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CONTRACTS, CONSENSUS AND GROUP DECISIONS*

by

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Introduction:

A group of individuals make one choice from a set of feasible states. This situation occurs frequently in real life and is addressed by Economic Theorists, Game Theorists and Political Scientists.

When the participants try to deal with the situation individually and selfishly, without the aid of some social mechanism or an arbitrator, inefficient outcomes often result. One observes this type of phenomena in strikes, wars, excessive competition between individuals and free rider type of situations. The classical game theoretic example is the prisoners' dilemma game where the only equilibrium of the game is inefficient.

When the situation is repetitive and is analyzed as an infinitly repeated game, the problem becomes somewhat less severe (see Aumann [1978]). One observes new equilibrium outcomes which are group efficient, but there are still many inefficient ones.

Purely cooperative game theory deals with the problem of which efficient outcome would, or should, result in such situations. But it does not address the issue of the cooperation inducing mechanism or the game that the "selfish" players play with the arbitrator. Some game theorists dealt with the question of the mechanism itself and its performance, for example Nash [1953], Raiffa [1953], and Kalai-Rosenthal [1978]. Some of the difficulties with their mechanisms is that they are too complicated, equilibrium behavior is difficult to compute, and the informational requirements imposed on the players and the arbitrator are very strong. Recently a preocedure to incorporate pre-play negotiations into the game was proposed by Kalai [1981], however, this procedure does not always guarantee efficient outcomes.

The purpose of this paper is to formalize a simple procedure of sequential contract signing that enables the group to reach a group efficient agreement. Every "regular" Nash equilibrium of the noncooperative extensive form game induced by this procedure is individually rational and Pareto optimal. Thus, individually, and as a group there is a strong incentive to follow this procedure.

Conversely, almost every individually rational Pareto optimal state is a possible regular equilibrium outcome of this procedure. I view this multiplicity of equilibrium outcomes as a positive characteristic of the procedure. If procedures of this type result in small and restricted sets of outcomes then there would be disagreement among the participants as to which procedure to follow. Thus the same problem would arise again in the form of a conflict about which procedure to follow or which arbitrator to use. In this proposed contract signing procedure with its multiple equilibria it is left to the bargaining ability of the individuals and there should be no problem in agreeing to adopt the procedure.

This proposed contract-signing procedure is attractive in its simplicity. This is true for two crucial aspects, the description of the procedure, and the optimal equilibrium behavior computations that have to be done by the participants. While simplicity is not a well defined mathematical property I feel that it is an important feature. Simplicity of description gives the procedure hope of being implemented. Simplicity of computing optimal behavior increases the chances that the individuals would actually behave as the theory prescribes.

There are three major contributions in the paper. The first is as a descriptive theory. The procedure suggested here is a close approximation to procedures that actually take place in real life. Thus the paper supplies a game theoretic analysis of such situations. A second major contribution lies in the hope that for certain real life situations in which non Pareto optimal outcomes occur this procedure will be implemented. The third contribution is within the theory of equilibrium notions for extensive form games. While analysing the equilibrium of our procedure I suggest some additional criteria for restricting the equilibrium notion to be more reasonable. I believe that these ideas can easily be incorporated into the general theory of extensive form games in the direction initiated by Selten [1975].

I. THE CONTRACT SIGNING PROCEDURE AND ITS APPLICATIONS.

Let $N = \{1,2,\ldots,n\}$ be a set of players $(n > 1)$ and $C = \{c_0,c_1,c_2,\ldots,c_m\}$ be a finite set of states. For $i=1,2,\ldots,n$ $U_i: C \to \mathbb{R}$ represents a utility function of player i which expresses his ordinal preferences over C.

For a predetermined commonly known positive integer k and a predetermined commonly known state c_0 the k attempts (or iterations) contract-signing game $G(c_0,k)$ is defined as follows. There are k identical iterations numbered k, $k-1,\ldots,1$. In every such iteration j, the players, simultaneously, each proposes a state $c^{j,i} \in C$. The proposals $(c^{j,1}, c^{j,2}, c^{j,n})$ are then made common knowledge.

To define the outcome of the contract-signing game we distinguish between two types of outcomes at iteration j. If $c^{j,1} = c^{j,2} = \ldots = c^{j,n} = c^j$ then we say that an agreement c^j was reached at iteration j. Otherwise we say that no agreement was reached at the j^{th} iteration. The outcome of the game is defined to be c_0 if there was no agreement reached at any iteration. If an agreement was reached at some iteration then we defined the outcome to be the last agreement, i.e., the c^j with minimal j.

Two intuitive examples of the model are the following. In a game theoretic context we consider an n-person strategic form game. Each $c_i (i > 0)$

may be thought of as a joint (n-tuple) pure strategy of the n players. Thus all joint pure strategies may be agreed upon. c_0 is the noncooperative (possibly mixed) Nash equilibrium strategy which would result if no agreement was reached.

In a social choice context we may think of the c_i's as social states with c_0 being the forecasted outcome in the case of no agreement, possibly the status quo state.

An alternative and possibly more intuitive description of the contract-signing procedure is the following inductive process. For j=0 we let for every $c_i \varepsilon C$ the degenerate game $G(c_i,0)$ be the game with no strategies whose outcome is c_i. For $k \geq j \geq 1$ and $c_i \varepsilon C$ we define the game $G(c_i,j)$ to be the following. Each player i chooses $c^i \varepsilon C$. If $c^1 = c^2 = \ldots = c^n = c_p$ then the outcome is the game $G(c_p,j-1)$. Otherwise the outcome is $G(c_i,j-1)$. The k attempts contract signing game is then $G(c_0,k)$. With this description we can view the contract signing game as a sequence of signing binding contracts. The initial contract is the noncooperative contract c_0. Earlier contracts are binding and they can be cancelled by a mutual agreement to sign a new contract later on.

II. REGULAR EQUILIBRIUM OF EXTENSIVE FORM GAMES

In this section a notion of regular equilibrium of extensive form games is described. It is assumed that the reader is familiar with the theories of extensive form games and normal form games (see Owen [1968] and Selten [1975]). The games described here will have no "chance" moves and will involve only "pure" strategies. Also the payoffs at the end of the game will be states rather than utilities. The players are assumed to have preferences over the states (as in the previous sections). Thus these are really "ordinal" extensive form games. The concepts presented here can easily be extended in the obvious way to "cardinal" games with probabilities. Since this would serve no purpose for the present paper I will ignore this extension.

The motivation in defining regular Nash equilibrium for these games is to restrict the set of equilibria to "reasonable" ones. This idea was suggested in a seminal paper by Selten [1975] and was recently extended further by Kreps-Wilson [1980]. Related ideas for games in the normal form were presented by Myerson [1978].

Given a game in extensive form of the type described above and Nash equilibrium strategies of the n-players, the equilibrium will be called regular if it satisfies the following three conditions.

1. Subgame Symmetry
2. Subgame Optimality
3. Reduction Compatibility

Weaker versions of these concepts have been introduced earlier by others. Subgame optimality is an obvious extension of Selten's [1975] idea of subgame perfectness. A somewhat different idea of reducing a game through domination is mentioned in Luce-Raiffa [1957] and a version of it was used by Moulin [1979].

To explain these notions it is necessary to first extend the outcomes of an extensive form game from the set of terminal nodes of the set to all decision nodes of the tree. But for given pure strategies of the n-players there is a unique natural way to do this extension. Namely, given a decision node of the game tree, the pure strategies of the players define a unique path from this node leading to one terminal node. It is the terminal node that will result if the game starts at the prescribed decision node and the specified strategies are really followed. Thus the outcome associated with a decision node relative to the given pure strategies is defined in this manner.

Given an n-person extensive form game as described at the beginning of this section, and pure strategies of the players, we are interested in the optimality of these strategies over logical complete subgames. A subtree of the game tree is a subset of the nodes and arcs of the game which constitutes a tree with the following properties.

1. There is a unique initial node of the subtree.
2. For every nonterminal node of the subtree the following two conditions hold.
   a. All the arcs coming out of the node are in the subtree, and
   b. All the nodes that belong to the same information set in the original tree as the specified node are also nonterminal nodes in the subtree.

A subgame of the original game is a restriction of the original game, with its information sets and players names, to such a subtree. The outcomes associated with the terminal nodes of the subtree are the outcomes associated with these nodes (by the given pure strategies) in the original game.

Two subgames are identical if there is a one to one correspondence between their nodes which preserves the tree structures, the names of the players and strategies, the information sets, and the outcomes.

The strategies of the players are subgame symmetric if the induced strategies on identical subgames are the same.

The strategies of the players are subgame optimal if the induced strategies on every subgame constitute a Nash equilibrium in the subgame.

The notion of reduction consistency is more involved. I first define it for games in the normal form. Let $(S_1,S_2,\dots,S_n) = S$ be a list of finite nonempty sets of <u>strategies</u>, one for every player and let $XS = \underset{i\varepsilon N}{X} S_i$. There is a list of utility functions $U = (U_1, U_2,\dots U_n)$ where $U_i: XS \to R$ expresses i^{th} ordinal preferences over XS. I first define, inductively, the notion of a <u>reduced version</u> of the game (S,U). In every reduced version of the original game the set of strategies available to individual players will be restricted by successively eliminating dominated strategies. The utilities over the remaining strategies will be the ones induced by the original utilities when restricted to the smaller set. More precisely the following method defines reduced versions inductively. (S,U) is defined to be a reduced version of (S,U). Suppose now that (T,U) is a reduced version then $T = (T_1, T_2,\dots,T_n)$ with $\emptyset \neq T_i \subseteq S_i$ and $U = U|_{XT_i}$ (with some abuse of notation in order to keep the notation simple). We can get further reduced versions as follows. Choose a player j and let $\bar{T}_j$ have the following properties:

1. $\emptyset = \bar{T}_j \subseteq T_j$
2. For every $s_j \varepsilon T_j - \bar{T}_j$ there is an $\bar{s}_j \varepsilon \bar{T}_j$ such that $U_j(s_1, s_2,\dots,s_{j-1}, \bar{s}_j, s_{j+1},\dots,s_n) \geq U_j(s_1, s_2,\dots,s_j,\dots,s_n)$ for every $(s_1, s_2,\dots,s_{j-1}, s_{j+1},\dots,s_n) \varepsilon \underset{i\varepsilon N/j}{X} T_i$.

If $\bar{T}_j$ satisfies these properties then $((T_1, T_2,\dots,T_{j-1}, \bar{T}_j, T_{j+1},\dots,T_n),U)$ is also a reduced version. A pair (T,U) is a <u>maximally reduced version</u> if it is a reduced version that can be reduced no further.

A Nash equilibrium $s = (s_1,s_2,\dots,s_n)\varepsilon XS$ of the game (S,U) is defined to be <u>reduction consistent</u> if for some maximally reduced version of (S,U), (T,U) we have $s_i \varepsilon T_i$ for $i=1,2,\ldots,n$.

Now, returning to extensive form games, the notion of reduction consistency can be defined there. A *simple subgame* of an extensive form game is defined to be a subgame in which every player has at most one information set. A strategy choice of the players will be called *reduction consistent* if the following condition holds. Consider any simple subgame with its induced strategies. These strategies are reduction consistent in the normal form representation of the simple subgame.

III. The Regular Equilibrium Outcomes of the Contract Signing Procedure

In this section I characterize the outcomes that arise when a regular Nash equilibrium strategy is played in the contract signing procedure. For two states c_i, $c_j \varepsilon C$ $c_i \succsim c_j$ denotes that $U_t(c_i) \geq U_t(c_j)$ for $t = 1,2,\ldots,n$, $c_i \succ c_j$ denotes that $U_t(c_i) > U_t(c_j)$ for $1,2,\ldots,n$. c_i satisfies *sophisticated individual rationality relative to* c_j if either

1. $c_i \succ c_j$ or

2. $c_i \succsim c_j$ and for no $c_k \varepsilon C$ $c_k \succ c_j$.

c_i satisfies *sophisticated individual rationality* if it satisfies sophisticated individual rationality relative to c_o. c_i is *Pareto Optimal* (weakly) if there is no $c_j \varepsilon C$ with $c_j \succ c_i$.

Notice that the set of states that are Pareto optimal and satisfy sophisticated individual rationality is not empty. Thus the following theorem, in addition to characterizing the equilibrium outcomes, establishes the existence of a regular equilibrium. Also the proof of the "if" direction supplies a constructive description of optimal stategies. These optimal strategies are very intuitive and their computations are very simple.

Theorem For large enough k's (that is, there exists $K > 0$ such that for all $k > K$) a state is an outcome of some regular Nash equilibrium of the k attempts contract-signing game if and only if it is Pareto optimal and satisfies sophisticated individual rationality.

Proposition 1 For every $k \geq 1$ if c_i is an outcome resulting from a Nash equilibrium of the k attempts contract-signing game then c_i is individually rational, i.e., $c_i^i \succsim c_0$.

Proof This proposition follows immediately from the fact that in this game every player i has the option to play the constant strategy c_0 at all the iteration and thus guaranteeing himself an outcome at least as good as c_0.

Before proceeding with the proof of the "only if" direction of the Theorem it is necessary to digress to an analysis of the equilibrium outcomes of diagonal type n-person normal form games. In such a game all the players i have an identical non empty set of strategies $S = S_1 = S_2 = \dots = S_n$. $U_i: S^n \Rightarrow R$ represents the utility of players i. A joint strategy $(s_1, s_2, \dots, s_n) \in S^n$ will be called a diagonal strategy if $s_1 = s_2 = \dots = s_n$, otherwise it will be called a nondiagonal strategy. This game will be called a diagonal game if every player is indifferent between all the nondiagonal strategies, i.e. for every player $i \in N$ and two nondiagonal strategies q,r $U_i(q) = U_i(r)$.

Lemma 1 In a diagonal game every Nash equilibrium strategy r is individually rational relative to every nondiagonal strategy. Furthermore if r is reduction consistent then it satisfies sophisticated individual rationality relative to any nondiagonal strategy.

Proof The first part is obvious since every player can move the play to be off the diagonal. Now suppose r is a nondiagonal strategy and there are joint strategies (necessarily diagonal) $\bar{b} = (b,b,\ldots,b)$ with $\bar{b} \succ r$. Suppose $((T_1, T_2,\ldots,T_n), U)$ is a maximally reduced version of the game. Observe that for $i = 1,2,\ldots,n$ $b \in T_i$ (to get a contradiction consider the first time that b was eliminated by some player). Thus let $B = \{b \in S: (b,b,\ldots b) \succ r\}$ then $\emptyset \neq B \subseteq T_i$ for $i = 1,2,\ldots,n$. Now to see that $B = T_i$ for all players suppose $a \in T_i - B$ for some player i. There exists a player j for which $U_j(\bar{a}) \leq U_j(r) < U_j((b,b,\ldots,b))$ for some (and every) $b \in B$, where $\bar{a} = (a,a,\ldots,a)$. Thus $a \notin T_j$ by the maximal reduction of T_j. Thus player j does not play a in the maximally reduced version. But then the strategy a is dominated for every player by the strategy b. Therefore $T_i = B$ for $i = 1,2,\ldots,n$. Now it is clear that the only equilibrium strategies in this reduced version $((B,B,\ldots,B), U)$ are of the form $(b,b,\ldots,b) = \bar{b}$ with $\bar{b} \succ r$.

Now, turning to the proof of Theorem 1, we consider a sequence of decision nodes d_k $d_{k-1},\ldots,d_1$ described as follows, d_K is the initial decision node of the game. d_{k-i} is an initial decision node of iteration k-i which followed previous iterations of disagreements going though d_{k-i+1}. Let o_i be the outcome associated with d_i when a regular equilibrium strategy is played and let $o_o = c_o$. The only if part of the theorem would be completed when we argue the following two observations for $i = 0,1,2,\ldots,k-1$

1. $o_{i+1} \geqslant o_i$, and

2. $o_{i+1} > o_i$ whenever o_i is not Pareto optimal.

These two observations are direct consequences of Lemma 1. When we consider for every i the game starting at d_{i+1} and ending after every player made his choice at this iteration. The fact that each one of these games is a diagonal game follows from the structure of the contract signing procedure and the subgame symmetry property of the equilibrium strategies played (giving all the off diagonal strategies the same outcome). Notice that the above argument proves the sophisticated individual rationality property of the outcome for every number of iterations $k \geqslant 1$.

In order to prove the "if" part of the theorem we give a constructive proof of a somewhat stronger version.

Proposition 2 for every $k \geq 1$ and every Pareto optimal state c_i which satisfies sophisticated individual rationality there exists a regular Nash equilibrium of the k attempts contract signing game which induces c_i as its outcome.

Proof Consider functions $g: C \Rightarrow C$ which satisfy the following three properties for every $c_i \in C$.

1. $g(c_i)$ is Pareto optimal

2. $g(c_i)$ satisfies sophisticated individual rationality relative

to c_i

3. $g(g(c_i)) = g(c_i)$

For such a function g define the strategies of the players to be the following. At every iteration if the last agreement leading up to this iteration was c_i then every players plays $g(c_i)$. If no agreement was reached prior to this iteration (including the case of the first iteration) then every player plays $g(c_o)$. The proof of the proposition will be completed if we observe that this strategy induced by g is a regular Nash equilibrium.

I first argue that these strategies satisfy subgame optimality. Consider a subgame T starting at some decision node d. Suppose the strategy of player i is not a best response in the subgame. Then it follows by the structure of the contract-signing procedure that it is not a best response in the larger subgame Γ consisting of d and all the nodes and arcs that follow from it to the very end of the original game. Let c_j be the last agreement reached before getting to d (c_o if such agreement was never reached). Notice that by the definition of the g induced strategies that at every iteration an agreement on a Pareto optimal outcome is reached. In particular starting at d the players immediately agree on $g(c_j)$ and continue on agreeing. Thus the final outcome is $g(c_j)$. If player i modifies his strategy then at some iterations he causes disagreement but whenever there is an agreement it would be based on a previous last agreement of the type c_j or $g(c_j)$. Thus overall, after he modifies, the outcome would be c_j or $g(c_j)$ and he cannot improve relative to $g(c_j)$.

Now to see that the strategies induced by g satisfy subgame symmetry consider two identical subgames G and $\bar{G}$ with inital nodes d and

$\bar{d}$ respectively. Suppose the last agreements leading to d and respectivelyto $\bar{d}$ are c_j and $c_{\bar{j}}$ (again making the converntion that $j = 0$ if no agreement was reached earlier). If $j = \bar{j}$ then subgame symmetry holds since the induced strategies are always determined by the last past agreement. So assumeing that $j \neq \bar{j}$ we want to show that the plays induced by g in G and in $\bar{G}$ are the same. Consider a path p starting from d with the following strategies played. At every iteration the first player plays a strategy different from the one prescribed by g while the other players play their g induced strategy. Let ℓ be the last node on this path which still belongs to the subgame G. Let $\bar{p}$ and $\bar{\ell}$ be the corresponding objects in $\bar{G}$. Now the outcome associated with ℓ is either $g(c_j)$ or c_j(if ℓ is a terminal node in the original game). Similarly the outcome associated with $\bar{\ell}$ is either $g(c_{\bar{j}})$ or $c_{\bar{j}}$. Thus, since G and $\bar{G}$ are identical we have one of the following identities $c_j = c_{\bar{j}}$ or $g(c_j) = c_{\bar{j}}$ or $c_j = g(c_{\bar{j}})$ or $g(c_j) = g(c_{\bar{j}})$.

In any of these cases it follows that $g(c_j) = g(c_{\bar{j}})$. Therefore the players will play the same induced strategy at the iteration starting at d as they would at $\bar{d}$. The same argument holds for every other iteration whose initial node is included in G and subgame symmetry is proved.

Finally, it remains to be shown that reduction compatibility holds for the strategies induced by g. Consider a simple subgame G which starts at a decision node d. Let c_j be the last agreement reached before d was reached. The normal form version corresponding to the game G is a t-person diagonal game with $T = \{1,2,\ldots,t\} \subseteq N$. The nondiagonal outcomes are $g(c_j)$ if there are any iterations left in the game after the current one, otherwise the non-diagonal outcomes are c_j. The diagonal outcome corresponding to the diagonal stratey $\bar{g} = (g(c_j), g(c_j),\ldots,g(c_j)) \in \mathbb{C}^t$ is $g(c_j)$. For other diagonal strategies $\bar{c}_i = (c_i,c_i,\ldots,c_i)$ with $c_i \neq g(c_j)$ the outcome is either

1. c_i if $T = N$ and we are at the last iteration,

or 2. $g(c_i)$ if $T = N$ and we are not at the last iteration,

or 3. c_j if $T \neq N$ and we are at the last iteration,

or 4. $g(c_j)$ if $T \neq N$ and we are at the last iteration.

It is straight forward to verify in every one of these cases (using the Pareto optimality of $g(c_j)$) that $\bar{g}$ is an equilibrium strategy for some maximally reduced version of the normal form game.

To complete the proof of the proposition observe that for every C_i which satisfies Pareto optimality and sophisticated individual rationality one can construct a function g with the property that $g(c_o) = c_i$. Define $g(c_o) = C_i$ and for every other Pareto optimal state c_j define $g(c_j) = c_j$. For non Pareto optimal states c_j choose $g(c_j)$ to be any state which satisfies Pareto optimality and sophisticated individual rationality relative to it.

IV. Further Discussion

Several extensions of the contract-signing procedure will be of interest. One such extension would allow the participants to reach partial agreements among subsets of players, rather than the whole group. This is important especially for situations in which the number of participants is large. Also agreements restricting the set of states (or strategies) to a smaller set rather than to one state should be of interest. This may be easier to implement in very complex situations. Most real life agreements and contracts are of this type since the participants reach an agreement on the strategies relating to one situation of conflict or cooperation. Their actions in other aspects of their lives (the ones which do not relate to the specific situation) are not specified by the agreement. These issues are important if we hope to be able to describe a consensus achieving mechanism which would be more applicable for economies involving private and public goods.

Another issue to study is the performance of the contract signing mechanism, or ones like it, in situations of incomplete information. Unlike the one shot game in this extended extensive form there are many more opportunities for the participants to learn about their coplayers and to transfer information.

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