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EQUILIBRIUM CONTRACTS IN A MONETARY ECONOMY

by

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1. INTRODUCTION

There has been much recent work done on implicit contracts particularly in labor markets.[1] The insights gained from these models have not thus far been incorporated into macroeconomic models which explicitly recognize that contractual arrangements arise from maximizing behavior on the part of agents. A distinctive feature of many theoretical macroeconomic models has long been the emphasis placed on temporary rigidities in wages and prices. It is sometimes claimed (Fischer (1977)) that implicit contract theory provides a basis for these assumptions. Using results obtained in one context in an entirely different model can be a dangerous practice. This paper is a modest start at incorporating contracts in a monetary model.

The explicit incorporation of contracts as well as money into a model with optimizing agents presents many technical and some conceptual problems. The primary conceptual feature of money that is incorporated into this model is that when transactions take place in spatially separated markets some institutional mechanism is needed so that the different trades may be co-ordinated. Specifically, think of an economy with a large number of firms and an equally large number of agents. Each agent is endowed with labor services and each firm has access to a technology for producing a good unique to the firm. The agent derives utility from the consumption of a composite basket of the goods produced in the economy. The technology for producing each good is subject to random shocks in each period. Think of the agent in this economy as a shopper-worker pair. Each "day" the worker sells his or her labor services to a firm while the shopper spends the day purchasing different amounts of the goods depending on the relative prices on that day. Clearly, then, the value of the worker's services must, in equilibrium, be equal to the value of the shopper's purchases.[2] It is difficult, however, to see how an equilibrium like this can be implemented. About the only way one could think of this is that each firm reports to a central authority on the exact quantity of goods purchased and labor services sold by each agent. The adoption of a social institution such as money would greatly facilitate exchange in this economy. Then, agents could be endowed with claims to consumption. The shoppers would presumably sell these claims to firms in exchange for goods and the workers would receive claims in exchange for labor services. One can easily conceive of situations where this institutional mechanism would dominate a record-keeping procedure.

To continue with this example further, think of a situation where there are substantial costs for the shopper of returning to or communicating with the worker about the various relative prices that have been encountered. Suppose also that the worker must agree to a contract with the firm specifying the wage and labor supply prior to knowing about prices elsewhere in the economy. Money here plays a crucial role in permitting one trade to be realized before other trades are made. To be sure, any security or durable good would perform the same function. However, the fact that the shopper may purchase goods from a large number of firms or that the worker may change firms practically mandates that the durable good in the economy be acceptable to all agents: namely that it be money. While nothing has been said about how this institutional mechanism may come into being, it is clear that nothing in the foregoing implies either that the money in question be fiat money or, for that matter, commodity money. What it does imply is that there is a good which is universally exchanged for other goods and services. Thus, money in this model obviates some of the problems that arise from moral hazard. If, indeed, money is exchanged for goods and goods are exchanged for money but goods are not exchanged for goods it is not immediately apparent how this notion can be incorporated into the individual agent's decision problem. One formalization, due originally to Clower (1967) and used by Lucas (1979) is to impose a constraint on the nominal value of expenditures by an agent. In this construction, it is assumed that the nominal value of expenditures cannot exceed the nominal quantity of money carried over from the previous period by the agent. In the context of the present model, that is tantamount to assuming that the worker is paid for labor services at the end of the day. We see, therefore, that money allows certain trades to be consummated before other trades are completed. In this sense, therefore, one can talk about wages and employment being "predetermined" with respect to prices. The labor contract is agreed to, prior to the agent knowing about relative prices or the general price level.

The contracts discussed in this paper arise out of maximizing behavior on the part of agents as well as the presence of asymmetric information. Firms are assumed to know the shocks to technology that affect them. Worker-consumers, however, do not. This, of course, requires that the workers and firms agree to a schedule relating labor supply to compensation prior to the revelation of the technology shock. It is this schedule that we will refer to as a contract. Given the asymmetry in the information structure the contract must induce truth-telling on the part of the firm. In an environment where there is asymmetric information, the allocations achieved are quite different from those achieved by a competitive mechanism. In this sense the model could be said to capture elements of price-setting behavior where one party sets both the price and quantity to be traded. It is my belief that such arrangements, frequently observed in the "real world" are indeed manifestations of such a contractual arrangement between traders. The literature on such arrangements has grown increasingly rich and diverse (Harris and Townsend (1977), Harris and Raviv (1978), Prescott and Townsend (1979)) and bears important implications for macroeconomic theory. This paper is an attempt to incorporate such arrangements into macroeconomic models. An interesting feature of such contractual arrangements which emerges in this model as well as that once the information is made available there are gains to trade. These gains are, however, never exercised simply because the possibility of making such ex-post trades would induce deviation from truth-telling behavior and render the resulting allocations suboptimal. One could potentially describe such "off the contract curve" allocations within the present context as "involuntary overemployment" in the sense that at the prevailing wage rate, workers are willing to work less hours, firms are willing to employ less workers but both parties are constrained from acting on these desires by the contractual arrangement. In passing it must be noted that the kinds of contracts discussed in this paper are not the much-ballyhooed contracts between General Motors and the UAW but rather the implicit, unstated "rules of good behavior" that govern the employers and employees in most firms.

In section 2, we lay out a model which captures some of the features described in the discussion above. The model describes a stationary equilibrium with a constant price level. There is no aggregate uncertainty but there is uncertainty at the individual level since the firms to which workers supply labor services are subject to random shocks to their technology. Only firms observe the technology shock. Sections 3 and 4 deal with the equilibrium concept and are aimed at establishing the existence of an equilibrium. Section 5 concludes the paper.

2. THE MODEL

There are two types of agents in the economy: consumer-workers and firms. Workers supply labor to firms to produce a single perishable consumption good. It is convenient to think of firms as located in spatially separated positions. Workers are infinitely-lived, whereas firms live for only one period at a time. Thus, one can think of a new set of firms as being born in each period. The technology for producing the consumption good is affected by a productivity shock which is independent across firms and time. The productivity shock is observed only by firms. At the beginning of each period, workers and firms negotiate a contract specifying an employment-compensation schedule. We constrain the firms from paying workers in terms of the good that is produced. Instead firms are required to pay workers with an infinitely durable commodity called money. The goods produced by the firms are sold in a centralized market. The sequencing of the decisions is as follows: the worker and a firm agree to a contract prior to the realization of the productivity shock. The consumer half of the worker-consumer pair sets off to the centralized market with nominal money balances inherited from the previous period to buy the good at a Walrasian market-clearing price of the good in terms of money. The productivity shock is realized. The worker supplies a previously agreed upon quantity of labor in exchange for an agreed-upon compensation (both possibly contingent upon the realized value of the productivity shock). The firm ships the produced goods to the centralized market where the good is sold. The consumer half of the worker-consumer pair returns with the goods purchased and they consume the goods. We assume that communication between the shopper and worker after the realization of the productivity shock does not occur. Consumption, therefore, is not contingent upon the realized productivity shock. Each consumer-worker owns a share of each firm in the economy. Ownership is a private partnership agreement. Consumers are thus liable for their share of any losses of the firms. An alternative interpretation is that the firms are owned by a single mutual fund in which each consumer owns a share. Dividends are distributed at the end of the day.

The economy is described by a continuum of agents in the interval [0,1] on which a Lebesgue measure is induced. Each agent is infinitely-lived and has a preference ordering over consumption sequences $\{c_t\}_{t=0}^{\infty}$, labor supply sequences $\{n_t\}_{t=0}^{\infty}$ described by a separable utility function

$$\sum_{t=0}^{\infty} \beta^t [U(c_t) - g(n_t)]$$

with $0 < \beta < 1$

$U : R^+ \to R$ bounded, twice differentiable, strictly concave

$g : [0,\bar{n}] \to R$ twice differentiable, strictly convex.

$$\lim_{x \to 0} U'(x) = \infty$$

$$g'(0) \geq \delta > 0$$

There are a large number of firms each with access to a constant returns to scale technology for producing the single consumption good.[3] Each firm's production is affected by stochastic shocks which are independent across firms and time. The productivity shocks are drawn from a finite set $\theta \varepsilon \Theta$, $\Theta = \{\theta_1, \theta_2, \ldots \theta_N\}$ with probabilities $\pi_1, \pi_2, \ldots, \pi_N$ respectively. One interpretation is that the fraction of firms which have productivity θ_i is π_i. For convenience, we assume without loss of generality $\theta_1 < \theta_2 \ldots < \theta_N$. Let $F(\theta)$ be the c.d.f. of θ. The output of a firm from the labor supplied by an agent is

$$\theta n \; .$$

Total output of a firm given that it hires a measurable set A of agents is then[4]

$$\int_A \theta n(\mu) d\mu \text{ where } \mu \text{ is the Lebesque measure on } [0,1] \; .$$

Firms are assumed to be profit-maximizers. Only the firm observes the productivity shock. Worker-consumers do not. Prior to observing the productivity shock each agent i meets with a firm and agrees to a "contract." The contract specifies the total nominal compensation $w_i(\theta)$ paid to agent i in each state of nature θ and the labor to be supplied by the agent $n_i(\theta)$ in each state of nature θ.

Thus, the contract is a pair of functions

$$\{w_i(\theta),\ n_i(\theta)\} : \Theta \times \Theta \to R^+ \times R^+$$

It will be noted that we have used the true value of θ as an argument of the functions. It is straightforward to show that any contract between the worker and the firm can be represented as shown above (Myerson (1979), Harris and Townsend (1977)). The reasoning is straightforward. Suppose the firm were to misreport the true value of θ. At the time of negotiating the contract, however, the consumer knows that the firm will lie (if it can increase its profits by doing so). The contract designed between the worker and the firm will therefore be such that the firm will not have an incentive to lie. One can, therefore, assume that the contract will be of the form laid out above.

In such a bilateral monopoly problem, some conditions must be imposed to determine how the surplus is distributed. We will assume that the expected profits of the firm in each period from every contract offered to any agent is zero. If P is the price of the consumption good

$$\int_{\Theta}[P\theta n_i(\theta) - w_i(\theta)]dF(\theta) = 0 \qquad \text{all } i = [0,1] .$$

One may rationalize this by assuming that there is free entry of firms at the time of making the contract. However, once the productivity shock has been realized, the worker cannot join another firm.

It is clear that each firm faces a static decision problem. The ex-post profits of the firms are distributed equally to all agents. With the expected profit constraint imposed above and the assumption that there are a large number of firms, it is clear that each agent receives a zero dividend.

The worker-consumers face a dynamic decision making problem.

Each agent i begins each period t with nominal money balances M_{it}. Then, he negotiates a compensation-employment contract with a firm. A productivity shock is realized, the worker supplies labor. The shopper goes to a central market where the good is exchanged for money at a Walrasian market clearing price.

The problem may therefore be summarized in the following manner. First, we define the set of feasible contracts. A feasible contract is a pair of functions (dropping the subscript for agents) $w_t(\theta)$, $n_t(\theta)$ such that

$$E\{P_t \theta n_t(\theta) - w_t(\theta)\} \geqq 0, \tag{2.1}$$

$$P_t \theta n_t(\theta) - w_t(\theta) \geqq P_t \theta n_t(\gamma) - w_t(\gamma), \tag{2.2}$$

all $\theta, \gamma \in \Theta$

and $$n_t(\theta) \geqq 0,\ w_t(\theta) \geqq 0 \qquad \text{all } \theta \in \Theta \tag{2.3}$$

for all $t = 0,1,\ldots$

Define the set of feasible contracts offered to an agent as $D_t \equiv \{w_t(\theta), n_t(\theta) | 2.1,\ 2.2 \text{ and } 2.3 \text{ are satisfied}\}$.

Constraint 2.2 is what is often defined as an incentive compatibility constraint. It ensures that the firm will never have an incentive to lie about the true value of θ. Its profits, if it reports the true value of θ will be higher than if it reports a false value of θ. This ensures that the contract is implementable.[5]

Given the set of feasible contracts, we turn to the problem faced by a worker-consumer. The worker selects that contract which yields highest expected utility over the set of all feasible contracts. Clearly, the contract so chosen will yield zero expected profits for a firm. Given that there are a continuum of consumers, and a continuum of firms, it follows that the ex-post average profits of all firms is zero. Thus, no dividends are paid out in equilibrium.

Each agent is then faced with the problem of maximizing the following functional (given initial money balances M_o and consumption in period $0, C_o$)

$$\text{Max} \quad E\{ \sum_{t=0}^{\infty} \beta^t [U(c_t) - g(n_t)] \} \tag{2.4}$$

$$\{c_t(\theta_0, \ldots \theta_{t-1})\}_{t=1}^{\infty} \geq 0$$

$$\{w_t, n_t\}_{t=0}^{\infty} \in d_t$$

$$\{M_t(\theta_0, \ldots \theta_{t-1})\}_{t=1}^{\infty} \geq 0$$

subject to

$$P_t c_t + M_{t+1} \leq M_t + w_t \tag{2.5}$$

$$P_t c_t \leq M_t \tag{2.6}$$

2.6 is the Clower constraint discussed in the introduction. It states that the nominal value of expenditures cannot exceed the nominal money balances brought over from the previous period. Note also the dependence of the consumption function c_t on $\theta_0, \ldots \theta_{t-1}$ and not on θ_t.

This captures the notion that is is prohibitively costly to deviate from planned consumption decisions, or alternatively that communication between the shopper and the worker is impossible after the realization of the productivity shock.

If the aggregate quantity of money in the economy is constant, then the fact that there is no aggregate uncertainty leads to the suspicion that the price level is constant. Thus, the equilibrium that will be constructed will be an equilibrium with a stationary price level. The technique for constructing the equilibrium is discussed in the next two sections.

Some comments are in order about certain features of the model. It has been quite arbitrarily assumed that firms maximize expected profits. It would probably be more accurate to consider the preferences of managers and the principal-agent contract that would arise between stockholders (worker-consumers) and managers. This problem is obviously an extremely complex one. The route that we will adopt is to suppose that each firm is managed by a single risk-neutral manager, who reports to the mutual fund that owns the shares of all firms. The incentive contract between the manager and the fund is a simple linear function of reported profits. Then, the optimal contract from the manager's perspective is precisely as in 2.3. Each manager manages a firm with a large enough number of workers so that his consumption is negligible compared to that of the workers. The effect of the manager's consumption decisions is therefore ignored in what follows.[7] Profit maximization in the Arrow-Debreu model on the part of firms leads to allocations that are Pareto-optimal. Pareto-optimal and core allocations with asymmetric information are as yet poorly understood (though see Prescott and Townsend (1979)) so we can make no conjectures about the reasonableness of this assumption.

It is also of interest to note that money plays an extremely important role only in the presence of asymmetric information. With no asymmetries in information, there is complete risk sharing and agents consume the same amount in every period. There is no incentive to hold money for precautionary purposes.[8] Money can play an important role in such models only when there is some asymmetry or privateness of information.

3. CONSTRUCTION OF AN EQUILIBRIUM

The consumer's problem discussed in section 2 may be formulated in terms of the usual dynamic programming framework. The state variable for each agent is the quantity of real balances he holds $m_t = \frac{M_t}{P_t}$. With a constant price level, the problem may then be stated as

$$v(m) = \text{Max } E_\theta\{U(c) - g(n_\theta) + \beta v(m'_\theta)\} \qquad 3.1$$

$$c \geqq 0$$

$$m'_\theta \geqq 0$$

$$w_\theta \geqq 0$$

$$n_\theta \geqq 0$$

subject to

$$c + m'_\theta \leqq m + w_\theta \qquad 3.2$$

$$c \leqq m$$

and

$$E[\theta n_\theta - w_\theta] \geqq 0 \qquad 3.3$$

and

$$\theta n_\theta - w_\theta \geqq \theta n_\gamma - w_\gamma \qquad 3.4$$

The consumer's and firm's problems have thus been collapsed into a single problem. The subscript (θ or γ) for the state is a notationally convenient means of indicating the contingency plan in the event that θ is the true state of nature. Note that c is assumed not to depend upon θ. The expectation operator is taken with respect to the random variable θ. m'_θ indicates the level of real balances carried over into the next period. Note that we have made the assumption here that agents assume that the current price level will prevail forever. At the risk of some confusion, w from hereon refers to the real wage.

Assuming that this problem has a solution, denote the decision rules of the agents by

$$c = c(m);\ m'_\theta = h(m,\theta);\ w_\theta = w(m,\theta);\ n_\theta = n(m,\theta)$$

In market equilibrium in this economy, it must be that per capita demand for cash balances when averaged over agents must be equal to per capita balances supplied $\frac{M}{P}$ at the stationary equilibrium price level P.

To calculate the per capita demand for cash balances, clearly we need to know the initial distribution of cash balances across agents. Assume that this initial distribution is $\psi(m)$. Given ψ, per capita demand is

$$\iint h(m,\theta)d\psi(m)dF(\theta)$$

Since the θ drawings are independent across firms and time, m and θ are independently distributed.

The equilibrium condition is

$$\iint h(m,\theta)d\psi(m)dF(\theta) = \frac{M}{P} \qquad 3.5$$

It is seen therefore that the price level depends upon the distribution of cash balances. In order for the agents' expectations that P is constant over time be rational, we require that the distribution of money balances replicate itself.

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Thus, ψ must solve

$$\psi(m') = \iint_{A(m')} d\psi(m)dF(\theta) \tag{3.6}$$

where $$A(m') = \{m,\theta | h(m,\theta) \leq m'\}.$$

Note that the usual application of Walras's law yields clearing of the goods market if the money market clears.

Definition of an Equilibrium

An equilibrium is a number $P > 0$, a continuous bounded function $v(\cdot) : R^+ \to R$, three continuous functions; $h(\cdot,\cdot)$; $w(\cdot,\cdot)$ and $n(\cdot,\cdot)$: $R^+ \times \Theta \to R^+$, a function $c(\cdot)$ from $R^+ \to R^+$ and a c.d.f. $\psi : R^+ \to [0,1]$ such that

1. v solves 3.1
2. c,h,w and n solve the maximum problem 3.1 for each m and θ.
3. h, ψ, P satisfy 3.5
4. h and ψ satisfy 3.6

An equilibrium is defined as four unknown functions and a positive number. The system is of course simultaneous but may be solved sequentially. First we find a value function v. Then policy functions c, h, w and n are obtained. A c.d.f. ψ is then found to satisfy 3.6 which yields the price P from 3.5.

Standard techniques (see Harris (1979)) are used in deriving most of the results.

<u>Proposition 1</u>: There is a unique, continuous, bounded function v

satisfying 3.1. The solution is strictly increasing and strictly concave.

<u>Proof</u>: Let L be the Banach space of continuous functions $f : R^+ \to R$ normed by

$$||f|| = \sup_x |f(x)|$$

Define $$Tv(m) = \text{Sup } E\{U(m + w_\theta - m'_\theta) + \beta v(m'_\theta) - g(n_\theta)\}$$

$$m'_\theta \geqq 0$$

$$n_\theta \geqq 0$$

$$w_\theta \geqq 0$$

subject to $$E\{\theta n_\theta - w_\theta\} \geqq 0$$

$$\theta n_\theta - w_\theta \geqq \theta n_\gamma - w_\gamma$$

Since n is bounded and θ is bounded (by assumption), it follows that if $E\{\theta n_\theta - w_\theta\} \geqq 0$ then w must be bounded. The constraint set is then clearly compact and by the theorem of Berge (1963, p. 116) it follows that $T : L \to L$. It is readily verified that T is monotone and is of contraction modulus β. Thus, T has a unique fixed point v^* (Blackwell (1965) theorem 5 p. 232)).

To show that T takes nondecreasing functions into increasing strictly concave functions let $\bar{m} > \underline{m}$.

$$\text{Then } Tv(\bar{m}) = E\{U(\bar{m} + \bar{w}_\theta - \bar{m}'_\theta) + \beta v(\bar{m}'_\theta) - g(\bar{n}_\theta)\}$$

$$\geqq E\{U(\bar{m} + \underline{w}_\theta - m'_\theta) + \beta v(\underline{m}'_\theta) - g(\underline{n}_\theta)\}$$

$$> E\{U(\underline{m} + \underline{w}_\theta - m'_\theta) + \beta v(\underline{m}'_\theta) - g(\underline{n}_\theta)\}$$

because u is strictly increasing. Hence, $Tv(\bar{m}) > Tv(\underline{m})$.

To show that T maps concave functions into strictly concave functions, let

$$m_\alpha = \alpha\underline{m} + (1 - \alpha)\bar{m} \qquad 0 < \alpha < 1$$

$$\alpha Tv(\underline{m}) + (1 - \alpha)Tv(\bar{m}) < E\{U(\alpha\underline{m} + (1 - \alpha)\bar{m} + \alpha\underline{w}_\theta + (1 - \alpha)\bar{w}_\theta - \alpha\underline{m}'_\theta -$$

$$(1 - \alpha)\bar{m}'_\theta) + \beta v(\alpha\underline{m}'_\theta + (1 - \alpha)\bar{m}'_\theta) - g(\alpha\underline{n}_\theta + (1 - \alpha)\bar{n}_\theta)\} \leqq Tv(m_\alpha).$$

Note that the proof here relies on the fact that $(\alpha w_\theta + (1 - \alpha)\bar{w}_\theta$, $\alpha\underline{n}_\theta + (1 - \alpha)\bar{n}_\theta)$ is a feasible contract for any agent with real balances m_α and that U and -g are strictly concave functions.

It follows therefore that $\lim_{n \to \infty} T^n v = v^*$ is nondecreasing and concave and since $v^* = Tv^*$ that v^* is strictly increasing and strictly concave.

Proposition 2: There are unique, continuous (in the first argument) functions $c(\cdot)$, $h(\cdot,\cdot)$, $w(\cdot,\cdot)$ and $n(\cdot,\cdot)$ which attain the right hand side of 3.1 for each m and θ.

Proof: The maximum problem 3.1 involves maximizing a continuous, strictly concave function over a compact, convex non-empty constraint set. Hence, there is a unique solution.

The theorem of Berge (1963; p. 116) ensures that the solution is continuous.

<u>Proposition 3</u>: The value function v is differentiable on $(0,\infty)$ and

$$\frac{\partial v(m)}{\partial m} = \frac{\partial u(c)}{\partial c} \text{ for } m > 0 \qquad 3.7$$

<u>Proof</u>: The proof uses a theorem by Benveneste and Scheinkman (1975).

Consider the following (possibly suboptimal) plan for m in some neighborhood of $m^o > 0$

Let $$c(m) = \text{Max } \{m + w(m^o,\theta) - h(m^o,\theta), 0\}$$

Since $c(m^o) > 0$ (from the Inada conditions on U) it follows that there is a neighborhood of m^o with $m + w(m^o,\theta) - h(m^o,\theta) > 0$

Now, the value of the (possibly suboptimal) plan in this neighborhood is

$$f(m) = E\{U(m + w(m^o,\theta) - h(m^o,\theta)) - g(n(m^o,\theta)) + \beta v(h(m^o,\theta))\}$$

and $f(m^o) = v(m^o)$ and $f(m) \leq v(m)$.

Then f(m) is a strictly concave differentiable function and from the Benveneste-Scheinkman theorem, it follows that

$$\frac{\partial f(m^o)}{\partial m} = E\left\{\frac{\partial U(c(m))}{\partial c}\right\} = \frac{\partial v(m^o)}{\partial m}$$

Hence, $$\frac{\partial v(m)}{\partial m} = \frac{\partial U(c)}{\partial c}$$

Q.E.D.

The strict concavity of v and u guarantee that c is increasing in m. We now introduce some additional notation.

Define $$x = m - c,\ J : R^{+} \to R \text{ by}$$

$$J(x) = \text{Max } E\{-g(n_\theta) + \beta v(w_\theta + x)\} \qquad 3.8$$
$$n_\theta \geqq 0$$
$$w_\theta \geqq 0$$

Subject to $$E\{\theta n_\theta - w_\theta\} \geqq 0$$

and $$\theta n_\theta - w_\theta \geqq \theta n_\gamma - w_\gamma \quad \text{all } \theta, \gamma \in \Theta$$

<u>Proposition 4</u>: $x(m)$ is nondecreasing in m for all $m > 0$, and $0 \leqq \frac{\partial x}{\partial m} < 1$.

<u>Proof</u>: $J(\cdot)$ is clearly a strictly increasing, strictly concave differentiable function of x.

By definition, $$v(m) = \underset{c \geqq 0}{\text{Max}} \{U(c) + J(x)\} \qquad 3.9$$

subject to $$c \leqq m$$

and $$c + x \leqq m$$

Hence, $$\frac{\partial U}{\partial c} = \frac{\partial J}{\partial x} \qquad \text{if } c < m .$$

Since c is increasing in m, it follows that x is nondecreasing in m. The fact that c is strictly increasing in m and $c + x = m$ shows that $0 \leqq \frac{\partial x}{\partial m} < 1$.

Q.E.D.

A further restatement of the programming problem is instructive in characterizing the nature of the optimal contract.

Let $y_\theta = x + w_\theta$.

Of course, $y_\theta = m'_\theta$. The renaming avoids the necessity of carrying a number of primes around. The programming problem now reads:

$$\max_{\substack{\bar{n} \geqq n_\theta \geqq 0 \\ y_\theta \geqq x}} E_\theta\{\beta v(y_\theta) - g(n_\theta)\} \tag{3.10}$$

subject to $E_\theta\{\theta n_\theta - y_\theta\} \geqq -x$

and $\theta n_\theta - y_\theta \geqq \theta n_\gamma - y_\gamma \quad$ all $\theta, \gamma \in \Theta$.

The nature of the optimal contract with asymmetric information is characterized by the following proposition[9] which follows entirely from the incentive compatibility constraints.

<u>Proposition 5</u>: w and n are nondecreasing in θ. Furthermore, profits are nondecreasing in θ and the compensation-employment schedule is convex, i.e.

$$\frac{w_2-w_1}{n_2-n_1} \leqq \frac{w_3-w_2}{n_3-n_2} \leqq \ldots. \quad \text{if } \theta_1 < \theta_2 < \ldots < \theta_N.$$

Proof: From the incentive compatibility constraints we have that

$$\theta_i n_i - w_i \geqq \theta_i n_j - w_j \qquad i,j = 1,2,\ldots,N$$

and

$$\theta_j n_j - w_j \geqq \theta_j n_i - w_i$$

Hence,

$$\theta_i (n_i - n_j) \geqq w_i - w_j \geqq \theta_j (n_i - n_j) \tag{3.11}$$

Recalling that $\theta_1 < \theta_2 \ldots < \theta_N$

we have

$$n_1 \leqq n_2 \leqq \cdots \leqq n_N$$

$$w_1 \leqq w_2 \leqq \cdots \leqq w_N$$

From 3.11 it also follows that

$$\theta_1 \leqq \frac{w_2 - w_1}{n_2 - n_1} \leqq \theta_2 \leqq \frac{w_3 - w_2}{n_3 - n_2} \ldots.$$

It is easy to see that profits are nondecreasing. Let $\theta_i > \theta_j$.

Then

$$\theta_i n_i - w_i \geqq \theta_i n_j - w_j \geqq \theta_j n_j - w_j$$

Q.E.D.

These results are independent of preferences of labor suppliers and provide some justification for the observed use of incentive schemes, overtime pay, bonuses, etc. in place of the complete risk sharing that would accompany models without asymmetric information.

It is well known (Harris and Townsend (1977)) that optimal allocations in the presence of asymmetric information may lead to possibilities of ex-post gains to trade. That possibility exists in this model as well. Suppose that there are only two states θ_1 and θ_2, $\theta_1 < \theta_2$ with probabilities π and $(1-\pi)$. Then the first order conditions for problem 3.10 are (assuming an interior solution)

$$\pi\beta v'(y_1) - \pi\lambda - \mu_1 + \mu_2 = 0 \tag{3.12}$$

$$(1 - \pi)\beta v'(y_2) - (1 - \pi)\lambda + \mu_1 - \mu_2 = 0 \tag{3.13}$$

$$-\pi g'(n_1) + \pi\lambda\theta_1 + \mu_1\theta_1 - \mu_2\theta_2 = 0 \tag{3.14}$$

$$-(1 - \pi)g'(n_2) + (1 - \pi)\lambda\theta_2 - \mu_1\theta_1 + \mu_2\theta_2 = 0 \tag{3.15}$$

and

$$\pi(\theta_1 n_1 - y_1) + (1 - \pi)(\theta_2 n_2 - y_2) = -x$$

$$\mu_1(\theta_1 n_1 - y_1 - \theta_1 n_2 + y_2) = 0$$

$$\mu_2(\theta_2 n_2 - y_2 - \theta_2 n_1 + y_1) = 0$$

λ, μ_1, μ_2 are Lagrange multipliers associated with the three constraints. It is easy to prove[10] that both incentive compatibility constraints cannot be binding simultaneously. Both μ_1 and μ_2 cannot be zero since the allocation in that case would have to involve complete risk sharing which is clearly not incentive compatible. μ_1 cannot be zero from inspection of the first order conditions for y_1 and y_2 since we know that $y_1 \leq y_2$. Thus, $\mu_2 = 0$.

In this case, note that multiplying 3.13 by θ_2 and adding 3.15

gives
$$\frac{g'(n_2)}{\beta v'(y_2)} > \theta_2 \ .$$

The marginal rate of substitution is greater than the marginal rate of transformation. In a sense, the worker is "overemployed" ex-post, i.e. both firm and worker can gain by reducing n_2 and y_2.

Certain features of the optimal contract must be emphasized. The allocations under asymmetric information differ markedly from those under full information. With full information, there is complete risk sharing and compensation is independent of the state of nature

$$w(m,\theta) = w(m)$$

and
$$g'(m_\theta) = \theta\beta v'(y_\theta) = \theta\beta v'(x + w(m))$$

This allocation is optimal in an environment where labor mobility is restricted so that all workers cannot migrate to high productivity firms. The same does not hold for the asymmetric information environment. Quite apart from the ex-post gains to trade, it is clear that in an environment with asymmetric information, some fortunate workers will experience repeated realizations of high productivity and would wish to lend some of their money to their less fortunate brethren. However, it is only in the environment with asymmetric information that money has a role to play in the sense that allocations in the monetary economy with asymmetric information are <u>strictly</u> preferred to those that would arise if firms paid workers in terms of the consumption good. This is not true of the full-information environment.

4. EXISTENCE OF A STATIONARY DISTRIBUTION

We now proceed to establish that the real balances of agents are in an ergodic set. In other words, there is a value of $m = \bar{m}$ such that $0 \leq h(m,\theta) \leq \bar{m}$ for $0 \leq m \leq \bar{m}$ and all θ. This will aid in establishing the existence of a stationary distribution of money balances of agents. The following propositions are useful in establishing the existence of an ergodic set:

PROPOSITION 6: If $w_i = 0$ for some i, $i = 1,2,\ldots,N$ then profits = $\theta_i n_i - w_i = 0$ for all i.

PROOF: $0 \leq w_1 \leq w_2 \leq \cdots \leq w_N$ (from Proposition 5). Thus, if $w_i = 0$ then $w_1 = 0$. But (also from Proposition 5):

$$\theta_1 n_1 - w_1 \leq \theta_2 n_2 - w_2 \leq \cdots \leq \theta_N n_N - w_N$$

and
$$\sum_{i=1}^{N} \pi_i [\theta_i n_i - w_i] = 0$$

Thus,
$$\theta_1 n_1 - w_1 \leq 0$$

and $w_1 = 0$ implies $n_1 = 0$ and $\theta_1 n_1 - w_1 = 0$. Consequently, $\theta_i n_i - w_i = 0$ for all i.

Q.E.D.

It is also useful to note from the structure of the incentive compatibility constraints that the number of constraints can be considerably reduced. It turns out that only sequential pairwise constraints need be considered, i.e.

$$\theta_i n_i - y_i \geqq \theta_i n_{i+1} - y_{i+1} \tag{4.1}$$

and
$$\theta_{i+1} n_{i+1} - y_{i+1} \geqq \theta_{i+1} n_{i+2} - y_{i+2} \tag{4.2}$$

together imply that

$$\theta_i n_i - y_i \geqq \theta_i n_{i+2} - y_{i+2}$$

and similarly for constraints between (i, i-1, i-2). The proof is obvious.

We have, from 3.14 that

$$y_{i+2} - y_{i+1} \geqq \theta_{i+1}(n_{i+2} - n_{i+1})$$

$$\geqq \theta_i(n_{i+2} - n_{i+1})$$

Hence
$$\theta_i n_{i+1} - y_{i+1} \geqq \theta_i n_{i+2} - y_{i+2}$$

Therefore
$$\theta_i n_i - y_i \geqq \theta_i n_{i+2} - y_{i+2}$$

The essential property on the preferences of workers that is used to establish that there is an ergodic set is that the marginal utility from labor supplied is strictly positive at $n = 0$

$$g'(0) = \delta > 0$$

Noting that $v(\cdot)$ is a strictly concave, bounded, increasing, differentiable function, we have that $v'(\cdot)$ is a decreasing function and furthermore that $\lim_{m\to\infty} v'(m) = 0$.

It follows that there is a value of $x = \bar{x}$ such that

$$\frac{\pi_N g'(0)}{\beta v'(\bar{x})} > \theta_N \tag{3.15}$$

It is clear from this that for $x \geq \bar{x}$ the worker will not desire to supply labor in any state and that he will receive no compensation in any state. This fact is proved in

PROPOSITION 7: For $x \geq \bar{x}$, where $\bar{x}$ is defined in 3.15, $w_i = 0$, $n_i = 0$ and $m_i' = y_i = x$ for all i.

PROOF: The proof is by contradiction and is divided into two parts. Suppose that $w(\bar{x},\theta_i) = 0$ for some i and $w(\bar{x},\theta_i) > 0$ for some j. Then, from Proposition 6 we have that $w_i = \theta_i n_i$ for all i. We now demonstrate that this allocation cannot be optimal. Consider the alternative allocation of $w_j^* = n_j^* = 0$ all j. Suppose $w_j = \theta_j n_j > 0$. Then the difference in utilities between the two allocations for state j is

$$[\beta v(\bar{x}) - g(0)] - [\beta v(\bar{x} + w_j) - g(n_j)]$$

$$> -\beta v'(\bar{x}) w_j + g'(0) n_j$$

$$> \beta v'(\bar{x})(\theta_N - \theta_j) n_j$$

$$\geq 0$$

2 5

The first inequality follows from the strict concavity of v and the strict convexity of g. The second from 3.15. It is clear that an allocation with $w(\bar{x},\theta_j) > 0$ is not optimal.

We now turn to a situation where $w(\bar{x},\theta_i) > 0$ all i. To study this case, let θ_k be the smallest value of θ for which $n_k > 0$. Consider the alternative allocation of reducing n_i and y_i in the following manner. Define

$$n_i^* = n_i - \frac{\varepsilon}{\theta_N} \qquad \text{for } i \geqq k$$

$$n_i^* = n_i = 0 \qquad \text{for } i \leqq k - 1$$

$$y_i^* = y_i - \varepsilon \qquad \text{all } i$$

ε is a small positive number $0 < \varepsilon << y_1$. The incentive compatibility constraints are satisfied since

$$\begin{aligned} \theta_i n_i^* - y_i^* &= \theta_i n_i - y_i + \varepsilon\left(1 - \frac{\theta_i}{\theta_N}\right) \\ &\geqq \theta_i n_{i+1} - y_{i+1} + \varepsilon\left(1 - \frac{\theta_i}{\theta_N}\right) \\ &= \theta_i n_{i+1}^* - y_{i+1}^* \qquad i \geqq k \end{aligned}$$

$$\begin{aligned} \theta_i n_i^* - y_i^* &= \theta_i n_i - y_i + \varepsilon \\ &\geqq \theta_i n_{i+1} - y_{i+1} + \varepsilon \\ &\geqq \theta_i n_{i+1}^* - y_{i+1}^* \qquad i \leqq k - 1 \end{aligned}$$

since $\quad n_{i+1} \geqq n_{i+1}^*$

and

$$\theta_{i+1} n^*_{i+1} - y^*_{i+1} = \theta_{i+1} n_{i+1} - y_{i+1} + \varepsilon \left(1 - \frac{\theta_{i+1}}{\theta_N}\right)$$

$$\geqq \theta_{i+1} n_i - y_i + \varepsilon \left(1 - \frac{\theta_{i+1}}{\theta_N}\right)$$

$$= \theta_{i+1} n^*_i - y^*_i \qquad i \geqq k$$

$$\theta_{i+1} n^*_{i+1} - y^*_{i+1} = \theta_{i+1} n_{i+1} - y_{i+1} + \varepsilon$$

$$\geqq \theta_{i+1} n_i - y_i + \varepsilon$$

$$= \theta_{i+1} n^*_i - y^*_i \qquad i \leqq k - 1$$

It is clear that the firm's profits strictly increase. The change in utility ΔJ in going to the "star" allocation can be computed by performing a Taylor's series expansion around the original allocation. This is given by

$$\Delta J \simeq - \sum_{i=1}^{N} \pi_i \beta v'(y_i)\varepsilon + \sum_{i=k}^{N} \pi_i g'(n_i) \cdot \frac{\varepsilon}{\theta_N}$$

$$> -\beta v'(\overline{x}) \cdot \varepsilon + \pi_N g'(0) \cdot \frac{\varepsilon}{\theta_N}$$

$$> 0$$

The last inequality follows from 3.15. It is evident from the structure of the proof that $y = x$ for $x \geqq \overline{x}$.

Q.E.D.

Recall that x is a function of m. Define $m_1 > 0$ by $x(m_1) = \overline{x}$.

By definition, $h(m,\theta_i) = y_i$. Thus, we have for $m \geqq m_1$

$$h(m,\theta_i) = x = m - c$$

It follows that $h(m,\theta_i) \geqq m$ for $m \geqq m_1$ (since $c(m) > 0$ for $m > 0$). From Proposition 4, we see that $\frac{\partial x}{\partial m} > 0$ for $x > 0$.

Thus, $h(m,\theta_i)$ is an increasing function of m for $m \geqq m_1$. The fact that $U'(0) = \infty$ implies that

$$h(0,\theta_N) = y_N = w_N > 0$$

Since $h(\cdot,\cdot)$ is a continuous function of m, this collection of facts implies that there is a value[11] of $m = \bar{m} > 0$, such that $0 \leqq h(m,\theta_N) \leqq \bar{m}$ for $0 \leqq m \leqq \bar{m}$. Since we have from Proposition 5 that h is nondecreasing in θ

$$0 \leqq h(m,\theta) \leqq \bar{m} .$$

We turn now to the question of the existence of a stationary initial distribution of real balances. The proposition is proved in the appendix.

<u>PROPOSITION 8</u>: There exists a stationary initial distribution on $[0,\bar{m}]$ which satisfies 3.6.

<u>PROOF</u>: Given the existence of a stationary initial distribution ψ, the left side of 3.5 is constant. With a given constant per capita quantity of money the price level P is constant and can be computed from 3.5.

5. CONCLUSIONS

The central theme of this paper is that asymmetric information can help explain a number of phenomena of interest to macroeconomists. Some of these are "underemployment," quantity rationing and price setting behavior. The incentive compatibility constraints which capture the idea of asymmetric information also ensure that the allocations will not be ex-post optimal. Subsequent to the realization of the productivity shock, firms and workers may well wish that a Walrasian auctioneer would come along and move them to an ex-post efficient allocation. Moves like this will not of course be ex-ante efficient.

The most compelling reason for studying models of this kind are, of course, to examine the implications for policy changes. We advance some conjectures here about the effects on contracts of changes in the aggregate quantity of money. Clearly, if all monetary injections are pure proportional transfers, then there is an equilibrium with no real effects. Suppose, however, that periodically the government steps into the centralized market and buys (or sells) a random amount of the consumption good in exchange for money. This will induce a rise (or fall) in the price of the commodity. The primary initial effect is a reduction (rise) in real balances. Under complete information it can be shown that real compensation is a decreasing function of real balances. Real compensation and real output will therefore rise (fall). This is a peculiar artifact of the pure risk sharing model--and possibly of the asymmetric information model as well--that there is overindexation!

Other extensions of this work are worth contemplating. It would be worthwhile to make the role of managers and the contracts between managers and stockholders explicit. This issue is tied in closely with issues of Pareto-optimality with asymmetric information. Another extension would be to allow for multiperiod contracts thereby breaking the close connection between the shopping "day" and the contract period. Perhaps the most interesting extension of the type of models discussed here is to drop the assumption of separable production across workers thereby allowing for layoffs as well as changes in the number of hours worked.

APPENDIX

Much, though not all, of the material covered here is to be found (in greater generality and rigor) in Futia (1979) and Green (1976).

Let S denote the set of states for the economy $S \equiv [0,\bar{m}]$.

The law of motion is given by a continuous function $h(\cdot,\cdot) : [S] \times \Theta \to [S]$, where $\Theta = \{\theta_1,\theta_2,\ldots\theta_N\}$. θ is a random variable drawn from a fixed distribution with c.d.f. $F(\theta)$ and probabilities $\pi_1,\pi_2,\ldots\pi_N$ where π_i is the probability that θ_i is drawn.

The law of motion h and the c.d.f. of θ together define a Markov process with transition probabilities given by

$$P(m,A) = \int_{B(m)} dF(\theta) \qquad \text{A1}$$

where $B(m) = \{\theta \in \Theta | h(m,\theta) \in A\}$, where A is a measurable set in S.

$P(m,A)$ is the probability of ending in set A if one starts off at m.

Define: $$\Psi \equiv \left\{ \psi | \psi \text{ is a countably additive set function: } \mathcal{B} \to R \text{ where } \mathcal{B} \text{ is the smallest } \sigma\text{-algebra containing all closed subsets of } S. \right\}$$

It is well known that Ψ is a Banach space under the total variation norm (see Futia (1979), p. 17)).

Define $V : \Psi \to \Psi$ by

$$(V\psi)(A) = \int_S P(m,A)d\psi(m) = \sum_{i=1}^{N} \pi_i \psi(h_{\theta_i}^{-1}(A))$$

If ψ is the distribution of agents' money balances in the current period, $V\psi$ is the distribution next period.

A stationary initial distribution exists if V has a fixed point. To establish that V is continuous, it is convenient to operate in the dual space of Ψ which turns out to be the space of continuous functions on S. Some preliminary results are established now.

Define $\quad C \equiv \{f : S \to R | f \text{ is continuous}\}$

$$C' \equiv \{g : C \to R | g \text{ is a continuous linear functional}\}$$

A bilinear form $< \cdot,\cdot > : C \times C' \to R$ is defined by $< f,g > = g(f)$, $f \in C$, $g \in C'$. If $T : C \to D$ is a continuous linear transformation, there is a linear transformation $T^* : C' \to C'$ which satisfies $< Tf,g > = < f,T^*g >$ for all f and g, i.e., $g(Tf) = (T^*g)(f)$.

T^* is known as the adjoint operator of T (see Kolmogorov and Fomin (1970) p. 232)). The following theorem shows the linkage between Ψ and C.

<u>THEOREM</u> (Riesz): If C is the set of continuous functions on S, a compact subset of R^n, $\mathcal{B}$ is the smallest σ-algebra containing all closed subsets of S and M is the set of countably additive set functions $\mu : \mathcal{B} \to R$ then $M = C'$.

Here, $< f,\mu > = \int_S f d\mu$ and continuity is defined with respect to the sup norm on f.

Within the set-up of our problem,

Define $T : C \to C$ by

$$(Tf)(m) = \int_S f(t)P(n,dt)$$

$$= E(f(m')|m_0 = m)$$

$$= \sum_{i=1}^{N} \pi_i f(h(m,\theta_i))$$

Here m' refers to next period's state and m_0 to the current period's state.

T is well-defined since the composition of continuous functions is continuous (we make use here of the fact that h is continuous) and the linear combination of continuous functions is continuous. Furthermore $T(\alpha f_1 + \beta f_2) = \alpha Tf_1 + \beta Tf_2$ for α and β real and $f_1, f_2 \varepsilon C$. Thus, T is a linear operator. Since f is bounded, T takes bounded functions into bounded functions and is continuous (see Kolmogorov and Fomin (1970) p. 223)).

THEOREM A1: $V = T^*$ where T^* is the adjoint operator of T.

PROOF: Let $f \varepsilon C$, $\psi \varepsilon \Psi$

Then, $$< Tf, \psi > = \int_S E(f(m')|m_0 = m)d\psi(m)$$

If ψ is the initial distribution on S, then

$$RHS = E(f(m'))$$

But, $$E(f(m')) = \int_S fd(V\psi)$$

since $V\psi$ is the distribution on S in the next period.

Thus,
$$\langle Tf, \psi \rangle = \langle f, V\psi \rangle$$

and
$$V = T^*$$

Q.E.D.

THEOREM A2: If T is a continuous linear operator, its adjoint T^* is continuous.

PROOF: See Kolmogorov and Fomin (1970), p. 233.

THEOREM A3: There is a stationary initial distribution on S.

PROOF: If S is compact, the set of probability measures on S is a convex subset of Ψ and is compact in the weak topology (see Parthasarathy (19) p. 45, theorem 6.4)).

Thus, we may apply Tychonoff's theorem (Dugundji (1970) p. 414)) which states that a continuous mapping from a compact, convex subset of a linear topological space into itself has a fixed point. Clearly, V maps probability measures into probability measures and has a fixed point. Hence, there is a stationary initial distribution on S.

Q.E.D.

It should be noted that the only crucial ingredients in this proof are the assumed compactness of the ergodic set and the continuity of h.

FOOTNOTES

[1]The most frequently quoted in this context are Azariadis (1975), Bailey (1974), Gordon (1974). For a recent survey, see Azariadis (1979). Holmström (1980) considers generalizations of Azariadis's model.

[2]This statement is not strictly true. It is true that the discounted stream of earnings must equal the discounted stream of expenditures. The difficulties in implementing this concept when markets are decentralized remains.

[3]The constant returns to scale assumption is not a serious restriction. Suppose the production function is

$$q = f(n) \quad f' > 0 \quad f'' < 0$$

Then, replace the disutility from labor term in the utility function by $-g(f^{-1}(q))$ which is strictly concave and the same analysis goes through.

[4]Note that we do not aggregate the labor supplied and then assume that the total quantity of labor yields output. The output produced by any agent is assumed to be uncorrelated with that produced by any other agent.

[5]That there are mechanisms which will lead to optimal allocations which necessarily must satisfy such incentive compatibility constraints has been proved by Harris-Townsend (1977).

[6]For example, if there are a countably infinite number of firms and hence of managers, they are of Lebesgue measure zero and have no

measurable impact on the market.

[7]The idea that managers are risk-neutral though fits in well with the notion that entrepreneurs are self-selected individuals who are less risk-averse than the population at large.

[8]In such a case, the Clower constraint (and the need for money) are difficult to justify.

[9]These results are similar to those in Chapter 2.

[10]If $\frac{g'(n)}{U'(y)} < \theta_1$ then increase n_1 by ε, y_1 by $\theta_1\varepsilon$. If $\frac{g'(n)}{n'(y)} > \theta_1$, then increase n_2 by ε, y_2 by $\theta_2\varepsilon$.

[11]One technique for defining $\bar{m}$ is as follows:

Let $$m' = \max_{0 \leqq m \leqq m_1} h(m,\theta_N)$$

Then, $\bar{m} = \text{Max}\{m', m_1\}$ and it is clear that $0 \leqq h(m,\theta_N) \leqq \bar{m}$.

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