Discussion Paper #464

A DIFFICULTY WITH THE "COMMAND" ALLOCATION MECHANISM

by

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March 1981

* Research support by the NSF under Grant SES 8014296 gratefully acknowledged.

1. Introduction

The allocation of intermediate products among producers by administrative means ("the command mechanism") is an essential characteristic of centrally planned economies of the Soviet-type. Though there has been much study of how the optimal allocation could or should be determined (the "theory of planning"), the usual assumption in the analysis of centrally planned economies is that whatever allocation is decided on can, and will, be administratively implemented. There has been some discussion of the incentive problem, i.e., inducing subunits to accept and implement this allocation, but even with regard to incentives the emphasis has been on the planning process itself, on the incentives to cooperate in the process of determining the optimal allocation. [1/] Very little attention has been given to the process of implementation, to the problems that might arise in trying to carry out a planned allocation. Of course there would be no problem of implementation in a world of complete information and perfect certainty: a feasible allocation could always be carried out exactly as planned. In the presence of any uncertainty, however, proper implementation can no longer be assumed to result from a feasible plan. The economic outcome of any feasible plan will depend not only on the behavior of the various subunits/agents but also on the nature of the implementation mechanism.

In this paper I will deal with the command mechanism which incorporates centralization of both planning and the administration of the plan. It substitutes "commands" and administrative authority for market interaction as a means of implementing desired allocations. [2/] Its functioning will be studied in a very specialized and highly simplified model of the centralized allocation of production inputs among a number of users, the basic structure of which is presented in Section 2. In each time period the central authorities have available a fixed (non-random) amount of each of several material inputs which they divide, according to plan, among the various producers under their immediate control. It is assumed that the plan is perfectly feasible and that the total amount of inputs available is precisely that needed to exactly fulfill the plan, so that the center need only assign the planned allocation to each producer. In the absence of uncertainty, each producer would then exactly fulfill his production plan though he has no control over the supplies he receives. Thus I implicitly assume the only constraints in the system to be the availability of material inputs and the given, known technology.

Uncertainty enters this sytem as a slight i.i.d. perturbation of the central materials allocation. That is, there is a slight "mistake" made in dividing each of the available inputs among the users, though there is nothing lost (or gained) to the system. The allocation is "on average" perfect i.e. exactly what is needed by each user, but its realization in each time period deviates slightly from the plan. In aggregate there is no uncertainty; there is only a slight perturbation of the allocation <u>within</u> the system. And for the individual producer the uncertainty is "neutral" i.e. "pure noise" or i.i.d. with expected value of zero.

How will this uncertainty affect the functioning of this production system? Clearly the producers can no longer be expected to precisely fulfill their production plan each period. Each producer will, at some times, face a shortage of needed inputs, causing production to be cut back, while at other times there will be general surpluses, allowing overfulfillment of ("catching up to") the production plan. This means each producer will have to hold inventories as a buffer against supply uncertainty, containing at least surpluses of inputs complementary to scarce materials. In view of the "neutrality" of the uncertainty faced and the fact that total material availability to the system is non-random and fully consistent with needs, we might expect that the inventories of both individual producers and in the system as a whole would be well behaved. That is, we might expect that, on average, the misallocations would cancel out. While some material stocks would be necessary, their distribution would be stable (invariant over time) and the production (output) plan would, on average, be fulfilled over a sufficiently long period. In addition we would expect aggregate inventories in the system to be bounded as the total amount of each material supplied to the system as a whole (i.e. to be allocated by the center) each period is finite, non-random, and precisely consistent with aggregate needs.

This situation is analysed in Section 3 of the paper. In spite of the very limited amount of uncertainty in this model we shall see that inventories in, and the output of, the production system behave quite perversely. Under the assumption of a Leontief technology (no input substitutability) inventories both within the individual production units and in aggregate behave essentially as a random walk with aggregate holdings of some materials growing without bound. This occurs in spite of the best efforts of the producers to hold down their own inventory levels. As a result expected total output falls further and further behind the planned level. The introduction of a mild amount of uncertainty renders this "command" production system unstable and increasingly inefficient.

In section 4 the cases of more than two inputs and more than two enterprises are discussed. Not surprisingly, the system becomes only more unstable when there are more things that can go wrong.

Section 5 concludes the paper with a discussion of the sources of this instability and relates these results to some recent developments in the Soviet economy. It is argued that we should intuitively expect production instability to result from the institutional framework of a "command mechanism" if there is any irreducible uncertainty to the outcomes of economic (in this case, allocation) decisions. The existence of an effectively administered plan and the allocation of authority inherent in central planning rigidify the system and render it incapable of properly responding to sufficiently diverse contingencies. As argued in Ericson (1979) a proper response seems to require a decentralization of the allocation decision to those who have immediate knowledge of the contingency, the enterprises. Without such decentralization, the enterprises are unable to counter the individual random shocks, which therefore add up, generating the random walk that characterizes production instability. Thus we should expect that recent reforms in the Soviet supply system (GOSSNAB), centralizing supply functions at a lower, regional level, and the creation of production associations (ob'edinenie) will not improve the functioning of supply in any significant sense, but will merely shift the locus of instability. This conclusion and the general relevance of these results are discussed at the end of the section.

A final question which is not directly addressed in this paper is how one might stabilize inventories, and hence production, in this model. Evidently a direct feedback must be generated between the actual inventory holdings within production units and the proportions in which available materials are allocated to those production units. Obviously an omniscient center could command the necessary reallocations. This can also easily be done as in Ericson (1979), and the same sorts of arguments with regard to "extra-systemic" (e.g. Second Economy or informal) allocation mechanisms could be presented. What would be more interesting, however, would be to model a market mechanism with flexible prices which generates the necessary stabilizing reallocations of inputs. This would seem to require introducing financial aspects, which significantly complicates the model. I do not yet have a workable model of such a market mechanism within the framework of intermediate product allocation.

2. The Simplest Model

In this section I consider the simplest possible allocation problem for a command economy. There are two users (enterprises) A and B, of two different material inputs, 1 and 2, producing the same type of output, y. The technology of production is represented by the input coefficient vectors $a, b \in R^2$. The enterprises are allocated inputs by a central (sectoral) authority and face production (output) plans, $\bar{y}^A, \bar{y}^B$, respectively. In order to fulfill the plan they require inputs

$$(1) \qquad \bar{x}^A = a\bar{y}^A, \ \bar{x}^B = b\bar{y}^B; \qquad \bar{x}^A, \bar{x}^B \in R^2_+; \qquad \bar{y}^A, \bar{y}^B \in R^1_+ .$$

The plan is assumed to be perfectly consistent and it is assumed that the center/sector receives precisely $\bar{x} = \bar{x}^A + \bar{x}^B$ to allocate to its two subordinate enterprises. Clearly if the sector can precisely allocate $\bar{x}^A$ to A and $\bar{x}^B$ to B both enterprises will precisely fulfill their plans, and hence the sector will fulfill its plan: $\bar{y} = \bar{y}^A + \bar{y}^B$. Thus in the absence of uncertainty no problems can arise.

I want to investigate, however, the consequences of introducing a slight mistake or perturbation into the allocation process, while leaving the environment (i.e. technology and the overall availability of inputs) perfectly deterministic. I assume that in dividing the available resources between the two users a slight error occurs, either at the center (i.e. in the decision), during preparation and loading for shipment, or during transportation itself. This error is purely allocative; materials are neither created or destroyed by the disturbance. Thus the supplies received by the enterprises in any period t are:

$$(2) \qquad x_t^A = a\bar{y}^A + \varepsilon_t \qquad x_t^B = b\bar{y}^B - \varepsilon_t$$

where $\varepsilon_t \in R^2$ and ε_{it}, $i = 1,2$, are i.i.d. with mean zero and variance σ^2. Clearly the enterprises can no longer precisely meet their plans in each period so that inventories, at least of that material complementary in production to the deficit input, will appear. These inventory holdings will be designated s_t^A, $s_t^B \in R^2$, at the end of period t.

With the existence of inventories I need to be more precise about the production behavior of enterprises and their desired inventory holdings. Each enterprise, i, faces a perfectly certain per period demand $\bar{y}^i$ and an input supply which is uncertain but <u>on average</u> precisely what is needed to meet demand. Further, it has no control over that supply (from the "command" mechanism) and can thus only alter inventory levels by changing its rate of output, y_t^i. This output in any period is constrained by received supplies and existing inventories:

$$(3) \qquad y_t^i \leq \min_j \,(c_j^i)^{-1}(s_{j,t-1}^i + x_{jt}^i) \qquad c_j^i = a_j \text{ or } b_j \text{ as } i = A \text{ or } B.$$

This output decision will result in end-of-period inventories

$$(4) \qquad s_t^i = s_{t-1}^i + x_t^i - c^i y_t^i \;\underline{\Delta}\; s_{t-1}^i - c^i z_t^i \pm \varepsilon_t \qquad i = A,\, B$$

where $z_t^i = y_t^i - \bar{y}^i$ and the sign before ε_t is positive for A and negative for B by convention. This yields inventory dynamics:

$$(5) \qquad \Delta s_t^A = - a z_t^A + \varepsilon_t \qquad \Delta s_t^B = - b z_t^B - \varepsilon_t \,.$$

Due to tautness of the plan (and the existing incentives in centrally planned economies to keep inventories as low as possible), I will assume that the enterprises try to minimize their holding of inventories by producing the maximum amount possible in each period. [3/] Thus desired inventories are $\bar{s}_t \equiv 0$ and the production decision becomes:

$$y_t^{*i} = \min_j (c_j^i)^{-1}(s_{j,t-1}^i + x_{jt}^i) \qquad i = A, B; \quad j = 1,2. \tag{6}$$

This implies that in every period t at least one inventory stock will be zero, that of the relatively deficit input. Only complementary stocks will be held in inventory, and only until they can be used in the proper combination with the deficit input. If we now substitute $z_t^{*i} = y_t^{*i} - \bar{y}^i$ into equation (5), the model is completely described formally by (1), (2), (5), and (6). The questions raised in Section 1 deal with the dynamic behavior of this model, to the analysis of which I now turn.

3. The Main Results

As argued in Section 1, because of the "neutrality" of the disturbance (i.e. i.i.d., zero mean), we might expect that over the long run the effects of these allocative disturbances would wash out: underdelivery implying reduced output, $y_t^i < \bar{y}^i$, would eventually be matched by overdelivery allowing "catch-up" production, $y_{t+t'}^i > \bar{y}^i$. This clearly requires that the stocks of any input be eventually matched by shipments of its complementary input so that inventory can be drawn down to zero in some finite time. Of course stocks of that, or the other, good may immediately build up again, but we would then know that in finite time they will again be drawn down to zero and production will again have caught up with the plan, i.e., there exists some T such that $\sum_{t=1}^{T} y_t^i = T \cdot \bar{y}^i$.

What has just been described is called <u>positive</u> <u>recurrent</u> or, somewhat loosely, <u>ergodic</u> behavior of the inventory stocks: with probability one their configuration will return <u>in finite time</u> to any open neighborhood of the desired inventory levels, $\bar{s} = 0$. 4/ It is a very weak stability condition which implies all misallocations eventually cancel out. It also implies that there exists an invariant distribution of inventory levels which can be used to determine the physical (e.g. warehouse size) and cost parameters of an optimal inventory policy. However even in the two input, two user case this weak form of qualitative stability, so necessary for smooth production, fails to hold.

<u>Proposition 1:</u> Inventory levels in both firms constitute a non-ergodic stochastic process.

Proof: Look at enterprise A, dispensing with the superscript, as the argument for B is identical. At the beginning of any period at most one input will be in inventory: call it k. Letting j be the other input, we see there are two possible inventory changes (from (5) and (6)):

$$\Delta s_{kt} = \varepsilon_{kt} - \frac{a_k}{a_j}\varepsilon_{jt} \text{ and } \Delta s_{jt} = 0 \Rightarrow s_{j,t+1} = 0$$

(7) or

$$\Delta s_{kt} = - s_{kt} \text{ and } \Delta s_{jt} = \varepsilon_{jt} - \frac{a_j}{a_k}(\varepsilon_{kt} + s_{kt}) \Rightarrow s_{k,t+1} = 0$$

depending on whether $\varepsilon_j/a_j < (s_k + \varepsilon_k)/a_k$ or $\varepsilon_j/a_j > (s_k + \varepsilon_k)/a_k$. Note that the second part of (7) only becomes relevant when there is sufficiently little of the surplus good for a small allocation error to make it deficit.

Due to (6) inventories will always be on the boundary of the positive orthant, which in this case is homeomorphic to the real line. So when measured in commensurate units, the state of inventories at any time, t, can be described by the real random variable $\zeta_t = a_1 s_{2t} - a_2 s_{1t}$, where surpluses of the second input are taken to be the positive direction. ζ is a measure of the amount of output that could be produced from the inventory if the other input were available ($= \pm a_1 a_2 \tilde{y}$, where $\tilde{y}$ is the possible output and the sign shows which input is in surplus).

Now note that, as an immediate consequence of (7),

(8) $$\Delta\zeta_t = a_1 \Delta s_{2t} - a_2 \Delta s_{1t} = \eta_t$$

where $\eta_t \underline{\Delta} a_1 \varepsilon_{2t} - a_2 \varepsilon_{1t}$, so that, when initial stocks are s_o,

(9) $$\zeta_t = s_o + \sum_{n=1}^{t} \eta_n \underline{\Delta} s_o + a_1 \sum_{n=1}^{t} \varepsilon_{2n} - a_2 \sum_{n-1}^{t} \varepsilon_{1n} .$$

Hence, as ε_t are i.i.d., ζ_t is a pure random walk which is non-positive (Feller (1966) p. 174) and hence non-ergodic.

Q.E.D.

Though non-ergodic, the inventories in each enterprise are recurrent (as random walk in 2 or fewer dimensions; Dynkin and Yushkevich (1969)), so there is still some hope for overall sectoral production. Further, as the total inflow/supply to the system is constant and non-random, it would seem possible that aggregate inventories in the sector would be well behaved. Finally, what is delivered as a surplus to one user is automatically a deficit to the other, so one might hope for a cancellation of surpluses and deficits when looking at overall sectoral inventory holdings. Yet even here inventories turn out to be badly behaved, with devastating consequences for sectoral plan performance. For ease of exposition I present the case where the enterprises produce identical output. For heterogeneous outputs the same argument would work with output measured in value terms at the fixed plan prices. 5/

Proposition 2: If $a \neq b$ (technology differs between enterprises) then aggregate sectoral inventories, $s_t = s_t^A + s_t^B$, form a non-ergodic stochastic process.

Proof: As in the proof of Proposition 1, make inputs commensurable by measuring each in terms of the output it could produce if a sufficient amount of the complementary input were available. Total sectoral inventories are then:

$$(10) \qquad \xi_t \triangleq \begin{bmatrix} a_2 s_1^A + b_2 s_1^B \\ a_1 s_2^A + b_1 s_2^B \end{bmatrix} \equiv \begin{bmatrix} (a_1 a_2 + b_1 b_2)\, \tilde{y}^1 \\ (a_1 a_2 + b_1 b_2)\, \tilde{y}^2 \end{bmatrix}$$

where $\tilde{y}^i$ shows the total potential output from the available stocks. As in the case of the individual enterprise, the behavior of the random variable

$$\Xi \equiv \xi_{2t} - \xi_{1t}$$

is studied. The increments of Ξ, after some algebraic manipulation, can be seen to be:

$$(11) \qquad \Delta\Xi \triangleq \Delta\xi_{2t} - \Delta\xi_{1t} = \Delta\zeta_t^A + \Delta\zeta_t^B = \eta_t^A - \eta_t^B$$

$$= (a_1 - b_1)\,\varepsilon_{2t} - (a_2 - b_2)\,\varepsilon_{1t}$$

where $\Delta\zeta_t^i$ and η_t^i, $i = A,B$, are defined by (8). This follows immediately from an analysis of each of the four possible inventory states in the sector, defined by which good is in surplus in which sector. Hence.

$$(12) \qquad \Xi_t = s_o + (a_1 - b_1) \sum_{n=1}^{t} \varepsilon_{2n} - (a_2 - b_2) \sum_{n-1}^{t} \varepsilon_{1n} ,$$

which shows that Ξ_t is non-ergodic unless $a = b$. But then

$$(13) \qquad \| s_t \| > \| \xi_t \| = (a_1a_2 + b_1b_2) \| \tilde{y}_t \| \geq (a_1a_2 + b_1b_2) | \tilde{y}_t^2 - \tilde{y}_t^1 | =$$

$$> | \Xi_t |$$

implies that s_t cannot be an ergodic process.

Q.E.D.

Remark: It should be noted that $\Xi_t \equiv (a_1a_2 + b_1b_2)(\tilde{y}_t^2 - \tilde{y}_t^1)$ (from (10) actually shows that the difference in output in principle produceable from the sum of stocks of each input held by the enterprises is a non-positive process. This is a stronger result than claimed in the proposition which states that even if the enterprise holding the scarcer of the two inputs (in terms of y) were to give its inventory remaining after production to the other, so that one enterprise always holds zero inventories, then the total inventories of the sector will still change in an unstable manner.

Since there are only two inputs, the inventory process is still recurrent, though the expected time to return to any neighborhood of the origin (planned inventory levels) is infinite. This means that the sector will, in general, not be able to fulfill its production plan over the long run, in spite of getting exactly what it needs for plan fulfillment. We can rather expect that an increasing portion of the resources supplied to the sector will be tied up ("frozen" in Soviet parlance) in inventory even though each producer does everything possible to minimize inventory levels. While there may still be some positive probability of fulfilling the plan, we should expect to observe increasingly serious underfulfillment of plan assignments. That this is so follows immediately from non-ergodicity of inventories, implying that of $y^*_t \not\leq y^{*A}_t + y^{*B}_t$ from (6).

To see this analytically, look at the special case where the ε_{jt} are independently uniformly distributed on $(-c, c)$. The choice of bounded interval is non-essential and only made for notational convenience in computations. For this case it is easy to calculate a lower bound on the expected underfulfillment of the production plan.

Proposition 3: The expected shortfall in sectoral production grows without bound i.e.

(14) $$E(Y^A_T + Y^B_T - T\bar{y}) \leq -kT$$

where $k > 0$ is some constant, $Y^i_T = \sum_{t=1}^{T} y^i_t$, $i = A,B$, and $\bar{y} = \bar{y}^A + \bar{y}^B$.

Proof: From (2) and (6) it is clear that, assuming w.l.o.g. that $s_o = 0$,

$$(Y^A_T - T\bar{y}^A)a = \sum_{t=1}^{T} \varepsilon_t - s^A_T \text{ and } (Y^B_T - T\bar{y}^T)b = - \sum_{t=1}^{T} \varepsilon_t - s^B_T$$

and hence

$$(Y^A_T - T\bar{y}^A) = a_i^{-1} \Sigma \varepsilon_{it} - a_i^{-1} s^A_{it} \qquad i = 1,2,$$

and

$$(Y^B_T - T\bar{y}^B) = - b_i^{-1} \Sigma \varepsilon_{it} - b_i^{-1} s^B_{jt} \qquad i = 1,2 .$$

Summing over all inventories, now measured in terms of potential output, we get

$$(15)\qquad 2(Y_T - T\bar{y}) = (a_1^{-1} - b_1^{-1}) \sum_{t=1}^{T} \varepsilon_{1t} + (a_2^{-1} - b_2^{-1}) \sum_{t=1}^{T} \varepsilon_{2t} - [a_1^{-1} s_{1T}^A + a_2^{-1} s_{2T}^A + b_1^{-1} s_{1T}^B + b_2^{-1} s_{2T}^B]$$

Since $\min(s^j_{1t}, s^j_{2t}) = 0$ for $\forall\, t,\ j = A,B$, $a_1^{-1} s_{1T}^A + a_2^{-1} s_{2T}^A = |\zeta_T^A|(a_1\ a_2)^{-1} = |a_2^{-1} \sum_{t=1}^{T} \varepsilon_{2t} - a_1^{-1} \sum_{t=1}^{T} \varepsilon_{1t}|$, and similarly $b_1^{-1} s_{1T}^B + b_2^{-1} s_{2T}^B = |b_1^{-1} \sum_{t=1}^{T} \varepsilon_{1t} - b_2^{-1} \sum_{t=1}^{T} \varepsilon_{2t}|$. Therefore

$$2(Y_T - T\bar{y}) = (a_1^{-1} - b_1^{-1}) \Sigma\, \varepsilon_{1t} + (a_2^{-1} - b_2^{-1}) \Sigma\, \varepsilon_{2t} - |a_2^{-1} \Sigma \varepsilon_{2t} - a_1^{-1} \Sigma\, \varepsilon_{1t}| - |b_1^{-1} \Sigma\, \varepsilon_{1t} - b_2^{-1} \Sigma\, \varepsilon_{2t}| \leq$$

$$(16)\qquad \leq (a_1^{-1} - b_1^{-1}) \Sigma\, \varepsilon_{1t} + (a_2^{-1} - b_2^{-1}) \Sigma\, \varepsilon_{2t} - |(b_1^{-1} - a_1^{-1}) \Sigma\, \varepsilon_{1t} + (b_2^{-1} - a_2^{-1}) \Sigma\, \varepsilon_{2t}|$$

Define $\Omega_t \triangleq \alpha\, \varepsilon_{1t} + \beta\, \varepsilon_{2t}$ where $\alpha = (a_1^{-1} - b_1^{-1})$ and $\beta + (a_2^{-1} - b_2^{-1})$. Then clearly

$$2(Y_T - T\bar{y}) \leq \sum_{t=1}^{T} \Omega_t - \left|\sum_{t=1}^{T} -\Omega_t\right| \equiv \sum_{t=1}^{T} \Omega_t - \left|\sum_{t=1}^{T} \Omega_t\right|$$

To show (14), we show that $E(\sum_{t-1}^{T} \Omega_t) = 0$ while $E(|\sum_{t=1}^{T} \Omega_t|) = kT$, $k > 0$. First note that, by the independence of ε_{it}, Ω_t are i.i.d. Hence the behavior of its sums can be studied through the products of the characteristic function of ε:

$$\Phi(u) = \frac{i[\sin(\alpha+\beta)uc + \sin(\alpha-\beta)uc]}{-2\alpha\beta c^2 u^2} \quad , \quad i = \sqrt{-1} \quad .$$

$$\Phi_{\Sigma\Omega_t}(u) = \Phi^T(u) \quad .$$

We investigate the general case: $\alpha \neq \pm \beta$. If $\alpha = \pm \beta$, trivial modification yields the same result. As the distribution of $\Sigma\,\Omega_t$ is clearly symmetric, its density is an even function, so that:

$$(17) \qquad \Phi_{|\Sigma\,\Omega_t|}(u) = \left[\frac{(e^{i\alpha cu} - 1)(e^{i\beta cu} - 1)}{i^2\alpha\beta c^2 u^2}\right]^T .$$

Now $E(|\Sigma\,\Omega_t|) = -i \cdot \left[\frac{d}{du}\Phi_{|\Sigma_1\Omega_t|}(0)\right]$ and $E(\Sigma\,\Omega_t) = -i \cdot \left[\frac{d}{du}\Phi_{\Sigma_1\Omega_t}(0)\right]$.

(Billingsley (1979), p. 298). Therefore, letting

$$\frac{e^{i\alpha cu} - 1}{i\alpha cu} \underline{\Delta}\, A \qquad \text{and} \qquad \frac{e^{i\beta cu} - 1}{i\beta cu} \underline{\Delta}\, B \ ,$$

$$E\left|\sum_{t=1}^{T} \Omega_t\right| = (-i)T(AB)^{T-1}\left\{\left[\frac{e^{i\alpha cu} - 1 - i\alpha cue^{i\alpha cu}}{i\alpha cu^2}\right]A + \left[\frac{e^{i\beta cu} - 1 - i\beta cue^{i\beta cu}}{i\beta cu^2}\right]B\right\}\Bigg|_{u=0}$$

$$= \frac{c(\alpha+\beta)}{2} \cdot T$$

as can be checked by repeated application of l' Hospital's rule.

It is completely straightforward to check that

$$- i\left[\frac{d}{du}\left\{\frac{i[\sin(\alpha+\beta)uc + \sin(\alpha-\beta)uc]}{-2\alpha\beta c^2u^2}\right\}^T\right]\Bigg|_{u=0} = 0 .$$

Hence $E\{2(Y_T - T\bar{y})\} \leq - E\left|\sum_{t=1}^{T} \Omega_t\right| = \frac{-c(\alpha+\beta)}{2} \cdot T$, so that

$E(Y_T^A + Y_T^B - T\bar{y}) \leq - kt$, where $k = \frac{c(\alpha+\beta)}{2} > 0$.

Q.E.D.

Remark: This result can clearly be generalized to arbitrary distributions of $\varepsilon_t \in R^2$ as long as the ε_t remain independent across t. It can also be shown that the variance of the production shortfall grows without bound. In fact for the process $\upsilon_t = Y_t - t\bar{y}$ we have

$$\lim\sup \upsilon_t = 0 \qquad \lim\inf \upsilon_t = -\infty$$

These three propositions show that the introduction of even the slightest allocative disturbance renders the production system highly unstable. Waste increases without bound and average output falls unboundedly behind the plan. Of course this result depends crucially on three assumptions of the model: the unobservability of local inventories by the allocating authority, the tautness of the plan, and the absolute lack of technological substitution of this paper . The third deserves some further discussion. While in the short run it can be defended as a reasonable approximation to reality, particularly as I am interested in uncovering general tendencies rather than making predictions, over the long run it is clearly untenable. This then raises the question as to how robust these results are to the introduction of input substitution in production. While that will be the subject of another paper, let me indicate some preliminary results derived from working with a CES production function. First it is clear that with any substitution, an extreme policy in the spirit of (6) can always hold $s_t \equiv 0$. Thus the problem of inventory stability disappears. However, unless inputs have identical productivity (i.e. are perfect substitutes in production) so that no output is lost from temporary disproportions in inputs, the problem of output stability becomes even more severe. In fact, output losses can <u>never</u> be made up as materials are not sitting in inventory to be used in their most productive combination when their complements become available. In the limit ($\rho \to -\infty$ in the CES function) output behaves as if surpluses of complementary inputs were just thrown away (in the model without substitution above), rather than being held in inventory until they can be used. Thus the general result of instability of production performance under the slightest allocative uncertainty seems quite robust with respect to allowing input substitution.

4. Two extensions

In the preceeding section we saw that, under taut planning, even the slightest perturbation of the input allocation between users causes both inventories and output to behave in an unstable manner when the production technology is of a fixed-coefficients type. Here I will show that this result extends quite directly to the situation when two complementary inputs, $i = 1, 2$, are allocated among n users, indexed by $j = 1,\ldots,n$. Then I will study the generalization to m complementary inputs, $i = 1,\ldots,m$, used by each of the enterprises, $j = A, B$. In both cases the extension is straightforward.

In order to handle n enterprises using two material inputs the only significant change that must be made in the equations of Section 2 relates to the handling of the disturbance term. There are now n perturbation vectors, ε_t^j, at each time t, the components of each vector are independent of each other and across time, and the vectors sum to the zero vector: $\varepsilon_{it} = (\varepsilon_{it}^i \ldots \varepsilon_{it}^n)$ is viewed as an independent drawing from a simplex. Let $a^j \in R_+^2$ be the vector of technological coefficients in the j-th enterprise. Then the basic equations describing inventory and output behavior (5) and (6) are:

$$(19) \qquad \Delta s_t^j = - a^j z_t^j + \varepsilon_t^j \qquad\qquad y_t^{*j} = \min_i \, (a_i^j)^{-1}(s_{i,t-1}^j + x_{it}^j)$$

where x_t^j is as defined in (2), mutatis mutandis. Clearly (7) holds within each enterprise so that we can define ζ_t^j precisely as in (8) and (9). Proposition 1 then follows immediately for each of the n enterprises.

To show that Proposition 2 also holds in this case, define

$$\xi_t \in R^2, \ \xi_{it} = \sum_{j=1}^{n} a_k^j s_i^j, \ k \neq 1,2, \text{ and } \Xi_t = \xi_{2t} - \xi_{1t}.$$

Then, as in (11)

$$(19) \qquad \Delta\Xi_t = \sum_{j=1}^{n} \Delta\xi_t^j = \sum_{j=1}^{n} \eta_t^j = \sum_{j=1}^{n} (a_1^j \varepsilon_{2t}^j - a_2^j \varepsilon_{1t}^j).$$

Since $\sum_j \varepsilon_t^j \equiv 0$, this can be normalized to

$$(20) \qquad \Delta\Xi_t = \sum_{j=1}^{n-1} (a_1^j - a_1^n)\varepsilon_{2t}^j - (a_2^j - a_2^n)\varepsilon_{1t}^j$$

so that, summing over t, it is immediate that Ξ_t is a random walk and hence non-ergodic if the a^j are linearly independent.

It is also easy to demonstrate Proposition 3 for this case, though the computations become quite tedious, under the assumption that the ε_{it} are i.i.d. uniform random variables. The key to the argument lies in bounding the deviation from planned output from below, as in (16), and then computing the expectation of that random variable through use of its characteristic function, as in (17) and the argument following. This yields the expectation of increasingly poor (i.e. linearly deteriorating) output performance relative to the plan.

The generalization to the use of m inputs by enterprises A and B is just as direct. The only change required of the basic model in Section 2 is to consider all vectors elements of R^m rather than R^2. Behavior of the system is then described by equations (5) and (6). Combining these we can see that inventory behavior of an enterprise (say A for definiteness) is given by the equation

$$(21) \qquad \Delta s_t^A = - a a_{i*}^{-1} (s_{i*,t-1}^A + \varepsilon_{i*,t}) + \varepsilon_t \quad .$$

To see that (21) describes a non-ergodic stochastic process, note that $\| s_t^A \| \geq \| \Pi_A s_t^A \|$ where $\Pi_A \triangleq I - a(a^T a)^{-1} a^T$ is the projection onto the hyperplane orthogonal to the technology coefficient vector. $\Pi_A s_t^A$ thus measure the distance of inventory holdings from the closest proportional stocks that could be completely used in production. It is a continuous bounded function of the amount of production lost due to inventory disproportions (like $\tilde{y}$ in (10)) and is hence a generalization of ζ_t. Direct calculation shows $\Pi_A s_t^A = \Pi_A s_o^A + \Pi_A \sum_{n=1}^{t} \varepsilon_n$, which is clearly a random walk, thus demonstrating Proposition 1.

Propositions 2 and 3 follow just as directly once we notice that the $\tilde{y}$ defined in Section 3(p. 12) is precisely the Euclidean distance along the ray defined by the technological coefficients, a and b. This is given for the A enterprise by $(I - \Pi_A) s_t^A = \tilde{y}^A \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$, and similarly for B. The disproportion of inventories in terms of foregone output is thus legitimately measured by $\Pi_j s_t^j$ for j = A, B, and sector-wide disproportions become $\Xi_t = \Pi_A s_t^A + \Pi_B s_t^B$. Remembering that the disturbance in enterprise B is just the negative of that in A, and assuming w.l.o.g. that $s_o^A = s_o^B = 0$, we see

$$\Xi_t = \sum_{n=1}^{t} (\Pi_A - \Pi_B) \varepsilon_n = \left[b(b^T b)^{-1} b^T - a(a^T a)^{-1} a^T \right] \sum_{n=1}^{t} \varepsilon_n$$

is a random walk if a and b are not colinear. If the rank of $(\Pi_A - \Pi_B)$

is greater than two, then the process is not only nonpositive but also nonrecurrent, so that sectoral inventory behavior is more perverse than in the basic case of Section 2. Again, under simplifying assumptions about the distribution of ε_{it} (e.g. uniformity over some interval) yielding a tractible characteristic function, it is relatively straightforward to show that expected realized sectoral output falls linearly behind the plan.

The combination of these generalizations of the basic model creates no further problems, but yields no further insights. As noted in Section 3, the problem of inventory stability disappears if input substitution is allowed but this only further aggravates the problem of long run plan fulfillment. One might deal with that by eliminating the assumption of tautness and allowing sufficient slack in aggregate supply. That I will investigate in another paper. However, the general result seems quite robust: in a tautly planned command economy a slight perturbation of an administered allocation generates a general instability rendering long-run fulfillment of the plan increasingly unlikely.

5. Conclusion

This paper addresses issues of both implementation by command and modeling centrally planned economic processes. On the latter issue the main point is quite simple. There is a major difference between models of planned resource allocation (the command mechanism) which explicitly recognize the uncertain consequences of the allocation decision and models which treat these consequences as certain outcomes. The presence of such ex-post uncertainty means that centralized allocation decisions cannot implement an economic plan which is in principle a perfectly feasible one. Unexpected shocks accumulate, disrupting the production process and causing an incresing divergence between plan and performance. Optimal allocation decisions no longer imply that the desired results will be achieved, even "on average." Thus it is important in the study and modeling of economic planning that uncertainty in implementation be taken explicitly into account.

Here this point was argued through an extremely simple model of administrative resource allocation. The presence of ex-post uncertainty rendered the dynamic behavior of important production indices (i.e. inventories and output) highly unstable. Should we have expected this? I would argue, yes. It seems to me that instability is generated by the institutional nature of the "command mechanism." Only the center has the power and authority to act, to allocate materials and to alter that allocation. Uncertainty affects the consequences of that allocation, but only on the most disaggregate level, well beyond the ken of the central authorities. Only the local producers can know the true resulting state of affairs, and then only their own small part of it (e.g. their own inventory levels). In any realistic situation the information is too vast and detailed to be transmitted, and even if transfered would overwhelm any conceivable storage, processing and/or computational capabilities of the center. 6/ Yet in a command system the center must make the decisions, and hence must do so without the requisite detailed information. Thus the separation of the loci of detailed information and of decision making authority is an essential structural characteristic of a command resource allocation mechanism. 7/

This separation necessarily rigidifies the system in the face of uncertainty. It prevents the generation of a feedback response to growing disproportions in the economy; the center lacks the information to reallocate and the producers cannot trade among themselves. Hence it reinforces the inflexibility of technology in generating inventory instability and thus loss of output to the system as a whole. Therefore the elimination of such separation would seem to be a necessary condition for effectively dealing with even so trivial a source of uncertainty as that studied in this model. Yet that would seem to imply a serious weakening of the "command mechanism." 8/

This result can be looked on as an extension of some of the results in Ericson (1979). There the source of instability might have been supposed to be the aggregate random behavior of the whole system, the fact that total material availabilities behave (loosely speaking) as a "random walk." Yet here the same result is derived for a similar system with fixed aggregate availability. 9/ Of course I am here only looking at a part of a command production system, rather than the full system, but the formulation here allows interpretations which were not open to that model.

In particular we can consider this analysis as dealing with localized allocation processes such as seem to result from the partial decentralization of command economies. Such decentralization does not affect the essential character of the "command mechanism," but merely moves its functioning closer to the operational level (e.g., basic production units) in either a territorial or "functional" sense. Examples of this type of decentralization are the organization of the supply network of Territorial Directorates (UMTS) and supply bases of GOSSNAB and the concentration of supply functions in production associations (ob'edinenija) in the Soviet Union. The results of this paper would seem particularly relevant to the functioning of the former, as one would expect informational feedback to be better developed in the latter.

In each of these cases the model of this paper represents a situation in which the supply center for a territorial unit or the supply department of a production association is given a perfectly feasible plan and precisely the material allocations needed to implement it. These must be properly allocated among the enterprises within the territory or association. The results above show that, if there is any uncertainty in the consequences of the allocation decision, implementation of the plan becomes infeasible. The core of the problem lies in the fact that the central authority in each case is implementing a plan, rather than dealing with the uncertain and developing production situation. Yet this seems an essential characteristic of the command mechanism and administrative allocation in general. Thus I would not expect any noticeable improvement in the economic system from the introduction of these changes.

Finally, I would argue that the results are more robust than the specific model used would indicate. They are essentially based on the rigidity inherent in any plan and the separation of information and authority inherent in any administered system. The existence of informational feedback and the ability to flexibly respond to contingencies where they occur are shown in Ericson (1979) to be sufficient to solve the problem in a similar model. Those conditions would clearly also be sufficient in this even simpler model. There is, however, one aspect of the model which would seem to vitiate the generality of the result: the Leontief specification of technology. Clearly inventory instability depends crucially on the total lack of input substitution in production. But inventory instability is a necessary step toward failure to fulfill the production plan only in the case of fixed coefficient technology. As noted in Section 3 (p. 18), when input substitution is allowed the instability of production/output is in general aggravated: in a sense, input substitution transfers the instability from inventories to the output flow. Yet it is final output in which we are ultimately interested. Inventories are only a means to the end. So the central point of this paper remains unaffected: centralized allocation decisions cannot implement an economic plan which is in principle perfectly feasible.

FOOTNOTES

1. This is particularly true of the "incentives in teams" approach Groves (1973) and the literature on inducing correct/truthful revelation of information, e.g. Groves and Ledyard (1977), Groves and Loeb (1975), Green and Laffont (1977), and the papers in the Review of Economic Studies Symposium on the incentive compatibility problem, June 1979. Work relating to inducing subunits to implement some central decision (an "optimum" according to some central criterion) includes Domar (1974), Tam (1979), Weitzman (1974 and 1976). This work does not, however, deal with the issue of implementing a comprehensive plan.

2. Grossman (1963) lays out the basic framework of the concept of a Command Economy and discusses its relevance to the Soviet Union.

3. This assumption is also made for analytical simplicity, though it is in no way essential to the results. It in fact provides the best chance for stabilizing inventories and thus fulfilling the production plan over the long run. For a discussion of this see Section 3 of Ericson (1979), where this is referred to as the "MP rule." On tautness and the incentives to maximize output see Hunter (1961), Portes (1969), and Keren (1972 and 1979). A quick perusal of any part of the Soviet literature on the material supply system will convince the reader of the strong pressure applied (largely unsuccessfully, however) to minimize inventory holdings.

4. Here the process being described in clearly not stationary so that ergodicity is being used in a wider sense, i.e. a process will be called <u>ergodic</u> if it converges a.s. to an ergodic stationary proces. On the latter see, for example, Loeve (1963), Chapter 9. This implies that the process is governed, in the limit,by an invariant probability distribution.

5. Any prices will work as long as the input coefficient vectors are not linearly dependent. The choice of fixed prices affects the magnitude of the effects but not the qualitative fact of instability.

6. Here I am implicitly assuming what has been called "bounded rationality" (Radner (1975)). In modeling questions of practical implementation an assumption of full rationality throws out the most interesting issues.

7. This point is beautifully illustrated in Nove (1977) in his discussion of almost every aspect of the Soviet economy.

8. I have, of course, ducked the issue of the information to be gained by the center in the process of repeating the planning cycle which includes bargaining with subunits. It would be of interest to investigate whether information on the state of inventories so gained (given reasonable assumptions about incentives and bargaining behavior) is sufficient for the center to stabilize inventories by periodic shifts in allocation policy, and to analyse the resulting loss to such policies. I would conjecture that there exists a finite amount of

material slack that could be introduced into the system to yield stable plan-fulfilling behavior in this very simple model. Hence the command mechanism could work, but at the cost of a finite level of waste.

9. These results also demonstrate that the continuous time formulation in that model was purely a mathematical convenience, and not essential.

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