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A THEORY OF PRICE FORMATION AND EXCHANGE IN ORAL AUCTIONS*

by

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One of the major justifications for the study of equilibrium situations in economics is the argument that there are forces at work in any economy that tend to drive the economy toward an equilibrium if it is not in equilibrium already. Although the concept of economic equilibrium has proven useful in many situations, attempts to model and explain the forces that drive an economy to equilibrium have met with little success. Most of the literature on stability of equilibrium uses the fiction of an auctioneer who adjusts a single, known price for each good in response to stated excess demand resulting from agents' equilibrium plans. The limitations and defects of this approach are well known; for a survey of the literature see Arrow and Hahn [1971].

The recent work on experimental auction markets provides an opportunity to examine the question of price formation and exchange in a new light. These experiments simulate the auction-trading process characterstic of organized commodity or stock exchanges. In these experiments prices and quantities exchanged typically converge to at or near their competitive equilibrium values. However, the experiments are characterized by a small number of traders, with imperfect information, who determine prices through interactive behavior rather than taking them as given. As a result, although the supply and demand model of perfect competition yields reasonably correct predictions of long-run average prices and quantities, it cannot yield any insights into the process by which these prices are obtained.

This paper attempts to explain the price formation and exchange process of experimental auction markets. We view the explanation of this process as a first step toward understanding equilibrium price and quantity formation in real markets.

SECTION I: THE EXPERIMENT

In an auction experiment a pool of subjects (usually 8 to 12) is divided at random into a group of buyers and a group of sellers. The buyers are given payoff schedules telling them the amount in dollars that they will receive, from the experimenter, for each unit of the good that they purchase. The sellers are given cost schedules telling them their cost in dollars for each unit of the good they sell. Sellers keep the difference between their selling price and cost on each unit sold. These payoff and cost schedules induce demand and supply curves for the good; see Smith [1976] for a complete explanation of induced demand and supply curves. Each trader knows his own payoff schedule but is given no information about others' payoffs or about the induced demand and supply curves.

After they receive their payoff information subjects are allowed to trade during a market period of some fixed length. Buyers can make bids (which become public information) to buy a unit of the good and sellers can make offers (which also become public information) to sell a unit. If a bid or offer is accepted a binding trade occurs and all traders are informed of the contract price. Once a trade has been completed, bids and offers can be made for another unit of the good. No information other than bids, offers, acceptances, and contract prices is transmitted.

Once a market period ends the subjects are given new payoff schedules, identical to their schedules for the previous period, and the experiment is repeated. Market demand and supply conditions are typically held constant across periods so that any equilibrating process that exists has a chance to establish an equilibrium. After a small number of repetitions, prices and quantities are close to the competitive equilibrium values for the demand and supply curves induced by the subjects' payoffs. For a more detailed explanation of auction experiments and the usual results see Smith and Williams [1980]

These experiments provide a unique opportunity to examine price formation for two reasons. The first is that unlike real-world markets, actual equilibrium values for prices and quantities are known for the experimental markets. Second, complete data on bids, offers, and contracts is available.

In order to examine the price formation process we will construct two alternative models of the experiments. Each model yields the principal observable experimental result - that contract prices converge to the competitive equilibrium price. The essential difference between the models is in the degree of aggregation. The first model is an aggregate model of the trading process and it contains no explicit theory of individual behavior. The second model provides a description of individual behavior and deduces convergence from the assumed behavior. The assumptions on behavior may be thought of as the result of prior learning about how to behave in the experiment. This model predicts convergence and places strict bounds on the time path of prices and quantities traded prior to convergence. Since this model places stringent bounds on individual behavior its predictions are more often violated by actual trades than are the predictions of the aggregate model. However, the number of violations is amazingly small and the model seems to do a good job of explaining how prices get adjusted.

SECTION II: DEFINITIONS AND MODEL I

The payoffs or values given to the buyers are ranked as $V^1 \geq V^2 \geq \ldots \geq V^n \geq 0$ where V^i is the i-th highest value and there are n units. The costs given to sellers are ranked as $0 \leq M^1 \leq M^2 \leq \ldots \leq M^m$ where M^j is the cost of the j-th unit and there are m units. If a buyer or seller has payoffs for a number of different units we will assume that he decides on strategies for each of them separately. Prices, bids, offers, and contracts are in integer units in the interval $[0,\bar{P}]$ (where $\bar{P} < \infty$ is some arbitrarily selected upperbound above V^1 and M^m) and are indexed by P, b, o and c respectively.

Market periods or days for an experiment are indexed by $d \in \{1,2,\ldots\}$. The time remaining in any given day is indexed by $t \in \{0,1,\ldots,T\}$. In order to insure that an experiment gets started we assume that the highest value, V^1, is greater than the lowest cost, M^1.

Model 1 will be an aggregate model in which trading occurs between randomly slected agents. We assume that the prices at which these trades occur in day d+1 are influenced by prices, bids and offers observed in day d. To this end we first present

Definition: Let c^d be the set of observed contract prices in day d. Let b^d be the set of observed bids in d, and let 0^d be the set of observed offers in d.

(i) $\bar{p}^{d+1} = \text{MAX } \{c^d, b^d\}$

(ii) $\underline{p}^{d+1} = \text{MIN } \{c^d, 0^d\}$, where

$c^o = \emptyset$, $b^o = \{0\}$, $0^o = \{\bar{P}\}$ and $\infty > \bar{P} > \text{MAX } \{V^1, M^m\}$,

(iii) $S^d = [\underline{p}^d, \bar{p}^d] \subseteq R^1$.

Although it is unnecessary for the theory, one can view S^d as the assumed support of buyers' and sellers' expectations about prices in day d + 1. With these expectations, at least early in day d + 1, buyers would be unwilling to pay more than $\bar{p}^d$ and sellers would be unwilling to sell for less than $\underline{p}^d$. Hence we are interested in the collection of buyers and sellers who can trade in S^d at any time t.

Definition: Let $R^d(t) = \{(i,j): \underline{p}^d \leq V^i \leq M^j \leq \bar{p}^d$ and i and j are not traded by t$\}$. Let N(t) be the cardinality of $R^d(t)$.

$R^d(t)$ is the collection of buyers and sellers who can trade in $[\underline{p}^d, \bar{p}^d]$ at time t (subject to the restriction imposed in the experiment that $c^d \leq V^i$ and $c^d \geq M^j$, i.e. no one can make a trade that results in a loss). We shall assume that $T \geq N(T)$; that is, there are enough time periods to enable all feasible trades in $[\underline{p}^d, \bar{p}^d]$ to be made. In Model 1 we assume that trade takes place randomly among the buyers and sellers of $R^d(t)$.

If $R^d(T) \neq \phi$, select $\tau_1 \varepsilon [T, N(T)]$, $(i^*, j^*) \varepsilon R^d(T)$ and $p_1^* \varepsilon [\underline{p}^d, \bar{p}^d]$ such that $M^{j^*} \leq p_1^* \leq V^{i^*}$. The time, pair and price are selected randomly from the sets of feasible times, pairs and prices according to probability distributions on these feasible sets. The only assumption that we make on these probabilities is that each point in the sets of feasible times, pairs and prices has positive probability. We assume that a trade occurs at τ_1, between i^* and j^*, and at price p_1^*. Further, we assume that for all $\tau \geq \tau_1$, $b^d(\tau) \leq p_1^*$ and $0^d(\tau) \geq p_1^*$. Hence some buyers and sellers who can trade at a price in S^d do so and by the time of their trade bids have not risen above the contract price and offers have not fallen below it. Hence, either i^* has made the maximum bid of $b^d(\tau_1) = p_1^*$ and j^* accepts or j^* has made the minimum offer of $0^d(\tau_1) = p_1^*$ and i^* accepts j^*'s offer.

The next trade occurs at some randomly selected feasible price and involves a randomly selected pair of buyers and sellers in $R^d(\tau_1)$. If $R^d(\tau_1) \neq \phi$, select $\tau_2 \varepsilon (\tau_1, N(\tau_1)]$, $(i^*, j^*) \varepsilon R^d(\tau_1)$ and $p_2^* \varepsilon [\underline{p}^d, \bar{p}^d]$ with $M^{j^*} \leq p_2^* \leq V^{i^*}$. Again positive probability of selection is placed on each feasible time, pair and price. A trade occurs at τ_2, between i^* and j^*, and at price p_2^*. Further, for all $\tau \varepsilon (\tau_1, \tau_2]$, $b^d(\tau) \leq p_2^*$ and $\sigma^d(\tau) \geq p_2^*$. This process of random trading continues until there are no more buyers and sellers who can trade in S^d, i.e. until τ where $R^d(\tau) = \phi$.

Finally, we assume that any buyer (seller) who is left without having traded by the time where $R^d(\tau) = \phi$ will eventually raise his bid above $\bar{p}^d$ if $V^i > \bar{p}^d$ (lower his offer below $\underline{p}^d$ if $M^j < \underline{p}^d$). Formally, let τ^* be the first time at which $R^d(\tau) = \phi$, then if there is an untraded i such that $V^i > \bar{p}^d$ we assume that there is a $\tau \varepsilon (\tau^*, 0]$ at which $b^i(\tau) > \bar{p}^d$. We make a symmetric assumption on sellers. Essentially we are assuming that any trader who has been unable to complete a trade in S^d will eventually be willing to trade outside of S^d if he can make a profit by doing so.

This model of the trading process has a number of relatively weak implications for the data generated by market experiments. Let p^e and q^e be respectively the competitive equilibrium price and quantity for the induced demand and supply curves. We assume that there are traders with values at p^e and that the number of buyers with $V^i = p^e$ is equal to the number of sellers with $M^j = p^e$. Under this assumption whenever all contracts take place at p^e we have demand equal to supply. For experiments in which this assumption is satisfied, Model I has four principal implications:

1. If $p^e \varepsilon S^d$, at least q^e units are traded in day d + 1.
2. If $p^e > \bar{p}^d$ ($p^e < \underline{p}^d$) then in day d + 1 there is an offer or contract price below $\bar{p}^d$ (bid or contract price above $\underline{p}^d$).
3. The time of trade h, τ_h, is in $(\tau_{h-1}, N(\tau_{h-1})]$ for each trade h.
4. With probability one there is a finite time at which S^d stabilizes at exactly p^e.

Theorem 1 provides formal statement of implication 4.

<u>Theorem 1</u>: Assume

(i) The trading process of Model I.

(ii) There exists a $\bar{k}$ such that $V^{\bar{k}} = M^{\bar{k}} \equiv p^e$ and $\# \{i:V^i = p^e\} = \# \{j^i:M^j = p^e\}$.

Then with probability one there exists a $d^* < \infty$, such that for all $d \geq d^*$, $S^d = p^e$.

All proofs will be presented in the Appendix.

Although this model yields the principal observable result of convergence it provides little explanation of why convergence occurs. In order to explain how prices are adjusted we need to construct a model of individual behavior.

SECTION III: MODEL II

A subject in an auction has a complex decision problem: he must decide when to bid, how much to bid, and whether or not to accept the trades offered by other subjects. Further, all of these decisions must be made with very imperfect information. The subject does not know the payoffs or strategies of other agents and he does not know the terms of trade that will be available to him in the future. Formally this structure defines a very complex incomplete information game in which the individual must choose bidding and acceptance strategies. We will not attempt to model or solve the individual optimization problem; instead we directly make assumptions about individual behavior. The criteria that we use for the model are; first, that it be reasonably consistent with optimal behavior but simple enough to allow a wide range of "rule of thumb" strategies; second, that it be general enough to allow differing expectations and learning behavior; third, that it be reasonably consistent with the data.

We break the individual decision problem down into three elements. First, the individual is assumed to have expectations about his trading opportunities which are based on the contracts, bids and offers that he observed last period. Second, the individual is assumed to have a reservation price strategy which for a buyer gives the maximum offer price that he will accept. Third, individuals are assumed to have bidding strategies which for a buyer gives the maximum bid that the buyer would like to enter.

The reservation price and bidding strategies are obviously dependent on the individuals' expectations and will be adjusted over time based on how successful they are. Any buyer who observes that he has paid the highest price in a market period should learn to reduce both his reservation price and his bids. Similarly any buyer who was unable to make a purchase, i.e.

was rationed-out, should raise his reservation price if possible. Sellers should behave symmetrically in adjusting to having sold for the lowest price or having been rationed-out of the market. Rather than formally describing the learning process outlined above we will describe reservation price and bidding strategies that could be the outcome of such a learning process. The following assumption specifies the key unobserved primitives of our theory of individual behavior.

Assumption 0: For each untradedunit of a buyer $(i = 1,\ldots,n)$ or seller $(j = 1,\ldots,m)$ there is an (unobservable) reservation price at each d and t which is denoted by $v_d^i(t) \varepsilon R^1$ for buyers and $w_d^j(t) \varepsilon R^1$ for sellers.

We next formulate two assumptions which specify bidding behavior given reservation prices and reservation price behavior given observable data.

Assumption 1: Bidding and Contracts

(i) $b^d(t)$, the current outstanding bid in day d with time t left, is held by a buyer i* where $v_d^{i*}(t) \geq v_d^i(t)$, for all untraded $i = 1,\ldots,n$.

(ii) $b^d(t) \leq v_d^{i*}(t)$

(iii) $b^d(t) \geq v_d^i(t)$, for all $i \neq i^*$ with i untraded.

(iv) i* accepts the current outstanding offer, $0^d(t)$, if and only if $0^d(t) \leq v_d^{i*}(t)$. No other i accepts $0^d(t)$. (Notice that i* is a function of d and t.)

This assumption is really the reduced form of an assumption that auctions behave as English auctions and that action occurs sufficiently smoothly so that "surprises" don't occur. In particular, we are assuming that if buyer i is willing to pay $v_d^i(t)$, and the outstanding bid $b(t)$ is less than that and is not held by i, then i should offer a higher bid. Any buyer who does not do so takes a chance of being rationed out even though he is willing to pay more than the current high bid. Hence bids should quickly rise to $\underset{i \neq i^*}{\text{MAX}} \{v_d^i(t)\}$

and i* should hold the highest bid. We further assume that only this highest bid is accepted. Since i* has the highest reservation price he will be the only buyer to accept an offer provided that offers fall smoothly. That is if the offers start above $v_d^{i^*}(t)$ and do not jump down to $\max_{i \neq i^*} \{v_d^i(t)\}$ or less then only i* can accept an offer.

If reservation prices change smoothly over time, parts (iii) and (iv) of Assumption 1 are not likely to be violated at any time other than the beginning of a market day. At t = T it is possible to have a number of buyers with reservation prices above the reservation prices of some sellers. In this case, even if bids follow the pattern of Assumption 1, trading need not be between the highest reservation price buyer and the lowest reservation price seller. However, such overlap of reservation prices can occur only at the beginning of the market day and any perverse trading that results will not affect convergence if only infra-marginal units trade.

We make assumptions on the reservation price and bids of sellers that are symmetric with those made for buyers. The inequalities are reversed in the obvious way.

<u>Assumption 1'</u>: Offers and Contracts

(i) $0^d(t)$, the current outstanding offer in day d with time t left, is held by a seller j* where $w_d^{j^*}(t) \leq w_d^j(t)$, for all j = 1,...,m, for all untraded j.

(ii) $0^d(t) \geq w_d^{j^*}(t)$.

(iii) $0^d(t) \leq w_d^j(t)$, for all $j \neq j^*$, j untraded.

(iv) j* accepts $b^d(t)$ if and only if $b^d(t) \geq w_d^{j^*}(t)$. No other j accepts $b^d(t)$.

The assumptions made to this point in Model II place no restrictions on the data since for any observed sequence of bids, offers, and acceptances, one can construct reservation prices consistent with Assumption 1 and those observations. Therefore, in order to have a theory, we must connect the reservation prices to the observable data for each i and j. This link is made in Assumption 2.

Assumption 2: Reservation Price Formation

For all $i = 1,\ldots,n$:

(i) $v_d^i(t) \leq V^i$ for all t,d.

(ii) There exists $\hat{t}_d^i > 1$ such that;

(a) For all $t \geq \hat{t}_d^i$, if $\underline{p}^d \leq V^i$ then $v_d^i(t) \in S^d$

(b) For all $t \geq \hat{t}_d^i$, if $\bar{p}^d - \underline{p}^d > 1$ then $v_d^i(t) < \bar{p}^d$

(c) $v_d^i(t) = \text{MIN}\ \{\bar{p}^d - 1, V^i\}$ for all t, $\hat{t}_d^i \geq t \geq \hat{t}_d$

where $\hat{t}_d = \text{MAX}\ \{t | b^d(t) \geq \bar{p}^d - 1$ and $b^d(t)$ unaccepted$\}$, if this maximum exists, $\hat{t}_d = 0$ otherwise.

(d) if $\hat{t}_d > 0$ then $v_d^i(t) = \text{MIN}\ \{\bar{p}^d, V^i\}$ for all t, $\hat{t}_d > t \geq \bar{t}_d$, where $\bar{t}_d = \text{MAX}\ \{t | b^d(t) \geq \bar{p}^d$ and $b^d(t)$ unaccepted$\}$, if this maximum exists, $\bar{t}_d = 0$ otherwise.

(e) if $\bar{t}_d > 0$, then $v_d^i(t) \geq \text{MIN}\ \{\bar{p}^d + 1, V^i\}$ for all t, $\bar{t}_d > t \geq 0$.

(iii) (ranking) Let $k^* = \text{MIN}\ \{k | V^k < M^k\}$. For all d, t if $V^i > V^{k^*}$, $V^i \geq \bar{p}^d$, $V^k \leq V^{k^*}$, and $V^k \leq \bar{p}^d$, $v_d^i(t) > v_d^k(t)$ whenever $\bar{p}^d - \underline{p}^d > 1$.

Assumption 2 (i) is that no buyer can be willing to pay more than the true value for any unit. Assumption 2 (ii)(a) would be a natural result of having expectations with a support of $S^d(t)$. Part 2 (ii)(b) requires the buyer to not be willing to pay $\bar{p}^d$ or more too early in the day (i.e., for $t \geq \hat{t}^i$). Any buyer who deviates from this assumption will be willing to pay the highest price; may end up doing so; and hence should learn to reduce his reservation price next time. If there is a buyer who has $v_d^i(t) \geq \bar{p}^d$ and who does not learn, he may continually pay the highest price and hence prevent convergence to the competitive equilibrium.

Assumption 2 (iii) is a minimal assumption on the relationhip of reservation prices to true values. If $V^{k*} < M^{k*}$, then k* is larger than the competitive equilibrium quantity, or what is more important for our purposes, the maximum quantity that can be transacted at any single price. The assumption is that buyers with values above V^{k*} and above $\bar{p}^d$ have higher reservation prices than those with values at or below V^{k*}, provided that $||S^d|| > 1$. Any such buyer who does not have $v_d^i(t)$ high enough to keep the potential extra marginal buyers (those with $V^i \leq V^{k*}$) out of the market may either be rationed out of the market or be forced to pay a high price at the end of the day in order to trade with an extra marginal seller. Such a buyer should learn to raise his reservation price for the early part of the market day. Any buyer who does not do so will either frequently be rationed and hence frequently move the market away from p^e or will end up paying a price above $\bar{p}^d$ at the end of the period and hence keep the upper limit of the contract prices from falling to the competitive equilibrium.

Part (ii) (c-e) is a sequence of assumptions requiring that the buyers eventually cave in to $\bar{p}^d$ and above but that they not cave in too quickly. Part (c) requires that for some $\hat{t}^i > 1$ buyer i must cave in to $\bar{p}^d - 1$, if possible, provided that he has not yet traded. Presumably a buyer holds out for a low price early in the day but eventually he should be willing to pay higher and higher prices as the end of the day approaches. This characteristic will result from a finite horizon search problem. Parts (d) and (e) require that near the end of the day buyers who see their bids of $\bar{p}^d - 1$ or $\bar{p}^d$ rejected must increase their reservation prices if it is possible for them to do so. Buyers who do not learn to do so may end up being continually rationed out and not signalling to the market that the prices are too low.

If this happens prices can get stuck below the competitive equilibrium. Buyers who cave in too soon, that is, raise $v_d^i(t)$ before a bid is rejected may end up paying an unnecessarily high price and should learn to reduce their reservation prices. Again if such buyers do not learn then $\bar{P}^d$ can get stuck above the competitive equilibrium.

We make a symmetric assumption for sellers and call that Assumption 2'. Whether the assumptions of Model II on individual behavior are consistent with rational maximizing behavior or not is an open question. We believe that these assumptions result in a reasonable description of the actual behavior that occurs in the experimental markets that we have examined. In Section V a number of testable implications that result from this model of behavior are confronted with data from a group of nine market experiments.

Figure 1 illustrates a set of time paths for reservation prices, bids and offers that is consistent with our assumptions. In this example buyer 1 and seller 2 contract the first unit at t_1 since 1's acceptance price is exceeded by 2's offer at this time. At t_2, seller 1 accepts buyer 2's bid and at t_3 buyer 3 accepts seller 4's offer. Note that buyer 4 and seller 3 may not trade since there are no competitive pressures driving their bids and offers together.

Figure 2 illustrates the observable data from figure 1. This type of pattern is consistent with the data in experiments.

INSERT FIGURE 1

INSERT FIGURE 2

FIGURE 1

Day d

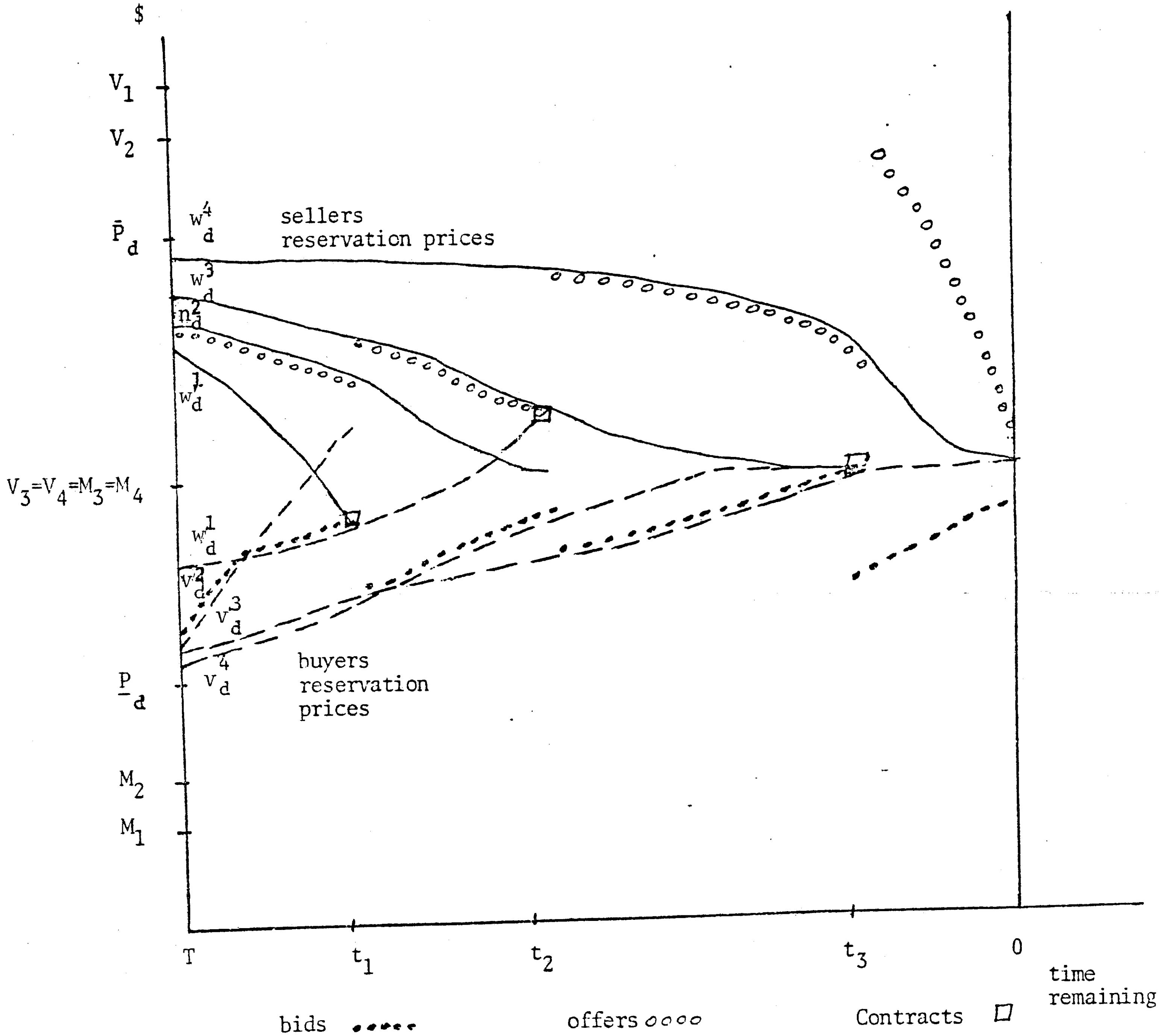
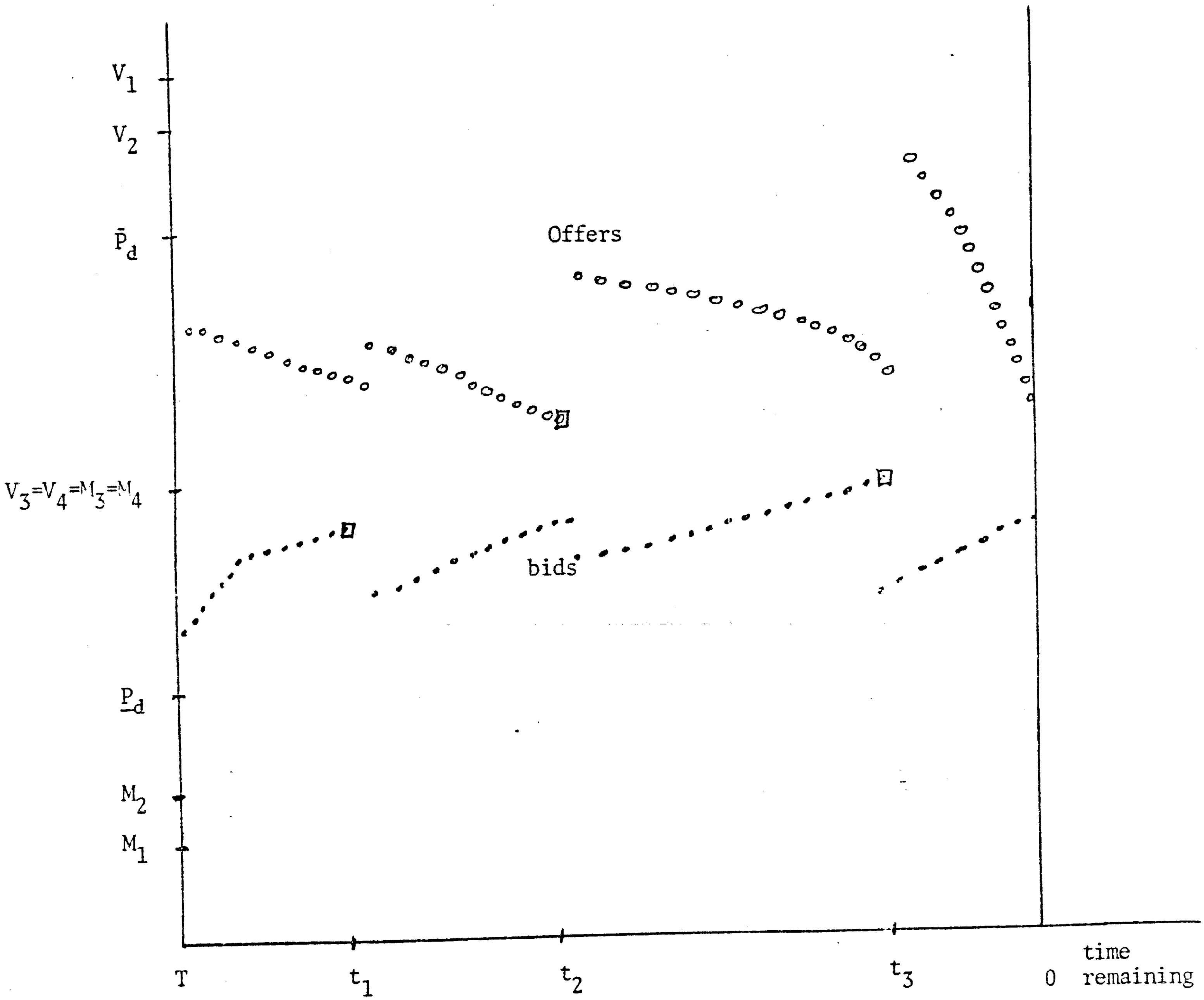


FIGURE 2

Observations Day d



SECTION IV: THEOREMS FOR MODEL II

In this section we trace through some of the implications of Model II. As will become apparent, most of the action will occur when there is an "excess demand or supply" of two or more units remaining in the auction since in this case there are competitive pressures on bids and offers. Thus we are interested in the following concept.

<u>Definition</u>: Let $D(P) = \# \{V^i \geq P\}$ and $S(P) = \# \{M^j \leq P\}$. We let $P^* = \inf \{P|S(P) - D(P) \geq 2\}$ and $P_* = \sup \{P|D(P) - S(P) \geq 2\}$.

To see the role of P^* and P_* we consider the following propositions.

<u>Lemma 1</u>: For Model II:

(a) If $\bar{P}^d < P_*$ then $\bar{P}^{d+1} > \bar{P}^d$.

(b) If $\underline{P}^d > P^*$ then $\underline{P}^{d+1} < \underline{P}^d$.

<u>Lemma 2</u>: For Model II:

(a) If $\bar{P}^d \geq P_*$ then $\bar{P}^{d+1} \geq P_*$.

(b) If $\underline{P}^d \leq P^*$ then $\underline{P}^{d+1} \leq P^*$.

Thus there are competitive forces driving the maximum contract prices above P_* and the minimum contract prices below P^*. There are also competitive pressures driving the maximum prices down and the minimum prices up. These pressures are exerted by extra-marginal units as can be seen in the following propositions.

<u>Lemma 3</u>: In Model II, suppose $\bar{P}^d - \underline{P}^d > 1$:

(a) If $\bar{P}^d > M_{k*}$ then $\bar{P}^{d+1} < \bar{P}^d$.

(b) If $\underline{P}^d < V_{k*}$ then $\underline{P}^{d+1} > \underline{P}^d$.

At this point we know that the maximum price paid will fall if it is greater than M^{k^*} and increase if it is less than P_*.[1] In order to establish that "prices converge to an equilibrium" we must consider what occurs when $P_* \leq \bar{P}^d \leq M^{k^*}$. Unfortunately, under the assumed behavior of Model II, it is possible for prices to bounce around inside this interval and, in some cases, to bounce outside. The case where this won't happen is described in the next lemma.

<u>Lemma 4</u>: For Model II:

(a) If $V^{k^*} \leq \bar{P}^d \leq M^{k^*}$ then $\bar{P}^{d+1} \leq M^{k^*}$.

(b) If $V^{k^*} \leq \underline{P}^d \leq M^{k^*}$ then $\underline{P}^{d+1} \geq V^{k^*}$

Notice that if $P_* \leq \bar{P}^d \leq V^{k^*}$ then it is possible that $\bar{P}^{d+1} > M^{k^*}$. Thus we cannot insure, under our model, that "convergence to equilibrium" occurs for all demand-supply configurations. However, we can say something for most configurations and, in particular, for those which have been used in the experiments reported in the next section.

<u>Theorem 2</u>: For Model II

If $M^{k^*} \geq P^* \geq P_* \geq V^{k^*}$ then there exists d^* where $0 < d^* < \infty$ such that for all $d \geq d^*$:

(a) $S^d \subseteq [V^{k^*} - 1, M^{k^*} + 1]$

(b) $\bar{P}^d \geq P_*$, $\underline{P}^d \leq P^*$

and

(c) $\bar{P}^d - \underline{P}^d \leq 1$.

(That is, prices converge to some "small" interval contained by the extra-marginal units and centered by competitive pressures around P^* or P_*).

1. It is as if Walras' auctioneer were setting $\bar{P}^{d+1}$ and $\underline{P}^{d+1}$.

Further, the number of contracts q^d in each day $d > d^*$ is constrained by $q^e - 1 \leq q^d \leq q^e$ where $q^e = \max\{k | V^k \geq M^k\}$ is the Walrasian equilibrium quantity.

For purposes of completeness we should note that convergence is <u>not</u> assured (although possible) when either $P^* > M^{k^*}$ or $V^{k^*} > P_*$. The following configuration is consistent with this situation.

INSERT FIGURE 3

A more precise characterization of convergence can be had if one narrows the preference configuration somewhat.

<u>Corollary 1</u>: If $M^{k^*} \geq P^* = P_* \geq V^{k^*}$ then $\exists d^*$ with $0 < d^* < \infty \ni \forall\, d \geq d^*$

(a) $S^d \subseteq [P_e - 1, P_e + 1]$

and

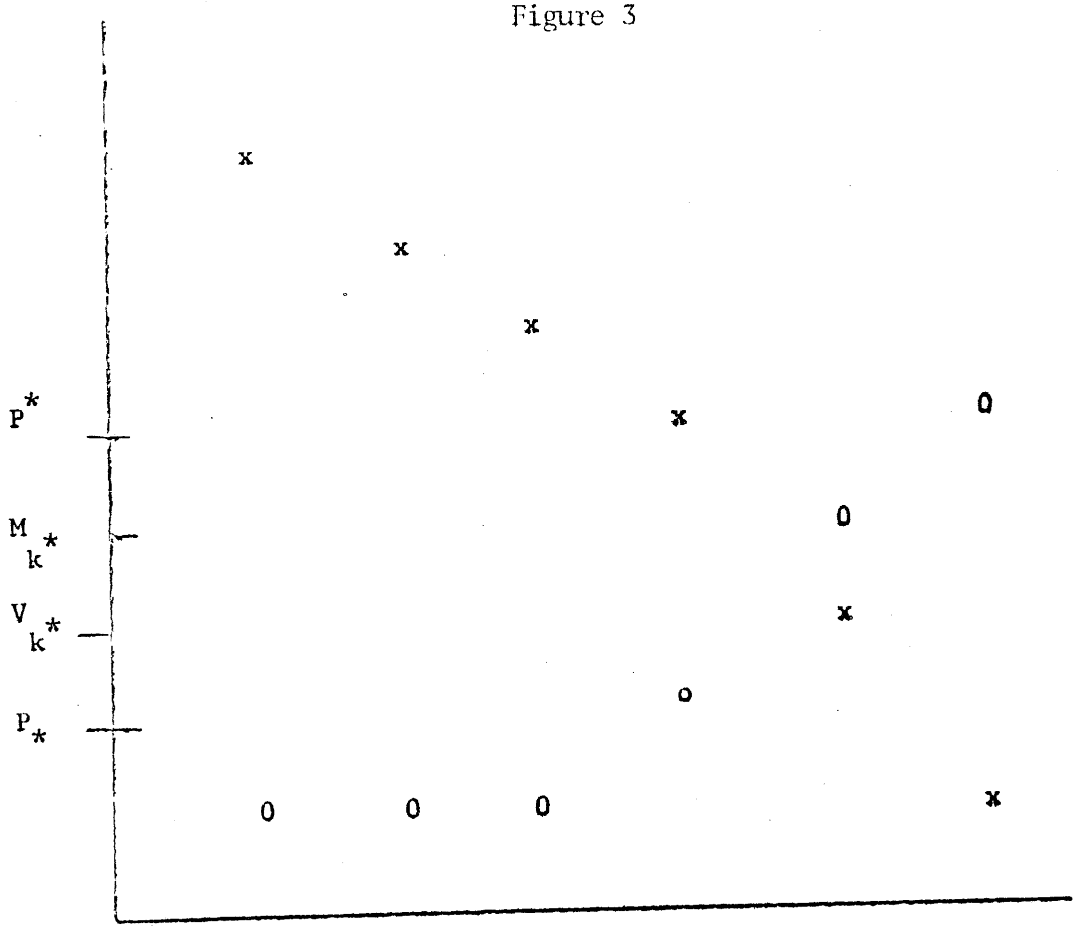
(b) $\bar{P}^d \geq P_e$, $\underline{P}^d \geq P_e$

where $P^e = P^* = P_*$ is the Walrasian equilibrium price.

There are at least two classes of experiments which have been reported for which the hypothesis of the corollary is true. If there are at least two marginal units on each side of the market, (i.e., $V^{k^*-2} = V^{k^*-1} = M^{k^*-2} = M^{k^*-1}$) then $P^* = P_*$.

Another case in which Corollary 1 applies occurs in the so-called "swastika" experiments. On these $V^1 = V^2 = \ldots = V^n > M^1 = \ldots = M^m$ and either $M \geq N + 2$ or $N \geq M + 2$. If $N = M + 1$ or $M = N + 1$, then Corollary 1 would not apply.

Figure 3



[x are V^i, 0 are M^i.]

SECTION V: COMPARISON OF PREDICTIONS WITH THE DATA

The prediction of convergence, which results from both models, is not testable since the number of necessary replications d^* may be large. Hence in order to compare the models with the data it is necessary to describe the potential dynamics implied by the theories. There are four categories of data for which one or both of the theories have implications: the sequence of contract prices, the sequence of trading partners, the total quantity traded, and the behavior of untraded units at the end of the day. We will carry out the analysis of the dynamics for Model I under an assumption that $\# \{i:V^i = p^e\} = \# \{j:M^j \cdot p^e\} \neq 0$. This assumption is satisfied in all but one of the experiments for which we have data. In terms of the variables used to discuss convergence for Model II a similar assumption is $M^{k^*} \geq P^* \geq P_* \geq V^{k^*}$. This assumption is implied by the assumption to be used for Model I, but they are not equivalent. There is one experiment (PDA 22) in which this condition is satisfied but the condition used for Model I is violated. For all of the other experiments for which we have data both conditions are satisfied and hence violation of the predictions of the alternative models are directly comparable.

MODEL I:

The implications from Model I can be represented in two cases.

<u>Case 1:</u> $\bar{p}^d < p^e$

(a) First, $q^d = S(\bar{p}^d)$ contracts occur at all prices $c^d \in S^d$.

(b) Second, after the q_d contracts there is a bid above $\bar{p}^d$.

<u>Case 2:</u> $\bar{p}^d \geq p^e$

(a) First, $q^d = \text{Min}\ \{S(\bar{p}^d), D(\underline{p}^d)\}$ contracts occur at prices $c^d \in S^d$.

(b) There are no implications for bids or offers after the q^d trades.

The implications for $\underline{p}^d$ are symmetric. Model I has implications for prices, quantities, and the final bids (or offers) if $\bar{p}^d < p^e$ (if $\underline{p}^d > p^e$); but it has no implications on the sequence of trading.

MODEL II:

Model II has much more precise implications than does Model I. The implications of Model II for prices, total quantity, and the sequence of trading can be represented in three cases.

<u>Case 1(a)</u>: $\bar{p}^d < P_*$, $\bar{p}^d < V^{k*}$.

(1) First all $M^i < \bar{p}^d$ trade with some $V^i \geq \bar{p}^d$ at prices less than $\bar{p}^d$ if $||S^d|| > 1$ or at prices $\bar{p}^d$ if $||S^d|| < 1$.

(2) Then all $M^i = \bar{p}^d$ trade with some $V^i \geq \bar{p}^d$ at $\bar{p}^d$.

(3) At this point further trades may occur, at prices above $\bar{p}^d$, but this is not nessary. However, there will be at least one bid above $\bar{p}^d$.

(4) Finally, note that the number of contracts is $q_d \geq S(\bar{p}^d)$.

<u>Case 1(b)</u>: $\bar{p}^d < P_*$, $\bar{p}^d \geq V^{k*}$.

(1) First all $M^i < \bar{p}^d$ trade with some $V^i > V^{k*}$ at prices below $\bar{p}^d$ if $||S^d|| > 1$ and at prices less than or equal to $\bar{p}^d$ if $||S^d|| \leq 1$.

(2) Next all $M^i = \bar{p}^d$ trade with some $V^i > \bar{p}^d$ at $\bar{p}^d$.

(3) Same as case 1(a).

(4) Same as case 1(a).

Case 2: $\bar{P}^d > M^{k*}$

(1) All trades occur at prices below $\bar{P}^d$ if $||S^d|| > 1$ or at prices less than or equal to $\bar{P}^d$ if $||S^d|| \geq 1$.

(2) All $V^i \geq \bar{P}^d$ trade.

(3) All $V^i \geq \bar{P}^d$ trade before those $V^i \leq V^{k*}$.

(4) q_d depends on $\underline{P}^d$ as follows: (a) if $\underline{P}^d \leq M^{k*}$ then $q_d \geq q_e - 1$, (b) if $\underline{P}^d > M^{d*}$ then $q_d \geq S(P^d)$.

Case 3: $M^{k*} \geq \bar{P}^d \geq P_*$

(1) All contract prices are no more than $\bar{P}^d$ and at least one is at or above P_*

(2) All $V^i \geq \bar{P}^d$ trade before those $V^k \leq V^{k*}$.

(3) If $\underline{P}^d \geq V^{k*}$ all trades are at prices below $\bar{P}^d$ if $||S^d|| < 1$ and at prices $\leq \bar{P}^d$ if $||S^d|| \leq 1$.

(4) If $\underline{P}^d < V^{k*}$, all initial trades (at least k*-2) are at prices below $\bar{P}^d$ if $||S^d|| > 1$ and at prices no more than $\bar{P}^d$ if $||S^d|| \leq 1$. The (k* + 1)st trade can occur at a price ε $[\underline{P}^d, M^{k*}]$.

(5) $q^e - 1 \leq q_d \leq q^e$.

The analysis of each case follows from the earlier lemmas. Case 1 comes from lemma 1, Case 2 from lemma 3 and Case 3 from lemmas 2 and 4. Symmetric results hold for $\underline{P}^d$.

Model II also has more precise implications for final bids and offers. Although reservation prices are unobservable, A.2(ii) does imply that, near the end of each day, reservation prices of untraded units should be equal to true values if those values are less than $\bar{P}^d$ for buyers or greater than $\underline{P}^d$ for sellers. Thus, in strict terms at t = 0 our theory implies

(a) $b_d(0) \geq V^{(2)}$, $0_d(0) \leq M^{(2)}$

and

(b) $V^{(1)} < 0_d(0)$, $M^{(1)} > b_d(0)$

where $V^{(1)}$ is the largest non-traded value and $V^{(2)}$ is the next highest. Implication (a) follows from the fact that $v_d^i(0) = V^i$ and A.1 on bidding. Implication (b) follows from the fact that $v_d^i(0) = V^i$. If $0_d(0) \leq V^{(1)}$ then buyer (1) should accept at t = 1.

Data:

The following table summarizes the number of violations of the predictions of the two models as a percentage of total possible predictions for nine Plato experiments for which we have data.

	Exp	Marg	Com	Unit	Qe	NY	Que	Model I Prices	Model I Quantity	Model I Dayend	Model II Prices	Model II Sequence	Model II Dayend
I PDA8	N	1	5	8	6	Y	N	18	9	25	17	4.2	22
I PDA9	N	1	5	8	6	Y	N	10	6	0	22	11.1	18.8
I PDA10	Y	1	5	10	8/6	Y	N	10	3	0	16.5	0	6.2
I PDA11	N	1	5	10	8	Y	N	4	4	-	12.7	2.8	18.8
I PDA14	Y	1	5	8	6	Y	N				16.1	4.1	12
II PDA14	N	3	10	21	15	Y	N				2.9	16.7	0
II PDA57	N	3	10	21	15	Y	Y				6.5	10	33.0
II PDA22	N	0	10	16,11	11	N	N				9.3	NA	0
II PDA25	Y	2	10	12	7	N	Y				16.7	9.1	50.0
					TOTALS			10	5	17	11.4	7.9	18

{Note: exp Y means experienced subjects, marg = number of marginal units, com = commission in cents, NY = NYSE rules, Que = queing of bids and offers].

To see the price violations in perspective the following table illustrates the margin of error.

Errors of x ¢ or less not counted

	Model I	Model II
x = 1	9 %	5.2 %
x = 5	5 %	4 %
x = 10	2 %	2 %

To put the dayend errors in context, there are two major violators. In II PDA57, seller 2 consistently held to a high reservation price on a marginal unit and, in fact, lost money on this strategy. No other dayend errors occured in II PDA57. Second, the experiment II PDA25 had a time queue in which bids entered are accepted chronologically and leave the queue at 3-second intervals. This institution clearly destablizes bidding. If we could ignore these two pathologies, the total percentage dayend errors for Model II would be 7.5%.

SECTION VI: CONCLUSION

The two models presented here are deterministic and, although they do not describe precise paths of bids, offers, and contracts, they do place fairly tight bounds on these data. One observation not in accord with these bounds is grounds for rejection of these theories and, in fact, there are a number of such observations. However, the percentage of observations which violate the crucial implications of the theories is, we feel, amazingly low. The implications of Model I are violated less often than the implications of Model II for two reasons. One is simply that Model I has weaker implications for the time paths of prices and trades than does Model II. Secondly, Model I provides almost no explanation of which buyers and sellers trade so it has no implications for the sequence of trading. Hence, although there are more violations of the implications of Model II, we feel that it contains a useful theory as it provides an explanation of how and why prices and quantities adjust to their competitive equilibrium values.

The potential importance of this theory is not that it seems to describe what happens in DOA experiments, but that it is the beginning of a theory of how market prices are formed and of how they adjust to changes in demand and supply conditions. The question of price formation has a long history of ad-hoc and unsuccessful attempts at an answer. Our theory is also ad-hoc in the sense that we directly make assumptions on individual behavior which are not derived from an optimizing model. However, our assumptions seem reasonably consistent with rational behavior and more importantly they seem to do a reasonable job of describing actual behavior.

APPENDIX: PROOFS

Proof of Theorem 1:

We first show: (1) If $S^{d^*} = p^e$ then $S^{d^*+n} = p^e$ for all $n = 1,2,\ldots$. First consider day d^*+1. Since $S^{d^*} = p^e$ all contracts at any t before τ where $R^{d^*+1}(\tau) = \phi$ must occur at p^e. Now since $S^{d^*} = p^e$ we have $R^{d^*+1}(T) = \{(i,j): V^i \geq V^{\bar{k}} \text{ and } M^j \leq M^{\bar{k}}\}$. Hence by (ii), $\#\{i: (i,j) \in R^{d^*+1}(T)$, for some $j\} = \#\{j: (i,j) \in R^{d^*+1}(T)$, for some $i\}$. Since all trades consist of one buyer and one seller we must have $\#\{i: (i,j) \in R^{d^*+1}(t)$, for some $j\} = \#\{j: (i,j) \in R^{d^*+1}(t)$, for some $i\}$, for any t, $T \geq t \geq 0$. Therefore if $R^{d^*+1}(\tau) = \phi$ all i with $V^i > V^{\bar{k}} = p^e$ and all j with $M^j < M^{\bar{k}} = p^e$ are traded. Then by the signalling assumption of Model I no bids above p^e or offers below p^e will result. Therefore $\bar{P}^{d^*+1} = \text{MAX}\{c^{d^*+1}, b^{d^*+1}\} = p^e$ and $\underline{P}^{d^*+1} = \text{MIN}\{c^{d^*+1}, \sigma^{d^*+1}\} = p^e$ and $S^{d^*+1} = p^e$. The same argument clearly works for d^*+n for arbitrary n.

Secondly, we show: (2) If $p^e \in S^d$ then Prob $\{S^{d+1} = p^e\} > 0$. If $p^e \in S^d$ then it follows from the trading process assumption that exactly $\bar{k}$ trades, all at price p^e, and between buyers $i = 1,2, \ldots , \bar{k}$ and sellers $j = 1, 2, \ldots , \bar{k}$ has positive probability. If this event occurs then no further trades at any price other than p^e can result and there can be no bids above p^e or offers below p^e. Hence, if this event occurs $S^{d+1} = p^e$ and we have Prob $\{S^{d+1} = p^e\} > 0$ from the trading process assumption.

Finally, we show: (3) If $p^e \notin S^d$ then there exists a finite integer $\bar{d}$ (indeoendent of d) such that $p^e \in S^{\hat{d}}$, for some $\hat{d} \leq d + \bar{d}$. It is sufficient to show that if $\bar{P}^d < p^e$ then $\bar{P}^{d+1} > \bar{P}^d$ and similarly if $\underline{P}^d > p^e$ then $\underline{P}^{d+1} < \underline{P}^d$, since there are a finite number of possible prices $(\bar{P})$. If $\bar{P}^d < p^e$, then $\#\{i:(i, j) \in R^d(T)$, for some $j\} > \#\{j: (i, j) \in R^d(T)$, for some $i\}$. Hence, it follows from the assumption on the trading process that there exists a $\tau > 0$ with $R^d(\tau) = \phi$ and since all trades are two sided there is at least one buyer $\hat{i}$ with $V^{\hat{i}} > \bar{P}^d$ and $\hat{i}$ untraded. Then by the signalling assumption there will be a bid above $\bar{P}^d$. Hence, $\bar{P}^{d+1} > \bar{P}^d$. The argument for $\underline{P}^{d+1} < \underline{P}^d$ if $\underline{P}^d > p^e$ is symmetric.

Now let d^* be the first d at which $S^d = p^e$. We have $p^e \varepsilon S^d$ infinitely often by (3) and Prob $\{S^{d+1} = p^e\} > 0$ if $p^e \varepsilon S^d$ by (2). Therefore, d^* is finite with probability one. Then it follows from (1) that for all $d \geq d^*$, $S^d = p^e$.

Proof of Lemma 1: We show (a); the proof of (b) is symmetric. If all contracts are at pricess less than or equal to $\bar{P}^d$ then there exist at least two $V^i > \bar{P}^d$ which are untraded. By A.2(ii) at some time before t = 0 there will be a bid b greater than or equal to the second highest $v^i_d(t) > \bar{P}^d$.

Proof of Lemma 2: We show (a); the proof of (b) is symmetric. If all contracts are less than P_* then there exist at least two untraded V^i such that $V^i \geq P_*$. Thus there will be a bid $b \geq P_*$ before t = 0.

Proof of Lemma 3: We show (a); the proof of (b) is symmetric. We will show that whenever there is an untraded $V^i \geq \bar{P}^d$, there will be an offer $0 < \bar{P}^d$ which i can accept. Thus no contracts at prices $\geq \bar{P}^d$ will be seen.

Suppose there are r untraded V^i with $V^i \geq \bar{P}^d$. Since all $V^i \geq \bar{P}^d$ must be traded before any $V^k \leq V^{k^*} < M^{k^*}$, by A.2(iii), there must be at least r + 1 untraded M^i with $M^i \leq M^{k^*} < \bar{P}^d$. If $r \geq 1$, then at some time before $\hat{t}$ there will be an offer $0 < \bar{P}$ which some $V^i > P^d$ will accept. If r = 0, then there are no $V^i \geq \bar{P}^d$ remaining.

Proof of Lemma 4: We show (a); the proof of (b) is symmetric. In order to have $\bar{P}^{d+1} > M^{k^*}$ there was a $V^i > M^{k^*}$ who at $\hat{t}_d$ of day d bids above or accepted a contract at a price above M^{k^*}. If this were true then at $\hat{t}_d$, since $\bar{P}^d \geq V^{k^*}$ and by A.2(iii), there were at most $k^* - 2$ contracts. Thus there exist untraded $M^i \leq M^{k^*}$. Therefore there was an offer $0 \leq M^{k^*} < V^i$ which i would have accepted. Thus there was no such V^i.

Proof of Theorem 2: The proof of Theorem 2 follows directly from Lemmas 1 through 4 and the observation that there are a finite number of possible prices, $0, 1, \ldots, \bar{P}$.

Proof of Corollary 1: The proof of Corollary 1 follows directly from Theorem and the observation that if $P_* = P^*$ then $P_* = P^* = p^e$.

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