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WHEN DOES THE J-CURVE HAVE A J-SHAPE?
A RECONSIDERATION
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Introduction

The current theory of the J-curve states that because of contracts outstanding during an exchange rate adjustment, the balance of trade first moves in a direction opposite to its long-run change, brought about by adjustments in the quantities exported and imported (assuming the Marshall-Lerner condition). Hence the response to the adjustments has a J-shape over time: e.g. in case of devaluation the trade balance deteriorates before it gets better. (see Magee, 1973 and McKinnon, 1979 p. 151) Under flexible exchange rates this has a destabilizing effect: depreciation in an initial trade balance deficit makes the deficit greater, causing further depreciation, and vice versa for appreciation.

This theory is based on the implicit assumption that the currency of invoicing is always the exporter’s currency; i.e. the country’s exports are invoiced in domestic, and its imports in foreign currency. However, empirical evidence on all the countries studied shows this assumption to be grossly violated in reality. Therefore the theory of the of the J-curve itself needs reconsideration.

Magee (1973), using a taxonomic approach of the four cessionsations of the currencies of invoice in exports and imports, showed that a J-shaped J-curve

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is inevitable in only one of the four cases, and possible in another. If the "pass-through", which he concentrates on, is taken into account, it can have a variety of shapes. Carse et al. (1980, p. 102) examine the stability of the foreign exchange market under flexible rates, when all invoicing is either in the exporter's or the importer's currency, and conclude that the system is unstable in the former, and stable in the latter case.

The purpose of this paper is to examine the joint effects of the immediate impact of exchange rate adjustments on the trade balance through the changes it causes in the domestic currency prices of exports and imports already contracted for, and the long-run effect of quantity adjustments, assuming the Marshall-Lerner condition and a monotonic adjustment in quantities. The problems of pass-through will thus be ignored, since they are independent of the currency of invoicing. We will first look at the empirical evidence on the currency of invoicing, and then develop a simple model to derive the critical condition for the J-curve to have a J-shape. It turns out to depend both on the currencies of invoice and the initial state of the trade balance. The curve can in general be expected to have the conventional shape in an initial trade balance deficit - the case on which the current theory is based. In an initial surplus, however, this is far from guaranteed.

Accordingly, for the majority of countries, an initial surplus changes the shape of the J-curve so that the impact and longer-run effects strengthen each other. The J-curve effect being symmetrical for devaluation and revaluation, given the initial state of the trade balance, this thus has particular significance for revaluation, and under flexible rates, for the stability of the system in an initial surplus.
The Empirical Evidence

Curze et al. (1980, p. 18) summarize the existing evidence on the currency of invoice. It suggests that for all the countries, the share of exports invoiced in foreign currency is lower than the respective share of imports. The shares appear to increase, the smaller the role of the country’s currency is in international trade. Accordingly, 59 percent of Danish exports, and 81 percent of her imports is invoiced in foreign currency (Grassman 1973, 1976), while most of the trade between the United States and Canada is invoiced in U.S. dollars (Fieke 1971). The authors suggest that there exists a hierarchy of currencies in international trade. The higher the currency is in this hierarchy, the higher the probability that it be used as the currency of invoice, other things equal. They suggest that currencies on the bottom of the hierarchy— all but some ten currencies— have no international role. Since the evidence reported only includes studies of the top ten countries, one should exercise caution when extrapolating it beyond these countries. Indeed, Nars (1986) reports that less than 3 percent of Finnish exports and practically none of her imports is invoiced in Finnish currency. Casual empiricism suggests that the same can be said about most other convertible-currency countries, which supports the above hypothesis. I.e., most countries in the world both invoice and pay in foreign currency.

This procedure appears to be rational, as well, due to the information costs of keeping track and forecasting currency developments, the possibilities of internal hedging as the number of invoice currencies increases, the thinness of forward markets in most currencies, and the thinness of many national money markets, which affects the costs of flexibility in the timing of payments. Therefore one would expect this pattern to hold into the future, as well.
Implications for the J-curve

The expression of the trade balance (T) is as follows:

\[ T = X(e)p^X(e) - M(e)p^M(e) \]

where \( X \) and \( M \) are exports and imports, respectively, \( p^X \) and \( p^M \) the respective domestic-currency prices, and \( e \) the price of foreign currency in terms of the domestic currency.

Totally differentiating (1), treating \( e \) as a variable, we get, setting the initial values of \( p^X \), \( p^M \), and \( e \) at unity by an appropriate choice of units:

\[ dT = \left( \frac{X}{e} - \frac{M}{e} + X'p^X - M'p^M \right) de \]

where \( X' \) is the partial of \( X \) with respect to \( e \) etc. The Marshall-Lerner condition implies that the expression in the brackets is positive.

The export and import price indices, and their partials with respect to \( e \), can be expressed as weighted averages of the prices of goods invoiced in domestic and in foreign currencies:

\[ \frac{p^X}{e} = a \frac{p^F}{e} + (1-a)p^{X0} \]

\[ \frac{p^M}{e} = b \frac{p^F}{e} + (1-b)p^{M0} \]

where superscripts \( F \) and \( D \) refer to foreign currency and domestic currency, and \( a \) and \( b \) are the shares of exports and imports, respectively, invoiced
in foreign currency.

In the short run, the quantities are fixed and the respective values show in the balance of payments statistics as the contracts outstanding are delivered. Thus $X_e = M_e = 0$. Furthermore, $p^{X}_e = p^{M}_e = 0$ and $p^{X}_e = p^{M}_e = 1$; the domestic prices of domestic currency-denominated contracts remain unchanged, and those denominated in foreign currency change by the amount of the exchange rate change, remaining constant in terms of the foreign currency. Therefore, in the short run we get, by substituting these values into (2), (3), and (4), and equations (3) and (4) into (2):\(^1\)

\[
(5) \quad dT = (aX - bt)de
\]

Adding and subtracting $(bXde)$ to the right-hand-side we get:

\[
(6) \quad dT = [(a-b)X + b(X-M)]de
\]

The J-curve has a J-shape only if the expression in the brackets is negative. The critical condition for a J-shaped J-curve can thus be written as:

\[
(7) \quad \frac{X-M}{X} < \frac{1-a}{b}
\]

If the reverse inequality holds, the J-curve effect strengthens the quantity adjustment, and if equality holds, the J-curve effect is zero.

First, in the conventional case, which was shown not to be supported by empirical evidence, $a=0$ and $b=1$. The right-hand-side is one so that the J-curve has a J-shape as long as the trade balance surplus is smaller than
exports, i.e. as long as imports are positive. For Denmark the trade balance surplus has to be less than 27 percent of her exports for the J-curve to have a J-shape, whereas the critical limit for Britain is 70 percent. For Finland and other countries with equal a's and b's, if the trade balance is at all in surplus the J-curve has the unconventional shape. This apparently applies to most countries outside the Group of Ten.

It is notable that this applies to revaluation, as well: revaluation has (an inverted) J-shaped J-curve effect only if the above critical condition is met. As proposed, for most countries the condition is violated whenever the trade balance is in surplus.

Our findings have implications also for the stability of the system under flexible exchange rates. The critical condition for instability is, of course, (7) so that whenever equality or the reverse inequality holds, the system is stable.2

The J-Curve and Linking of Currencies

Small countries often link their currencies to their major trading partners' currencies. If the trading partner (a large country, in whose currency the invoicing between the countries generally takes place) adjusts its exchange rate, the small country often follows suit also in an initial trade balance surplus. As will be shown in the Appendix, if the share of its exports invoiced in the partner's currency, out of total exports invoiced in foreign currency exceeds the corresponding share of imports, the critical condition in (7) is partly relaxed, if it falls short of it, it gets more stringent, and if equality holds, it remains unchanged. The same is of course true of the trading partner, though, as suggested, the share of trade invoiced in the small countries' currencies is likely to be close to zero, leaving the
Critical condition unchanged.

Conclusion

The shape of the J-curve—and the stability of the system under flexible rates—depends on the country's currency of invoicing in exports and imports and on the initial state of its trade balance. Therefore the J-curve effect does not lend itself for the kinds of generalizations made about it.

Available empirical evidence suggests that the share of countries' exports invoiced in their own currencies is higher than or equal to the respective share of their imports. Therefore the J-curve can in general be expected to have a J-shape whenever the country devalues or revalues in an initial trade balance deficit. However, for countries whose foreign currency shares are equal—which apparently constitutes a majority of trading countries—an initial surplus is enough to guarantee that the J-curve effect strengthens the longer-run effect of quantity adjustments, making the J-curve obtain a shape quite different from J. Countries that do a higher percentage of their export invoicing in their own currencies may sometimes require substantial initial trade balance surpluses before the J-curve changes its shape.

The above has implications also for the stability of the system under flexible rates. When the J-curve has the unconventional shape, the conventionally unstable system becomes stable.
Footnotes

1. Magee derives the equivalent of equation (5). His analysis is, however, limited to an initial trade balance deficit. He concludes that the higher $b$, compared to $a$, the more likely the J-curve is to have a J-shape. He also states on p. 313, "...it also depends on the relation of these shares to the patterns of deficits and devaluations [across foreign countries]." The statement concerning the distribution of trade balances across countries is inaccurate, since the exchange rate is adjusted against all other currencies. Our analysis is then a generalization of Magee's, where the tradeoff between $a$'s and $b$'s and the initial trade balance is derived.

2. The special cases studied by Carse, p. 102, referred to in the Introduction, are $a = 0; b = 1$ (all invoicing in the exporter's currency), and $a = 1; b = 0$ (all invoicing in the importer's currency). They state, "The evidence indicates quite decisively that in Britain, as in other industrial countries, exports are denominated predominantly in domestic currency, while imports are denominated predominantly in foreign currency. One can therefore conclude that, in the absence of capital mobility or central bank intervention, the foreign exchange market would be dynamically unstable. This instability is global." As we have shown, this statement can be expected to hold for Britain in all conceivable circumstances, but not for Denmark, let alone the countries whose currencies are not in the top ten.
Appendix

If a major trading country adjusts its exchange rate and a small country follows suit, the domestic currency values of trade invoiced in the large country's currency remain unchanged for the small country in the short run. This trade is thus equivalent to trade invoiced in domestic currency in (1) and (4). As a result, the equivalent of a decline in $a$'s and $b$'s follows:

$$a_n = a - a_1 \quad ; \quad b_n = b - b_1$$

where $a_n$ and $b_n$ are the net values of these shares and $a_1$ and $b_1$ the shares of trade invoiced in the large country's currency. Substituting $a_n$ and $b_n$ into (7) we get:

$$(7') \quad \frac{X-M}{X} < 1 - \frac{a(1 - \frac{a_1}{a})}{b(1 - \frac{b_1}{b})}$$

It is seen that if

$$\frac{a_1}{a} > \frac{b_1}{b}$$

i.e. the share of exports invoiced in this country's currency out of total exports invoiced in foreign currency exceeds the corresponding share for imports, the critical condition in (7) is partly relaxed. If it falls short of it the condition becomes more stringent. It equality holds, it remains unchanged.
The same holds of course for the large country. Since typically none of its trade is invoiced in the small country’s currency, its critical condition is likely to remain unchanged.
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