $C_\omega(v)$. For $S \leq I_o$ and $-1 < \tau < 0$ the function $\chi_I + \tau\chi_S$ is in F. Since

$$\chi_I + \tau\chi_S = \chi_{I_1} + (\chi_{I_0} + \tau\chi_S)$$

we can use lemma 4.1 for $B = I_1$ and $g = \chi_{I_0} + \tau\chi S$ and get

$$(4.2) \qquad \nu^*(\chi_I + \tau\chi_S) \geqslant v^*(\chi_I + \tau\chi_s).$$

Also

$$(4.3) \qquad v^*(\chi_I) = \nu^*(\chi_I) = \nu(I).$$

From (4.2) and (4.3) we have for $-1 < \tau < 0$

$$\nu(S) = \frac{\nu(I) + \tau\nu(S) - \nu(I)}{\tau} = \frac{\nu^*(\chi_I + \tau\chi_S) - \nu^*(\chi_I)}{\tau} \leq$$

$$\leq \frac{v^*(\chi_I + t\chi_S) - v^*(\chi_I)}{\tau}.$$

When $\tau \to 0^-$ we get by lemma 4.2

$$\nu(S) \leq (\phi v)(S)$$

Q.E.D.

The last inequality was proved using the fact that for $t = 1$

$$(*) \; \nu^*(tX_I + \tau X_S) \geq v^*(tX_I + \tau X_S)$$

and by differentiating $v^*(1 \cdot X_I + \tau X_S)$ with respect to τ, when τ gets only negative values. When the inequality (*) holds also for $0 < t \leq 1$ we can differentiate $v^*(tX_I + \tau X_S)$ using also positive values of τ proving in that way that $v(S) \geq (\phi v)(S)$ and so $v(S) = (\phi v)(S)$. The condition in theorem 3.7 enables us to do it.

<u>Lemma 4.3</u> Let ξ be a non-atomic vector measure such that I_1 is of ξ-type. If g_1 and g_2 are two functions in F which vanish on I_0 and satisfy $\xi^*(g_1) = \xi^*(g_2)$, and f is a function in F which vanishes on I_1 then

$$v^*(f_1 + g) = v^*(f_2 + g).$$

Proof: For $\varepsilon > 0$ chose a polynomial in measures $p \circ \mu$ such that

$$(4.4) \quad |v^*(h) - p(\mu^*(h))| < \varepsilon$$

for each h is F. By Lyapunov's theorem for g_i $(i=1,2)$ and the vector measure (ξ,μ) as a non-atomic vector measure on I_1 there exist sets $S_i \leq I_1 (i=1,2)$ for which

$$\xi(S_i) = \int_{I_1} g_i d\xi = \int_I g_i d\xi = \xi^*(g_i)$$

$$\mu(S_i) = \int_{I_1} g_i d\mu = \int_I g_i d\mu = \mu^*(g_i) \qquad i = 1,2$$

Again by Lyapunov's theorem there exists $T \leq I_0$ such that

$$\mu(T) = \int_{I_0} f d\mu = \int_I f d\mu = \mu^*(f)$$

Now

$$(4.5) \quad p(\mu^*(f + g_i) = p(\mu(T \cup S_i) \qquad i = 1,2$$

and by (4.4),

$$(4.6) \quad |p(\mu(T \cup S_i)) - v(T \cup S_i)| < \varepsilon \qquad i = 1,2$$

But

$$\xi(S_1) = \xi^*(g_1) = \xi^*(g_2) = \xi(S_2)$$

Since I_1 is of ξ-type if follows that

$$v(T \cup S_1) = v(T \cup S_2)$$

and by (4.5) and (4.6),

$$|p(\mu^*(f + g_1) - p(\mu^*(f + g_2))| < 2\varepsilon$$

From the last inequality and (4.4) we have:

$$|v^*(f + g_1) - v^*(f + g_2)| < 4\varepsilon$$

which is true for any $\varepsilon > 0$, and thus the lemma is proved.

Q.E.D.

<u>Proof of Theorem 3.2:</u> Denote $A' = \bigcup_j A_{i_j}$ and $A'' = I_1 \setminus A'$.

We have:

$$\xi(A') = \alpha\, \xi(I_1)$$

and

$$\xi(A'') = (1-\alpha)\xi(I_1).$$

Let ν be a measure in $C_\omega(v)$.

$$\nu(A') + \nu(A'') = \nu(I_1) = \alpha\nu(I_1) + (1-\alpha)\ \nu(I_1)$$

and therefore

$$\nu(A') \leqslant \alpha\nu(I_1)$$

or

$$\nu(A'') \leqslant (1-\alpha)\nu(I_1).$$

Without loss of generality we can assume $\nu(A') \leqslant \alpha\nu(I_1)$. Let $S \subseteq I_0$ and let τ be a small enough positive real number such that the function $\alpha\chi_I + \tau\chi_S$ is in F. (Observe that $\alpha < 1$).

Let us write

$$\alpha\chi_I + \tau\chi_S = \alpha\chi_{I_1} + \alpha\chi_{I_0} + \tau\chi_S$$

and denote

$$f = \alpha\chi_{I_0} + \tau\chi_S.$$

Now,

$$(4.7)\qquad \nu^*(\alpha\chi_I + \tau\chi_S) = \nu^*(\alpha\chi_{I_1} + f) = \alpha\nu^*(\chi_{I_1}) + \nu^*(f) \geq$$

$$\geq \nu^*(A') + \nu^*(f) = \nu^*(\chi_{A'} + f)$$

Using lemma 4.1 for B = A' and g=f we get

$$(4.8)\qquad \nu^*(\chi_{A'} + f) \geq v^*(\chi_{A'} + f).$$

But $\qquad \xi^*(A') = \xi(A') = \alpha\xi(I_1) = \xi^*(\alpha\chi_{I_1})$

and therefore by lemma 4.3 (for $f_1 = \chi_{A'}$, $f_2 = \alpha\chi_I$)

$$(4.9)\qquad v^*(\chi_{A'} + f) = v^*(\alpha\chi_I + f)$$

From (4.7), (4.8) and (4.9) it follows that

$$\nu^*(\alpha\chi_I + \tau\chi_S) \geq v^*(\alpha\chi_I + \tau\chi_S).$$

Hence for $S \subseteq I_0$ and a small enough positive τ

$$\nu(S) = \frac{\alpha\nu(I) + \tau\nu(S) - \alpha\nu(I)}{\tau} = \frac{\nu^*(\alpha\chi_I + \tau\chi_S) - \nu^*(\alpha\chi_I)}{\tau} \geq$$

$$\geq \frac{v^*(\alpha\chi_I + \tau\chi_S) - v^*(\alpha\chi_I)}{\tau}.$$

When $\tau \to 0^+$ the last expression tends to $(\phi v)(S)$ (lemma 2.1) and therefore

$$\nu(S) \geq (\phi v)(S).$$

The reverse inequality holds by theorem 4.1 and thus $\nu(S) = (\phi v)(S)$ for each $S \subseteq I_0$.

We have to show now that $\nu(A') = (\phi v)(A')$ and $\nu(A'') = (\phi v)(A'')$. Let $\varepsilon > 0$ be given, and let $p\circ\mu$ be a polynomial in measures such that $|v^*(h) - p(\mu^*(h))| < \varepsilon$ for each $h \in F$. By Lyapunov's theorem there are coalitions $S \subseteq I_0$ and $T \subseteq I_1$, such that S is an α proportion of I_0 for μ and ϕv and T is an α proportion of I_1 for $\mu, \phi v$ and ξ.

Therefore

$$p(\mu^*(\alpha\chi_I)) = p(\mu(S \cup T))$$

and hence

$$(4.10) \qquad |v^*(\alpha X_I) - p(\mu(S \cup T))| < \varepsilon.$$

Also

$$(4.11) \qquad |p(\mu(S \cup T)) - v(S \cup T)| < \varepsilon.$$

But by the definition of T,

$$\xi(T) = \alpha\xi(I_1) = \xi(A')$$

and therefore, since I_1 is of ξ-type

$$(4.12) \qquad v(S \cup T) = v(S \cup A').$$

From (4.10) (4.11) and (4.12) if follows that

$$(4.13) \qquad |v^*(\alpha X_I) - v(S \cup A')| < 2\varepsilon.$$

By the efficiency of ϕ, $(\phi v)(I) = v(I)$ and because of the homogeneity of v

$$\alpha(\phi v)(I) = \alpha v(I) = v^*(\alpha X_I)$$

and thus

$$(4.14) \quad (\phi v)(S \cup T) = v^*(\alpha X_I).$$

Since I_1 is of ξ type and $\xi(T) = \xi(A')$, it follows that any automorphism of $(I_1, \mathcal{C})$ that maps T onto A' and A' onto T and keeps all the other points fixed, does not change v and therefore the symmetry of ϕ implies

$$(\phi v)(T) = (\phi v)(A').$$

By (4.14)

$$(\phi v)(S \cup A') = (\phi v)\ (S \cup T) = v^{*}(\alpha X_I)$$

Using (4.13) we conclude

$$|(\phi v)(S \cup A') - v(S \cup A')| < 2\varepsilon$$

which proves that for any ν in $C_\omega(v)$

$$\nu(S \cup A') > (\phi v)(S \cup A') - 2\varepsilon$$

Since, by the first part of the proof $\nu(S) = (\phi v)(S)$, it follows that

$$\nu(A') > (\phi v)(A') - 2\varepsilon$$

for each $\varepsilon > 0$ and therefore

$$\nu(A') > (\phi v)\ (A')$$

In the same way

$$\nu(A'') > (\phi v)(A'')$$

which shows (since $v(I) = (\phi v)(I)$) that

$$\nu(A') = (\phi v)(A')$$
$$\nu(A'') = (\phi v)(A'').$$

Q.E.D.

<u>Proof of Corollary 3.3</u> Observe first that for each i with $\xi(A_i) = 0$, A_i is a null coalition (i.e for each S, $v(S) = v(S\backslash A_i)$) and therefore $(\phi v)(A_i) = 0$ [Note 4, p 18, A-S]. Denote $K = \{i \mid 0 < \xi(A_i) < \xi\ (I_1)\}$ By the assumption $K \neq \emptyset$. By theorem 3.2 for each ν in $C_\omega(\nu)$, $\nu(s) = (\phi\nu)(s)$ for each $S \leq I_0$ and $\nu(A_i) = (\phi v)(A_i)$ for each $i \varepsilon K$. Since $\nu(I) = \nu(I_0) + \sum_i \nu(A_i)$ and $(\phi v)(I) = (\phi v)(I_0) + \sum_{i \varepsilon K} (\phi v)(A_i)$ we conclude that whenever $\xi(A_i) = 0$, $\nu(A_i) = (\phi v)\ (A_i) = 0$.

Therefore for any ω-coalition S, $\nu(S) = (\phi v)(S)$, i.e. $C_\omega(v) = \{\phi v\}$.

Part II

Core and Competitive Equilibrium in Monetary Markets with Synidicates

5. Preliminaries

The space of traders in the Monetary Market is a measure space $(I,\mathcal{C},\mu)$ where μ is a non-atomic probability measure. Denote by Ω the non-negative orthant of the Euchidian space E^m. The points of Ω are commodity bundles. An integrable function $\underline{x}(t)$ from I to Ω is called an assignment. The initial assignment is denoted by $\underline{a}(t)$. For an assignment $\underline{x}$ and a coalition S we shall denote the integral $\int_S \underline{x}(t)d\mu$ by $\underline{x}(S)$. An assignment $\underline{x}$ for which $\underline{x}(I) = \underline{a}(I)$ is called an allocation. Each trader t has a utility function $u_t(x)$, defined on Ω and having the following properties:

(5.1) For each t, u_t is non-negative and increasing on Ω.

(5.2) For each t, $u_t(x)$ is measurable, as a function of both t and x, on the product space $(I \times \Omega,\ \mathcal{C} \times \mathcal{B})$ where $\mathcal{B}$ is the Borel σ - field on Ω and $\mathcal{C} \times \mathcal{B}$ is the product σ - field.

(5.3) For each t, $u_t(x)$ is $o(\sum_{j=1}^{m} x_j)$ when $\sum x_j \to \infty$ intergably in t.

(5.4) For each t, u_t is continous in Ω and for each $1 \le j \le m$ the derivative $\frac{\partial u_t}{\partial x_j}$ exists and it is continuous for each x in Ω with $x_j > 0$.

The meaning of (5.3) is that for each $\varepsilon > 0$ there exists an integrable function $\eta(t)$ such that $u_t(x) < \varepsilon \sum x_j$ whenever $\sum x_j > \eta(t)$.

We require also

(5.5) $\underline{a}(I) > 0$

Define a set function v on (I,) by:

$$v(S) = \max \{\int_S u_t(\underline{x}(t))d\mu: \underline{x}(S) = \underline{a}(S)\}.$$

Under assumptions (5.1) - (5.4), v in well defined, [A-S, proposition 31.7]. We call v(S) the <u>worth</u> of S. A <u>monetary competitive equilibrium</u> (m.c.e.) is a pair $(p,\underline{x})$ where $\underline{x}$ is an allocation, p is in Ω and for almost all t, $u_t(x) - p(x - \underline{a}(t))$ attains its maximum at $x = \underline{x}(t)$

The measure σ defined by

$$\sigma(S) = \int_S [u_t(\underline{x}(t)) - p(\underline{x}(t) - \underline{a}(t))]d\mu$$

is called the <u>monetary competitive payoff</u> (m.c.p) of $(p,\underline{x})$.

<u>Lemma 5.1</u> Let M be a market which satisfies (5.1) - (5.4). Then the worth function v is a market game in pNAD $\cap$ pNA' and the core of v contains only one element which is the value ϕv. If (5.5) is also satified, then there exists a unique monetary competitive payoff and this m.c.e coincides with ϕv.

For proof see propositions 31.7, 32.3 and section 45 in [A-S].

We denote by C(M) the core of the game v associated with M and we shall call it also the core of the market M. By $C_\omega(M)$ we shall denote the ω-core of v.

We say that two traders t and s are of the same type if $\underline{a}(s) = \underline{a}(t)$ and $u_t = u_s$. A syndicate A_i is called atom if all the traders of A_i are of the same type. For each coalition S define U_S by

$$U_S(b) = \max\{\int_S u_t(\underline{x}(t))d\mu : \underline{x}(S) = b\}$$

U_S is well defined under conditions (5.1) - (5.4). [A-S, proposition 31.7]. Two coalitions S and T are similar if $\mu(T),\mu(S) > 0$

and

$$\frac{1}{\mu(S)} U_S(\mu(S)x) = \frac{1}{\mu(T)} U_T(\mu(T)x)$$

for each x in Ω, and

$$\frac{1}{\mu(S)} \underline{a}(S) = \frac{1}{\mu(T)} \underline{a}(T).$$

A trader t and a coalition S are called similar if $\mu(S) > 0$

and

$$u_t(x) = \frac{1}{\mu(S)} U_S(\mu(S)x)$$

for each x in Ω, and

$$\underline{a}(t) = \frac{1}{\mu(S)} \underline{a}(S).$$

For simplicity we assume from now on that each syndicate has a positive measure.

6. The Main Results

The first theorem shows that the formation of syndicates hurts, in general, the unorganized traders.

Theorem 6.1 Let M be a market which satisfies (5.1) - (5.5), and let $\omega = \{A_1,A_2,\ldots,\}$ be a syndicate structure. Then for each element ν in the

ω-core of M, and for each coalition S of unorganized players, $v(S)$ does not exceed the monetary competitive payoff of S.

Theorem 6.2 and 6.3 provide conditions under which the formation of syndicates does not hurt the unorganized traders. For the unorganized traders it seems, under these conditions, that perfect competition prevails in the market, i.e. the payoff they get from each element of the core is the competitive payoff.

Theorem 6.2 If there is a non-trivial partition of the set $I_1 = \bigcup_i A_i$ (i.e. a partition with at least two non empty sets) into ω - coalitions B_1, B_2,... which are all similar, then every element in the ω-core gives to coalitions of unorganized traders and to each of the coalitions $B_1, B_2, \ldots$ their monetary competitive payoff.

The condition in theorem 6.2 deals only with the organized traders. The condition in theorem 6.3 deals with the relationship between the unorganized traders and the syndicates. For each ω-coalition B with $\mu(B) > 0$, let us denote by B' the set of all traders in I_o that are similar to B.

Theorem 6.3 If there is a partition $B_1, B_2, \ldots$ of $I_1 = \bigcup_i A_i$, such that

$$\inf \frac{\mu(B_k')}{\mu(B_k)} > 0$$

then coalitions of unorganized traders and the coalitions B_k get their monetary competitive payoff at each element of the ω-core.

The following propositions do not concern directly markets with syndicates but are immediate results of the discussion in the next section.

<u>Proposition 6.4</u> Let M' be a market generated from M by reallocating the initial bundles of a given coalition S between its members. Then the restriction of the monetary competitive payoff to $I \setminus S$ is the same in M and M'.

The meaning of this proposition is, that in a market with side payments the competitive payoff to a certain coalition is not changed if the distribution of resources of the complementary coalition is changed.

<u>Proposition 6.5</u> Let C_1 and C_2 be two disjoint coalitions such that $C_1 \quad C_2 =$ I. Denote $\omega_1 = \{C_1\}$, $\omega_2 = \{C_2\}$. Then

$$C(M) = C_{\omega_1}(M) \cap C_{\omega_2}(M)$$

This proposition means that a sufficient condition for a measure ν to be in the core, is that $\nu(S) \geq v(S)$ only for those coalitions which include or exclude one of the coalitions C_1 and C_2.

7. <u>Atomization of Syndicates</u>

We shall show in this section how syndicates can be atomized without considerably changing the market. We shall use this procedure in the proofs of theorem 6.2 ad 6.3.

Let M be a market satisfying (5.1) - (5.5) and let $\omega = \{A_1, A_2, \ldots\}$ be a syndicate structure. We define now a market M_ω on the space of traders (I,C). The initial assignment in M_ω is

$$\underline{a}'(t) = \begin{cases} \underline{a}(t) & , t \in I_o \\ \frac{1}{\mu(A_i)} \underline{a}(A_i) & , t \in A_i \end{cases}$$

The utility functions in M_ω are:

$$u'_t(x) = \begin{cases} u_t(x) & , t \in I_o \\ \frac{1}{\mu(A_i)} U_{A_i}(\mu(A_i)x) & , t \in A_i \end{cases}$$

Proposition 7.1 The market M_ω satisfies (5.1) - (5.5).

Proof: Properties (5.1), (5.2) and (5.5) are easy to verify. Let us prove (5.4).

The function U_S is continuous and concave in Ω ([A-S], propositions 37.13, 36.3). By proposition 38.1 of [A-S] for each $1 \leq i \leq M$ the derivative $\frac{\partial U_S}{\partial x_i}$ exsits at each x for which $x_i > 0$. From these properities it follows by proposition 39.1 of [A - S] that U_S satisfies property (5.4). Let us prove now property (5.3). For each $\varepsilon > 0$ there is an integrable function $\eta(t)$ for which

$$(7.1) \quad u_t(x) \leq \frac{\varepsilon}{2} \sum x$$

whenever $\sum x \geq \eta(t)$.

Without loss of generality assume $\eta(t) > 0$ for each t in I.

Let us define $\eta'(t)$ by

$$\eta'(t) = \begin{cases} \eta(x) & , \quad t \in I_o \\ \frac{1}{\mu(A_i)} \eta(A_i) & , \quad t \in A_i. \end{cases}$$

Clearly η' is integrable and it suffices to show that for each t in A_i

$$u'_t(x) < \varepsilon \sum x.$$

whenever $\sum x > \eta'(t)$.

Let t be in A_i, b in Ω and

$$(7.2) \qquad \sum b > \eta'(t).$$

Let $\underline{x}$ be the allocation for which the maximum in the definition of $U_{A_i}(\mu(A_i)b)$ is attained, i.e.,

$$(7.3) \qquad \int_{A_i} u_t(\underline{x}(t)d\mu = U_{A_i}(\mu(A_i)b)$$

and

$$(7.4) \qquad \underline{x}(A_i) = \mu(A_i)b.$$

By (7.3) we have:

$$(7.5) \qquad u'_t(b) = \frac{1}{\mu(A_i)} U_{A_i}(\mu(A_i)b) = \frac{1}{\mu(A_i)} \int_{A_i} u_t(\underline{x}(t))d\mu.$$

Denote by B_i the set of traders in A_i for which $\sum \underline{x}(t) > \eta(t)$, by C_i the set of traders in A_i for which $0 < \sum \underline{x}(t) < \eta(t)$ and let $D_i = A_i \setminus (C_i \cup B_i)$. By (7.1) we have for each t in B_i,

$$(7.6) \qquad u_t(\underline{x}(t)) < \frac{\varepsilon}{2} \sum \underline{x}(t).$$

Since u_t in increasing it follows that for each t in C_i,

$$(7.7) \quad u_t(\underline{x}(t)) < u_t(\frac{\eta(t)}{\Sigma\, \underline{x}(t)}\, \underline{x}(t))$$

and for each t in D_i,

$$(7.8) \quad u_t(\underline{x}(t)) = u_t(0) < u_t\, (\frac{\eta(t)}{n}\, e)$$

where e is the vector that has 1 in all its coordinates.

But

$$(7.9) \quad \sum (\frac{\eta(t)}{\Sigma\, \underline{x}(t)}\, \underline{x}(t)) = \eta(t)$$

and

$$(7.10) \quad \sum \frac{\eta(t)}{n}\, e = \eta(t).$$

Thus from (7.1), (7.2), (7.8), (7.9) and (7.10) it follows that for each t in $C_i \quad D_i$

$$(7.11) \quad u_t(\underline{x}(t)) < \frac{\varepsilon}{2}\, \eta(t).$$

From this, using (7.2), (7.4), (7.6) and (7.8) and the positivity of η:

$$\int_{A_i} u_t(\underline{x}(t))d\mu < \int_{B_i} (\frac{\varepsilon}{2}\, \Sigma\, \underline{x})d\mu + \int_{C_i} \int_{D_i} \frac{\varepsilon}{2}\, \eta(t))d\mu <$$

$$< \frac{\varepsilon}{2}\, \mu(A_i)\, \Sigma b + \frac{\varepsilon}{2}\, \mu(A_i)\, \eta'(t) <$$

$$< \varepsilon\, \mu(A_i)\, \Sigma b$$

and therefore by (7.5)

$$u_t'(b) = \frac{1}{\mu(A_i)} \int_{A_i} u_t(\underline{x}(t))d\mu < \varepsilon\, \Sigma b.$$

Q.E.D.

By proposition 7.1, M_ω is a market that satisfies properties (5.1)-(5.5) and in which each syndicate A_i is an atom. The following proposition shows how similar are the markets M and M_ω.

Proposition 7.2

For each S in C_ω,

(i) The coalition S has the same worth in M and in M_ω.

(ii) The monetary competitive payoff to S is the same in M and in M_ω.

Proof: (i) Denote by v and v_ω the worth functions of M and M_ω respectively. Let S be an ω-coalition and assume that v(S) is attained at $\underline{x}$.

Define:

$$\underline{x}'(t) = \begin{cases} \underline{x}(t) & ,t\varepsilon I \\ \frac{1}{\mu(A_i)}\,\underline{x}(A_i) & ,t\varepsilon A_i \end{cases}$$

Now

$$v(S) = \int_S u_t\big(\underline{x}(t)\big)d\mu \leq \int_{S\cap I_o} u_t\big(\underline{x}(t)\big)d\mu + \sum_{A_i \subseteq S} U_{A_i}\big(\underline{x}(A_i)\big) =$$

$$= \int_{S\cap I_o} u'_t\big(x'(t)\big)d\mu + \sum_{A_i \subseteq S} \int u'_t\big(\underline{x}'(A_i)\big) = \int_S u'_t\big(\underline{x}'(t)\big)d\mu \leq v_\omega(S).$$

In order to prove the reverse inequality assume $v_\omega(S)$ is attained at $\underline{x}'(t)$. Since for each t in A_i, $u'_t = U_{A'_i}$ and U_{A_i} is concave we can assume without loss of generality that x' is constant over A_i and

$\int_{A_i} u'_t\big(\underline{x}'(t)\big)d\mu = U_{A_i}\big(\underline{x}'(A_i)\big)$. Let us assume that $U_{A_i}\big(x'(A_i)\big)$ is attained at $\underline{y}_i$, i.e.:

$$U_{A_i}\left(\underline{x}'(A_i)\right) = \int_{A_i} u_t\left(\underline{y}_i(t)\right)d\mu$$

and

$$\underline{x}'(A_i) = \underline{y}_i(A_i)$$

Define an assignment $\underline{x}$ by:

$$\underline{x}(t) = \begin{cases} \underline{x}'(t) & , t\varepsilon T_o \\ \underline{y}_i(t) & , t\varepsilon A_i \end{cases}$$

Clearly $\underline{x}(S) = \underline{x}'(S) = \underline{a}(S)$ and

$$v_\omega(S) = \int_S u'\left(\underline{x}'(t)\right)d\mu = \int_S u_t\left(\underline{x}(t)\right)d\mu \leq v(S).$$

(ii) Let $(p,\underline{x})$ be an m.c.e in M and let σ be the m.c.p in M. Define

$$\underline{x}'(t) = \begin{cases} \underline{x}(t) & , t\varepsilon I_o \\ \frac{1}{\mu(A_i)} x\ (A_i) & , t\varepsilon A_i \end{cases}$$

We will show that $(p,\underline{x}')$ is a an m.c.e in M_ω. It is sufficient to show that for each A_i and t in A_i, $u_t'(x) - p\left(x - \underline{a}'(t)\right)$ attains its maximum at $x = \underline{x}'(t)$. Let $x\varepsilon\Omega$ and assume that $U_{A_i}\left(\mu(A_i)x\right)$ is attained at $\underline{y}(t)$. It follows that

$$u_t\left(\underline{x}(t)\right) - p\left(\underline{x}(t) - \underline{a}(t)\right) \geq u_t\left(\underline{y}(t)\right) - p\left(\underline{y}(t) - \underline{a}(t)\right)$$

and by integration

$$(7.12)\quad \int_{A_i} u_t\left(\underline{x}(t)\right)d\mu - p\left(\underline{x}(A_i) - \underline{a}(A_i)\right) \geq U_{A_i}\left(\mu(A_i)x\right) - p\left(\mu(A_i)x - \underline{a}(s_i)\right) = \\ = \mu(A_i)\left[u_t'(x) - p\left(x - \underline{a}'(t)\right)\right].$$

But

$$(7.13)\quad \mu(A_i)\left[u_t'\left(\underline{x}'(t)\right) - p\left(\underline{x}'(t) - \underline{a}'(t)\right)\right] = U_{A_i}\left(\underline{x}(A_i)\right) - p\left(\underline{x}(A_i) - \underline{a}(A_i)\right) \geq$$

$$\geq \int_{A_i} u_t\left(\underline{x}(t)\right)d\mu - p\left(\underline{x}(A_i) - \underline{a}(A_i)\right)$$

From (7.12) and (7.13) we deduce:

$$u_t'\left(x'(t)\right) - p\left(\underline{x}'(t) - \underline{a}'\ (t)\right) \geq u_t'(x) - p\left(x-\underline{a}'(t)\right).$$

By that we have shown by that $(p,\underline{x}')$ is an m.c.e in M_ω. Moreover, if σ_ω is the m.c.p in M_ω then by (7.13), for each A_i,

$$\sigma_\omega(A_i) \geq \sigma(A_i).$$

But $\sigma_\omega(S) = \sigma(S)$ for each $S \subseteq I_o$ and $\sigma_\omega(I) = \sigma(I)$ and therefore

$$\sigma_\omega = \sigma.$$

Q.E.D.

8. Proofs of the main results

Proof of Theorem 6.1: By Lemma 5.1 the worth function v of the market M is in pNAD pNA'. It follows by Theorem 3.1 that for each $S \subseteq I_o$ and $\nu\varepsilon C_\omega(M)$, $\nu(S) \leq (\phi v)(S)$. Since, by Lemma 5.1 ϕv is the monetary competitive payoff, the theorem is proved.

<u>Proof of Theorem 6.2</u>: Let us denote $\bar{\omega} = \{B_1, B_2, \ldots,\}$ and consider the market $M_{\bar{\omega}}$, generated by atomization of the sets $B_1, B_2, \ldots,$. Denote by $\bar{v}$ the worth function of $M_{\bar{\omega}}$. Since the sets $B_1, B_2, \ldots,$ are similar in M it follows that all the traders in $\bigcup_k B_k$ are of the same type in $M_{\bar{\omega}}$ and have the same (concave) utility function and the same initial assignment. It follows therefore that if $S_1, S_2 \leq_k B_k$ and $\mu(S_1) = \mu(S_2)$ then for each T which is disjoint to S_1, S_2, $\bar{v}(T \cup S_1) = \bar{v}(T \cup S_2)$ and thus $\bigcup_k B_k$ is a coalition of type) with respect to $\bar{v}$. Since $\mu(B_k) > 0$ for each k it follows by corollary 3.3 that for each ν in $C_{\bar{\omega}}(M_{\bar{\omega}})$ and each S in $C_{\bar{\omega}}$,

$$\nu(S) = (\phi\bar{v})(S).$$

But $\phi\bar{v}$ is the m.c.p in $M_{\bar{\omega}}$ and by proposition 7.2 it follows that the monetary competitive payoffs of $M_{\bar{\omega}}$ and M coincide on $C_{\bar{\omega}}$, and therefore for each S in $C_{\bar{\omega}}$, $\nu(S)$ is the m.c.p for S in M. Since $C_{\bar{\omega}} \leq C_{\omega}$ and for each $S \in C_{\bar{\omega}}$, $\bar{v}(S) = v(S)$ (by proposition 7.2) it follows that $C_{\omega}(M) \leq C_{\bar{\omega}}(M_{\bar{\omega}})$. We conclude therefore that for each ν in $C_{\omega}(M)$ and each S in $C_{\bar{w}}$, $\nu(S)$ in the monetary competitive payoff to S, in M.

<u>Proof of Theorem 6.3:</u> By the stipulation of the theorem and since μ is non-atomic, we can find for any sufficient large n, coalitions $B_{k,n}$ such that:

$$B_{k,n+1} \leq B_{k,n} \leq B'_k$$

and

$$\mu(B_{k,n}) = \frac{1}{n}\mu(B_k).$$

Let us denote and $\bar{B}_n = \cup_k B_{k,n}$. Consider the syndicate structure $\omega_n = \{I_1, \bar{B}_n\}$ It is easy to see that I_1 and $\bar{B}_n$ are similar coalitions and therefore by Theorem 6.2 for each S in $I_0 \setminus \bar{B}_n$ the payoff $\nu(S)$ for ν in $C_{\omega_n}(M)$ is the m.c.p to S. This last statement is true also for $\nu \varepsilon C_\omega(M)$ since $C_\omega(M) \leq C_{\omega_n}(M)$. By the construction of the sequence $\bar{B}_n$ it follows that $\bar{B}_{n+1} \leq \bar{B}_n$ and $\mu(\bar{B}_n) = 0$. We conclude therefore by Theorem 6.2, that ν coincides with m.c.p on $I_o \setminus \cup_n \bar{B}_n$ and on each B_k. Since these two measures are absolutely continuous with respect to μ it follows that they coincide on I_o, and are the same for each B_k.

<u>Proof of Proposition 6.4:</u> Consider the syndicate structure $\omega = \{S\}$. Clearly M_ω and M'_ω are the same market. By proposition 7.1 for $T \leq I$ S the m.c.p of T is the same in M and M_ω and the same in M' and M'_ω. Therefore T has the same m.c.p. in M and M'.

<u>Proof of Proposition 6.5:</u> Let v be the worth function of M and let ν be a measure in $C_{\omega_1}(M)$ $C_{\omega_2}(M)$. By Theorem 6.1 for each $S \varepsilon C_1$, $\nu(S) \leq (\phi v)(S)$ and similarly $\nu(S) \leq (\phi v)(S)$ for each $S \varepsilon C_2$. But since $\nu(I) = (\phi v)(I)$ it follows that $\{\nu\} = \{\phi\nu\} = C(M)$.

9. <u>Comparison to Non-Transferable Utility Markets</u>

Shitovitz studied in [S] non-transferable utility (n.t.u) markets in which the big traders are atoms of the measure space. The core of these markets can equivalently be studied in a market with atomless space of traders and in which the set of big traders ω, is a set of disjoint coalitions such that traders in the same coalition are of the same type. (This is the way atoms were defined in this paper). The equivalent of the core in [S] will be then the ω -core. Shitovitz has shown that in general, in core allocations small traders may be monetarily exploited. He also bring some sufficient conditions that prevent this exploitation. In general, for each allocation $\underline{x}$ in the core, there exsits a price vector p, such that $(p,\underline{x})$ is an efficiency equilibrium and for almost all small traders $p\,\underline{x}(t) < p\,\underline{a}(t)$ where $\underline{a}(t)$ is the intial allocation [Theorem A,S]. The exploitation of the small traders is equivocal however, since although monetarily exploited, the small traders may have higher utility of their core allocations then of their inital bundles, as was shown by Aumann [A]. In the transferable utility (t.u) market the exploitation of small trader is unequivocal. In these markets as we have shown in Theorem 6.1, the small traders are exploited in the sense that the core payoffs to them cannot exceed their monetary competitive payoff. Since in t.u.e the monetary payoff measurs the utility, no distinction can be made between monetary loss and utility loss. For the subsequent discussion we need the following definitions. Two coalitions S_1 and S_2 are of the <u>same type</u> if there exist a measurable one to one mapping ϕ from S_1 onto S_2 and a real number $\alpha > 0$ such that for almost all traders t in S_1, t and $\phi(t)$ are of the same type, and for each T S_2, $\mu(T) = \alpha\mu\phi^{-1}(T)$. Two coalitions are of the <u>same kind</u> if they are of the same type and $\alpha = 1$. Observe that two coalitions that are of the same kind or of the same type are similar according to the definition in section 5. Theorem B in [S] says that if there are at least two atoms and all the atoms are of the same type than the small traders are not exploited in the core, (in this case the efficiency equilibrium of theorem A is the competitive equilibrium). Theorem C in [S] claims that if the sets of atoms can be partitioned into coalitions (at least two) which are of the same kind then the small traders are not exploited. The conditions in Theorems B and C in [S] are stronger then the condition in Theorem 6.2 of this work and therefore for t.u. markets the propositions of these theorems are special cases of Theorem 6.2. Moreover, by this theorem we can conclude that the small traders are not exploited even when the requirement of "the same kind" in theorem C is replaced by the weaker requirement "The same type". Similarly we can also replace "atoms" in Theorem B by the wider concept of "syndicates". This strengthing of theorem B and C is typical to the t.u. markets and is impossible for n.t.u.e as was shown by Chamsaur and Laroque [C-L].

Theorem 6.3 is comparable with a theorem proved by Gabszewicz and Mertens [G-M]. They discuss n.t.u markets in which the A_i's are atoms (rather than syndicates) and $\{B_k\}_k$ is a partition of the atoms according to their types. Let us denote by $\bar{B}_k$ the set of traders in the market which are of the same type as the atoms in B_k. (Clearly $B_k \leq \bar{B}_k$).

It is shown in [G-M] that a sufficient condition for the coincidence of the core and the competitive equilibrium is

$$\sum_k \frac{\mu(B_k)}{\mu(\bar{B}_k)} < 1$$

when the number of types is greater than 1 and $\mu(B_1)/\mu(\bar{B}_1) < 1$ when there is only one type. By Theorem 6.3, the coincidence of the core and the m.c.p. in the transferable utility case is reached by a weaker condition. Indeed the condition of Theorem 6.3 can be rewritten as

$$\operatorname{Sup} \frac{\mu(B_k)}{\mu(\bar{B}_k)} < 1$$

(Observe that when the A_i's are atoms then the core coincides with the m.c.p. not only on the B_i's but also on the A_i's).

Proposition 6.5 is analogous to a theorem that was proved recently by Shitovitz and Okuda [O-S] for n.t.u markets. In this theorem the sets C_1 and C_2 of Proposition 6.5 are replaced by sets $C_1, C_2, \ldots, C_{n+1}$ where n is the dimension of the linear subspace spanned by the efficiency price vectors.

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