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A THEORY OF PRICE FORMATION AND EXCHANGE IN ORAL AUCTIONS

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1. INTRODUCTION

One of the main justifications for the use of equilibrium models in economics is the argument that there are forces at work in any economy which tend to drive the agents and their decisions towards an equilibrium, if they are not at one already. Equilibrium models have proven to be extremely powerful in the analysis of many situations; however, attempts to model and explain the forces that might drive an economy to equilibrium have met with little success. Most of the literature on the stability of equilibrium uses the fiction of a disinterested auctioneer who adjusts a single known price for each good in response to stated excess demand resulting from agents' equilibrium plans. The limitations and defects of this approach are well-known; for a survey of the literature see Arrow and Hahn [1]. In addition, as far as we know, the <u>only</u> institutional arrangement that even approximates this idealized model of price formation is the London gold market.

Now, however, a body of data has been generated which provides detailed information on the disequilibrium behavior of traders in auction markets similar to those of organized commodity or stock exchanges.[2] Further, these data are difficult to ignore since they are generated experimentally under controlled conditions. They cannot be explained away by reference to measurement error, unobserved variables, or other fudge factors. In these experiments, a small number of traders (four seems to be large enough to generate competitive-like behavior), each with limited imperfect information, determine prices and quantities transacted through interactive bargains. There is neither a single price nor a single price quoter. Nonetheless, the quantities exchanged and the prices at which transactions take place typically converge to, or near to, the values predicted by the competitive equilibrium model. However, in spite of the fact that the traditional demand-supply model appears to yield reasonably accurate predictions of long-run average prices and quantities, it fails to yield any insights into the process by which these prices and quantities are obtained.

In this paper we provide a theory of the price formation and exchange process in a class of experimental exchange markets called Double Oral Auctions. The ability of this theory to explain price formation and exchange in other markets such as the New York Stock Exchange depends on the degree of parallelism that exists between the two. (See Smith [6].) An astronomer's maintained hypothesis is that the physics of the lab is the same as that of the sun; our working hypothesis is that behavior in experimental markets is similar to that in other markets, and that insights discovered in the evidence generated in the lab are potentially transferable to non-experimental markets with similar institutional structure. Thus, we view the theory in this paper as a first step towards contructing a positive theory of the process of exchange and price formation in many other markets.

2. THE EXPERIMENTAL MARKET

In a Double Oral Auction (DOA) experiment, a pool of subjects (usually eight to twelve) is divided at random into a group of buyers and a group of sellers. The buyers are given payoff schedules telling them the amount in cents that they will receive, from the experimenter, for each unit of the good they purchase. The sellers are given cost schedules telling them their cost in cents for each unit of the good they sell. Sellers keep the difference between their selling price and cost on each unit sold. These payoff and cost schedules induce demand and supply curves for the good; see Smith [4] for a complete explanation of induced demand and supply curves. Each trader knows his own payoff schedule, but is given no information about others' payoffs or about the induced demand and supply curves. See Appendix A for an example of these induced demand and supply curves for one experiment.

After they receive their payoff information, subjects are allowed to trade during a market period of some fixed length. Buyers can make bids to buy a unit of the good and sellers can make offers to sell a unit. If a bid or offer is accepted, a binding trade occurs and all traders are informed of the contract price. Once a trade is completed, bids and offers can be made for another unit of the good. No information other than bids, offers, acceptances, and contract prices is transmitted or known by the participants.

When a market period ends, the subjects are given new payoff schedules, identical to their schedules for the previous period, and the experiment is repeated.[3] Market demand and supply conditions are typically held constant across periods so that any equilibrating process that exists has a chance to establish an equilibrium. For a more detailed explanation of auction experiments and the usual results see Smith and Williams [7].

These experiments provide a unique opportunity to examine price formation for two reasons. The first is that unlike real-world markets, actual competitive equilibrium values for prices and quantities are known for the experimental markets. Second, complete data on bids, offers, contracts, and their timing is available. An example of a typical design and the data generated is provided in Appenxicies A and B. Demand and supply functions can be calculated from the subjects' valuations, and competitive equilibrium prices and quantities can then be computed. The first obvious fact from these experiments is that <u>actual exchange prices are not equal to those predicted by the competitive model</u>. In a strict sense, demand-supply theory is rejected by these data. The second obvious fact, however, is that after a very few replications, <u>transaction prices and quantities converge to near those predicted by the competitive model</u>. These observations have been replicated many times.[4] The only conclusion one can draw is that the traditional theory needs refining before one has a compelling explanation of the observed behavior in these markets. Not only must "equilibrium" be explained, but we must also explain the "disequilibrium" values, the sequence in which they occur, and the process by which participants are "learning".

3. THE THEORY - SOME PRELIMINARY THOUGHTS

Our goal is to explain how these markets really work. Although there are a variety of models which purport to explain price adjustments, the existence of experimental data provides many opportunities to reject obviously incorrect theories. The set of feasible theories for these markets can now be severely constrained by the data in a way that is unusual for economics. Before we present our theory let us consider several other obvious candidates.

Since both the institutional description and the data from the experimental DOA markets reject the Walrasian auctioneer as the appropriate model of price formation, a natural alternative might be a Marshallian theory. In a naive version of this theory, the trading sequence depends on the differences in buyers' prices (willingness to pay) and sellers' prices. In particular, this theory predicts that the first trade will occur between the buyer with the highest induced value (Buyer 1 in the example in Appendix A) and the seller with the lowest induced cost (Seller 1 in the example in Appendix A). The second trade occurs between the buyer and seller with the second values and costs, and so on. This theory does not predict which prices will occur, but it does predict that the total quantity transacted will be the competitive equilibrium quantity. Although data on prices and quantities seem to support this model, when we look closely at the microdata we see that the theory is soundly rejected. A cursory glance at Appendix B should convince even the most skeptical reader that the predictions of the naive Marshallian theory are not at all consistent with the data. This is an excellent example of a case in which the experimental setup allows us to test more hypotheses than would be possible if we only had access to non-experimental market data. In particular we are able to test the prediction concerning the order in which participants are involved in transactions. This would be impossible without explicit knowledge of the individual valuations.

For our second candidate we turn to a more sophisticated theory based on game theoretic models. There is a complete information Nash equilibrium (with price-quantity offers or bids as strategies) for most of the experimental DOA markets in which all trades take place at the competitive equilibrium. However, the use of a complete information Nash equilibrium concept to describe the experimental market has two difficulties. First, as Appendix B illustrates, the data are not consistent with this equilibrium. Second, the participants in the experiments do not have enough information to calculate the strategies required to support this equilibrium. (They would have to be able to calculate the competitive equilibrium price.)

In the experiments which have been run details on others valuations (and thus on the competitive equilibrium) may only be inferred by the subjects from the public market data on bids, offers, and contracts. Thus, we must recognize that the structure in which the participants find themselves is an iterated game with incomplete information. The standard theory for such a game involves the use of Bayesian learning and the assumption of a lot of rationality. Aside from the facts that it is extremely difficult to explicitly solve for the "equilibria" in these games, and that game theorists are still debating which, among many, the appropriate equilibrium concept is, we feel there are at least two other reasons why this theory would be inadequate as an explanation of the observed data in Double Oral Auctions. The first is that, as far as we can tell, participants seem to learn "faster" than any Bayesian theory would predict. (This is not a sound scientific argument concerning the rejection of a well stated hypothesis, but the data seem to lead to such a conclusion.) The second reason is that even if agents' learning is approximately Bayesian, the theory of incomplete information games requires that agents use the correct initial likelihood function (at some level of abstraction) to make inferences from their observations. There appears to us to be absolutely no theoretical or empirical basis on which to choose one possible function over another prior to observing behavior and as this choice is crucial to the predictions of the theory, we think this approach is unlikely to yield insights into the DOA experiments.

Finally, there is an existing literature on price adjustment; see, for example, the discussion in Arrow and Hahn [1]. There are two prominent approaches in this literature: tâtonnement and nontâtonnement adjustment. It is difficult to know if, or how, this literature applies to the DOA experiment as the experiment is a combination of nontâtonnement adjustment in a market period and tâtonnement adjustment across market periods. Further, the goal of most of the previous research is presumably to build a theory which yields convergence to a competitive equilibrium. Hence, little, if any, attention is given to explaining how individuals act, and learn to modify their actions, in disequilibrium situations. Thus, other than "predicting" convergence to a competitive equilibrium for certain preference configurations, this literature yields no predictions about the sequence of prices and trades in a DOA.

4. THE THEORY

4.a Preliminaries

A participant in a Double Oral Auction experiment has a complex decision problem. He must decide when to bid; how much to bid; and, whether or not to accept the trades offered by other subjects. Further, all of these decisions must be made with very imperfect information. The subject does not know the payoffs or expectations of other agents, he does not know the terms of trade that will be available to him in the future, and he does not know the effect of his actions on the actions of others. This is a very complex incomplete information game in which individuals choose bidding and acceptance strategies. To place some structure on this problem, we first introduce some notation and definitions concerning data known to the experimenter.

The payoffs or values given to buyers are integers, and are ranked as $V^1 \geqslant V^2 \geqslant \ldots \geqslant V^n \geqslant 0$, where V^i is the ith highest value and there are n units. The costs given to sellers are integers, and are ranked as $0 \leqslant M^1 \leqslant M^2 \leqslant \ldots \leqslant M^m$, where M^j is the cost of the jth unit and there are M units. To avoid trivial cases of no trade, we assume that the highest value, V^1, is greater than the lowest cost, M^1.

Market periods or days for an experiment are indexed by $d = 1,2,\ldots$. The time remaining in any given day is indexed by $t = 0,1,\ldots,T$.

Contract prices, bids, and offers are in integer units in the interval $[0,\overline{P}]$, where $\overline{P} < \infty$ is some arbitrarily selected upperbound above V^1 and M^m. We let c_d, b_d, and o_d represent the sets of, respectively; contract prices, bids, and offers occuring in day d. During any particular day d, each participant observes all contract prices, bids, and offers.[5] It is these data alone, along with the buyer's own value, on which the buyer can base his decision to bid and accept.

If a buyer or a seller has multiple units, she decides on strategies for each of them separately. Thus, we identify units with buyers (n of them) and with sellers (m of them). This assumption seems relatively innocous to us except in the case of a very small number of buyers or sellers (1 or 2) and many units per trader. In such an experiment, the competitive-like behavior we postulate might be inappropriate and we would not expect convergence to the competitive equilibrium. We feel, however, that the simplicity it buys is well worth the price.

4.b An Intuitive look

To avoid having to model or solve the complex game theoretic problem, we adapt the spirit of revealed preference theory and demand-supply analysis by placing assumptions on individual behavior which we believe are consistant with optimal or approximately optimal behavior. We do this by decomposing the decision problem into three main elements; expectations, reservation prices, and bidding strategies. These are most easily explained in reverse order.

At any point in time, given reservation prices, buyers are assumed to be willing to bid up to that price or to accept any offer up to that price. This means that the auction will procede like an English auction, with reservation prices substituting for true values, and that after some period of time the outstanding bid will be held by the buyer with the highest reservation price and that bid will be at least as high as the second highest reservation price. In this paper we will ignore the time it takes for this to occur and instead assume that all observed bids are the reduced form results of the above English auction. This intuitive view of the bidding is formalized in assumption 1 below. Sellers are viewed symmetrically in assumption 1'.

Since bids are viewed to depend on reservation prices in a somewhat mechanistic way, the driving force of our theory will be the reservation price formation process. This in turn is assumed to be driven by two principles of learning in these auctions. First, whenever the bids and acceptance prices of a buyer are higher than were necessary to complete a transaction, the buyer completes a trade but overpays. That buyer should realize that he overpaid and should, during the next auction, lower his reservation price. If this is not lowered too much the buyer should still complete a transaction but at a better price. Second, if a buyer waits too long to bid or, what is the same thing, maintains too low an acceptance price duing the day, then that buyer may not complete any contract even though profitable ones are available. If a buyer could have purchased a unit at less that V_i but did not, then that buyer should realize he underbid and should, during the next auction, raise his reservation price. It is the delicate balance between "paying too much" and "not offering to pay enough" which the buyers must learn in order to be successful in the auction. Unfortunately, we do not model this learning process; instead, we place restrictions on allowable reservation price behavior which reflect these learning principles, and which also represent the possible reduced form of some complicated learning dynamic. We summarize this rather simple intuition in assumption 2 below.

The readers must decide for themselves whether or not the reduced form assumptions we make are justified. We present a fairly detailed analysis of the experimental evidence and its relation to the implications of our theory in section b. We feel the theory "fits" the data very well but not perfectly. We leave it as a challange to others to improve upon the precision.

4.c Bidding Behavior

We start our description of the formal theory with the introduction of a hypothesis concerning the existence of the key unobservable of our model.

ASSUMPTION 0: (Reservation Prices)

For each buyer ($i = 1,\ldots,n$) and seller ($j = 1,\ldots,m$) there is a (unobservable) reservation price at each day d and time t, denoted $r^i_d(t) \in \mathbf{R}^1$ for buyers and $s^j_d(t) \in \mathbf{R}^1$ for sellers.

This only contains notation. To link these unobservables to the data, we need to tie bids and acceptances to these reservation prices. As we indicated in the previous section we do this by assuming that, given reservation prices, bids and acceptances are the reduced form of English auction behavior.

ASSUMPTION 1: (Bidding and Contracts) (English Auction)

(i) $b_d(t)$, the current outstanding bid in day d, with time t left, is held by buyer i^* where $r^{i^*}_d(t) \geq r^i_d(t)$, for all $i = 1,\ldots,n$.

(ii) $b_d(t) \leq r^{i^*}_d(t)$.

(iii) $b_d(t) \geq r^i_d(t)$, for all $i \neq i^*$.

(iv) i^* accepts the current outstanding offer, $o_d(t)$, if and only if $o_d(t) \leq r^{i^*}_d(t)$. No other i accepts $o_d(t)$.

Simply stated, at each point in time, the current bid is held by the buyer with the highest reservation price. This bid lies below that reservation price and above the second highest reservation price. We simply ignore the time it might take for this state to occur since there seem to be few empirical problems with doing so. (Nonetheless, this is one point of possible difference between reality and the theory which could be adjusted.) Under

Assumption 1, and 1' below, trades always occur between the buyer with the highest reservation price and the seller with the lowest reservation price. However, these need not be the buyer with the highest value and the seller with the lowest cost. It is important to notice that we assume the English auction is based on reseration prices and not on the "true values", V_i. Thus, at this point, we really do not yet have a testable theory since, given any sequence of bids and contracts, it is possible to construct a sequence of reservation prices which, under assumption 1, would imply the given data precisely. Until we place some restrictions on the reservation prices, we can therefore explain anything.

For completeness, we make an assumption on the offers and acceptances of sellers that is symmetric with that made for buyers. The only difference is that we have arbitrarily assumed that if seller j^* is willing to accept $b_d(t)$ and buyer i^* is willing to accept $o_d(t)$ then the buyer accepts first.

ASSUMPTION 1': (Offers and Acceptances) (English Auction)

(i) $o_d(t)$, the current outstanding offer in day d with time t left, is held by seller j^* where $s_d^{j^*}(t) \leq s_d^j(t)$, for all $j = 1,\ldots,m$.

(ii) $o_d(t) \geq s_d^{j^*}(t)$.

(iii) $o_d(t) \leq s_d^j(t)$, for all $j \neq j^*$.

(iv) j^* accepts $b_d(t)$ if and only if $b_d(t) \geq s_d^{j^*}(t)$ and i^* does not accept $o_d(t)$. No other j accepts $b_d(t)$.

4.d Reservation Price Formation

In this section we tie the theory down by restricting allowable reservation price behavior in a way which relates it to the observable data. This is the way we connect bids, contract prices, and the sequence of trades to the initial data known by the experimenter and, thus, provide testable propositions about these auctions.

Reservation prices are assumed to be formed in accordance with the intuitive principles outlined in Section 4.b. We begin by assuming that a buyer's expectations in any period are based on last periods prices. In particular, we assume that the support of the buyer's expectations is the set of prices bounded by the maximum of last period's highest contract price and highest bid and the minimum of last period's lowest contract price and lowest offer. Based on these expectations, reservation prices are formed over time as follows: (a) for most of a trading day, one's reservation price lies below V_i and within the support of the expectations (when this is feasible), (b) if possible, the reservation price is actually below the maximum price in the support since the buyer does not want to "overpay", (c) eventually, if no contract is agreed to, buyers will cave in and let the reservation price approach the maximum price in the support, and (d) if still no contract is completed, the reservation price will rise higher than even the maximum in the support of the expectations. This sequence of actions seems to us to be consistent with behavior known to be optimal in finite time stocastic search models and therefore somewhat uncontroversial. Part (e) is a bit different but is included to allow adjustment to events which happen even though they were originally believed by the trader to occur with zero probability.

ASSUMPTION 2: (Reservation Price Formation)

For all $i = 1,\ldots,n$:

(i) $r^i_d(t) \leq V^i$ for all t, d and if i has traded (accepted an offer or had a bid accepted) in d, before t, then $r^i_d(t) = 0$.

(ii) For each d there exists $\hat{t}^i_d > 1$ such that, if i has not traded in d before t, then:

(a) For each $t \geq \hat{t}^i_d$, if $\underline{P}_d < V^i$ then $r^i_d(t) \in [\underline{P}_d, \overline{P}_d]$.

(b) For each $t \geq \hat{t}^i_d$, if $\overline{P}_d - \underline{P}_d > 1$ then $r^i_d(t) < \overline{P}_d$.

(c) $r^i_d(d) = \text{MIN}\ \{\overline{P}_d - 1, V^i\}$ for each t, $\hat{t}^i_d \geq t \geq \hat{t}_d$,

where $\hat{t}_d = \text{MAX}\ \{t \mid b_d(t) \geq \overline{P}_d - 1$ and $b_d(t)$ unaccepted$\}$,

if this maximum exists, $\hat{t}_d = 0$ otherwise.

(d) If $\hat{t}_d > 0$ then $r^i_d(t) = \text{MIN}\ \{\overline{P}_d, V^i\}$ for each t,

$\hat{t}_d > t \geq \overline{t}_d$, where $\overline{t}_d = \text{MAX}\ \{t \mid b_d(t) \geq \overline{P}_d$ and $b_d(t)$ unaccepted$\}$, if this maximum exists, $\overline{t}_d = 0$ otherwise.

(e) If $\overline{t}_d > 0$, then $r^i_d(t) \geq \text{MIN}\ \{\overline{P}_d + 1, V^i\}$ for each t,

$\hat{t}_d > t \geq 0$.

The conditions in assumption 2(i), 2(i)(a), and 2(i)(b) embody the intuition that, as a result of learning, reservation prices will not be "too high". The conditions in 2(ii)(c), 2(ii)(d), and 2(ii)(e) embody the intuition that, towards the end of the day, if the trader has not completed a transaction then that trader will learn to raise his reservation price. Towards the end of the day, reservation prices will not be "too low". Finally, to ensure that reservation prices are also not "too low" during the earlier part of the trading day, we need an assumption which ties together reservation prices and true values. Drawing from our understanding of optimal behavior in sealed-bid auctions (see, e.g., Meyerson and Satterthwaite (2) and Wilson (8)), we feel that a natural assumption would be that individuals with higher true values have higher reservation prices at any point in time. That is, if $V_i \geq V_j$ then $r_d^i(t) \geq r_d^j(t)$. However, the data reject this hypothesis, in the presence of our other assumptions. In particular, under this hypothesis the highest true value should trade first with the lowest true cost. As we indicated earlier, this naive Marshallian model does not stand up to the evidence. A second, weaker hypothesis is that if $V_i \geq \overline{P}^d$ and $V_j \leq \underline{P}^d$ then $V_i(t) > V_j(t)$. This simply ignores those whose true values lie in the support of expectations and requires only that those with especially high true values have higher reservation prices than those with especially low true values. Whether this weaker hypothesis is rejected by the data depends on one's standard of acceptance. Fortunately it is possible to construct an acceptable theory with an even weaker assumption on the relative ranking of reservation prices.

ASSUMPTION 2(iii): The Ranking Hypothesis

Let $k^* = \text{MIN}\ \{k \mid V^k < M^k\}$. For all d, t if $V^i > V^{k^*}$, $V^i \geq \overline{P}_d$, $V^k \leq V^{k^*}$, and $V^k \leq \overline{P}_d$, then if i and k have not traded in d before t, $r_d^i(t) > r_d^k(t)$ whenever $\overline{P}_d - \underline{P}_d > 1$.

It may appear that an individual trader would "need to know" both the values of others as well as k^* in order to satisfy this assumption. However, this is simply not true. We view this assumption as a natural reduced form implication of the intuitive learning process we described earlier. If those with values higher than $\overline{P}_d$ do not keep their reservation prices high enough then they will be rationed out of the market and not be able to complete a trade. Assumption 2(iii) embodies what is necessary to be "high enough". If one believes that with common information reservation prices will be ordered similiarly to true values then one must believe our strictly weaker hypothesis.

To complete our model we make symmetric assumptions concerning sellers' reservation prices which we call Assumption 2'.

Some final remarks are in order. We believe that optimal behavior, or some close approximation to it, would satisfy the assumptions we have made but, as we have not fully modeled the iterated incomplete information game, we have no way to formally address this issue. We also believe that there are traders in the experiments whose behavior is, at least for a few iterations, vastly different from the behavior which would be consistant with our assumptions. In particular there are traders, such as seller 2 in IIPDA 57 (see section VI) who continually hold out for a highly profitable trade even though they never complete one. These traders usually modify their behavior after a few iterations. Those who don't lose a considerable amount of "opportunity" reveue; however, we need to "explain" this "irrationality".

We turn now to the derivation of a number of testable implications of the theory. We then confront these with the data from a small number of representitive experiments. The reader may decide for herself whether or not we have captured something real in our model.

5. THEOREMS

In this section we trace through some of the implications of our theory. As will become apparent, most of the action will occur when there is an "excess demand or supply" of two or more units remaining in the auction as there are then competitive pressures on bids and offers. Thus we are interested in the following concept.

DEFINITION: Let $D(P) = \# \{V^i \geq P\}$ and $S(P) = \# \{M^j \leq P\}$. We let $P^* = \inf \{P | S(P) - D(P) \geq 2\}$ and $P_* = \sup \{P | D(P) - S(P) \geq 2\}$.

To see the role of P^* and P_* we consider the following propositions. All propositions are stated under Assumptions 0, 1, 1', 2, 2'. (All proofs are given in Appendix C.)

LEMMA 1: (a) If $\overline{P}_d < P_*$ then $\overline{P}_{d+1} > \overline{P}_d$. (b) If $\underline{P}_d > P^*$ then $\underline{P}_{d+1} < \underline{P}_d$.

LEMMA 2: (a) If $\overline{P}_d \geq P_*$ then $\overline{P}_{d+1} \geq P_*$. (b) If $\underline{P}_d \leq P^*$ then $\underline{P}_{d+1} \leq P^*_d$.

Thus there are competitive forces driving the maximum contract prices above P_* and the minimum contract prices below P^*. There are also competitive pressures driving the maximum prices down and the minimum prices up. These pressures are exerted by extra-marginal units as can be seen in the following propositions. Remember $* = \min \{k | V^k < M^k\}$.

LEMMA 3: Suppose $\overline{P}_d - \underline{P}_d > 1$: (a) If $\overline{P}_d > M^{k*}$ then $\overline{P}_{d+1} < \overline{P}_d$.

(b) If $\underline{P}_d < V^{k*}$ then $\underline{P}_{d+1} > \underline{P}_d$.

At this point we know that the maximum contract price will fall if it is greater than M^{k*} and increase if it is less then P_*. In order to establish that "prices converge to an equilibrium" we must consider what occurs when $P_* \leq \overline{P}_d \leq M^{k*}$. Unfortunately, under our behavioral assumptions, it is possible for prices to bounce around inside this interval and, in some cases, to bounce outside. The case where this won't happen is described in the next lemma.

LEMMA 4: (a) If $V^{k*} \leq \overline{P}_d \leq M^{k*}$ then $\overline{P}_{s+1} \leq M^{k*}$. (b) If $V^{k*} \leq \underline{P}_d \leq M^{k*}$ then $\underline{P}_{d+1} \geq V^{k*}$.

Notice that if $P_* < \overline{P}_d \leq V^{k*}$ then it is possible that $\overline{P}_{d+1} > M^{k*}$. Thus we cannot insure, under our model, that "convergence to equilibrium" occurs for all demand-supply configurations. However, we can say something for most configurations and, in particular, for those which have been used in the experiments reported in the next section.

THEOREM: If $M^{k*} \geq P^* \geq P_* \geq V^{k*}$ then there exists d^*, where $0 < d^* < \infty$, such that for all $d \geq d^*$: (a) $[\underline{P}_d, \overline{P}_d] \subseteq [V^{k*} - 1, M^{k*} + 1]$; (b) $\overline{P}_d \geq P_*$, $\underline{P}_d \leq P^*$; and, (c) $\overline{P}_d - \underline{P}_d \leq 1$. Further the number of contracts q_d in each day $d > d^*$ is constrained by $q^e - 1 \leq q_d \leq q^e + 1$, where $q^e = \text{MAX}\ \{k \mid V^k \geq M^k\}$ is the Walrasian equilibrium quantity.

That is, prices converge to some "small" interval contained by the extra-marginal units and centered by competitive pressures around P^* or P_*. Refering to Appendix A, it can be seen for experiment IPDA14 that $k* = 7$, $P^* = 4.30$, $P_* = 4.10$, MAX = 4.30, $V_{k*} = 4.00$ and, therefore, the theorem is applicable. For completeness we note that convergence is not assured (although possible) if either $P^* > M^{k*}$ or $V^{k*} > P_*$.

A more precise characterization of convergence can be obtained for narrower preference configurations.

COROLLARY: If $M^{k*} \geq P^* = P_* \geq V^{k*}$ then there exists d^*, where $0 < d^* < \infty$, such that for all $d \geq d^* L$ (a) $[\underline{P}_d, \overline{P}_d] \subseteq [P^e + 1]$, and (b) $\overline{P}_d \geq P^e$, $\underline{P}_d \leq P^e$, where $P^e = P^* = P_*$ is the Walrasian equilibrium price.

For IPDA14 in Appendix A, $P^* \neq P_*$ and, therefore the corollary is not applicable. But there are at least two classes of experiments which have been reported for which the hypothesis of the corollary is true. If there are at least two marginal units on each side of the market, (i.e., $V^{k*-2} = V^{k*-1} = M^{k-2} = M^{k*-1}$) then $P^* = P_*$ and the Corollary applies. Another case in which the Corollary applies occurs in the so-called "swastika" experiments. In these experiments all buyers have a common value V and all sellers have a common value M, with V > M; that is, $V^1 = V^2 = \ldots = V^n > M^1 = \ldots = M^m$, and either $m \geq n + 2$ or $n \geq m + 2$. If $|m-n| < 2$ then $P^* = V > M = P_*$ so the Corollary does not apply. The preceding theorem does apply to this case, but here it only yields $[\underline{P}_d, \overline{P}_d] \subseteq [M,V]$. The specific prices are determined solely by bargaining. There are no competitive forces.

6. COMPARISON OF PREDICTIONS WITH THE DATA

The prediction of convergence, which results from many potential theories, is not testable with the experimental data as the number of replications necessary for convergence may be large. In order to compare a model with the data, it is necessary to describe the potential dynamics that it implies. There are four categories of data for which our theory has implications: The sequence of contract prices; the sequence of trading partners; the total quantity traded; and, the behavior of untraded units at the end of the day. We will carry out the analysis of the dynamics under the assumption of the Theorem that $M^{k*} \geqslant P^{*} \geqslant P_{*} \geqslant V^{k*}$. This assumption is satisfied in all of the experiments for which we have data.

The implications of our theory for prices, total quantity, and the sequence of trading can be represented by three cases.

CASE 1(a): $\overline{P}_d < P_*$, $\overline{P}_d < V^{k*}$.

(1) First, all $M^j < \overline{P}_d$ trade with some $V^i \geqslant \overline{P}_d$ at prices less than $\overline{P}_d$, if $\overline{P}_d - \underline{P}_d > 1$, or at prices $\overline{P}_d$, if $\overline{P}_d - \underline{P}_d \leqslant 1$.

(2) Next, all $M^j = \overline{P}_d$ trade with some $V^i \geqslant \overline{P}_d$ at $\overline{P}_d$.

(3) At this point further trades may occur, at prices above $\overline{P}_d$, but this is not necessary. However, there will be at least one bid above $\overline{P}_d$.

(4) Finally, the number of contracts is $q_d \geqslant S(\overline{P}_d)$.

CASE 1(b): $\overline{P}_d < P_*$, $\overline{P}_d \geqslant V^{k*}$.

(1) First, all $M^j < \overline{P}_d$ trade with some $V^i > V^{k*}$ at prices below $\overline{P}_d$,

if $\overline{P}_d - \underline{P}_d > 1$, and at prices less than or equal to $\overline{P}_d$, if $\overline{P}_d - \underline{P}_d \leq 1$.

(2) Next, all $M^j = \overline{P}_d$ trade with some $V^i > \overline{P}_d$ at $\overline{P}_d$.

(3) Same as case 1(a).

(4) Same as case 1(a).

CASE 2: $\overline{P}_d > M^{k*}$.

(1) All trades occur at prices below $\overline{P}_d$, if $\overline{P}_d - \underline{P}_d > 1$, or at prices less than or equal to $\overline{P}_d$, if $\overline{P}_d - \underline{P}_d \geq 1$.

(2) All $V^i \geq \overline{P}_d$ trade.

(3) All $V^i \geq \overline{P}_d$ trade before those $V^i \leq V^{k*}$.

(4) q_d depends on $\underline{P}_d$ as follows: (i) If $\underline{P}_d \leq M^{k*}$ then $q_d \geq q_e - 1$; (ii) If $\underline{P}_d > M^{k*}$ then $q_d \geq S(\underline{P}_d)$.

CASE 3: $M^{k*} \geq \overline{P}_d \geq P_*$.

(1) All contract prices are no more than $\overline{P}_d$ and at least one is at or above P_*.

(2) All $V^i > \overline{P}_d$ trade before those $V^i \leq V^{k*}$.

(3) If $\underline{P}_d \geq V^{k*}$ all trades are at prices below $\overline{P}_d$, if $\overline{P}_d - \underline{P}_d > 1$, and at prices less than or equal to $\overline{P}_d$, if $\overline{P}_d - \underline{P}_d \leq 1$.

(4) If $\underline{P}_d < V^{k*}$, all initial trades (at least k^*-2) are at prices below $\overline{P}_d$, if $\overline{P}_d - \underline{P}_d > 1$, and at prices no more than $\overline{P}_d$, if $\overline{P}_d - \underline{P}_d \leq 1$. The ($k^*$1)st trade can occur at a price in $[\underline{P}_d, M^{k*}]$.

(5) $q^e - 1 \leq q_d \leq q^e$.

The analysis of each case follows from the earlier lemmas. Case 1 comes from Lemma 1, Case 2 from Lemma 3 and Case 3 from Lemmas 2 and 4. Symmetric results hold for $\underline{P}_d$.

The theory also has implications for final bids and offers. Although reservation prices are unobservable, Assumption 2.ii does imply that near the end of each day, reservation prices of untraded units should be equal to true values if those values are less than $\overline{P}_d$ for buyers or greater than $\overline{P}_d$ for sellers. Thus, in strict terms at t = 0 our theory implies:

(a) $b_d(0) \geq V^{(2)}$, $o_d(0) \leq M^{(2)}$, and

(b) $V^{(1)} < o_d(0)$, $M^{(1)} > b_d(0)$,

where $V^{(1)}$ is the largest non-traded value and $V^{(2)}$ is the next highest. Implication (a) follows from the fact that $r_d^i(0) = V^i$ and A.1 on bidding. Implication (b) follows from the fact that $r_d^i(0) = V^i$. If $o_d(0) \leq V^{(1)}$ then buyer (1) should accept at t = 1.

Data:

The following table summarizes violations of the predictions about prices, quantity or sequence of trades, and dayend bids and offers as a percentage of total possible violations for nine Plato Double Oral Auction experiments. This table is based upon unpublished data which was made available by Vernon Smith.

TABLE I

Experiment		Exp	Marg	Com	Unit	Qe	NY	Que	Errors as % of Possible Prices	Sequence	Dayend
I	PDA8	N	1	5	8	6	Y	N	17	4.2	22
I	PDA9	N	1	5	8	6	Y	N	22	11.1	18.8
I	PDA10	Y	1	5	10	8/6	Y	N	16.5	0	6.2
I	PDA11	N	1	5	10	8	Y	N	12.7	2.8	18.8
I	PDA14	Y	1	5	8	6	Y	N	16.1	4.1	12
II	PDA14	N	3	10	21	15	Y	N	2.9	16.7	0
II	PDA57	N	3	10	21	15	Y	Y	6.5	10	33.0
II	PDA22	N	0	10	16,11	11	N	N	9.3	NA	0
II	PDA25	Y	2	10	12	7	N	Y	16.7	9.1	50.0
					TOTALS				11.4	7.9	18

[Note: Exp Y means experienced subjects, Marg = number of marginal units, Com = commission in cents, Unit = number of units in the market, Qe = competitive equilibrium quantity, NY = NYSE rules, Que = electronic queing of bids and offers.]

To see the total number of price violations in perspective, the following tabel illustrates the margins of error. This table reports the total number of violations of our price predictions (over all nine experiments) which were more than x cents, as a percentage of the total number of possible violations of our price predictions.

TABLE II

Price violations of x¢ or less not counted:	Percentage of price violations over 9 DOA's:
x = 1	5.2
x = 5	4
x = 10	2

To put the dayend errors in context, there are two principal sources of violations. In II PDA57, seller 2 consistently held to a high reservation price on a marginal unit and, in fact, lost money on this strategy. No other dayend errors occurred in II PDA57. Second, the experiment II PDA25 had a time queue in which bids entered are accepted chronologically and leave the queue at 3-second intervals. This institution clearly destablizes bidding. If we ignore these two pathologies, the total percentage of dayend errors is 7.5%.

7. CONCLUSION

The theory presented here is deterministic and, although it does not completely describe precise paths of bids, offers and contracts, it does place fairly tight bounds on these data. One observation not in accord with these bounds is grounds for rejection of the theory and, in fact, there are a number of such observations. However, the percentage of observations which violate the crucial implications of the theory is, we feel, amazingly low.

The potential importance of this theory is not only that it seems to describe what happens in DOA experiments, but also that it is the beginning of a theory of how market prices are formed and of how they adjust to changes in demand and supply conditions. The question of price formation has a long history of ad-hoc and unsuccessful attempts at an answer. Our theory is also ad-hoc in the sense that we make assumptions on individual behavior which are not derived form an optimizing model. However, our assumptions seem possibly consistent with rational behavior and, more importantly, they seem to do a reasonable job of describing actual bids, offers, and contracts.

Cornell University

and

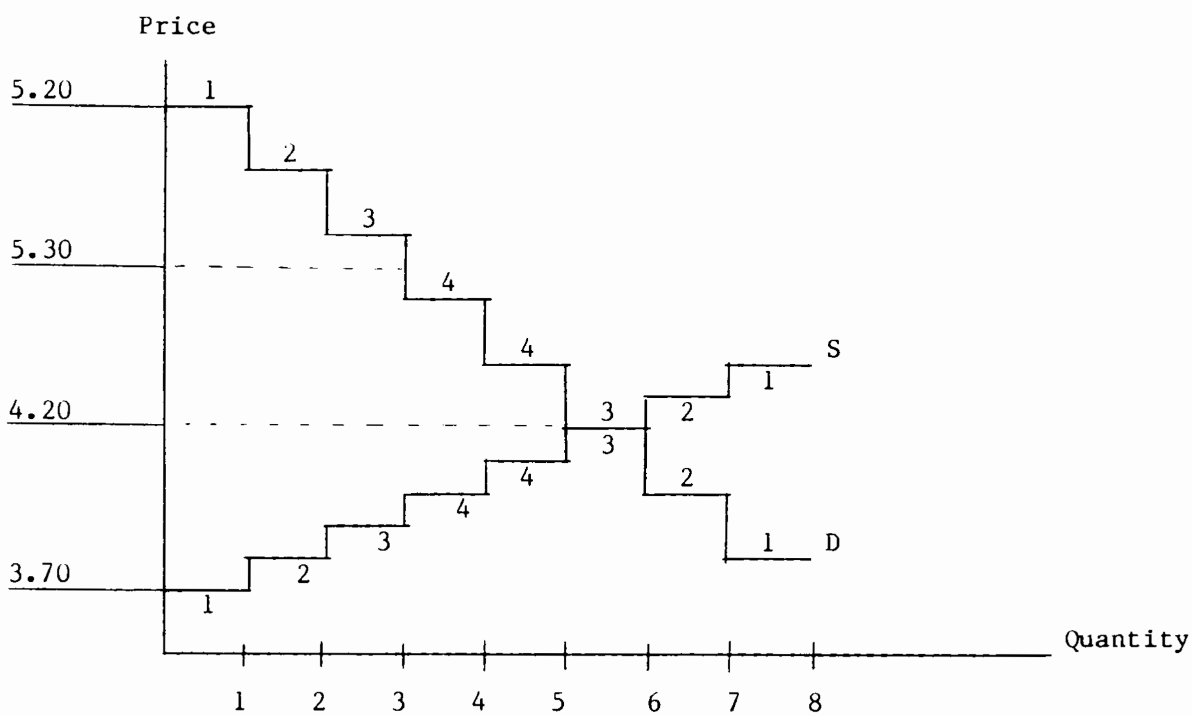
Northwestern University

Appendix A

Induced Demand-Supply Schedule

DOA # IPDA14 (10/18/77)

Week 1



The number indicates the holder of the unit with that value or cost.

Appendix B

The following information is provided for a typical design:

1) Instructions from the standard DOA experiments run without the aid of a computer.

2) The values, V_i, and costs, M_j, from a Plato computer-assisted experiment IPDA14 run on 10/18/77 at the University of Arizona. (A week is 5 days.)

3) The record sheets of Buyer 4 and Seller 1 from IPDA14.

4) The data saved by the computer for Day 9. (MKR = maker of bid or offer, TM= time left in day in seconds, TKR = accepter of that bid or offer. A * in the TKR column indicates that bid or offer occured before the acceptance.)

5) A list of contracts in the order agreed to each day including price, buyer, and seller.

INSTRUCTIONS

General

This is an experiment in the economics of market decision making. Various research foundations have provided funds for this research. The instructions are simple and if you follow them carefully and make good decisions you might earn a considerable amount of money which will be paid to you in cash.

In this experiment we are going to simulate a market in which some of you will be buyers and some of you will be sellers in a sequence of market days or trading periods. Attached to the instructions you will find a sheet, labeled Buyer or Seller, which describes the value to you of any decisions you might make. You are not to reveal this information to anyone. It is your own private information.

Specific Instructions to Buyers

During each market period you are free to purchase from any seller or sellers as many units as you might want. For the first unit that you buy during a trading period you will receive the amount listed in row (1) marked 1st unit redemption value; if you buy a second unit you will receive the additional amount listed in row (5) marked 2nd unit redemption value. The profits from each purchase (which are yours to keep) are computed by taking the difference between the redemption value and purchase price of the unit bought. Under no conditions may you buy a unit for a price which exceeds the redemption value. In addition to this profit you will receive a 5 cent commission for each purchase. That is

[your earnings = (redemption value) - (purchase price) + .05 commission]

Suppose for example that you buy two units and that your redemption value for the first unit is $200 and for the second unit is $180. If you pay $150 for your first unit and $160 for the second unit, your earnings are:

$ earnings from 1st = 200 - 150 + .05 = 50.05

$ earnings from 2nd = 180 - 160 + .05 = 20.05

total $ earnings = 50.05 + 20.05 = 70.10

The blanks on the table will help you record your profits. The purchase price of the first unit you buy during the first period should be recorded on row (2) at the time of purchase. You should then record the profits on this purchase as directed on rows (3) and (4). At the end of the period record the total of profits and commissions on the last row (9) on the page. Subsequent periods should be recorded similarly.

Specific Instructions to Sellers

During each market period you are free to sell to any buyer or buyers as many units as you might want. The first unit that you sell during a trading period you obtain at a cost of the amount listed on the attached sheet in the row (2) marked cost of 1st unit; if you sell a second unit you incur the cost listed in the row (6) marked cost of the 2nd unit. The profits from each sale (which are yours to keep) are computed by taking the difference between the price at which you sold the unit and the cost of the unit. Under no conditions may you sell a unit at a price below the cost of the unit. In addition to this profit you will receive a 5 cent commission for each sale. That is

[your earnings = (sale price of unit) - (cost of unit) + (.05 commission)]

Your total profits and commissions for a trading period, which are yours to keep, are computed by adding up the profit and commissions on sales made during the trading period.

Suppose for example your cost of the 1st unit is $140 and your cost of the second unit is $160. If you sell the first unit at $200 and the second unit at $190, your earnings are

$ earnings from 1st = 200 - 140 + .05 = 60.05

$ earnings from 2nd = 190 - 160 + .05 = 30.05

total $ earnings = 60.05 + 30.05 = 90.10

The blanks on the table will help you record your profits. The sale price of the first unit you sell during the 1st period should be recorded on row (1) at the time of sale. You should then record the profits on this sale as directed on rows (3) and (4). At the end of the period record the total of profits and commissions on the last row (9) on the page. Subsequent periods should be recorded similarly.

Market Organizations

The market for this commodity is organized as follows. We open the market for a trading period (a trading "day"). You will be warned when the end of the trading period is approaching. Any buyer (or seller) is free at any time during the period, to raise his hand and make a verbal bid (offer) to buy one unit of the commodity at a specified price. Any seller (or buyer) is free to accept or not accept the bid of any buyer (or seller). If a bid is accepted a binding contract has been closed for a single unit and the buyer and seller will record the contract price to be included in their earnings. Any ties in bids or acceptances will be resolved by a random choice of buyer or seller. Except for the bids and their acceptance you are not to speak to any other subject. There are likely to be many bids that are not accepted, but you are free to keep trying, and as a buyer or a seller you are free to make as much profit as you can.

Are there any questions?

	Week 1		Week 2	
	Unit 1	Unit 2	Unit 1	Unit 2
BYR 1	5.20	3.80	3.70	3.60
BYR 2	5.00	4.00	3.80	3.50
BYR 3	4.80	4.20	3.90	3.40
BYR 4	4.60	4.40	4.00	3.30
SLR 1	3.70	4.40	3.10	3.30
SLR 2	3.80	4.30	2.90	3.50
SLR 3	3.90	4.20	2.70	3.70
SLR 4	4.00	4.10	2.50	3.90

Touch the parameter to be changed.

BACK for last page; BACK1 to replot this page;
LAB for monitor; DATA to update specific vc
HELP1 for Week 2 D-shift.
HELP to shift back to original

Record Sheet for Buyer 4

	Trading Period	1	2	3	4	5	6	7	8	9	10
1	1st Unit Resale Value	4.60	4.60	4.60	4.60	4.60	4.00	4.00	4.00	4.00	4.00
2	Purchase Price	4.20	4.30	4.25	4.30	4.30	3.35		3.40	3.40	3.45
3	Profit (row 1 - row 2)	0.40	0.30	0.35	0.30	0.30	0.65		0.60	0.60	0.55
4	Profit + $.05 Commission	0.45	0.35	0.40	0.35	0.35	0.70	0.00	0.65	0.65	0.60
5	2nd Unit Resale Value	4.40	4.40	4.40	4.40	4.40	3.30	3.30	3.30	3.30	3.30
6	Purchase Price	4.40	4.30	4.30	4.30	4.25					
7	Profit (row 5 - row 6)	0.00	0.10	0.10	0.10	0.15					
8	Profit + $.05 Commission	0.05	0.15	0.15	0.15	0.20	0.00	0.00	0.00	0.00	0.00
9	Total Profit (row 4 + row 8)	0.50	0.50	0.55	0.50	0.55	0.70	0.00	0.65	0.65	0.60

Subject: ott

Profit: periods 1 - 5 = $ 2.60

periods 6 - 10 = $ 2.60

TOTAL PROFIT = $ 5.20

Experiment finished: 10/18/77

Record Sheet for Seller 1

	Trading Period	1	2	3	4	5	6	7	8	9	10
1	Selling Price of 1st Unit	4.20	4.35	4.25	4.30	4.30	3.35	3.35	3.40	3.43	3.45
2	Cost of 1st Unit	3.70	3.70	3.70	3.70	3.70	3.10	3.10	3.10	3.10	3.10
3	Profit (row 1 - row 2)	0.50	0.65	0.55	0.60	0.60	0.25	0.25	0.30	0.33	0.35
4	Profit + $.05 Commission	0.55	0.70	0.60	0.65	0.65	0.30	0.30	0.35	0.38	0.40
5	Selling Price of 2nd Unit						3.32	3.38	3.40	3.45	3.50
6	Cost of 2nd Unit	4.40	4.40	4.40	4.40	4.40	3.30	3.30	3.30	3.30	3.30
7	Profit (row 5 - row 6)						0.02	0.08	0.10	0.15	0.20
8	Profit + $.05 Commission	0.00	0.00	0.00	0.00	0.00	0.07	0.13	0.15	0.20	0.25
9	Total Profit (row 4 + row 8)	0.55	0.70	0.60	0.65	0.65	0.37	0.43	0.50	0.58	0.65

Subject: mecham

Profit: periods 1 - 5 = $ 3.15

periods 6 - 10 = $ 2.53

TOTAL PROFIT = $ 5.68

Experiment finished: 10/18/77

PERIOD 9

Per. 9 now in progress. TIME: 0 1

	MKR	TM	BIDS	OFFERS	TKR	TM
1	B2	297	3.30			
2	B1	295	3.34			
3	B2	292	3.35			
4	S4	289		3.45	B3	285
5	B2	281	3.30			
6	B4	278	3.35			
7	B2	275	3.36			
8	S3	274		3.50		
9	B1	268	3.39			
10	B4	258	3.40		S2	252
11	S1	254		3.45	*	
12	B2	247	3.30			
13	B1	245	3.40			
14	S3	244		3.50		
15	S1	236		3.45		
16	B2	231	3.41			
17	S3	191		3.44		
18	S1	181		3.43	B2	154
19	B4	151	3.30			
20	B2	147	3.31			
21	S2	146		4.50		
22	B1	146	3.40			
23	B2	140	3.41			
24	S1	136		3.45	B1	68
25	B2	122	3.42		*	
26	B1	112	3.43		*	
27	B2	106	3.44		*	
28	S2	63		4.50		
29	B4	62	3.30			
30	B1	61	3.40			
31	S3	57		3.49		
32	B2	55	3.41			
33	B1	45	3.43			
34	B2	41	3.44			
35	B2	23	3.45		S3	17
36	B4	11	3.30			
37	S2	10		3.50	B1	0
38	B1	8	3.40		*	

Summary DATA from IPDA14

	Contract Price	Buyer	Seller
Day 1	4.25	2	3
	4.20	4	1
	4.50	3	2
	4.40	4	4
	4.30	1	2
Day 2	4.35	3	1
	4.30	4	2
	4.30	4	2
	4.30	1	4
	4.25	2	4
Day 3	4.25	4	1
	4.35	3	3
	4.30	4	2
	4.27	2	4
	4.25	1	3
Day 4	4.30	4	1
	4.39	3	4
	4.30	4	4
	4.26	2	2
	4.25	1	3
	4.20	3	2
Day 5	4.30	4	1
	4.26	2	2
	4.35	3	4
	4.26	1	3
	4.25	4	4
Day 6	3.35	4	1
	3.30	3	2
	3.35	3	3
	3.32	2	1
	3.35	1	4
Day 7	3.31	2	2
	3.35	3	1
	3.40	3	4
	3.38	2	1
	3.45	1	3
Day 8	3.40	4	2
	3.40	1	1
	3.40	1	1
	3.40	3	4
	3.40	2	3

	Contract Price	Buyer	Seller
Day 9	3.45	3	4
	3.40	4	2
	3.43	2	1
	3.45	1	1
	3.45	2	3
	3.50	1	2
Day 10	3.41	2	2
	3.44	1	4
	3.45	4	1
	3.45	1	3
	3.50	3	1

Appendix C: Proofs

Proof of Lemma 1: We show (a); the proof of (b) is symmetric. If all contracts are at prices less than or equal to $\overline{P}_d$ then there exist at least two $V^i > \overline{P}_d$ which are untraded. By A.2(ii) at some time before $t = 0$ there will be a bid $b_d(t)$ greater than or equal to the second highest $r_d^i(t) > \overline{P}_d$.

Proof of Lemma 2: We show (a); the proof of (b) is symmetric. If all contracts are less than P_* then there exist at least two untraded V^i such that $V^i \geq P_*$. Thus there will be a bid $b_d(t) \geq P_*$ before $t = 0$.

Proof of Lemma 3: We show (a); the proof of (b) is symmetric. We will show that whenever there is an untraded $V^i \geq \overline{P}_d$, there will be an offer $o_d(t) < \overline{P}_d$ which i can accept. Thus no contracts at prices greater than or equal to $\overline{P}_d$ will be seen.

Suppose there are ℓ untraded V^i with $V^i \geq \overline{P}_d$. Since all $V^i \geq \overline{P}_d$ must be traded before any $V^i \leq V^{k*} < M^{k*}$, by A.2(iii), there must be at least $\ell + 1$ untraded M^j with $M^j \leq M^{k*} < \overline{P}_d$. If $\ell \geq 1$, then at some time before $\hat{t}_d$ there will be an offer $o_d(t) < \overline{P}_d$ which some $V^i > P_d$ will accept. If $\ell = 0$, then there are no $V^i \geq \overline{P}_d$ remaining.

Proof of Lemma 4: We show (a); the proof of (b) is symmetric. In order to have $\overline{P}_{d+1} > M^{k*}$ there was a $V^i > M^{k*}$ who at $\hat{t}_d$ of day d bid above or accepted a contract at a price above M^{k*}. If this were true at $\hat{t}_d$, then as $\overline{P}_d > V^{k*}$ and by A.2(iii), there were at most $K* - 2$ contracts. Thus there exist untraded

$M^j \leqslant M^{k*}$. Therefore there was an offer $o_d(t) < M^{k*} < V^i$ which i would have accepted. Thus there was no such V^i.

<u>Proof of Theorem</u>: The proof of the Theorem follows directly from Lemmas 1 through 4 and the observation that there are a finite number of possible prices, 0, 1, ... , $\overline{P}$.

<u>Proof of Corollary</u>: The proof of the Corollary follows directly from the Theorem and the observation that if $P_* = P^*$ then $P_* = P^* = p^e$.

FOOTNOTES

1. This paper has benefitted from discussions in seminars at Cornell, Northwestern, Stonybrook, and a NSF Conference on Experimental Economics at the University of Arizona. We would like to thank Vernon Smith for making unpublished data on his Plato DOA experiments available to us. Helpful comments from Maureen O'Hara and a referee are gratefully acknowledged.

2. These markets are described in detail in Appendix A and a description of some representative data is provided in Appendix B.

3. Other designs are also used. See Smith [4] for some of these.

4. See Smith and Williams [7] for a description of the usual results.

5. Each participant also observes the timing of each contract, bid, and offer. It is highly probable that the timing of these events is an important piece of information which affects the actions of the buyers and sellers. However, the level of complexity required to incorporate timing into the model seems to outweigh the gains to be achieved. Thus, we ignore it throughout the paper.

REFERENCES

[1] Arrow, K.J. and F.H. Hahn, General Competitive Analysis, Holden-Day Inc., 1971.

[2] Myerson, R. and M. Satterthwaite, "Efficient Mechanisms for Bilateral Trading", Discussion Paper # 469 S, Center for Mathematical Studies in Economics and Management Science, Northwestern University, May 1981.

[3] Plott, C.R. and V.L. Smith, "An Experimental Examination of Two Exchange Institutions", Review of Economic Studies, 45, 1978, 133-153.

[4] Smith, V.L., "Experimental Economics: Induced Value Theory", American Economic Review, 66, 1976, 274-279.

[5] Smith, V.L., "A Survey of Experimental Market Mechanisms for Classical Environments", October 1979.

[6] Smith, V.L., "Microeconomic Systems as an Experimental Science", Discussion Paper 81-32, College of Business and public Administration, University of Arizona, October 1981.

[7] Smith, V.L. and A.W. Williams, "Price Adjustment Processes, 1: PLATO Double Auction Experiments", 1977.

[8] Wilson, Robert, "Double Auctions", Working Paper, Stanford University, July 1982.