Discussion Paper No. 466

CONJECTURAL VARIATIONS

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Revised March 1981 *

Managerial Economics and Decision Sciences

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This paper was supported in part by the Sloan Foundation Grant for the Applied Microeconomics Workshop at Northwestern University.

* Revision of Discussion Paper No. 441

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A feature common to the perfectly competitive model and the monopoly model is each firm's disregard of other firms' reactions to its price or quantity decisions. In the former case, each firm regards itself as too small to influence the market price and therefore to attract the attention of rivals; in the latter case, the monopolist regards itself as having no rivals. (In some dynamic monopoly models, the incumbent firm does take into account the effect of its actions on potential entrants.) In many markets the assumptions necessary for application of either of these two polar models are not satisfied. The firms serving a market are neither so numerous that each contributes only a negligible fraction of the total output, nor does just one firm serve the market. Instead there are relatively few firms, each of whom has some influence over the total quantity supplied and the market price. In these circumstances each firm can anticipate that its price or quantity decisions may call forth a response from rivals. All this, as we know, was recognized by Cournot almost one-hundred-and-fifty years ago. In the model of oligopoly that he formulated, firms do not take reactions into account but maximize myopically. That model has become widely employed to explain competition among the few.

One of the most appealing features of the Cournot model of oligopoly is that it yields the monopoly solution when there is just one firm and yields the perfectly competitive solution when the number of firms increases indefinitely, assuming average production costs are nondecreasing; see also Ruffin. Thus, in terms of the traditional industrial organization characterization of a market by its structure (number of firms), conduct (their response to each others' actions), and performance (proximity of the actual market equilibrium to the one that would prevail under perfect competition), the Cournot model provides a direct link between structure and performance. This link, however, rests on Cournot's assumption that each firm regards the current output of its rivals as fixed in deciding its profit maximizing level of output (under the supposition that firms choose quantities, not prices). That is, each firm behaves as if its rivals will not alter their levels of output in response to change in its own choice of output. The firm's belief about the rivals' response to change in its own decision variable was named the conjectural variation by Frisch. The Cournot assumption is that the conjectural variation is zero.

A couple of definitions are of use here. Suppose, for ease of discussion until indicated otherwise, that firms choose output levels. The firm's profit maximizing output depends on the output levels chosen by its rivals. The firm's <u>reaction function</u> specifies this relationship, giving its profit maximizing output as a function of rivals' output levels. The slope of this reaction function is the rate at which the firm's profit maximizing output will change with a change in a rival's output. The firm may believe that its rivals similarly have reaction functions. The rival's reaction function is not known to the firm but the firm may nonetheless have conjectures about it. In particular, the firm may make conjectures about the slopes of the rivals' reaction functions. These conjectured slopes are called <u>conjectural variations</u>. Constant conjectural variations imply that the conjectured reaction function is linear. The particular Cournot assumption of zero conjectural variations implies that it is horizontal. The conjectural variation may depend on the output, rather than being constant. A nonconstant conjectural variation implies a conjectured reaction function with a nonconstant or variable slope.

The assumption of zero conjectural variation has been found objectionable for some time for several reasons; see Fisher. First, if reactions of rivals are thought to be sequential, as Cournot viewed them, then firms involved in the process are bound to observe that rivals do in fact react to their actions. Only if all other rivals produce their equilibrium output will they not change their output when the firm produces its equilibrium output. But this is really a very special event. In all other circumstances, one's change of output will lead to a change in rivals' output levels. In particular, a deviation from the equilibrium output (rather than to it) will generate a change in rivals' output. If rivals ignore the observed reactions of rivals, then the equilibrium is achieved for the wrong reasons, as Fellner pointed out.

Second, the assumption of zero conjectural variation is suspect since it leads to a logical inconsistency even if the equilibrium is attained through a simultaneous rather than sequential process. To see this, suppose that each participant has zero conjectural variations and determines his reaction function, i.e. his profit maximizing output as a function of rivals' output levels. Suppose further that the reaction functions of all participants are then fed into a computer that determines the equilibrium output level. The equilibrium is achieved nonsequentially in this situation. A logical inconsistency occurs, however, because the firm's reaction function will indicate that the firm itself should react to changes in rivals' output. Thus the firm is in the position of believing that it is optimal to respond to its rivals' actions while also believing that its rivals will not react to its choices. This requires the belief that its rivals are either very different from itself or not as smart. In the absence of such beliefs, the firm faces logical inconsistency.

Third, the zero conjectural variations assumption has been criticized indirectly because it leads to an equilibrium that does not maximize the joint profits of the firms involved. This means that there are opportunities for additional profits, either through exploitation of rivals' naivete, as suggested by Stackelberg, or through tacit collusion as suggested by Chamberlin. The supposition that tacit collusion occurs is appealing because it is difficult to believe that rivals would not recognize and act on this possibility for additional profit. Just as nature is thought to abhor a vacuum, so a market may be thought to abhor an unexploited profit opportunity. The emphasis on tacit collusion stems from the fact that, at least in the U. S., overt collusion is unlawful.

Modelling of tacit collusion has taken two paths. The first is through dropping Cournot's assumption of zero conjectural variation. Thus Fama and Laffer and later Kamien and Anderson noted that an industry of any fixed number (greater than one) of firms may produce the entire range of outputs between the competitive and the monopolistic, depending on the conjectural variations of the participants. Smithies and Savage seemed partially aware of this result. Reinganum proved an analogous result in the context of a dynamic differential game of research and development.

The second path to modelling tacit collusion has been through a dynamic framework in which each firm seeks to maximize the discounted present value of its profits. The firm chooses an entire sequence of outputs (or prices) in these formulations instead of just one, as in the static models. Dynamic oligopoly models have been presented by Friedman, Cyert and DeGroot, Marschak and Selten, and Shapiro, among others. The underlying theme is that firms having to compete with each other through time will recognize the mutual advantage to tacit cooperation and that the resulting equilibrium will have higher prices and lower quantities than predicted by the static Cournot equilibrium. Moreover, because each firm has to face its rivals repeatedly, any short run profit advantage it may gain by departing from the tacitly collusive equilibrium strategy must be weighed against the adverse reactions of its rivals in the future.

Fourth, the classic criticism of the Cournot model due to Bertrand is that the firm's decision variable is price, not quantity. The variable that is subject to choice affects the predicted equilibrium outcome. For under the usual assumptions of nondecreasing costs and zero conjectural variations, a model of differentiated goods with price as decision variable yields an equilibrium with a higher output and lower price than is predicted if quantity is the decision variable. The formal relationship between the two equilibria has been developed by Levitan and Shubik for linear demand functions and by Hathaway and Rickard for more general demand functions. It has been recognized, however, that the equilibrium attained depends not only on which variable is subject to choice (price or quantity) but also on the assumption regarding the conjectural variations. Zero conjectural variation in quantity (price) does not correspond to zero conjectural variation in price (quantity). Circumstances under which the zero conjectural variation assumptions can be viewed as dual, arising in related but differing circumstances, have been identified by Sonnenschein and Bergstrom.

Fifth, empirical studies by Iwata of the Japanese glass industry and by Gollop and Roberts of the U. S. paint industry indicate that the conjectural variations in these industries are not zero. Experimental studies by Dolbear et. al. also support the finding of nonzero conjectural variations.

These five objections to the assumption of zero conjectural variations have led us to study the implications of nonzero conjectural variations. Our study involves three related topics. First we explore the role of nonzero conjectural variations in the link between market structure and performance. Since the market equilibrium depends importantly on the conjectural variations held by the producers, a measure or index of industry structure should reflect more than the number or size distribution of firms if it is to capture elements impinging upon market performance. Cowling and Waterson, Hause, Dickson, and Dansby and Willig have addressed the question of industry structure or performance indices, taking account of possible nonzero conjectural variations. In this paper, we first show in a particularly simple format how the industry and firm output depend on the conjectural variations of the industry members. We also present some other industry structure or performance indices within this simple format.

The dependence of the market equilibrium on the conjectural variations of the industry members dilutes the predictive power of the theory linking the number of firms with market performance. Any market equilibrium between the perfectly competitive and the monopolistic is possible for any fixed number of firms. Several independent attempts have been made to narrow the possible market equilibria by imposing the additional constraint that the conjectural variations be "consistent." These independent efforts include work by Friedman, Laitner, Bresnaham with follow-ups by Perry and Farley, Holt, Capozza and Van Order, Boyer, and this paper. The basic notion is that if conjectural variations are consistent, then the firm's beliefs about rivals' rates of response will coincide with the actual rates of response, at least at the equilibrium. The formulations of this notion by the various contributors differ in conceptual detail, as for instance by whether one's rivals are viewed as a single composite firm or as a number of individual rivals, by whether all actual responses are to be profit maximizing, by whether the response to a firm's action involves only the direct response by its rivals or somehow includes indirect responses of each to the others' responses as well, and by whether the consistency is to hold at equilibrium or in a larger neighborhood of equilibrium as well.

The requirement that the conjectural variations be consistent is, of course, appealing in its own right, apart from any restrictions it may impose on the possible market equilibria. For the absence of consistency is, as we indicated above, a major flaw in the Cournot-based theory of oligopoly. However, as we will show, the requirement of consistent conjectural variations need not significantly restrict the possible equilibria. What it does do is imply a specific functional form for the industry demand function. This is an important result in bridging theory and estimation.

In the second part of this paper we develop the implications of the requirement that conjectural variations be consistent under the supposition that all the firms are identical. We define consistency by the requirement that at the equilibrium, each firm's conjecture about rivals' rates of change of output in response to its own output change be equal to the actual profit-maximizing output change in response to that exogenous output change. The method of calculating the consistent conjectural variation is due to Holt. Our analysis of consistent conjectural variations involves two phases. In the first, we assume that the conjectural variations are constants, independent of the total output level. Bounds on the conjectural variations restrict the equilibrium between the perfectly competitive and the monopolistic. We display the class of demand functions that gives rise to constant consistent conjectural variations. The relationship between the conjectural variation and the particular demand function is shown. In the second phase of this analysis, we relax the assumption of constant conjectural variations, allowing them instead to be functions of total output. We display the relationship between the functional form of the conjectural variations function and the functional form of the demand function that yield consistency at equilibrium. The results are local, not global. That is, the functional forms are required at the consistent equilibrium only; no restrictions are implied (or relevent) on functional forms in portions of the domain away from the equilibrium.

In the third part of the paper, the results in a differentiated market when firms choose price are compared with those that obtain if they all choose output instead. Specifically, we decribe the relationship between the constant conjectural variations in quantity and in price that yield identical equilibria. This is done because some oligopoly models are posed with quantity as the decision variable while other employ price. For certain conjectural variations, Levitan and Shubik explored various solutions that result in differentiated markets, depending on which is the choice variable. However, the general relationship between the conjectural variations in the respective models has not been shown previously. This has resulted in some apparent incomparability of the outcomes from the alternative formulations in the literature. Consistency of conjectural variations is also discussed. It is shown that study of the homogeneous good case does not provide very good guidance for when to expect constant consistant conjectural variations to exist in the differentiated good case. It is also shown that if demand is linear, then the constant consistent conjectural variation in prices leads to the same equilibrium in price and quantity as does the constant consistent conjectural variation in quantities. This result need not hold for other demand functions.

Homogeneous Product--Firm and Industry Output

Let the industry inverse demand function be $p(Q)$, where $Q = \Sigma_{i=1}^{n} q_i$ is industry output and q_i is the output of the i^{th} firm in the n firm industry. The demand function is twice continuously differentiable, downward sloping, and has a downward sloping associated marginal revenue function. The unit cost of production is c, constant. The profit function of firm i, to be maximized by choice of q_i, is

(1) $V(q_i) = p(Q)q_i - cq_i$

Before discussing the profit maximization, we take up conjectural variations and their properties. Let

(2) $w_i = \partial Q/\partial q_i = 1 + \sum_{\substack{j=1 \\ j \neq i}}^{n} \partial q_j/\partial q_i \qquad i = 1,\ldots,n$

be firm i's belief of the rate at which industry output will change with increase in its own production. The term $\partial q_j/\partial q_i$ is the conjectural variation, the rate of change in firm j's output anticipated by firm i in response to its own change. The pertinent information about the conjectural variations of firm i is summarized in w_i. We assume that an increase in one's own output is expected to raise industry output. Further, other firms are expected to expand their own output <u>at most</u> at the same rate as does firm i in response to i's increase. That is, each w_i satisfies $0 < w_i \leq n$. The conjectural variations may, but need not, be constants; they may depend on industry members' output.

Define W by

(3) $1/W = \sum_{i=1}^{n} (1/w_i)$

W is $1/n^{th}$ of the harmonic mean of the w_i's and will be called the harmonic sum. Since $0 < w_i \leq n$, we have $\Sigma_{i=i}^{n}(1/w_i) \geq n/n$ so that $0 < W \leq 1$. The harmonic mean (sum) is always smaller than the arithmetic mean (sum) unless the components are identical. Thus for a fixed arithmetic sum, the harmonic mean is largest (and equals the arithmetic mean) when the firms hold identical beliefs: $w_i = w$ for all i. The harmonic sum W tends to zero if <u>any single</u> component w_i tends to zero. And the harmonic sum $W = 1$ only if <u>all</u> components $w_i = n$.

An optimal positive finite output q_i (that we assume to exist) satisfies

(4) $V'(q_i) = q_i w_i p'(Q) + p(Q) - c = 0$

for $i = 1,\ldots,n$. Since (4) holds for all firms, the product $q_i w_i$ must be the same for each firm. Hence $q_i = w_1 q_1/w_i$ so that

$$Q = \sum_{i=1}^{n} q_i = w_1 q_1 \sum_{i=1}^{n} (1/w_i) = w_1 q_1/W .$$

Therefore

(5) $q_i/Q = W/w_i \qquad\qquad i=1,\ldots,n.$

Equation (5) tells us that the market share of each firm depends only on the beliefs of all the firms. Specifically, a firm's share of total output equals its share in the harmonic sum of beliefs. Differing beliefs generate differing market shares. A firm's market share varies inversely with its own conjectural variation and directly with the conjectural variation of its rivals.

From (5), it follows that $w_i q_i = WQ$ so that (4) can be written as

(6) $QWp'(Q) + p(Q) - c = 0.$

Industry output depends on the harmonic sum W as well as on the unit cost and the demand function. It does not depend on the number of firms as such. Of course n affects the harmonic sum W . It is clear from (6) that the entire impact of market structure and of market conduct (conjectural variations) upon market performance--industry output--is captured in the harmonic sum W.

Dansby and Willig have introduced an industry performance gradient index defined by

$$(7) \quad \phi = [\sum_{1}^{n} ((p_i - c_i'(q_i))/p_i)^2]^{1/2}.$$

In view of (6), this index becomes

$$(8) \quad \phi = n^{1/2} W/\varepsilon$$

where

$$(9) \quad \varepsilon = -p/p'(Q)Q$$

is the elasticity of industry demand. While the industry output does not depend on the number of firms as such, recall (6), the Dansby-Willig performance index (8) does.

Other indices of market structure include the Lerner index

$$L = (p-c)/p = W/\varepsilon$$

and the Herfindahl index

$$H = \sum_{i=1}^{n} (q_i/Q)^2 = W^2 \sum_{i=1}^{n} (1/w_i^2) .$$

Both depend on the conjectural variations. The former depends on the demand function while the latter does not. Note that if all firms have identical beliefs, $w_i = w$, then $H = 1/n$, independent of that belief.

The conjectural variations of each firm can be inferred from market data. From (5), $w_i = W(q_i/Q)$. The w_i are proportional to market shares in equilibrium, where the constant of proportionality W is found from (7) to be

$$W = \varepsilon(p-c)/p.$$

Essentially this observation was employed by Iwata in his empirical estimation.

The results to this point hold whether W is a constant or depends on the outputs of the industry members. If W is constant, then we may ask how the industry output varies with W. Differentiating (6) implicitly, we find that

$$(10) \quad \partial Q/\partial W = -QP'(Q)/[WQP''+(1+W)P'].$$

To sign the denominator in the right side of (10), note that

$$WQP''+(1+W)P' = W[QP''+(1+W)P'/W] \leq W(QP''+2P') < 0.$$

The first inequality holds since $W \leq 1$ implies that $(1+W)/W \geq 2$ and the second inequality holds since the marginal revenue function is downward sloping. Therefore, $\partial Q/\partial W < 0$; industry output varies inversely with the harmonic sum W. We noted above that for any given arithmetic sum of conjectural variations, the harmonic sum W will be largest when the firms hold identical beliefs. Thus the more homogeneous the firms' beliefs, ceteris paribus, the smaller the industry output.

Industry output Q is maximized if W=0; that occurs if $w_i=0$ for any i. Thus if any firm believes that its output will have no impact on industry output, then the industry produces the competitive output: $p(Q) = c$. (Review (6)). This result was noted by Fama and Laffer. The competitive assumption is usually stated as the firm believes it will have no impact on industry price, but so long as the industry demand is downward sloping, this is equivalent to the belief that it will have no impact on industry output.

At the other extreme, Q is minimized if W=1, which occurs if $w_i=n$ for all $i=1,\ldots,n$. The monopolistic output results if <u>each</u> firm expects its output changes to be matched by each firm in the industry.

The standard Cournot output results if W=1/n; one of many ways for this to happen is that $w_i=1$ for $i=1,\ldots,n$ so each firm expects others do not respond to its output changes.

Thus the oligopoly can produce the monopoly output, the competitive output, or any intermediate output, depending on the conjectural variations. The more responsive the industry is thought to be to one's own action and the more similar are the firms' beliefs about the others' responsiveness, the smaller the industry output and the larger the industry profit will be.

Homogeneous Product — Consistent Conjectural Variations

The Cournot assumption of zero conjectural variation is naive and experience usually shows it to be inappropriate. To illustrate, consider a simple two firm example with a linear industry demand $p = A-B(q_1+q_2)$. If firm 1 selects its output to maximize its own profit, taking q_2 as given, it chooses $q_1 = (A-c)/2B - q_2/2$. This implies that a change in firm 2's output will lead to a change in firm 1's output in the opposite direction and half as large; i.e. the slope of firm 1's reaction curve is $dq_1/dq_2 = -1/2$. Thus with a linear demand curve, a firm that makes the Cournot assumption will always respond to its rival's change in output and so will not itself satisfy the Cournot assumption.

Can some other conjectural variation lead to consistent expectations in symmetric equilibrium if demand is linear? If firm 1 of the previous paragraph believed that its rival's output would change with its own at rate $dq_2/dq_1 = k_1$, then it would select its profit maximizing output to be $q_1 = (A-c-Bq_2)/(2+k_1)B$. This means firm 1's output changes with firm 2's output at rate $dq_1/dq_2 = - 1/(2+k_1)$. At a symmetric equilibrium with consistent expectations, the actual rate of change of firm 1's output with firm 2's output should equal the rate conjectured by firm 2 so $-1/(2+k_1) = k_2$. With $k_1 = k_2 = k$ by the assumed symmetry, this condition can be satisfied by $k = -1$ only. Only if both firms act as though they are in a perfectly competitive situation will their expectations about each other be fulfilled in symmetric equilibrium!

The discussion above rests on the supposition that the duopolists face linear demand. If the firms are identical, is there any family of demand functions and any uniformly held conjectural variation for which expectations will be fulfilled in symmetric equilibrium?

Before answering this query, a few more words on the notion of consistent conjectural variations are appropriate. For a duopoly such as in the example above, a consistent conjectural variation is such that the actual profit maximizing rate of change of firm 1's output when firm 2's output changes--i.e. the slope of firm 1's reaction function--equals the rate conjectured by firm 2. The actual slope of firm 1's reaction function is the slope that firm 2 believes it to be.

For an oligopoly, the requirement of consistency is again that the actual rate of change of firm i's output with firm n's output equals the rate conjectured by firm n. However, it is not simply the slope of the reaction function, a partial derivative holding fixed the output of the other n-2 firms. Rather, as Holt pointed out, in responding to firm n's output change, firm i should take into account not only the instigating change in firm n's output but also the profit-maximizing adjustments of their n-2 rivals. Thus the rate of change of each firm's output in response to a change in firm n's output is profit-maximizing for each, given the exogenous change in firm n's output and given the simultaneous adjustments of all n-2 rivals. This common consistent rate of output change of any firm in response to a change in another's output is denoted k. To relate the present notation with that of the previous section, we observe that $w = 1+(n-1)k$. The rate at which industry output is expected to change with one's own is one's own output (rate 1) plus the contribution of each of the n-1 rivals at rate k (the conjectural variation). Since $0 < w \leq n$, we have $-1/(n-1) < k \leq 1$.

We will address in three parts the question of the existence of families of demand functions p(Q) and corresponding conjectural variations k for which the conjectural variations will be consistent in equilibrium. We will first discuss the results under the restriction that the conjectural variations be constants, independent of output. This is the hypothesis most frequently maintained in the emerging literature on consistent conjectural variations. Second we will discuss the results under the more general hypothesis that the conjectural variations depend on industry output. Third, the verification of the results will be provided.

In the limiting case that $k = -1/(n-1)$ (so $w = 0$), each firm believes that any output change it initiates will be exactly offset by its rivals so industry output will be unchanged. It will be shown that in this case, the conjectural variation is consistent for any demand function. If each firm believes it will have no impact on industry output, it will have none, and the industry will produce the competitive output.

We will also show that if the conjectural variations are constant and larger than $-1/(n-1)$ and are to lead to consistent expectations in an n-firm symmetric equilibrium, then at the equilibrium the demand function must be

(11) $p = A + BQ^{1-r}$

where

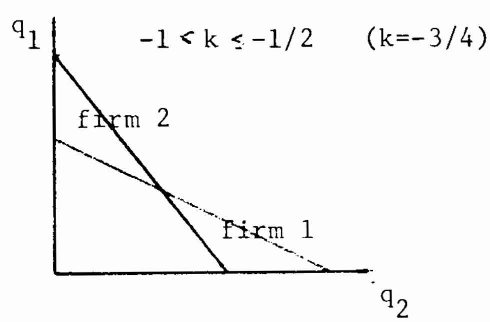
(12) $r = (1+k)n/(1+(n-1)k), \quad -1/(n-1) < k < 1, \quad A < c, \quad B > 0.$

Note that the demand function depends on the conjectural variation $\partial q_i/\partial q_j = k$ parametrically. The parameter $r \geq 2$ so the quantity demanded varies inversely with price. The bounds on k correspond exactly to the requirement that $0 < w < n$ (since $w = 1+(n-1)k$). The bound on the parameter A assures positive output at equilibrium. Note that this result is local. It relates the consistent conjectural variation and the demand function for which expectations will be satisfied at equilibrium. No restriction is placed on the demand function or conjectural variation away from the equilibrium.

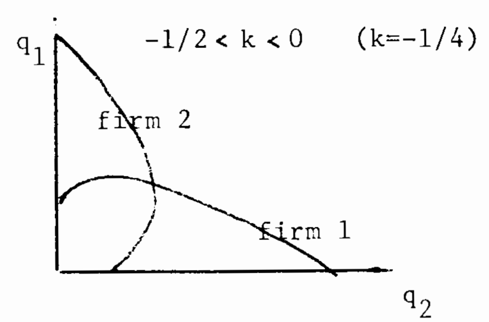
If $n = 2$, then $r = 2$, independent of k for duopoly. Thus, for a duopoly facing a unitary elastic demand $p(Q)=B/Q$, any conjectural variation less than one can be consistent in equilibrium. Firm 1's reaction function near equilibrium is

$$q_1 = [-(Bk+2cq_2) + (B^2k^2+4Bc(1+k)q_2)^{1/2}]/2c$$

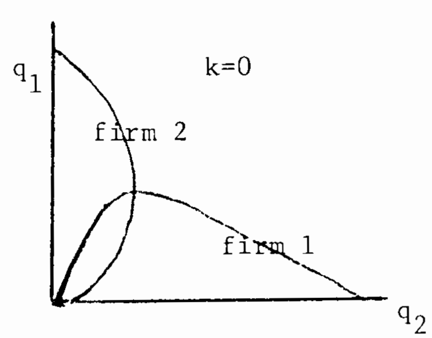
and firm 2's reaction function is analogous. These reaction functions are sketched for various values of k (with B=1, c=1/2). Note that our results are for the equilibrium only so no restriction is placed on the functional form away from the equilibrium.



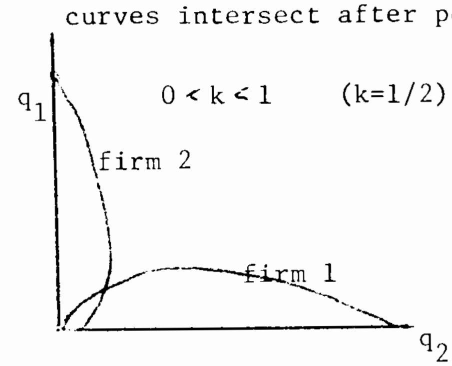
curves downward sloping



curves intersect after peak



curves intersect at peak



curves intersect before peak

If the conjectural variations are zero, then $r = n$. This--demand function (11) with r replaced by n--is the only case that the Cournot assumption will be appropriate in symmetric equilibrium.

If $A=0$, demand function (11) has constant price elasticity equal to $1/(r-1) \leq 1$. Since r is a decreasing function of k for $n > 2$ and it is an increasing function of n, it follows that the isoelastic demand function for which constant conjectural variations are consistant is less inelastic if the conjectural variation is larger or the number of rivals is smaller. As we have noted before, for duopoly it has unitary elasticity regardless of k.

The industry equilibrium that will be attained if the demand function is (11) and if firms have the corresponding constant consistent conjectural variations is

$$p = A + n(c-A)/(1-k), \qquad Q = [B(1-k)/n(c-A)]^{1/(r-1)}.$$

The second order condition for a profit maximum is always satisfied under the stated conditions and profit is positive.

Next we discuss the consequences for consistency of supposing that the conjectural variation need not be constant. Suppose now that k depends on industry output Q: $k = k(Q)$. Then, as we will show, if the conjectural variation is to be consistent in symmetric equilibrium, the industry demand function $p(Q)$ must have slope

$$\text{(13)} \quad p'(Q) = -(a/(1+(n-1)k))e^{-\int [n(1+k)/(1+(n-1)k)Q]dQ}, \quad a > 0.$$

This equation shows the slope of the demand function corresponding to any conjectural variation function $k(Q)$ that may be specified, such that the conjectural variation will be consistent in equilibrium. That is, if the conjectural variation is $k(Q)$ and if the industry demand function has the slope specified in (13), depending on that $k(Q)$, then the expectations of a firm regarding the others' profit maximizing responses to changes in its own output will be fulfilled in equilibrium. It is evident from (13) that a range of demand functions can be generated by specifying a range of conjectural variation functions.

Because of the complexity of (13), we will give some examples for duopoly only. With n=2, (13) reduces to

$$p' = -a/Q^2(1+k(Q)), \ a > 0.$$

Examples of consistent pairs of demand functions and conjectural variation functions for duopoly are therefore as follows:

$p(Q)$	$1+k(Q) = -a/p'(Q)Q^2$
$A + B/Q^2$	$aQ/2B$
$A + B/Q$	a/B
$A - B \ln Q$	a/BQ
$A - BQ$	a/BQ^2

All four demand functions are downward sloping. But k(Q) may be an increasing function (first example), constant (second example), or decreasing (third and fourth examples.)

To validate our claims, we use the technique proposed by Holt to determine the symmetric consistent conjectural variation. Firm i will choose its profit maximizing output q_i to satisfy

$$(14) \quad q_i(1+(n-1)k(Q))p'(Q) + p(Q) - c = 0.$$

This is (4) with $w = 1+(n-1)k$. The total differential of this first order condition is

$$(15) \quad (1+(n-1)k(Q))p'(Q)dq_i + (n-1)q_i p'k'\sum_1^n dq_j +$$

$$+ q_i(1+(n-1)k(Q))p''\sum_1^n dq_j + p'\sum_1^n dq_j = 0.$$

If firm n were to exogenously change its output from the equilibrium quantity by dq_n, the first n-1 firms would adjust their output to maximize their own profits, i.e. maintain their respective first order conditions (14) or, equivalently, (15). Divide (15) through by dq_n and denote the other n-1 firms' responses by $k(Q) = dq_j/dq_n$, $j=1,\ldots,n-1$. Then (15) gives

$$[1+(n-1)k(Q)][p'(1+k+(n-1)q_i k') + (1+(n-1)k)q_i p''] = 0.$$

Write $q_i = Q/n$ (by symmetry). Then this equation implies that

$$(16) \quad p'(1+k+k'(n-1)Q/n) + (1+(n-1)k)Qp''/n = 0.$$

Equation (16) is a differential equation for the demand function $p(Q)$; it depends on the conjectural variation function $k(Q)$. To solve, multiply through by $n/(1+(n-1)k)Qp'$ to get

$$n(1+k)/(1+(n-1)k)Q + (n-1)k'/(1+(n-1)k) + p''/p' = 0,$$

or equivalently, after noting that $f'/f = d \ln f/dx$ for any function $f(x)$ and separating variables,

$$[n(1+k)/(1+(n-1)k)Q]dQ + d \ln (1+(n-1)k) + d \ln (-p') = 0.$$

Integrating gives

$$\int (n(1+k)/(1+(n-1)k)Q)dQ + \ln (1+(n-1)k) + \ln (-p') = \ln a, \text{ where } a > 0.$$

Rearranging the results gives (13).

In the limiting case that $k = -1/(n-1)$, the first order condition (14) becomes $p - c = 0$. Its total differential is $(1+k(n-1))p' = 0$, which is satisfied for any demand function in this case. Therefore our assertion that $k = -1/(n-1)$ is always consistent is verified.

Now that we have derived (13) for the other cases, we can use it to obtain the remaining conclusions. If k is a constant, then (13) simplifies to

$$p'(Q) = -\,(a/(1+(n-1)k))e^{-r \ln Q} = -\,(a/(1+(n-1)k))Q^{-r}$$

where $r = (1+k)n/(1+(n-1)k)$. Separating variables and integrating produces $p(Q) = A + BQ^{1-r}$, as claimed (where $B = a/(1+(n-1)k)(r-1) > 0$). Finally, if k is not constant, then we have (13).

Differentiated Product

Suppose there are n symmetric firms, each selling a single good that is an imperfect substitute for the goods of the n-1 other firms. For instance, one may think of the products differing in color or flavor. The price each firm receives for its product can be written as a function of the quantity made available by each firm. For the first firm

$$(17) \quad p_1 = P(q_1, q_2, \ldots, q_n).$$

The price received by the i^{th} firm, $i=2,3,\ldots,n$ is written similarly, except that the positions of q_1 and q_i are interchanged on the right. Letting subscript j indicate the partial derivative with respect to the j^{th} argument, we assume the functions

$$(18) \quad P_j = P_2 \quad \text{for} \quad j = 2,\ldots,n$$

(symmetry) and

$$(19) \quad P_1 < P_2 < 0.$$

An increase in the quantity supplied by any firm will depress the price received by firm 1. Further, an increase in his own output will depress his price more than will an equal increase in the output of a rival. (If the goods were perfect substitutes, in the limit, the impacts would be equal.)

As examples of the general model, one may think of the linear form

$$(20) \quad p_i = A - Bq_i - D\sum_{j=1}^{n} q_j$$

or the isoelastic (loglinear) form

(21) $p_i = Aq_i^{-B}(q_1 \cdots q_n)^{-D}$,

where $B > 0$, $D > 0$. For (20), $P_1 = -(B+D)$ and $P_2 = -D$; for (21), $P_1 = -(B+D)p_i/q_i$ and $P_2 = -Dp_i/q_j$. It is readily checked that (18) and (19) are satisfied in each case in symmetric equilibrium.

Each firm has a constant unit cost of c. Firm 1 chooses his output q_1 to maximize his profit:

$$\max_{q_1} \; [P(q_1,q_2,\ldots,q_n)-c]q_1$$

A positive finite symmetric solution in which all n firms behave identically satisfies

(22) $P - c + (P_1+(n-1)P_2k)q_1 = 0$

where $k = dq_j/dq_1$, $j = 2,\ldots,n$ is firm 1's conjectural variation in quantities, the assumed rate at which each rival will adjust output in response to its own output change. The second order condition is assumed to be satisfied.

In symmetric equilibrium, each firm behaves identically, $q_j = q$ for $j=1,\ldots,n$ and (22) holds for each firm, i.e. with q_1 replaced by q in (22) and with $k = dq_j/dq_i$ for all $i \neq j$. The same price is received by each firm.

Alternatively, the firms may perceive the problem as one of choosing prices. The system of n price equations represented by (17) can be inverted to express the quantities that can be sold by each firm as a function of the prices charged by each. For firm 1, the demand function is

(23) $q_1 = Q(p_1,p_2,\ldots,p_n)$.

For example, corresponding to (20) is

$$q_1 = [A-p_1)B - \sum_{j=2}^{n} (p_1-p_j)D]/B(B+nD)$$

while corresponding to (21) is

$$q_1 = [\frac{A}{p_1} \prod_{2}^{n} (\frac{p_j}{p_1})^{D/B}]^{1/(B+nD)}$$

The demand functions for the other firms are similar; in the demand function for firm i, p_1 and p_i are interchanged on the right. The properties of demand functions (23) are obtained using the price equations from which they have been derived, together with the implicit function theorem. To determine the partial derivatives of (23), differentiate totally the n equations represented by (17), yielding in symmetric equilibrium ($q_j=q$, $j=1,\ldots,n$) the system

$$(24)\quad dp_j = P_1 dq_j + \sum_{\substack{i=1 \\ i\neq j}}^{n} P_2 dq_i , \qquad j = 1,\ldots,n.$$

Solving this system by Cramer's rule (see appendix) gives

$$(25)\quad \partial q_i/\partial p_i = (P_1+(n-2)P_2)/(P_1+(n-1)P_2)(P_1-P_2) < 0, \quad i=1,\ldots,n$$

$$(26)\quad \partial q_i/\partial p_j = -P_2/(P_1+(n-1)P_2)(P_1-P_2) > 0, \quad i,j=1,\ldots,n;\ i\neq j$$

where the signs follow from (19). As expected, the quantity demanded from firm i will increase if either firm i reduces its price or any rival raises price.

If firm 1 perceives the problem as one of selecting price, it chooses p_1 to maximize its profit

$$\max_{p_1} \ [(p_1-c)Q(p_1,p_2,\ldots,p_n)]$$

The first order condition satisfied by a positive finite price in symmetric equilibrium is

$$(27)\quad Q + (p_1-c)[Q_1 + (n-1)Q_2 m] = 0$$

where Q_1 and Q_2 denote the partial derivatives of Q with respect to its first and second arguments respectively (and $Q_2 = Q_j$, $j = 2,\ldots,n$ by the assumption of symmetric equilibrium) and where

(28) $m = \partial p_j/\partial p_1$, $\qquad j = 2,\ldots,n$

is firm 1's conjectural variation in prices, the rate at which it believes any rival will adjust price in response to a change in its own. By symmetry, equations (27)-(28) hold for each firm; p_1 may be replaced by p in (27) and by p_i with $i=1,\ldots,n$; $i\neq j$, in (28).

If the conjectural variation in quantities is zero, so each firm believes that its rivals will not change their quantities sold in response to an increase in its own sales, it then implicitly assumes that the rivals will lower price to maintain sales in the face of its own price reduction (that must accompany a planned increase in sales). Any conjectural variation in quantities or in prices implicitly implies a corresponding equivalent conjectural variation in prices or in quantities respectively that yields the same symmetric equilibrium price and quantity. To see more generally how the conjectural variations in prices and in quantities are related, we observe that (22) and (27) can give identical price-output prescriptions only if

(29) $-(p-c)/q = P_1+(n-1)kP_2 = 1/[Q_1+(n-1)mQ_2]$.

This was obtained by arranging both (22) and (27) to give expressions for $-(p-c)/q$ and equating the expressions.

Using (25)-(26) as expressions for Q_1 and Q_2 respectively,

(30) $Q_1+(n-1)mQ_2 = [P_1+(n-2)P_2-(n-1)mP_2]/[P_1+(n-1)P_2](P_1-P_2)$.

Substitute (30) into (29) and collect terms. Rearranging the result gives

(31) $1-m = (1-k)(1-Z)/[1+(n-1)kZ]$

where

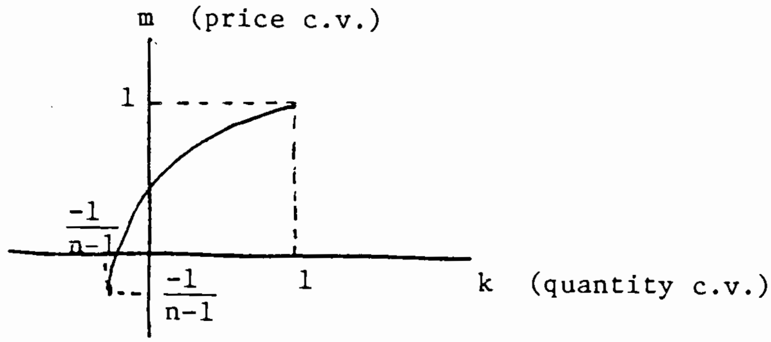
(32) $Z = P_2/P_1$, $\qquad 0 < Z < 1$

reflects the extent of perceived product differentiation. The bounds on Z follow from (19). The limiting case of Z=0 indicates independent products while the limiting case of Z=1 indicates perfect substitutes. For both the linear (20) and loglinear (21) forms of inverse demand function, Z = D/(B+D), a constant.

Equation (31) is our objective, relating the conjectural variation in prices m to the equivalent conjectural variation in quantities k. In our simple model, that relationship depends parametrically on the number of rivals and on the extent of perceived product differentiation. In the limit, if k=1, then m=1; if the firms expect their changes in output to be followed exactly, then they implicitly expect their price changes to be followed exactly, and vice versa. At the other extreme, if $k = -1/(n-1)$, then $m = -1/(n-1)$. If each firm believes that its increase in quantity will be just offset by reductions in quantity by each of its rivals so industry output is unchanged, then it implicitly expects that its price increase will be offset by price reductions by each of its rivals to maintain industry sales.

We can now check our assertions about the meaning of zero conjectural variations. If k=0, then m=Z. If the firm expects rivals to keep their quantities fixed despite an increase in its own output, it is implicitly expecting others to change their price in the same direction as it changes its own price, but by a lesser amount (since $0 < Z < 1$). On the other hand, if $m = 0$, then $k = -Z/(1+(n-2)Z) < 0$. To understand this, suppose we raise our price and thereby reduce sales. If rivals are expected to maintain their prices unchanged ($m = 0$), then they must increase their output (to accommodate the customers we lose by raising our price). Thus their output is expected to move in the opposite direction that ours does. That is, the conjectural variation in quantities is negative if the conjectural variation in prices is zero.

Having discussed (31) at several interesting points, we next look at the relationship between m and k, for given values of n and Z. It is an increasing, concave relationship from $(-1/(n-1),-1/(n-1))$ to $(1,1)$.



Thus an increase in one conjectural variation implies an increase in the other. Nonetheless, the conjectural variations need not always have the same sign, as we saw earlier. There is an interval in which a positive conjectural variation in prices implies a negative conjectural variation in quantities.

Comparative statics analysis applied to the first order condition (22) or (27) indicates, as expected, that an increase in the conjectural variation (either k or m) will lead to a reduction in the equilibrium quantity, an increase in the equilibrium price, and an increase in profits. The maximum profit is achieved by perfect coordination: $m = k = 1$.

In our earlier study of homogeneous goods, we saw that constant conjectural variations could be consistent only if the demand function were in a certain class. For differentiated goods, we will show that constant conjectural variations can be consistent in different circumstances. Consistant constant conjectural variations in quantities may, but need not, correspond to consistent constant conjectural variations in prices. To support this assertion, we show that in an oligopoly facing linear demand, consistency in price conjectures corresponds exactly to consistency in quantity conjectures. We will also show that in a duopoly facing loglinear demand, there may be consistent constant conjectural variations in quantities but none in prices. In this case, consistent conjectural variations may not be constant.

In order to perform our demonstration, it will be useful to develop the conditions satisfied by constant consistent conjectural variations and the demand function at symmetric equilibrium, if one exists. The total differential of the first order condition (22) is

$$[2P_1 + (n-1)P_2k + q_1(P_{11}+(n-1)P_{21}k)]dq_1 +$$
$$[P_2 + q_1(P_{12}+(n-1)P_{22}k)] \Sigma_2^n dq_j = 0.$$

Divide by dq_n, write $k = dq_j/dq_n$, $j \neq n$, and collect terms to get in symmetric equilibrium

$$(33) \quad (n-1)k^2[P_2 + q(P_{21}+(n-2)P_{22})]$$
$$+ k[2P_1 +(n-2)P_2 + q(P_{11} +(n-2)P_{12} + (n-1)P_{22})] + P_2 + qP_{12} = 0.$$

This equation relates the constant consistent conjectural variation in quantities with the characteristics (derivatives) of the demand function at symmetric equilibrium, if any. In symmetric equilibrium, the common output q and the consistent constant conjectural variation are simultaneously determined by the first order condition (22) and this consistency condition (33), provided that such solutions q and k exist. For economic sense, we require $q > 0$ and $-1/(n-1) < k \leq 1$ as well.

Similarly, the total differential of the first order condition (27) is

$$[2Q_1 + (n-1)Q_2m + (p_1-c)(Q_{11}+(n-1)mQ_{21})]dp_1$$
$$+ [Q_2 + (p_1-c)(Q_{12}+(n-1)mQ_{22})] \Sigma_2^n dp_j = 0.$$

Divide by dp_n, write $m = dp_j/dp_n$, $j \neq n$, and collect terms to get in symmetric equilibrium

$$(34)\quad m^2(n-1)[Q_2 + (p-c)(Q_{21}+(n-2)Q_{22})]$$
$$+ m[2Q_1 + (n-2)Q_2 + (p-c)(Q_{11}+(n-1)Q_{22} + (n-2)Q_{12})]$$
$$+ Q_2 + (p-c)Q_{12} = 0.$$

Equation (34) relates the constant consistent conjectural variation in prices with the derivatives of the demand function at the symmetric equilibrium, if such a consistent equilibrium exists. In symmetric equilibrium, the common price p and the consistent constant conjectural variation in prices m are simultaneously determined by the first order condition (27) and the consistency condition (34), if such p and m exist. For sense, we need $p > 0$ and $-1/(n-1) < m < 1$.

If industry demand is linear (20), then (33) specializes to a quadratic equation in k

$$(35)\quad (n-1)Dk^2 + (2B+nD)k + D = 0$$

with real roots $- Y \pm [Y^2 - 1/(n-1)]^{1/2}$ where

$$(36)\quad Y = (2B+nD)/2(n-1)D.$$

The smaller root is less than $-1/(n-1)$ and so is outside the relevant range. The larger root

$$(37)\quad k = - Y + (Y^2 - 1/(n-1))^{1/2}$$

lies between $- 1/(n-1)$ and zero; it is the relevant one. Since it is negative, a constant consistent conjectural variation with linear demand will be such that rivals expect their output changes to be partially but not wholly offset by rivals. This consistent conjectural variation will be discussed further after the consistent conjectural variation in prices is developed.

Substituting the demand function (20) and the value of k from (37) into the first order condition (22) yields the symmetric equilibrium output per firm at which the constant conjectural variations in quantities are consistent

$$(38)\quad q = (A-c)/[B + (n+2)D/2 + (B^2+nBD+(n-2)^2D^2/2)^{1/2}].$$

If firms face linear demand (20) but consider price as the decision variable, then (34) must be satisfied by a constant consistent conjectural variation in prices. For the linear demand, $Q_1 = -(B+(n-1)D)/B(B+nD)$ while $Q_2 = D/B(B+nD)$ so (34) specializes to a quadratic equation in m

$$(n-1)Dm^2 - (2B+nD)m + D = 0$$

with roots $Y \pm [Y^2 - 1/(n-1)]^{1/2}$ where Y is as defined in (36). The larger root exceeds one and so is outside the economic range. The smaller root

$$(39)\quad m = Y - [Y^2 - 1/(n-1)]^{1/2}$$

is the relevant one. It is positive and less than one. With linear demand, in an equilibrium with consistent conjectural variation in prices, firms expect that price changes will be partially followed by rivals.

It can be verified by substitution and lengthy algebra that (37) and (39) satisfy (31) when $Z = D/(B+D)$. This means that the consistent conjectural variations in quantities and the consistent conjectural variation in prices lead to the same equilibrium output (38) and hence the same equilibrium market price as well.

It is worth noting a few points about the consistent conjectural variation and the resulting equilibrium for this example. First, the reaction functions are linear and therefore have constant slope. This suggests that we might not have been surprised to find that there is a consistent constant conjectural variation. On the other hand, for linear demand and a homogeneous good, we found that the only constant consistent conjectural variation is -1. Second, comparing (37) and (39) indicates that $0 < m = -k < 1$; the consistent constant conjectural variations are equal in magnitude and opposite in sign. In the earlier sketch, the consistent conjectural variations are at the intersection of the curve with the 135^o line from the origin. A price increase will be followed, but less than fully. A quantity increase will be partially offset by the rival.

Third, the absolute value of the consistant constant conjectural variations is decreasing with B and increasing with D. The larger is D, i.e. the better substitutes the goods are, the greater is the responsiveness expected of the rival. The less differentiated the goods in the minds of the consumer, the more a price change will be followed and the more a quantity change will be offset. In the limit, as D approaches zero, reflecting nearly independent products, the consistent conjectural variations approach zero as well. If the products are perceived to be nearly unrelated by consumers, the rivals consistently expect no response from the rival to any change in their decision variable. The equilibrium approaches the monopoly price and quantity.

The simplicity and elegance of constant consistent conjectural variations in the case of linear demand need not carry over for other demand functions. To illustrate, we consider loglinear demand.

Suppose duopoly faces demand (21) with $B+D < 1$ so the inverse demand function for firm 1 is

(40) $p_1 = Aq_1^{-(B+D)}q_2^{-D}$ where $B + D < 1$.

Then (33) specializes to

$$k^2 + Rk + 1 = 0$$

where

$$R = [B-D^2-(B+D)^2]/D(1-B-D).$$

This has roots

$$k_1 = -(R + (R^2-4)^{1/2})/2, \qquad k_2 = (-R + (R^2-4)^{1/2})/2.$$

In order for a real k to exist, it is necessary that $R^2 > 4$. If $R \geq 2$, then the smaller root $k_1 < -1$, outside the relevant range. Only the larger root k_2 is relevant; k_2 is an increasing function of R and lies in the range $-1 \leq k_2 < 0$.

If $-2 < R < 2$, there is no real k since the argument of the square root is negative. If $R \leq -2$. the smaller root k_1 is increasing and lies in the range $0 < k_1 \leq 1$. The larger root k_2 exceeds unity and therefore is not relevant. Hence the magnitude of R and resulting consistent conjectural variations in quantities are related as follows:

parameters	R	k
$(1-B)/2 \leq D$	$R \leq -2$	$0 < k_1 \leq 1$
$B(1-B)/2 < D < (1-B)/2$	$-2 < R < 2$	none
$D \leq B(1-B)/2$	$R \geq 2$	$-1 \leq k_2 < 0$

Thus for a duopoly facing loglinear demand, there may or may not be consistent constant conjectural variations, depending on the relation of the parameters B and D. These conjectural variations, if they exist, may be positive or negative, again depending on the relation of these two parameters.

When we seek consistent constant conjectural variations in prices, we find that none exist. The first order condition and consistency condition for a duopoly facing loglinear demand give us

$$p = c/[1-(B+2D)^{1/2}], \qquad m = (B+D)/D - B(B+2D)^{1/2}/D$$

Since price must be positive, we have the restriction that $B+2D < 1$. But when $B+2D < 1$, we have $m > 1$ which is outside the economic range. Hence there is no constant consistent conjectural variation in prices in this case. Of course we know from (31) that there is a constant conjectural variation in prices that leads to the same equilibrium as does the consistent conjectural variation in quantities, but it will not itself be consistent. We may surmise (but have not verified) that corresponding to the constant consistent conjectural variation in quantities is a nonconstant consistent conjectural variation in prices.

Summary

We have discussed several facts about conjectural variations in static equilibrium. We showed that a harmonic sum of conjectural variations in a homogeneous market contains all the relevant information about market structure (numbers) and conduct (beliefs or coordination) for determining market performance (industry output). The role of similarity of beliefs in market performance was noted; the more homogeneneous the beliefs, the smaller the industry output. We also wrote several established structure and performance indices in our format using the harmonic sum of conjectural variations.

It was shown that even in symmetric equilibrium, firms' constant conjectural variations are unlikely to be correct since there is only a very narrow class of demand functions and associated interior conjectural variations for which this is possible. This class was derived. It was also shown that the limiting conjectural variation that leads to the competitive output can be consistent for any demand function. Of course, the conjectural variation need not be constant; it may for instance depend on industry output. In that case, conjectural variations can be consistent for a far broader range of demand functions. We developed and discussed the relationship between the functional form of the conjectural variations function and that of the demand function for which expectations can be consistent in equilibrium.

Finally, we have demonstrated the relationship between conjectural variations in prices and conjectural variations in quantities that are equivalent in the sense of yielding the same price and output within the context of a differentiated product market in symmetric equilibrium. Consistent conjectural variations were also examined. We found that if the demand is linear, then constant consistent conjectural variations in quantity correspond to constant consistent conjectural variations in price in the sense that the same equilibrium price and quantity will be attained if either conjectural variation is consistent. The consistent equilibrium is such that price changes will be partially followed and quantity changes will be partially offset, each to the same degree. For other demand functions, such a neat relationship need not hold. For instance, we showed that for the loglinear demand function, there is a constant conjectural variation in quantities but none in prices. There is a constant conjectural variation in prices that leads to the same equilibrium as does the consistent conjectural variation in quantity but it is not consistent. We surmise that there may be a corresponding nonconstant consistent conjectural variation in that instance.

Appendix

We will show how to solve the system of linear equations in (24) using Cramer's rule. To evaluate the determinant of an n x n matrix with a on the main diagonal and b elsewhere, subtract the first column from each of the other columns. Then add each of the last n-1 rows to the first row. Finally expand by the first row:

(A1)

$$\begin{vmatrix} a & b & b & \dots & b \\ b & a & b & \dots & b \\ & & & & \\ b & b & b & \dots & a \end{vmatrix} = \begin{vmatrix} a & b-a & b-a & \dots & b-a \\ b & a-b & 0 & \dots & 0 \\ & & & & \\ b & 0 & 0 & \dots & a-b \end{vmatrix} = \begin{vmatrix} a+(n-1)b & 0 & 0 & \dots & 0 \\ b & a-b & 0 & \dots & 0 \\ & & & & \\ b & 0 & 0 & \dots & a-b \end{vmatrix}$$

$$= [a+(n-1)b](a-b)^{n-1}.$$

If the first column is replaced with a vector with 1 in the first row and zero elsewhere, the determinant is readily evaluated by expanding by the first column and then applying the above result:

$$\text{(A2)} \qquad \begin{vmatrix} 1 & b & b & \dots & b \\ 0 & a & b & \dots & b \\ & \dots & & & \\ 0 & b & b & \dots & a \end{vmatrix} = [a+(n-2)b](a-b)^{n-2}$$

If the first column of the original matrix is replaced by a vector with 1 in the second row and zeros elsewhere, the determinant may be evaluated by expanding by the first column, subtracting the first column of the reduced determinant from each of the other columns, and then expanding by the first row:

(A3)

$$\begin{vmatrix} 0 & b & b & \cdots & b \\ 1 & a & b & \cdots & b \\ & & \cdots & & \\ 0 & b & b & \cdots & a \end{vmatrix} = -\begin{vmatrix} b & b & b & \cdots & b \\ b & a & b & \cdots & b \\ & & \cdots & & \\ b & b & b & \cdots & a \end{vmatrix} = -\begin{vmatrix} b & 0 & 0 & \cdots & 0 \\ b & a-b & 0 & \cdots & 0 \\ & & \cdots & & \\ b & 0 & 0 & \cdots & a-b \end{vmatrix}$$

$$(n \times n) \qquad ((n-1)x(n-1)) \qquad ((n-1)x(n-1))$$

$$= -b(a-b)^{n-2}.$$

Let $a = P_1$ and $b = P_2$. The determinant of the coefficient matrix on the right in (24) is given by (A1). Using Cramer's rule, (25) is the ratio of (A2) divided by (A1) and (26) is the ratio of (A3) divided by (A1).

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References

Anderson, F. J., "Market Performance and Conjectural Variation," Southern Economic Journal, Vol. 44, July 1977, pp. 173-178.

Bergstrom, Ted, "Cournot Equilibrium in Factor Markets," manuscript, University of Michigan Department of Economics Discussion Paper, June 1978.

Boyer, Marcel, "The Rational Strategic Duopoly Equilibrium," manuscript, University of Montreal, Oct. 1980.

Bresnaham, T. F., "Duopoly Models with Consistent Conjectures," manuscript, Stanford University, 1980.

Capozza, D. R. and R. Van Order, "Competitive Reactions in Spatial Monopolistic Competition," manuscript, University of British Columbia, 1980.

Chamberlin, E., The Theory of Monopolistic Competition, Cambridge: Harvard University Press, 1933.

Cournot, Augustin, The Mathematical Principles of the Theory of Wealth, Homewood, Ill.: Richard D. Irwin, 1963 (first published 1838).

Cowling, Keith and Michael Waterson, "Price-Cost Margins and Market Structure," Economica, Vol. 43, August 1976, pp. 267-274.

Cyert, R. and M. DeGroot, "Multiperiod Decision Models with Alternative Choice as a Solution to the Duopoly Problem," Quarterly Journal of Economics, Vol. 84, August 1971, pp. 410-429.

Dansby, Robert E. and Robert D. Willig, "Industry Performance Gradient Indexes," American Economic Review, Vol. 69, June 1979, pp. 249-260.

Dickson, V. A., "The Lerner Index and Measures of Concentration," Economics Letters, Vol. 3, 1979, pp. 275-279.

Dolbear, F. T., L. B. Lave, G. Bowman, A. Lieberman, E. C. Prescott, F. Rueter, and R. Sherman, "Collusion in Oligopoly: An Experiment on the Effects of Numbers and Information," Quarterly Journal of Economics, Vol. 82, May 1968, pp. 240-259.

Fama, Eugene F. and A. B. Laffer, "The Number of Firms and Competition," American Economic Review, Vol. 62, September 1972, pp. 670-674.

Farley, D. E., "Symmetric Oligopolistic Equilibria with Consistent Conjectural Variations," manuscript, Dept. of Economics, Princeton University, Nov. 1980.

Fisher, I., "Cournot and Mathematical Economics," Quarterly Journal of Economics, Vol. 12, Jan. 1898, pp. 119-132.

Friedman, J. W., Oligopoly and the Theory of Games, New York: North-Holland, 1977.

Frisch, Ragnar, "Monopoly--Polypole--La Notion de Force dans L'Economie," Nationalokonomisk Tidsskrift, Vol. LXXI, 1933, pp. 241-259.

Gollop, Frank M. and Mark J. Roberts, "Firm Interdependence in Oligopolistic Markets," Journal of Econometrics, Vol. 10, Aug. 1979, pp. 313-331.

Hathaway, J. and J. A. Rickard, "Equilibria of Price-Setting and Quantity-

Setting Duopolies," Economics Letters, Vol. 3, 1979, pp. 133-137.

Hause, John C., "The Measurement of Concentrated Industrial Structure and the Size Distribution of Firms," Annals of Economic and Social Measurement, Vol. 6, 1977, pp. 73-107.

Holt, C. A., Jr., "Equilibrium Models of Tacit Collusion in Oligopoly Experiments with Price Setting Firms," manuscript, Dept. of Economics D. P. 80-138, University of Minnesota, October 1980.

Iwata, Gyoichi, "Measurement of Conjectural Variations in Oligopoly," Econometrica, Vol. 42, September 1974, pp. 947-966.

Kamien, Morton I., Review of Industrial Market Structure and Economic Performance, by F. M. Scherer, in Journal of Economic Literature, Vol. XIII, June 1975, pp. 533-534.

Laitner, J., "'Rational' Duopoly Equilibria," Quarterly Journal of Economics, Vol. 95, December 1980, pp. 641-662.

Levitan, R. E. and M. Shubik, "Noncooperative Equilibria and Strategy Spaces in an Oligopolistic Market," in H. W. Kuhn and G. P. Szego, eds., Differential Games and Related Topics, Amsterdam: North-Holland Publishing Company, 1971, pp. 429-447.

Marschak, T. and R. Selten, "Restabilizing Responses, Inertia Supergames, and Oligopolistic Equilibria," Quarterly Journal of Economics, Vol. 92, Feb. 1978, pp. 71-94.

Perry, M. K., "Oligopoly and Consistent Conjectural Variations," manuscript, Bell Telephone Labs, 1980.

Reinganum, Jennifer F., Dynamic Games of R&D with Rivalry, Dissertation, Northwestern University, 1979.

Ruffin, R. T., "Cournot Oligopoly and Competitive Behaviour," Review of Economic Studies, Vol. 38, October 1971, pp. 493-502.

Shapiro, L., "Decentralized Dynamics in Duopoly with Pareto Optimal Outcomes," Bell Journal of Economics, Vol. 11, Autumn 1980, 730-744.

Smithies, A. and L. Savage, "A Dynamic Problem in Duopoly," Econometrica, Vol. 8, April 1940, pp. 130-143.

Sonnenschein, Hugo, "The Dual of Duopoly is Complementary Monopoly: or, Two of Cournot's Theories Are One," Journal of Political Economy, Vol. 76, March-April 1968, pp. 316-318.