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EQUILIBRIUM LIMIT PRICING: THE EFFECTS OF PRIVATE INFORMATION AND STOCHASTIC DEMAND*

by

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1. INTRODUCTION AND BACKGROUND

Economists have long believed that the price level in an industry influences firms contemplating entry. This belief is justified generally on the basis that current prices convey information about post-entry profitability to potential entrants. However, it has been difficult to logically describe the exact nature of such information and the way it is embodied in prices.

The limit pricing literature can be roughly separated into two strands, depending upon which of two types of determinants of post-entry profit that prices are assumed to indicate. Both types of determinants of post-entry profitability were alluded to in a seminal paper by Joe S. Bain:

> "Even if [the potential entrant] does not believe the observed price will remain there for him to exploit, he may nevertheless regard this price as an indicator both of the character of industry demand and of the probable character of rival policy after his entry."(Bain [1949], p. 453)

In the first models of limit pricing, current prices indicated "the probable character of rival policy" to be adopted by established firms. In particular, Bain [1956], Modigliani [1958], and Sylos-Labini [1962] constructed models in which an established firm threatened to maintain the output corresponding to the current price if entry occurred. A potential entrant who believed this threat would therefore be deterred by a low price (high output). But output maintenance is not in the best interest of an established firm if entry actually occurs. Consequently, output maintenance is not a believable threat and should not deter a rational potential entrant.[1]

[1] In game-theoretic terms, an unbelievable threat such as output maintenance cannot be sustained in a perfect Nash equilibrium (Selten [1975, 8]).

To sidestep this criticism, many studies have not attempted to investigate the decisionmaking of potential entrants, but rather have simply assumed entry is deterred by low prices (Gaskins [1971], Kamien and Schwartz [1971, 5], Pyatt [1971], Baron [1973], Deshmukh and Chikte [1976], Lippman [1980]). More recently, models have been constructed in which the threats of established firms are credible because they consist of irreversible decisions, such as advertising, plant investment, or technology choice, that truly influence future demand or cost conditions (Salop [1979], Spence [1977], Salop [1978], Flaherty [1980], Spulber [1981]). Thus, in these models the current price reflects a decision made by an established firm which determines its own future objective function and, as a result, its policy if entry does occur.

The second, more recent strand of the limit pricing literature has assumed that prices convey information about an exogenous, currently existing determinant of post-entry profitability, such as "the character of industry demand." The conveyance of this type of information is the subject of the signalling literature originated in Spence [1973]. Signalling techniques have been used by Reynolds and Salop [1978] (discussed in Salop [1979]), Milgrom and Roberts [1979], and Gal-or [1980] to construct models in which prices indicate exogenously given parameters of the established firm's cost function, which is another determinant of an entrant's profit.

In these signalling models, established firms with low costs attempt to distinguish themselves by setting low prices. Equilibrium prices are therefore lower than their pure monopoly level. However, as a consequence of the assumed rationality of all actors in a signalling equilibrium, a potential entrant cannot be fooled by low prices into not entering. In a separating equilibrium the potential entrant correctly infers the costs of the established firm, so that entry occurs exactly when it would in a world of perfect information in which the potential entrant could directly observe the costs of established firms. Furthermore, the same amount of entry occurs in these limit pricing equilibria as would occur if instead it were common knowledge that established firms set monopoly prices, since potential entrants can also make correct inferences from monopoly prices. Thus, according to these models, it is not in the public interest to (somehow) ban limit pricing, or to force established firms to verifiably publish their cost data, or to allow established firms to remain unaware of entry threats.

This is the point of departure for our paper. Going back to Bain's idea that prices convey information about exogenous determinants of post-entry profitability, we assume that a persistent industry characteristic exists which is observed only by an established firm. However, in our model prices cannot perfectly reveal the industry characteristic, since a random demand shock, unobservable to the entrant, occurs after the established firm makes its decisions.[1] Thus a price observation can only provide the potential entrant with statistical information.[2] Although potential entrants still cannot be fooled into not entering, in our model their ability to learn from prices can be diminished in a continuous fashion. We can ask questions in this model that are either trivial or cannot be formulated in the previous models. For example, we can ask for the conditions under which price observations in a limit pricing equilibrium convey more or less useful

[1] The idea of a persistent demand characteristic known only to established firms, and of a demand shock initially known to no one, is actually Bain's: "Industry demands are never certainly known, and they are probably known less fully be potential entrants than by established firms." ([1949], p. 453)

[2] Prices also do not reveal perfect information in the above signalling models if firms with different costs make the same pricing decisions. However, conditions for such pooling equilibria to exist are highly restrictive.

information to potential entrants than they would in a monopoly pricing regime.

The basic structure of the model is described in the next section. In Section 3 sufficient conditions for an equilibrium to exhibit limit pricing are presented. Relative to an appropriately defined monopoly situation, sufficient conditions for the potential entrant to be worse off and for entry to be deterred in a limit pricing equilibrium are found in Section 4. Section 5 contains a simple example illustrating that a pricing equilibrium can exist and that entry can be deterred without most of the sufficient conditions of the previous sections holding.

2. THE MODEL

Consider an industry which operates for two periods. Only firm I, the incumbent firm, operates in period one. The second agent, firm E, is a potential entrant. Firm E decides whether or not to enter in period two after it observes the price that occurs in period one.

We view firm I as choosing an expected price p in the first period. The actual price, however, will not be p but rather a realization P of a random variable $\tilde{P} = p + \tilde{\eta}$. The noise variable $\tilde{\eta}$ is due to random demand shocks that occur after firm I has made its decisions. The primary example is a situation in which firm I makes an output decision before it knows exactly what the demand curve is. An output decision translates into an expected price via an expected demand function. Therefore the expected price, rather than the output quantity, can be viewed as the decision variable.

We make the following assumptions regarding the noise $\tilde{\eta}$. It is distributed independently of the expected price p. Its distribution function is denoted by F and its density function by f. We assume, until Section 5, that f is continuous and positive on the entire real line. Finally, we assume that $E\tilde{\eta} = 0$.

The expected first period profit of firm I depends upon the cost and demand characteristics of the industry. The model hinges on two aspects of these industy characteristics: (1) that they will be present in period two as well as in period one, and (2) that some of them are known in period one only to the firm I. These persistent industry charateristics known only to firm I are represented by a profitability parameter θ. Firm E views θ as a random variable with a distribution function $\lambda(\cdot)$. The set of possible values of θ, denoted by Θ, is assumed to be an interval $[\underline{\theta}, \bar{\theta}]$, and $\lambda(\cdot)$ is assumed to be increasing on this interval.

Given an expected price p, let $R(p,\theta)$ denote the expected profit, or net revenue, of firm I in period one. To justify interpreting θ as a measure of profitability, we assume that $R_2(p,\theta) > 0$ and $R_{12}(p,\theta) > 0$, i.e., that both total and marginal net revenue increase in θ. We also assume that $R_{11}(p,\theta) < 0$, and that the expected monopoly price $p^M(\theta)$ is positive and well-defined by $R_1(p^M(\theta),\theta) = 0$.

At the time of its entry decision, firm E has observed the first period price but not the profitability parameter θ. Upon entering, firm E will engage in some type of game with firm I in period two. This game may, for example, be Cournot, Stackleberg or perfect competition, and it may or may not involve firm E learning more about θ. Rather than specifying the post-entry game, we merely assume that in its equilibrium, firm I receives expected profit $\pi^M(\theta) - L(\theta)$, the monopoly profit minus a positive loss due to entry, and firm E receives expected profit $\pi^E(\theta)$. If firm E does not enter, then firm I monopolistically prices in period two and receives the expected monopoly profit $\pi^M(\theta)$.

The expected profit function $\pi^E(\cdot)$ is taken net of entry costs. Thus it is positive for some θ and negative for others. If firm E knew θ at the time of its entry decision, it would enter if and only if $\pi^E(\theta) \geq 0$.[1] In accordance with θ being a profitability measure, we also assume $\pi^E(\cdot)$ is an increasing function. Therefore a <u>breakeven level</u> $\theta_o \in (\underline{\theta},\bar{\theta})$ exists such that $\pi^E(\theta)(\theta - \theta_o) > 0$ for every $\theta \neq \theta_o$.

Observe that the profit of neither firm in period two depends upon first period decisions made by firm I. That is, firm I cannot alter future demand or supply conditions in order to deter entry. Furthermore, we assume that firm I cannot deter entry by simply telling firm E that $\theta < \theta_o$. Such a statement would not be believed if it cannot be verified, and we assume that a verification technology does not exist. Hence firm I can influence entry only though its pricing decisions. Since the expected price it sets will generally depend on the true θ, an observation of the first period price will convey statistical information about θ to firm E.

A strategy for firm E is an <u>entry rule</u>, $e: R \to \{0,1\}$. An entry rule specifies that, after observing a price P in period one, firm E enters if $e(P) = 1$ and does not enter if $e(P) = 0$.

A strategy for firm I is a <u>pricing rule</u>, $p: \Theta \to R$, that specifies an expected price $p(\theta)$ for each θ. It should be observed that firm I does not actually choose the entire rule $p(\cdot)$. Rather, it chooses only the number $p(\theta)$ that is optimal for the true θ. However, since firm E does not know the true θ, it is necessary to specify the decision of firm I for each possible value of θ. The problem of firm E will be well-defined only if it can

[1] For simplicity, we assume firm E enters when it is indifferent.

conjecture what the action of firm I is for every θ.

Given an entry rule $e(\cdot)$ and an expected price p, the probability of entry is

$$G \equiv Ee(p + \tilde{\eta}) = \int_{-\infty}^{\infty} e(p + \eta)f(\eta)d\eta.$$

As a function of the expected price p, the probability of entry G, and the profitability parameter θ, the two-period expected profit of firm I is

$$\Pi^I(p,G,\theta) = R(p,\theta) - L(\theta)G + \pi^M(\theta).$$

A pricing rule $p(\cdot)$ establishes a stochastic relationship between the random variables $\tilde{\theta}$ and $\tilde{P}$. Consequently, when a realization P of $\tilde{P}$ occurs, the distribution $\lambda(\cdot)$ can be updated to $\lambda(\cdot|P)$, where

$$\lambda(\hat{\theta}|P) = \frac{\int_{\underline{\theta}}^{\hat{\theta}} f(P - p(\theta))d\lambda(\theta)}{\int_{\Theta} f(P - p(\theta))d\lambda(\theta)}.$$

If firm E observes the price P and conjectures that firm I prices according to the rule $p(\cdot)$, then it views its expected profit from entering as

$$\Pi^E[P,p(\cdot)] = \int_{\Theta}\pi^E(\theta)d\lambda(\theta|P).$$

The rules $p^*(\cdot)$ and $e^*(\cdot)$ constitute an equilibrium provided each is a best response to the other. That is, $(p^*(\cdot), e^*(\cdot))$ is an <u>equilibrium</u> provided the following conditions are satisfied:

$$\text{(E1)} \qquad p^*(\theta) \; \varepsilon \; \underset{p}{\operatorname{argmax}} \; \Pi^I(p, \; Ee^*(p + \tilde{\eta}), \; \theta)$$

$$\text{(E2)} \qquad e^*(P) = 1 \quad \text{iff} \quad \Pi^E[P, p(\cdot)] \geq 0.$$

An equilibrium is best viewed in terms of self-confirming conjectures. If firm I conjectures that firm E is using the entry rule $e^*(\cdot)$, then its best response, given the true value θ, is to set the expected price equal to $p^*(\theta)$. In turn, if firm E believes that the expected price is set according to the rule $p^*(\cdot)$, then its best response is $e^*(\cdot)$, i.e., to enter exactly when expected profit conditional on the first period price is nonnegative. Thus an equilibrium has the property that each agent's conjectures about the other's behavior is correct.

Our central concern is with limit pricing, i.e., with situations in which equilibrium prices are lower than monopoly prices. Given an equilibrium $(p^*(\cdot), e^*(\cdot))$, let $\delta(\theta) = p^M(\theta) - p^*(\theta)$ be the difference between the monopoly and the equilibrium prices. The equilibrium satisfies the <u>limit pricing property</u> if

$$\text{(E3)} \qquad \delta(\cdot) \geq 0 \text{ and, for some } \theta \; \varepsilon \; \Theta, \; \delta(\theta) > 0.$$

A <u>limit pricing equilibrium</u> is a pair $(p^*(\cdot), e^*(\cdot))$ of rules satisfying conditions (E1) - (E3).

3. CONDITIONS FOR LIMIT PRICING

In order for firm I to engage in limit pricing, it should be true that entry is more likely when prices are high rather than low. A simple way for this to occur is for $e^*(\cdot)$ to specify entry if and only if the observed price exceeds some critical level. The following assumption will help insure that $e^*(\cdot)$ is of this form.

$$\text{(A1)} \qquad \text{If } p_1 < p_2, \text{ then } \frac{f(P - p_1)}{f(P - p_2)} \text{ decreases in } P.$$ [1]

The ratio in (A1) is actually the likelihood ratio $f(P|p_1)/f(P|p_2)$. Therefore (A1) implies that the expected price is more likely to be high when a high price is observed. Most commonly used probability functions, e.g., uniform, normal, or exponential, satisfy (A1).

<u>Lemma 1:</u> Let $e(\cdot)$ be a best response to an increasing pricing rule $p(\cdot)$. Then (A1) implies that

$$e(P) = \begin{cases} 0 & P < P_E \\ 1 & P \geq P_E, \end{cases}$$

where P_E is defined by $\Pi^E[P_E, p(\cdot)] = 0$ and satisfies $(P - P_E)\Pi^E[P, p(\cdot)] > 0$ for all $P \neq P_E$.

<u>Proof:</u> Let $\theta_1 < \theta_2$ be such that $p(\theta_1) < p(\theta_2)$. Then (A1) implies that the likelihood ratio $f(P|\theta_1)/f(P|\theta_2) = f(P - p(\theta_1))/f(P-p(\theta_2))$ decreases in P. Thus Theorem 1 in Milgrom [1979] implies that if $P_1 < P_2$, the posterior distribution $\lambda(\cdot|P_2)$ dominates $\lambda(\cdot|P_1)$ in the sense of first degree stochastic domiance. Consequently, as $\pi^E(\cdot)$ is an increasing function, $\Pi^E[P, p(\cdot)]$ increases in P. Enough continuity assumptions have been made to insure that $\Pi^E[P,p(\cdot)]$ is continuous. Also

[1] Assumption (A1) is a specialization of the <u>monotone likelihood ratio property</u>, which is the subject of Milgrom [1979].

$$\lim_{P \to -\infty} \Pi^E[P,p(\cdot)] < 0 \quad \text{and} \quad \lim_{P \to \infty} \Pi^E[P,p(\cdot)] > 0$$

are implied by $\theta_o \in (\underline{\theta},\bar{\theta})$, $\lambda(\cdot)$ increasing on $[\underline{\theta},\bar{\theta}]$, and $p(\cdot)$ increasing on $[\underline{\theta},\bar{\theta}]$. Thus there is a unique solution P_E to $\Pi^E[P_E,p(\cdot)] = 0$, and $\Pi^E[P,p(\cdot)] \geq 0$ if and only if $P \geq P_E$. Hence (E2) implies $e(P) = 1$ if and only if $P \geq P_E$. ////

Provided that the equilibrium pricing rule increases in θ, Lemma 1 insures that $e^*(\cdot)$ specifies entry if and only if the observed price exceeds an <u>equilibrium entry price</u> P_E^*. This type of entry rule is tractable and will imply limit pricing.

The following assumption insures that $p^*(\cdot)$ actually is increasing. It is an assumption about how firm I's marginal rate of substitution between expected price and entry probability, $\Pi_1(p,G,\theta)/\Pi_2(p,G,\theta) = - R_1(p,\theta)/L(\theta)$, changes with the profitability parameter.

(A2) $\quad R_1(p,\theta)/L(\theta)$ increases in θ.

This condition implies that as the profitability level θ increases, firm I is willing to suffer a greater increase in the probability of entry in order to increase the expected first period price a given amount. If $L(\cdot)$ does not vary too much, then (A2) will be satisfied, since $R_{12}(p,\theta) > 0$ and $L(\theta) > 0$.

<u>Lemma 2:</u> Let $p(\cdot)$ be a best response to an entry rule $e(\cdot)$. Then $p(\cdot)$ is nondecreasing if (A2) holds.

Proof: Let $\theta_1 > \theta_2$. Denote $p(\theta_i)$, $Ee(p(\theta_i) + \tilde{\eta})$, and $L(\theta_i)$ by p_i, G_i, and L_i, respectively. Then, since $p(\cdot)$ is a best response to $e(\cdot)$,

$$R(p_i,\theta_i) - L_iG_i \geq R(p_j,\theta_i) - L_iG_j$$

for $(i,j) = (1,2)$ and for $(i,j) = (2,1)$. Rearrangement yields

$$\frac{R(p_2,\theta_1) - R(p_1,\theta_1)}{L_1} \leq G_2 - G_1 \leq \frac{R(p_2,\theta_2) - R(p_1,\theta_2)}{L_2},$$

which implies $\int_{p_1}^{p_2} \left(\frac{R_1(p,\theta_2)}{L_2} - \frac{R_1(p,\theta_1)}{L_1}\right) dp \geq 0$. Therefore (A2) implies that $p_2 \geq p_1$. ////.

Conditions (A1) and (A2) will henceforth be assumed. Consequently, lemmas 1 and 2 imply the following result.

Theorem 1: Any equilibrium $(p^*(\cdot),\ e^*(\cdot))$ is a limit pricing equilibrium, and it is characterized by expressions (1) and (2):

$$(1) \quad R_1(p^*(\theta),\theta) = L(\theta)f(P_E^* - p^*(\theta))$$

$$(2) \quad e^*(P) = \begin{cases} 0 & P < P_E^* \\ 1 & P \geq P_E^*, \end{cases}$$

where P_E^* is defined by $\Pi_E^E[P^*,\ p^*(\cdot)] = 0$ and satisfies $(P - P_E^*)\Pi^E[P,\ p^*(\cdot)] > 0$ for all $P \neq P_E^*$.

Proof: We know by lemma 2 that $p^*(\cdot)$ is nondecreasing. Therefore (A1) and Theorem 1 in Milgrom [1979] imply $\Pi^E[P,p^*(\cdot)]$ is nondecreasing. Hence (E2) implies $e^*(\cdot)$ is nondecreasing. If $e^*(\cdot)$ is constant, then $p^*(\cdot) = p^M(\cdot)$, since firm I cannot affect the probability of entry. But then $p^*(\cdot)$ is an increasing function, so that lemma 1 implies the contradiction that $e^*(\cdot)$ is not constant. Therefore $e^*(\cdot)$ is not constant and so must be of the form (2) for some P_E^*. Thus the probability of entry, as a function of p, is

$$Ee^*(p + \tilde{\eta}) = 1 - F(P_E^* - p).$$

Since the derivative of $Ee^*(p + \tilde{\eta})$ is $f(P_E^* - p)$ at all p, (1) is the necessary first order condition for (E1). Since $f(\cdot) > 0$, (1) implies $R_1(p^*(\theta),\theta) > 0$, which in turn implies $p^*(\cdot) < p^M(\cdot)$, i.e., limit pricing. Also, (1) and (A2) imply that $p^*(\theta_1) \neq p^*(\theta_2)$ if $\theta_1 \neq \theta_2$. Hence $p^*(\cdot)$ is increasing, and lemma 1 implies that P_E^* satisfies $\Pi^E[P_E^*, p^*(\cdot)] = 0$ and $(P - P_E^*)\Pi^E[P,p^*(\cdot)] > 0$ for all $P \neq P_E^*$. ////

4. Conditions for Entry Deterrence

Two natural questions arise about a limit pricing equilibrium. The first is whether or not entry is deterred. The second is whether or not the expected profits of either firm is increased by limit pricing. In order to answer these questions, we must specify what nonequilibrium situation serves as the comparison.

A possible benchmark is one in which firm E enters according to the rule $e^*(\cdot)$, but firm I myopically prices according to the monopoly rule $p^M(\cdot)$. Comparing this benchmark $(p^M(\cdot), e^*(\cdot))$ to an equilibrium

$(p^*(\cdot), e^*(\cdot))$ serves to identify the effects that limit pricing has upon entry when the entrant's decision rule is held fixed. This is essentially the comparison made in the nonequilibrium limit pricing literature, e.g., Gaskins [1971] and Kamien and Schwartz [1971]. In our model this comparison is trivial: because $p^*(\cdot) < p^M(\cdot)$, the probability that the first period price exceeds the entry price P_E^* is less if $p^*(\cdot)$ rather than $p^M(\cdot)$ is used. Therefore, in this comparison the probability of entry is decreased by limit pricing.

Another possible benchmark is one in which the potential entrant has access to the same information as the established firm, namely, the true value of θ. This symmetric information benchmark is analagous to the perfect information benchmark examined in Milgrom and Roberts [1979]. Comparing an equilibrium to this symmetric information benchmark serves to identify the effects of the assumed asymmetry in information. If firm E can observe θ, then entry occurs exactly when $\theta \geq \theta_o$. Since firm I cannot therefore influence entry, it uses the pricing rule $p^M(\cdot)$. Thus, relative to the symmetric information situation, asymmetric information results in limit pricing which decreases entry if $\theta \geq \theta_o$ and increases entry if $\theta < \theta_o$.

To us the most interesting benchmark is $(p^M(\cdot), e^M(\cdot))$, where $e^M(\cdot)$ is the best response of firm E to the monopoly pricing rule $p^M(\cdot)$. This benchmark, which shall be referred to as the <u>monopoly situation</u>, would occur if for some reason firm I set a monopoly expected price, and firm E knew that firm I was setting a monopoly expected price. Unlike the above benchmarks, the monopoly situation maintains two basic premises of the model, its information structure and the rationality of the potential entrant. Since $p^M(\cdot)$ is not a best response to $e^M(\cdot)$, the monopoly situation does require, under one interpretation, that firm I be irrational. However, monopoly pricing is rational for firm I if it is unaware that entry is being contemplated by any existing or potential firm.

The entry rule $e^M(\cdot)$ is easy to characterize. Because $p^M(\cdot)$ is increasing, lemma 1 implies

$$(3) \qquad e^M(P) = \begin{cases} 0 & P < P_E^M \\ 1 & P \geq P_E^M \end{cases},$$

where the <u>monopoly entry price</u> P_E^M is defined by $\Pi^E[P_E^M, p^M(\cdot)] = 0$ and satisfies $(P - P_E^M)\Pi^E[P,p^M(\cdot)] > 0$ for all $P \neq P_E^M$.

Comparisons of an equilibrium $(p^*(\cdot), e^*(\cdot))$ to the monopoly situation depend upon whether $\delta(\cdot) = p^M(\cdot) - p^*(\cdot)$ is increasing or decreasing. If $p^M(\cdot)$ is steeper than $p^*(\cdot)$, then the distribution of $\tilde{P}$ will vary more with θ in the monopoly situation than in the equilibrium. That is, the noise will obscure θ less when the pricing rule is steeper. Hence, if $p^M(\cdot)$ is steeper than $p^*(\cdot)$, then we expect firm E to be better off in the monopoly situation than in the equilibrium.[1] This is verified in the following theorem.

<u>Theorem 2:</u> The expected profits of firm E are larger (smaller) in the monopoly situation than in an equilibrium $(p^*(\cdot), e^*(\cdot))$ if $\delta(\cdot)$ is increasing (decreasing).

<u>Proof</u>: (Since the arguments are symmetric, we only give the proof for $\delta(\cdot)$ increasing.) Given a pricing rule $p(\cdot)$, the profits obtained by firm E, when it adopts the rule of entering if and only if the observed price exceeds P_E,

[1] It is not generally true that observing a price from the conditional density $f(P - p(\theta))$ is more informative about θ, in the sense of Blackwell [1953], the steeper is $p(\cdot)$. Theorem 2 is proved directly, without relying on the theorem of Blackwell.

are

$$\int_\Theta \pi^E(\theta)[1-F(P_E - p(\theta))]d\lambda(\theta).$$

Therefore, since both $e^*(\cdot)$ and $e^M(\cdot)$ are in the class of rules characterized by an entry price,

$$P_E^j \in \underset{P_E}{\operatorname{argmin}} \int_\Theta \pi^E(\theta)F(P_E - p^j(\theta))d\lambda(\theta)$$

for both j = "*" and j = "M". Hence

$$\int_\Theta \pi^E(\theta)F(P_E^M - p^M(\theta))d\lambda(\theta) \leq \int_\Theta \pi^E(\theta)F(P_E^* + \delta(\theta_o) - p^M(\theta))d\lambda(\theta)$$

$$= \int_\Theta \pi^E(\theta)F(P_E^* - p^*(\theta) + \delta(\theta_o) - \delta(\theta))d\lambda(\theta).$$

Now, since $F(\cdot)$ and $\delta(\cdot)$ are increasing, and $\pi^E(\theta)(\theta - \theta_o) > 0$ for $\theta \neq \theta_o$,

$$\pi^E(\theta)F(P_E^* - p^*(\theta) + \delta(\theta_o) - \delta(\theta)) < \pi^E(\theta)F(P_E^* - p^*(\theta))$$

for every $\theta \neq \theta_o$. Consequently,

$$\int_\Theta \pi^E(\theta)F(P_E^M - p^M(\theta))d\lambda(\theta) < \int_\Theta \pi^E(\theta)F(P_E^* - p^*(\theta)d\lambda(\theta).$$

Thus the expected profit of firm E is greater in the monopoly situation than it is in the equilibrium. ////

<u>Remark:</u> Theorem 2 answers the question of whether or not firm E should inform firm I that it is contemplating entry. If firm I is unaware, and is known to be unaware, of the entry threat, then informing it of the entry threat changes the monopoly situation to an equilibrium. Thus firm E prefers that its threat

of entry be kept secret if $\delta(\cdot)$ is increasing, whereas it prefers that its threat of entry be made common knowledge if $\delta(\cdot)$ is decreasing.

Conditional on a profitability level θ, the probabilities of entry in an equilibrium and in the monopoly situation are, respectively,

$$(4) \qquad G^*(\theta) = 1 - F(P_E^* - p^*(\theta))$$

$$(5) \qquad G^M(\theta) = 1 - F(P_E^M - p^M(\theta)).$$

Entry is deterred at θ in the equilibrium if $G^*(\theta) < G^M(\theta)$. In addition to the properties of $\delta(\cdot)$, the relationship of $G^*(\cdot)$ to $G^M(\cdot)$ depends on the nature of the density function f and on firm E's initial estimate of profit, $E\pi^E(\tilde{\theta})$. Rather than presenting an exhaustive analysis, only illustrative cases are presented in the next theorem.

Theorem 3:[1]

(a) If $\delta(\cdot)$ is increasing (decreasing), then $G^*(\cdot)$ cannot cross $G^M(\cdot)$ from below (above). That is, if $G^*(\hat{\theta}) = G^M(\hat{\theta})$, then for all $\theta \neq \hat{\theta}$,

$$(\hat{\theta} - \theta)(G^*(\theta) - G^M(\theta)) > (<)\ 0.$$

(b) If $\delta(\cdot)$ is increasing and $f(\cdot)$ decreasing, then $G^M(\theta) > G^*(\theta)$ for all $\theta > \theta_o$. If $\delta(\cdot)$ is decreasing and $f(\cdot)$ increasing, then $G^M(\theta) < G^*(\theta)$ for all $\theta > \theta_o$.

(c) If $\delta(\cdot)$ and $f(\cdot)$ are increasing and $E\pi^E(\tilde{\theta}) < 0$, then $G^M(\cdot) > G^*(\cdot)$. If

[1] When we say f is decreasing (increasing) in this and the next theorem, we mean that it is decreasing (increasing) in the

$\delta(\cdot)$ and $f(\cdot)$ are decreasing, and $E\pi^E(\tilde{\theta}) > 0$, then $G^M(\cdot) < G^*(\cdot)$.

Proof:

(a) Assume $G^*(\hat{\theta}) = G^M(\hat{\theta})$. Then (4) and (5) imply $P_E^* - p^*(\hat{\theta}) = P_E^M - p^M(\hat{\theta})$. Hence $P_E^* - P_E^M + \delta(\hat{\theta}) = 0$. Thus, if $\delta(\cdot)$ is increasing (decreasing),

$$(\hat{\theta} - \theta)\ (P_E^* - P_E^M + \delta(\theta)) < (>)\ 0$$

for $\theta \neq \hat{\theta}$. But now we are finished, since (4) and (5) imply that the sign of $G^*(\theta) - G^M(\theta)$ is opposite that of $P_E^* - P_E^M + \delta(\theta)$.

(b) We prove this only for the case of $\delta(\cdot)$ increasing and $f(\cdot)$ decreasing - the other case can be treated similarly. Since $\delta(\cdot)$ is increasing and $P_E^M - p^*(\theta) - \delta(\theta) = P_E^M - p^M(\theta)$, $\theta \neq \theta_o$ implies

$$(\theta - \theta_o)[(P_E^M - p^*(\theta) - \delta(\theta_o)) - (P_E^M - p^M(\theta))] > 0.$$

Hence, as $f(\cdot)$ is decreasing, $\theta \neq \theta_o$ implies

$$(\theta - \theta_o)[f(P_E^M - p^*(\theta) - \delta(\theta_o)) - f(P_E^M - p^M(\theta))] < 0.$$

Therefore, since $(\theta - \theta_o)\pi^E(\theta) > 0$ for $\theta \neq \theta_o$,

$$\int_\Theta \pi^E(\theta) f(P_E^M - p^*(\theta) - \delta(\theta_o))d\lambda(\theta) < \int_\Theta \pi^E(\theta) f(P_E^M - p^M(\theta))d\lambda(\theta).$$

relevant range. Note also tht if f is decreasing, then the first order condition (1) is sufficient for a local maximum of (E1).

The RHS of this inequality is proportional to $\Pi^E[P_E^M, p^M(\cdot)]$, which is zero. Hence $\Pi^E[P_E^M - \delta(\theta_o), p^*(\cdot)]$, which is proportional to the LHS, is negative. Therefore $P_E^M - \delta(\theta_o) < P_E^*$, or rather, $P_E^M - p^M(\theta_o) < P_E^* - p^*(\theta_o)$. Thus (4) and (5) imply $G^M(\theta_o) > G^*(\theta_o)$. Part (a) now implies $G^M(\theta) > G^*(\theta)$ for all $\theta \geq \theta_o$.

(c) We prove this only for the case of $\delta(\cdot)$ and $f(\cdot)$ increasing and $E\pi^E(\tilde{\theta}) < 0$. Let $b = P_E^M - \delta(\underline{\theta})$. Because $p^*(\cdot)$ and $f(\cdot)$ are increasing, and $(\theta - \theta_o)\pi^E(\theta) > 0$, we have

$$\int_\Theta \pi^E(\theta) f(P_E^M - \delta(\underline{\theta}) - p^*(\theta))d\lambda(\theta)$$

$$= \int_{\theta \leq \theta_o} \pi^E(\theta) f(b-p^*(\theta))d\lambda(\theta) + \int_{\theta > \theta_o} \pi^E(\theta) f(b-p^*(\theta))d\lambda(\theta)$$

$$< \int_{\theta \leq \theta_o} \pi^E(\theta) f(b-p^*(\theta_o))d\lambda(\theta) + \int_{\theta > \theta_o} \pi^E(\theta) f(b-p^*(\theta_o))d\lambda(\theta)$$

$$= f(b-p^*(\theta_o))E\pi^E(\tilde{\theta}) < 0.$$

Hence $\int_\Theta \pi^E(\theta) f(P_E^M - \delta(\underline{\theta}) - p^*(\theta))d\lambda(\theta) < 0$. Therefore $P_E^M - \delta(\underline{\theta}) < P_E^*$, or rather, $P_E^M - p^M(\underline{\theta}) < P_E^* - p^*(\underline{\theta})$. Consequently (4) and (5) imply that $G^M(\underline{\theta}) > G^*(\underline{\theta})$. Now, since $\delta(\cdot)$ is increasing, (a) implies $G^M(\theta) > G^*(\theta)$ for all θ. ////

In view of Theorems 2 and 3, it is important to know when $\delta(\cdot)$ is increasing or decreasing. The nature of $\delta(\cdot)$ depends upon properties of $R(\cdot,\cdot)$, $f(\cdot)$ and $L(\cdot)$. We conclude by presenting sufficient conditions for $\delta(\cdot)$ to be increasing or decreasing.

Below, Theorem 4 makes use of the function $\hat{P}(\theta,r)$ defined by

$$R_1(\hat{P}(\theta,r),\theta) = r.$$

Thus $\hat{P}(\theta,r)$ is the expected price which equates expected marginal revenue to r, for a given profitability level θ. Observe that the monopoly price is defined by $p^M(\theta) = \hat{P}(\theta,0)$. The derivatives of $\hat{P}$ satisfy $\hat{P}_1(\theta,r) > 0$ and $\hat{P}_2(\theta,r) < 0$. If $\hat{P}_2(\theta,r)$ decreases in θ, then the amount by which price must be lowered to achieve a given decrease in marginal revenue is less for a high θ than for a low θ. This is in accordance with the interpretation that a high θ corresponds to favorable profitability conditions. However, $R_2(p,\theta) > 0$ and $R_{12}(p,\theta) > 0$ do not imply $\hat{P}_2(\theta,r)$ decreases in r.

Theorem 4: $\delta(\cdot)$ is increasing (decreasing) if the following three conditions hold:

(a) $\hat{P}_2(\theta,r)$ decreases (increases) in θ,

(b) $f(\cdot)$ decreases (increases) in η,

(c) $L(\cdot)$ increases in θ.

Proof: Expression (1) (in Theorem 1) implies that $p^*(\theta) = \hat{P}[\theta,\ L(\theta)f(P_E^* - p^*(\theta))]$. Let $A(\theta) = L(\theta)f(P_E^* - p^*(\theta))$. Then $\delta(\theta) = \hat{P}(\theta,0) - \hat{P}(\theta,A(\theta))$. Let $\theta_1 < \theta_2$. Then

$$\delta(\theta_2) - \delta(\theta_1) = \int_0^{A(\theta_1)} \hat{P}_2(\theta_1,r)dr - \int_0^{A(\theta_2)} \hat{P}_2(\theta_2,r)dr.$$

If $f(\cdot)$ is decreasing and $L(\cdot)$ is increasing, then $A(\theta_1) < A(\theta_2)$, since $p^*(\theta_1) \leq p^*(\theta_2)$. If in addition $\hat{P}_2(\theta,r)$ decreases in θ, then

$$\delta(\theta_2) - \delta(\theta_1) > \int_{A(\theta_2)}^{A(\theta_1)} \hat{P}_2(\theta_2, r)dr \geq 0$$

follows from $A(\theta_1) < A(\theta_2)$ and $\hat{P}_2(\theta,r) < 0$. The proof that $\delta(\theta_2) - \delta(\theta_1) < 0$ if $\hat{P}_2(\theta,r)$ increases in θ, and $f(\cdot)$ increases and $L(\cdot)$ decreases, is similar. ////

Whether or not the conditions of Theorem 4 hold is an empirical question. However, examination of its logic indicates that if (a) and (c) hold, then its conclusion is valid as long as (b) is not violated severely. Thus, if θ measures profitability strongly ($\hat{P}_{12}(\theta,r) < 0$ as well as $R_2(p,\theta) > 0$ and $R_{12}(p,\theta) > 0$), and the loss due to entry increases with the profitability of the industry ($L'(\theta) > 0$), then $\delta(\cdot)$ most likely increases.

5. AN EXAMPLE

In this section we consider an example in which there exists an (essentially) unique limit pricing equilibrium. Simple conditions are found for the expected profit of the entrant to be decreased and for entry to be deterred by limit pricing. The example exhibits properties that are analogous to those established in Theorems 1-4, even though it violates many of their sufficient conditions. In particular, the noise $\tilde{\eta}$ does not have infinite support, and the monotonicity condition (A2) does not necessarily hold in this example.

The expected revenue function is assumed to be $R(p,\theta) = \theta p - \frac{1}{2} p^2$. This is the revenue function if, for instance, there are no production costs and, in order to set an expected price p, a quantity $q(p) = \theta - \frac{1}{2}p$ must be produced. With this net revenue function, the monopoly expected price is $p^M(\theta) = \theta$.

The domain of θ is assumed to be $\Theta = \{\underline{\theta}, \bar{\theta}\}$, where $0 < \underline{\theta} < \bar{\theta}$. Furthermore, to keep the example simple, it is assumed that the prior of firm E satisfies $\lambda(\underline{\theta}) = \lambda(\bar{\theta}) = 1/2$, and that its post-entry profit function satisfies $\pi^E(\underline{\theta}) = -\pi^E(\bar{\theta}) > 0$.

The density function of the noise $\tilde{\eta}$ is assumed to be

$$(6) \qquad f(\eta) = \begin{cases} (w + \eta)/w^2 & \eta \in [-w,0], \\ (w - \eta)/w^2 & \eta \in [0,w], \\ 0 & \text{otherwise}, \end{cases}$$

where $w > 0$ is a parameter. The important properties of this density are its symmetry and its continuity everywhere. Since the random variable $\tilde{\eta}$ has support $[-w,w]$ and variance $w^2/6$, the parameter w can be interpreted as a measure of noise.

We are interested in obtaining a limit pricing equilibrium, as is depicted in Figure 1. This requires, essentially, that prices sufficiently garble the actions of firm I. Therefore w, the measure of noise, is assumed to be large enough to insure that the following three assumptions hold:

$$(a1) \qquad |L(\bar{\theta}) - L(\underline{\theta})| < w(\bar{\theta} - \underline{\theta}),$$

$$(a2) \qquad \theta_2 - \theta_1 < 2w,$$

$$(a3) \qquad L(\underline{\theta}) < w^2 \text{ and } L(\bar{\theta}) < w^2.$$

Under these assumptions, there is a unique equilibrium pricing rule. It satisfies

$$0 < p^*(\bar{\theta}) - p^*(\underline{\theta}) < 2w.$$

The first inequality implies that in equilibrium, low prices are associated with low profitability levels. The second inequality implies that in equilibrium, a subinterval of prices exists that has positive probability of occurring regardless of whether $\theta = \underline{\theta}$ or $\theta = \bar{\theta}$. An equilibrium $(p^*(\cdot), e^*(\cdot))$ shall be derived under the assumption that $p^*(\cdot)$ satisfies these inequalities. This procedure will then be justified by showing that the derived equilibrium does in fact satisfy the inequalities. That both inequalities must be satisfied by an equilibrium is demonstrated in the Appendix.

Given that $p^*(\cdot)$ satisfies these inequalities, the only prices that can occur are in the interval

$$\mathbf{P}[p^*(\cdot)] = [p^*(\underline{\theta}) - w,\ p^*(\bar{\theta}) + w].$$

For any $P \in \mathbf{P}[p^*(\cdot)]$, the expected profit of firm E conditional upon observing the price P is

$$\Pi^E[P,p(\cdot)] = \frac{\pi^E(\underline{\theta})f(P-p(\underline{\theta}))\lambda(\underline{\theta}) + \pi^E(\bar{\theta})f(P-p(\bar{\theta}))\lambda(\bar{\theta})}{f(P-p(\underline{\theta}))\lambda(\underline{\theta}) + f(P-p(\bar{\theta}))\lambda(\bar{\theta})}$$

$$= \frac{\pi^E(\bar{\theta})[f(P-p(\bar{\theta})) - f(P-p(\underline{\theta}))]}{f(P-p(\underline{\theta})) + f(P-p(\bar{\theta}))}.$$

Therefore (E2) implies that firm E enters after observing P if and only if $f(P - p^*(\bar{\theta})) \geq f(P - p^*(\underline{\theta}))$. Given that $p^*(\underline{\theta}) < p^*(\bar{\theta})$, (6) can be used to show that $f(P - p^*(\bar{\theta})) \geq f(P - p^*(\underline{\theta}))$ if and only if P exceeds an entry price P_E^* defined by

$$(7) \qquad P_E^* = \frac{p^*(\underline{\theta}) + p^*(\bar{\theta})}{2}.$$

Thus, the equilibrium entry rule $e^*(\cdot)$ is determined on $\mathbf{P}[p^*(\cdot)]$ once $p^*(\cdot)$ is specified. Specifically, $e^*(\cdot)$ is defined on $\mathbf{P}[p^*(\cdot)]$ by (7) and

$$(8) \qquad e^*(P) = \begin{cases} 0 & P < P_E^* \\ 1 & P \geq P_E^*. \end{cases}$$

In order to have the maximization problem (E1) of firm I well-defined, $e^*(P)$ must be defined even for prices P that cannot occur in equilibrium. From the viewpoint of firm E, however, any extension of $e^*(\cdot)$ off of the price support $P[p^*(\cdot)]$ is irrelevant and <u>ad hoc</u>. We only consider the equilibrium in which $e^*(\cdot)$ is extended off of $\mathbf{P}[p^*(\cdot)]$ in the simplest possible way. That is, we examine only the equilibrium $(p^*(\cdot), e^*(\cdot))$ in which $e^*(P)$ is defined by (8) <u>for every P</u>. Techniques used in the Appendix, particularly Lemma A, can be used to show that any other equilibrium $(\hat{p}(\cdot), \hat{e}(\cdot))$ satisfies $\hat{p}(\cdot) = p^*(\cdot)$ and, on the price support $\mathbf{P}[p^*(\cdot)]$, $\hat{e}(\cdot) = e^*(\cdot)$. Thus the equilibrium is unique, in this example, in the sense of all equilibria being observationally equivalent.

The $e^*(\cdot)$ specified by (8) implies that the probability of entry, given an expected price p, is $1-F(P_E^* - p)$. Hence (E1) becomes

$$p^*(\theta) \in \underset{p}{\operatorname{argmax}} \{\theta p - \tfrac{1}{2} p^2 - L(\theta)[1-F(P_E^* - p)]\}.$$

Therefore, because F is differentiable,

$$\theta - p^*(\theta) = L(\theta)f(P_E^* - p^*(\theta)).$$

(See Figure 2.) Expression (7) and $p^*(\bar{\theta}) - p^*(\underline{\theta}) < 2w$ imply that for both θ, $|P_E^* - p^*(\theta)| = \frac{1}{2}(p^*(\bar{\theta}) - p^*(\underline{\theta})) < w$. Therefore, by (6), $p^*(\bar{\theta})$ and $p^*(\underline{\theta})$ solve the linear equations

$$\text{(9)} \qquad \underline{\theta} - p^*(\underline{\theta}) = \frac{L(\underline{\theta})}{w^2}\left(w - \frac{p^*(\bar{\theta}) - p^*(\underline{\theta})}{2}\right)$$

$$\text{(10)} \qquad \bar{\theta} - p^*(\bar{\theta}) = \frac{L(\bar{\theta})}{w^2}\left(w - \frac{p^*(\bar{\theta}) - p^*(\underline{\theta})}{2}\right).$$

Expressions (7) - (10) determine the equilibrium.[1] The remaining bits of proof, which rely on (a1) - (a3), are collected below.

Proof: The linear system (9) and (10) has a unique solution $(p^*(\underline{\theta}), p^*(\bar{\theta}))$ provided $L(\bar{\theta}) - L(\underline{\theta}) \neq 2w^2$, which is guaranteed by (a3). The resulting rule $p^*(\cdot)$ is actually a best response to the $e^*(\cdot)$ given by (7) and (8), since (a3) implies that the sufficient second order conditions for (E1),

$$-1 + \frac{L(\underline{\theta})}{w^2} < 0 \quad \text{and} \quad -1 - \frac{L(\bar{\theta})}{w^2} < 0,$$

are satisfied. By construction, the $e^*(\cdot)$ given by (7) and (8) is a best response to $p^*(\cdot)$ so long as $p^*(\underline{\theta}) < p^*(\bar{\theta})$. But (9) and (10) imply

[1] We have not shown that in equilibrium the observed prices are necessarily positive. They will be positive if we assume (a4) $\underline{\theta} > w + L(\bar{\theta})w^{-1}$. One can easily find values of w, $L(\underline{\theta})$, $L(\bar{\theta})$, $\bar{\theta}$ and $\underline{\theta}$ satisfying (a1) - (a4).

$$(11) \qquad p^*(\bar{\theta}) - p^*(\underline{\theta}) = \frac{w(\bar{\theta} - \underline{\theta}) - (L(\bar{\theta}) - L(\underline{\theta}))}{w - \frac{1}{2w}(L(\bar{\theta}) - L(\underline{\theta}))},$$

which is positive by (a1) and (a3). This justifies the assumption $p^*(\underline{\theta}) < p^*(\bar{\theta})$ used to derive the equilibrium. The second assumed inequality, $p^*(\bar{\theta}) - p^*(\underline{\theta}) < 2w$, is justified by noting that (11) and (a2) imply

$$p^*(\bar{\theta}) - p^*(\underline{\theta}) < \frac{2w^2 - (L(\bar{\theta}) - L(\underline{\theta}))}{w - \frac{1}{2w}(L(\bar{\theta}) - L(\underline{\theta}))} = 2w.$$

Thus (7) - (10) determine an equilibrium. ////

The first obvious property exhibited by this equilibrium is the limit pricing property (E3). Since $p^*(\bar{\theta}) - p^*(\underline{\theta}) < 2w$, (9) and (10) imply that $\delta(\theta) = \theta - p^*(\theta) > 0$ for both θ.

As in Section 4, the equilibrium can be compared to the monopoly situation $(p^M(\cdot), e^M(\cdot))$. The rule $p^M(\cdot)$ is given by $p^M(\theta) = \theta$. The rule $e^M(\cdot)$, by the argument used to derive $e^*(\cdot)$, specifies entry if and only if the observed price exceeds the monopoly entry price

$$(12) \qquad p_E^M = \frac{\underline{\theta} + \bar{\theta}}{2}.$$

As in Section 4, the comparisons depend crucially upon whether or not $\delta(\theta) = p^M(\theta) - p^*(\theta)$ is an increasing function. If δ is increasing, then the interval of prices that can occur regardless of the value of θ, $[p(\bar{\theta}) - w, p(\underline{\theta}) + w]$, is larger in equilibrium than in the monopoly situation. Observing a first period price will consequently be less useful to firm E in conveying information about the true value of θ in equilibrium than in the monopoly situation. As a result, firm E will be worse off in equilibrium, entering more often under low profitability conditions ($\theta = \underline{\theta}$) and less often under high profitability conditions ($\theta = \bar{\theta}$). In this sense, entry is deterred in the limit pricing equilibrium if δ is increasing. If δ is decreasing, however, these comparisons are reversed.

We now substantiate these propositions. Suppose that firm I sets the expected price according to an increasing rule $p(\cdot)$, and firm E enters if and only if the observed price exceeds a number P_E. The expected profit of firm E before it observes the first period price is then

$$(13) \qquad \lambda(\underline{\theta})[1 - F(P_E - p(\underline{\theta}))]\pi^E(\underline{\theta}) + \lambda(\bar{\theta})[1 - F(P_E - p(\bar{\theta}))]\pi^E(\bar{\theta}).$$

We know that in both the equilibrium and in the monopoly situation, the pricing rule is increasing and the entry rule is characterized by an entry price. Consequently, in either situation the entry rule of firm E must be its best rule within the subclass of rules characterized by an entry price. Hence, for j = "*" or for j = "M",

$$(14) \qquad \begin{aligned} P_E^j &\in \underset{P_E}{\operatorname{argmax}} \{\lambda(\underline{\theta})[1-F(P_E - p^j(\underline{\theta}))]\pi^E(\underline{\theta}) + \lambda(\bar{\theta})[1-F(P_E - p^j(\bar{\theta}))]\pi^E(\bar{\theta})\} \\ &= \underset{P_E}{\operatorname{argmin}} \{\lambda(\underline{\theta})F(P_E - p^j(\underline{\theta}))\pi^E(\underline{\theta}) + \lambda(\bar{\theta})F(P_E - p^j(\bar{\theta}))\pi^E(\bar{\theta})\}. \end{aligned}$$

Expression (14) implies that if δ is increasing, firm E has greater expected profits in the monopoly situation than in equilibrium. (A similar argument shows the reverse conclusion holds when δ is decreasing.) To see this, let $\hat{P}_E = P_E^* + \delta(\underline{\theta})$. Then (14) implies that

$$\begin{aligned} &\lambda(\underline{\theta})F(P_E^M - p^M(\underline{\theta}))\pi^E(\underline{\theta}) + \lambda(\bar{\theta})F(P_E^M - p^M(\bar{\theta}))\pi^E(\bar{\theta}) \\ &\qquad \leq \lambda(\underline{\theta})F(\hat{P}_E - p^M(\underline{\theta}))\pi^E(\underline{\theta}) + \lambda(\bar{\theta})F(\hat{P}_E - p^M(\bar{\theta}))\pi^E(\bar{\theta}). \end{aligned}$$

But $\hat{P}_E - p^M(\underline{\theta}) = P^*_E - p^*(\underline{\theta})$, and $\hat{P}_E - p^M(\bar{\theta}) = P^*_E - p^*(\bar{\theta}) + \delta(\underline{\theta}) - \delta(\bar{\theta})$. Therefore

$$\begin{aligned}
&\lambda(\underline{\theta})F(P^M_E - p^M(\underline{\theta}))\pi^E(\underline{\theta}) + \lambda(\bar{\theta})F(P^M_E - p^M(\bar{\theta}))\pi^E(\bar{\theta}) \\
&\qquad \leq \lambda(\underline{\theta})F(P^*_E - p^*(\underline{\theta}))\pi^E(\underline{\theta}) + \lambda(\bar{\theta})F(P^*_E - p^*(\bar{\theta}) + \delta(\underline{\theta}) - \delta(\bar{\theta}))\pi^E(\bar{\theta}) \\
&\qquad < \lambda(\underline{\theta})F(P^*_E - p^*(\underline{\theta}))\pi^E(\underline{\theta}) + \lambda(\bar{\theta})F(P^*_E - p^*(\bar{\theta}))\pi^E(\bar{\theta}),
\end{aligned}$$

where the last inequality follows from $\delta(\underline{\theta}) < \delta(\bar{\theta})$ and $\pi^E(\bar{\theta}) > 0$. In view of (13), this shows firm E to have greater expected profits in the monopoly situation than in equilibrium.

The comparisions of entry probabilities now follow. The probability of entry in equilibrium, given the true value θ, is $G^*(\theta) = 1-F(P^*_E - p^*(\theta))$. The entry probability is $G^M(\theta) = 1-F(P^M_E - p^M(\theta))$ in the monopoly situation. In situation j = "*" or "M", the expected profit of firm E before observing the first period price is

$$\lambda(\underline{\theta})G^j(\underline{\theta})\pi^E(\underline{\theta}) + \lambda(\bar{\theta})G^j(\bar{\theta})\pi^E(\bar{\theta}).$$

Since $\pi^E(\underline{\theta}) < 0 < \pi^E(\bar{\theta})$, this quantity is less for j = "*" than for j = "M" only if $G^*(\underline{\theta}) > G^M(\underline{\theta})$ or $G^*(\bar{\theta}) < G^M(\bar{\theta})$.

For the example, a stronger statement can be made. Let $x^j = (p^j(\bar{\theta}) - p^j(\underline{\theta}))/2$ for j = "*" and j = "M". Then (7) and (12) imply that $G^j(\underline{\theta}) = 1-F(x^j)$

and $G^j(\bar{\theta}) = 1 - F(-x^j)$. Consequently, as F is strictly increasing, $G^*(\underline{\theta}) > G^M(\underline{\theta})$ if and only if $G^*(\bar{\theta}) < G^M(\bar{\theta})$. Therefore

$$\delta\ (\cdot) \text{ is increasing iff } G^*(\underline{\theta}) > G^M(\underline{\theta}) \text{ and } G^*(\bar{\theta}) < G^M(\bar{\theta}),$$

and

$$\delta\ (\cdot) \text{ is decreasing iff } G^*(\underline{\theta}) < G^M(\underline{\theta}) \text{ and } G^*(\bar{\theta}) > G^M(\bar{\theta}).$$

Thus, if $\delta(\cdot)$ is increasing (decreasing) then, relative to the monopoly situation, in equlibrium entry occurs more (less) often when it is unprofitable and less (more) often when it is profitable.

The expected profits of firm I in situation j given θ are

$$\Pi(p^j(\theta),G^j(\theta),\theta) = R(p^j(\theta),\theta) - L(\theta)G^j(\theta) + \pi^M(\theta).$$

Therefore firm I prefers, if $\delta(\cdot)$ is increasing (decreasing), the monopoly situation to the equilibrium when $\theta = \underline{\theta}$ $(\theta = \bar{\theta})$. Of course, it must be remembered that within the context of this model there is no way that the established firm can choose whether to be in the equilibrium or in the monopoly situation.

What determines whether $\delta(\cdot)$ is increasing or decreasing? Because Θ contains only two elements, we can show that $\delta(\cdot)$ is increasing if and only if $L(\cdot)$ is increasing. This follows directly from (9) and (10), since they imply

$$\frac{\delta(\underline{\theta})}{\delta(\bar{\theta})} = \frac{\underline{\theta} - p^*(\underline{\theta})}{\bar{\theta} - p^*(\bar{\theta})} = \frac{L(\underline{\theta})}{L(\bar{\theta})}.$$

There is a <u>prima facie</u> case for believing $L(\underline{\theta}) < L(\bar{\theta})$ to be the more realistic case. Certainly, if entry is complete enough to drive profits to zero, then the loss due to entry is simply equal to the monopoly profit $R(p^M(\theta),\theta)$. But, almost by definition, this monopoly profit increases in the profitability measure θ. Even if only the one firm enters, learns θ, and then both firms obtain Cournot profit $\pi^C(\theta)$ in the second period, the loss $L(\theta) = \pi^M(\theta) - \pi^C(\theta)$ is an increasing function for standard types of demand curves, such as as the linear one we have used.

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APPENDIX

We show here that any equilibrium in the example of Section 5 satisfies $0 < p^*(\bar{\theta}) - p^*(\underline{\theta}) < 2w$. The arguments rely on assumptions (a1) and (a2). The following lemma, which depends on the density f declining continuously to zero, will be used to show this result.

Lemma A: Suppose P_1 and P_2 $(P_2 > P_1)$ are contained in the interval $I = [p_o - w, p_o + w]$, and that $e(\cdot)$ is any entry rule satisfying

$$e(P) = \begin{cases} 0 & P \varepsilon I \text{ and } P \notin [P_1,P_2] \\ 1 & P \varepsilon [P_1,P_2]. \end{cases}$$

Then $h(p) = Ee(p + \tilde{\eta})$ is differentiable at $p = p_o$, with $h'(p_o) = f(P_1 - p_o) - f(P_2 - p_o)$.

Proof: Let $e_u(\cdot)$ and $e_\ell(\cdot)$ be defined to agree with $e(\cdot)$ on I, and for $P \notin I$ to satisfy $e_u(P) = 1$ and $e_\ell(P) = 0$. Let $h_u(p) = Ee_u(p + \tilde{\eta})$ and $h_\ell(p) = Ee_\ell(p + \tilde{\eta})$. Then $h_\ell(p) \leq h(p) \leq h_u(p)$, with equality holding at $p = p_o$. Hence, if $h_\ell'(p_o)$ and $h_u'(p_o)$ can be shown to exist and equal $f(P_1 - p_o) - f(P_2 - p_o)$, then $h'(p_o)$ exists and is also equal to $f(P_1 - p_o) - f(P_2 - p_o)$. Observe that

$$h_u(p) = \int_{-\infty}^{\infty} e_u(p + \eta)f(\eta)d\eta$$

$$= \int_{-\infty}^{p_o - w - p} f(\eta)d\eta + \int_{P_1 - p}^{P_2 - p} f(\eta)d\eta + \int_{p_o + w - p}^{\infty} f(\eta)d\eta.$$

Since f is continuous, $h_u'(p) = -f(p_o - w - p) - f(P_2 - p) + f(P_1 - p) + f(p_o + w - p)$. Hence, since $f(w) = f(-w) = 0$, $h_u'(p_o) = f(P_1 - p_o) - f(P_2 - p_o)$. Similarly, $h_\ell'(p_o) = f(P_1 - p_o) - f(P_2 - p_o)$.

If an equilibrium does not satisfy $0 < p^*(\bar{\theta}) - p^*(\underline{\theta}) < 2w$, then one of the following four cases must hold. We rule each one out in turn.

<u>Case 1:</u> $p^*(\underline{\theta}) = p^*(\bar{\theta}) = p_o$.

In this case, prices do not reveal information to firm E. Therefore $\pi^E[P,p^*(\cdot)]$ is independent of P for P contained in the support $\mathbf{P}[p^*(\cdot)] = [p_o - w, p_o + w] \equiv I$. Hence (E2) implies that $e^*(\cdot)$ is constant on I, although it may vary off of I. Therefore, letting $h(p) = Ee^*(p + \tilde{\eta})$, lemma A implies $h'(p_o) = 0$. That is, firm I cannot change the probability of entry by small changes in p_o. But then the first order condition for (E1) implies that $p^*(\theta) = p^M(\theta) = \theta$. Since $\underline{\theta} < \bar{\theta}$, this controdicts $p^*(\underline{\theta}) = p^*(\bar{\theta})$.

<u>Case 2:</u> $p^*(\underline{\theta}) - p^*(\bar{\theta}) \geq 2w$.

Here $\mathbf{P}[p^*(\cdot)]$ is the union of two essentially disjoint intervals, $I_1 = [p^*(\bar{\theta}) - w, p^*(\bar{\theta}) + w]$ and $I_2 = [p^*(\underline{\theta}) - w, p^*(\underline{\theta}) + w]$. Hence (E2) implies $e^*(P) = 0$ if $P \in I_2$ and $e^*(P) = 1$ if $P \in I_1$. Thus $e^*(\cdot)$ is constant on I_1 and on I_2, so that again application of lemma A and the first order conditions for (E1) yield $p^*(\bar{\theta}) = \bar{\theta}$ and $p^*(\underline{\theta}) = \underline{\theta}$. This contradicts $p^*(\bar{\theta}) > p^*(\bar{\theta})$.

<u>Case 3:</u> $0 < p^*(\underline{\theta}) - p^*(\bar{\theta}) < 2w$.

Here an argument similar to that used in Section 5 shows that the best response rule $e^*(\cdot)$ is defined on $\mathbf{P}[p^*(\cdot)] = [p^*(\bar{\theta}) - w, p^*(\underline{\theta}) + w]$ by

$$e^*(P) = \begin{cases} 0 & P > P_E^* \\ 1 & P \leq P_E^*, \end{cases}$$

where $P_E^* = (p^*(\underline{\theta}) + p^*(\bar{\theta}))/2$. Therefore, if we let $h(p) = Ee^*(p + \tilde{\eta})$, Lemma A implies $h'(p^*(\theta)) = -f(P_E^* - p^*(\theta))$ for both θ. Consequently, the first order conditions for (E1) imply that for both θ,

$$p^*(\theta) = \theta + L(\theta)f(P_E^* - p^*(\theta)).$$

Thus, since $f(P_E^* - p^*(\underline{\theta})) = f(P_E^* - p^*(\bar{\theta}))$ and $p^*(\underline{\theta}) > p^*(\bar{\theta})$,

$$\bar{\theta} - \underline{\theta} < (L(\underline{\theta}) - L(\bar{\theta}))f(P_E^* - p^*(\bar{\theta})).$$

But this inequality contradicts (a1), since $f(\cdot) \leq w^{-1}$.

<u>Case 4:</u> $p^*(\bar{\theta}) - p^*(\underline{\theta}) \geq 2w$.

The same argument used in Case 2 implies that $p^*(\theta) = \theta$ for both θ. Hence $\bar{\theta} - \underline{\theta} \geq 2w$, contrary to (a2).

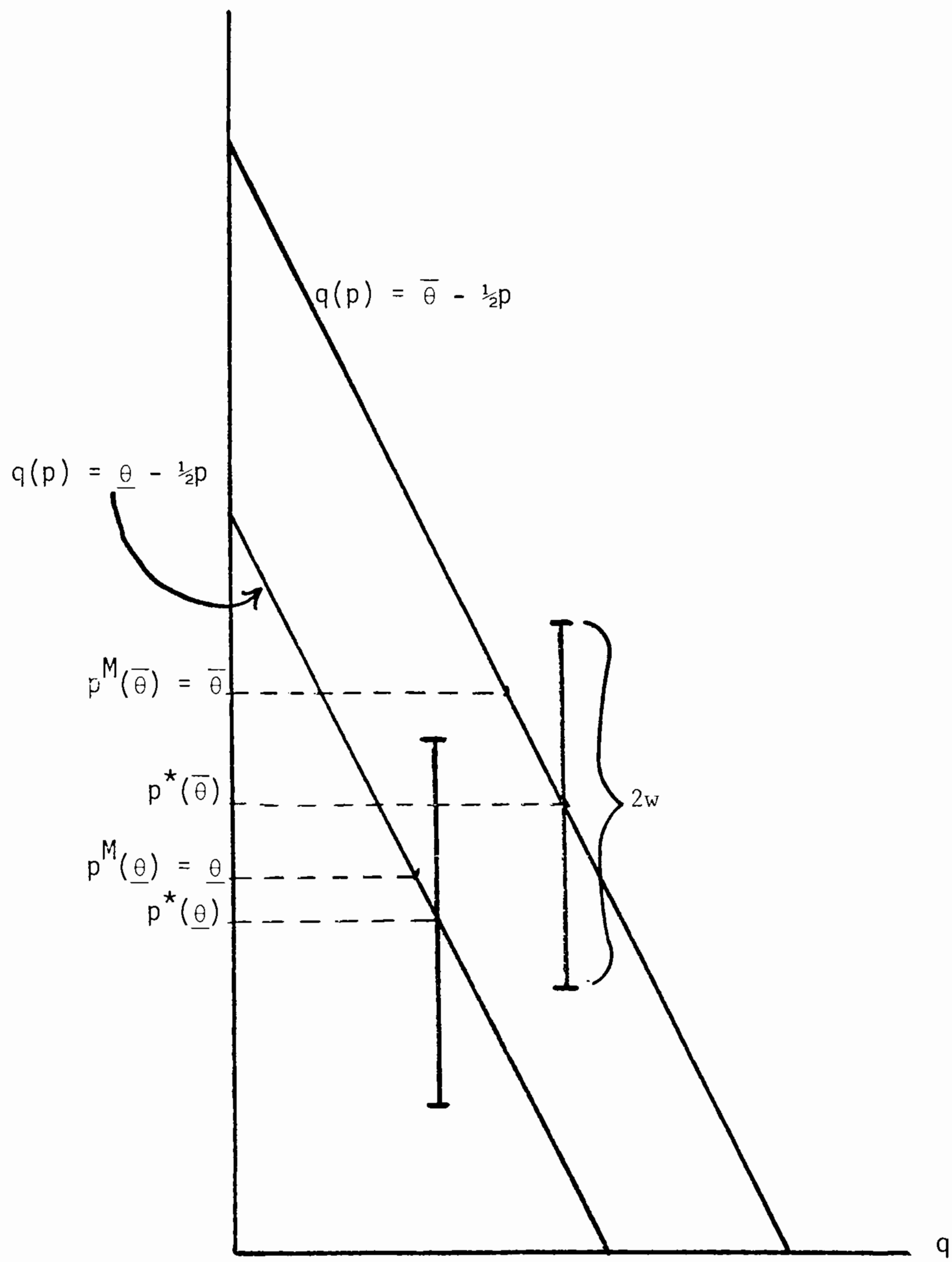
q(p) = $\bar{\theta}$ - ½p
q(p) = $\underline{\theta}$ - ½p
$p^M(\bar{\theta}) = \bar{\theta}$
$p^*(\bar{\theta})$
$p^M(\underline{\theta}) = \underline{\theta}$
$p^*(\underline{\theta})$
2w
q

Figure 1

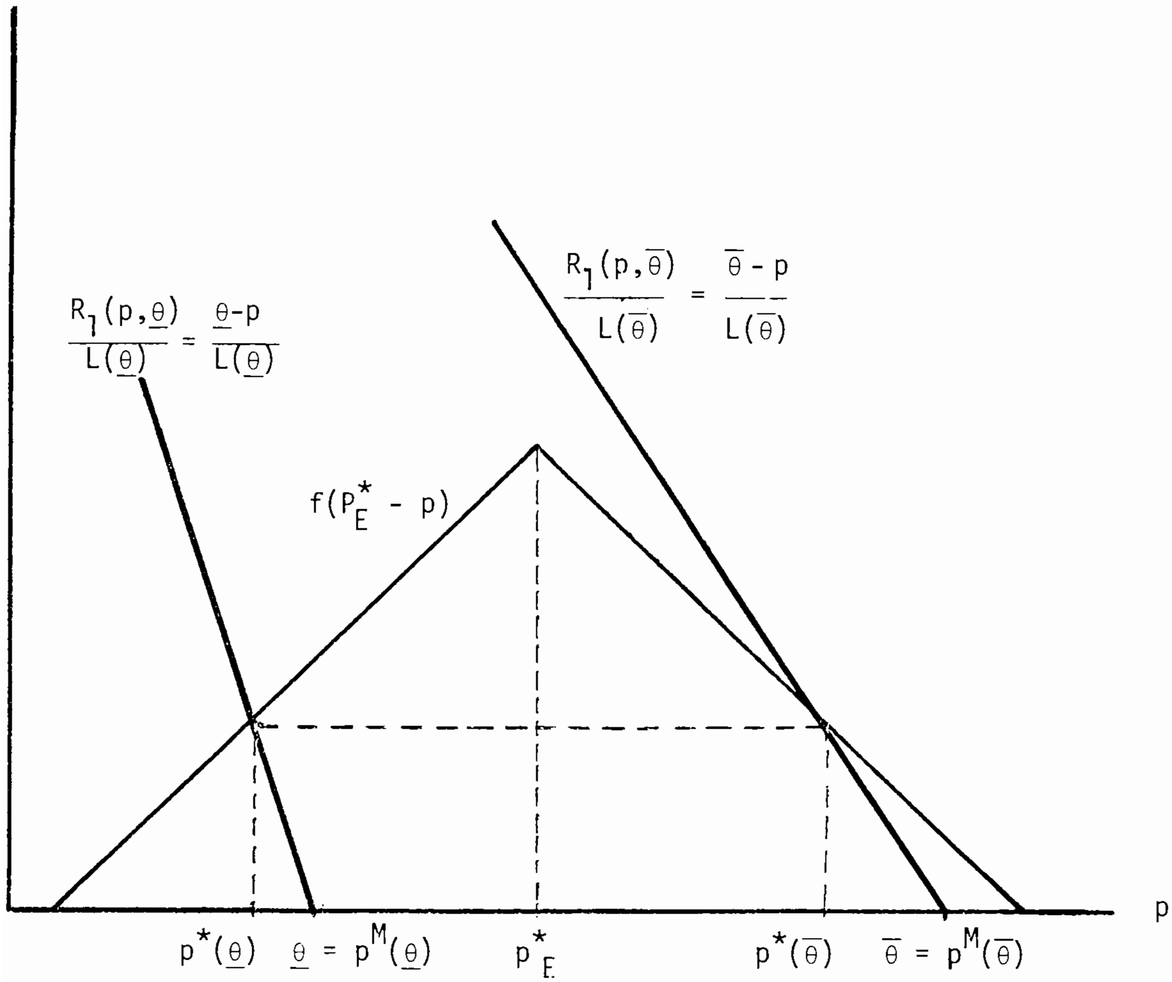
$\frac{R_1(p,\underline{\theta})}{L(\underline{\theta})} = \frac{\underline{\theta}-p}{L(\underline{\theta})}$
$\frac{R_1(p,\overline{\theta})}{L(\overline{\theta})} = \frac{\overline{\theta}-p}{L(\overline{\theta})}$
$f(P_E^* - p)$
p
$p^*(\underline{\theta})$
$\underline{\theta} = p^M(\underline{\theta})$
p_E^*
$p^*(\overline{\theta})$
$\overline{\theta} = p^M(\overline{\theta})$

Figure 2