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Discussion Paper #451

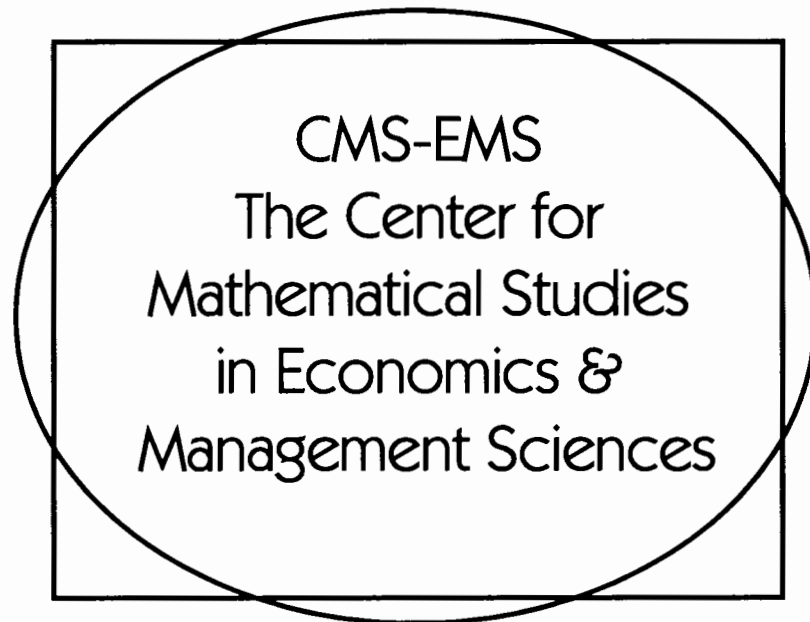
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“Post-War U.S. Business Cycles:
An Empirical Investigation”

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POST-WAR U.S. BUSINESS CYCLES:
AN EMPIRICAL INVESTIGATION ^{*}/₋

by

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Revised May 1981

*Support of the National Science Foundation is acknowledged. We also acknowledge helpful comments by the participants at 1979 Summer Warwick Workshop on Expectation and the money workshops at the Universities of Chicago and Virginia and at Carnegie-Mellon University. In particular, we would like to thank Robert Avery, V. V. Chari, Charles R. Nelson, Thomas J. Sargent and John H. Wood for comments. We also thank the Wharton Economic Forecasting Associates for providing the data.

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1. Introduction and Summary

In this article aggregate fluctuations in the post-war U.S. economy are investigated using quarterly data. The fluctuations studied are those that are too rapid to be accounted for by slowly changing demographic and technological factors and changes in stocks of capital. The principal concern of the study is the comovements between the rapidly varying components of real output and the rapidly varying components of other macroeconomic time series. These comovements are very different than those for the slowly varying components of the corresponding variables. Also reported are the serial correlation properties of the rapidly varying components of the data series.

The most striking findings, we think, are that fluctuations in output are positive associated with the fluctuations in hours and these fluctuations are of comparable (percentage) magnitude. As demographics change slowly, the rapidly varying component of hours is variations in hours of employment per person and not in the size of the population. The high frequency variations in capital and productivity (output per hour) are weakly associated with the high frequency variations in output. This is in sharp contrast with the slowly varying components. Growth in output per capita is associated with growth in the capital stocks and productivity and with little movement in hours of employment per person.

Another difference is that cyclical investment varies much more (in percentage terms) and cyclical consumption much less than does cyclical output. For the smoothed components, the corresponding movements all are of comparable percentage magnitudes as the variations in the shares of output allocated

to consumption and investment vary little over periods of time having large growths in output per household.

This study should be viewed as documenting some systematic deviations from the restrictions upon observations implied by neoclassical growth theory.¹ Our statistical approach does not utilize standard time series analysis. Our prior knowledge concerning the processes generating the data is not of the variety that permits us to specify a probability model as required for the application of that analysis. We proceed in a more cautious manner that requires only prior knowledge that can be supported by economic theory. The maintained hypothesis, based upon growth theory consideration, is that the growth component varies smoothly over time. The sense in which it varies smoothly is made explicit in Section 2.

Several researchers, using alternative methods, have added and are adding to our knowledge of aggregate economic fluctuations.² Our view is that no one approach dominates all the others and that it is best to examine the data from a number of different perspectives. We do think our approach documents some interesting regularities.

2. Decomposition Procedure

The observed time series are viewed as the sum of a cyclical and growth component. Actually, there is also a seasonal component, but as the data are seasonally adjusted, this component has already been removed by those preparing the data series. If growth accounting provided estimates of the growth component with errors that were small relative to the cyclical component, computing the cyclical component would be just a matter of calculating the difference between the observed value and the growth component. Growth theory accounting (cf. Denison [1974]), in spite of its considerable success, is far from adequate for providing such numbers. If our prior knowledge were sufficiently strong that we could model the growth component as a deterministic component, possibly conditional on exogenous data, plus a stochastic process and the cyclical component as some other stochastic process, estimating the cyclical component would be an exercise in modern time series analysis. Our prior knowledge is not of this variety, so these powerful methods are not applicable. Our prior knowledge is that the growth component varies "smoothly" over time.

Our conceptual framework is that a given time series y_t is the sum of a growth component g_t and a cyclical component c_t :

$$(1) \quad y_t = g_t + c_t \quad \text{for } t = 1, \dots, T .$$

Our measure of the smoothness of the $\{g_t\}$ path is the sum of the squares of its second difference. The c_t are deviations from g_t and our conceptual framework is that over long time periods, their average is near

zero. These considerations lead to the following programming problem for determining the growth components:

$$(2) \quad \text{Min}_{\{g_t\}_{t=-1}^T} \left\{ \sum_{t=1}^T c_t^2 + \lambda \sum_{t=1}^T [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2 \right\}$$

where $c_t = y_t - g_t$. The parameter λ is a positive number which penalizes variability in the growth component series. The larger the value of λ , the smoother is the solution series. For sufficiently large λ , at the optimum all the $g_{t+1} - g_t$ must be arbitrarily near some constant β and therefore the g_t arbitrarily near $g_0 + \beta t$. This implies that the limit of solutions to program (2) as λ approaches infinity is the least squares fit of a linear time trend model.

Our method has a long history of use, particularly in the actuarial sciences. There it is called the Whittaker-Henderson Type A method (Whittaker [1923]) of graduating or smoothing mortality experiences in constructing mortality tables. The method is still in use.³ As pointed out in Stigler's [1978] historical review paper, closely related methods were developed by the Italian astronomer Schiaparelli in 1867 and in the ballistic literature in the early forties by , among others, von Neuman.

Value of the Smoothness Parameter

The data analyzed, with the exception of the interest rates, are in logs so the change in the growth component, $g_t - g_{t-1}$, corresponds to a growth rate.

The growth rate of labor's productivity has varied considerably over this period (see McCarthy [1978]). In the 1947-53 period, the

annual growth rate was 4.20 percent, in the 1953-68 period 2.61 percent, in the 1968-73 period only 1.41 percent and in the subsequent period it was even smaller. Part of these changes can be accounted for by a changing capital/labor ratio and changing composition of the labor force. But, as shown by McCarthy, a sizable and variable unexplained component remains, even after correcting for cyclical factors. The assumptions that the growth rate has been constant over our thirty-year sample period, 1950-79, is not tenable. To proceed as if it were would result in errors in modeling the growth component and these errors are likely to be non-trivial relative to the cyclical component. For this reason, an infinite value for the smoothness parameter was not selected.

The following probability model is useful for bringing to bear prior knowledge in the selection of the smoothing parameter λ . If the cyclical components and the second differences of the growth components were identically and independently distributed, normal variables with means zero and variances σ_1^2 and σ_2^2 (which they are not), the conditional expectation of the g_t given the observations would be the solution to program (2).

If $\sqrt{\lambda} = \sigma_1 / \sigma_2$.

As this probability model has a state space representation, efficient Kalman filtering techniques can be used to compute these g_t .⁴ By exploiting the recursive structure one need not invert a $(T+2)$ by $(T+2)$ matrix (T is the number of observations in the sample) as would be necessary if one solved the linear first order conditions of program (2) to determine the g_t . The largest matrix that is inverted using the Kalman filtering computational approach is 2 by 2. If T is large, this is important for inverting large matrices is costly and there can be numerical rounding problems when implemented on computers. Kalman filtering can be performed with computer packages that are widely available.

Our prior view is that a five percent cyclical component is moderately large as is a one-eighth of one percent change in the growth rate in a quarter. This led us to select $\sqrt{\lambda} = 5/(1/8) = 40$ or $\lambda = 1600$ as a value for the smoothing parameter. One issue is how sensitive are the results to the value λ selected. To explore this issue various other values of λ were tried. Table 1 contains the (sample) standard deviations and autocorrelations of cyclical real GNP for the selected values of the smoothing parameter. These numbers change little if λ is reduced by a factor of four to 400 or increased by a factor of four to 6400. As λ increases, the standard deviation increases and there is greater persistence, with the results being very different for $\lambda = \infty$.

With our procedure for identifying the growth component ($\lambda = 1600$), the annual rate of change of the growth component varied between 2.3 and 4.9 percent over the sample period with the minima occurring in 1957 and in 1974. The maximum growth rate occurred in 1964 with another peak of 4.4 percent in 1950. The average growth rate over the period was 3.4 percent. The differences between our cyclical components and those obtained with perfect smoothing ($\lambda = \infty$) are depicted in Figure 1 along with the cyclical component. The smoothness of the variation in this difference relative to the variation in the cyclical component indicates that the smoothing parameter chosen is reasonable. We caution against interpreting the cyclical characteristic of the difference as a cycle of long duration. Such patterns can appear as artifacts of the data analysis procedure.

The same transformation was used for all series: that is, for each series j

$$(3) \quad g_{jt} = \frac{1}{\sum_{i=1}^T w_{it}} \sum_{i=1}^T w_{it} y_{ji},$$

where T is the length of the sample period. If the sample size were infinite,

TABLE 1

STANDARD DEVIATIONS AND SERIAL CORRELATIONS OF
CYCLICAL GNP FOR DIFFERENT VALUES OF THE SMOOTHING PARAMETER

	<u>$\lambda=400$</u>	<u>$\lambda=1600$</u>	<u>$\lambda=6400$</u>	<u>$\lambda=\text{infinity}$</u>
Standard Deviations	1.56%	1.80%	2.03%	3.12%
Auto Correlations				
Order 1	.80	.84	.87	.94
Order 2	.48	.57	.65	.84
Order 3	.15	.27	.41	.73
Order 4	-.14	-.01	.17	.61
Order 5	-.32	-.20	.00	.52
Order 6	-.39	-.30	-.11	.44
Order 7	-.42	-.38	-.20	.38
Order 8	-.44	-.44	-.27	.31
Order 9	-.41	-.44	-.31	.25
Order 10	-.36	-.41	-.32	.20

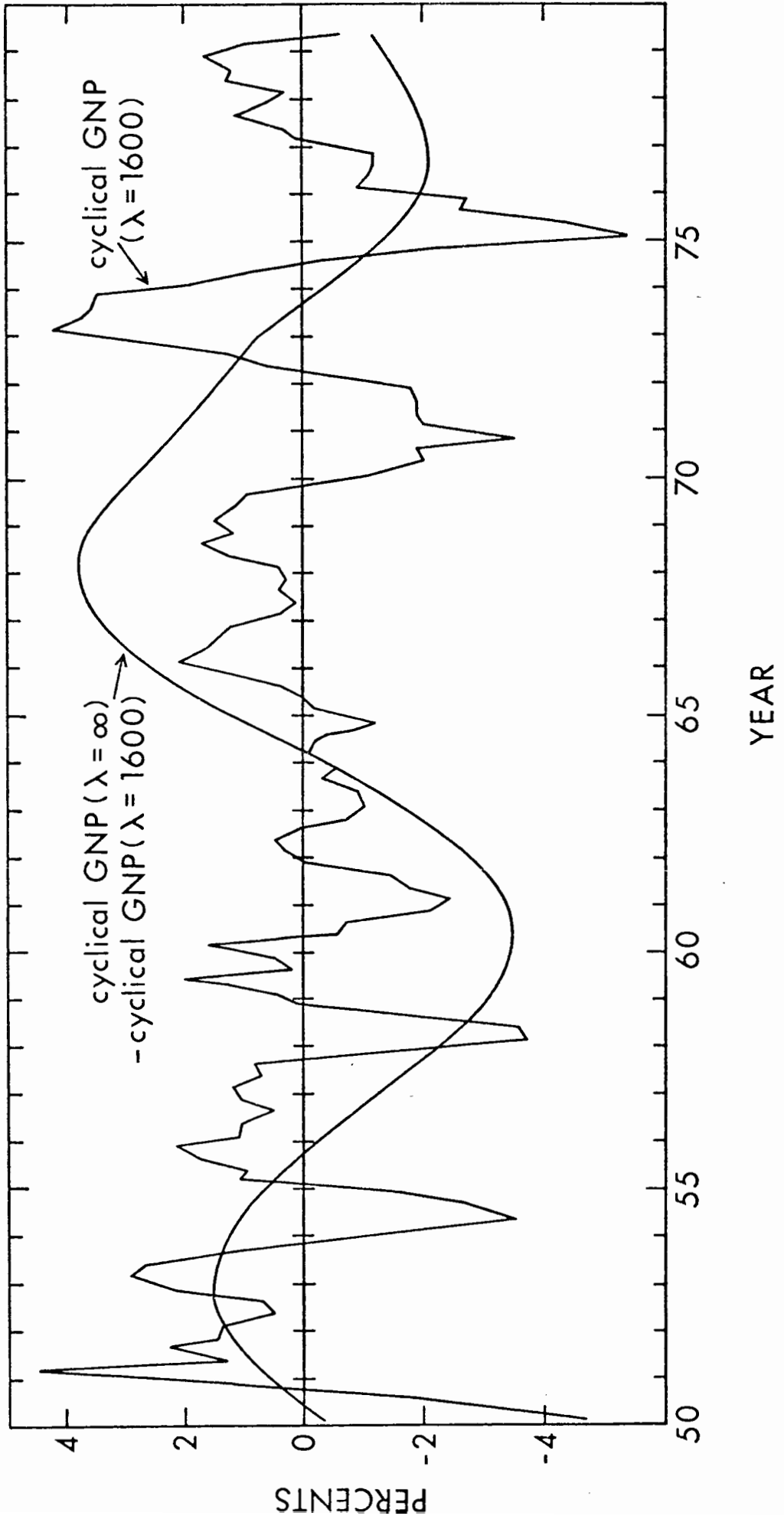


FIG. 2

it would not be necessary to index these coefficients by t and

$$(4) \quad g_{jt} = \sum_{i=-\infty}^{\infty} w_i^{\infty} y_{j,t+i}$$

where

$$(5) \quad w_i^{\infty} = .8941^i [.056168 \cos(.11168 i) + .055833 \sin(.11168 i)]$$

for $i \geq 0$ and $w_i = w_{-i}$ for $i < 0$.⁵ For t far from either the end or the beginning of the sample, the w_{it}^T are near w_{t-i}^{∞} so our method is approximately a two-way moving average with weights subject to a damped harmonic. The advantage of using the exact solution is that observations near the beginning and the end of the sample period are not lost.

The above makes it clear that the data is being filtered. As any filter alters the serial correlation properties of the data, the reported serial correlations should be interpreted with caution. The results do indicate that there is considerable persistence in the rapidly varying component of output. When using these statistics to test some model, the serial correlation of the rapidly varying component of the model's aggregate output series should be compared to these numbers. That is the model's output series should be decomposed precisely as was the data for the U.S. economy. Only then would the model's statistics and those reported here be comparable.

As the comovement results were not sensitive to the value of the smoothing parameter λ selected, in the subsequent analysis only the statistics for $\lambda = 1600$ are reported. With a larger λ , the amplitudes of fluctuations are larger but, the relative magnitudes of fluctuations of the series change little. We do think it is important that all series be filtered using the same parameter λ .

3. Variability and Covariability of the Series

The components being studied are the cyclical component and subsequently all references to a series relate to its cyclical component. The sample standard deviations of a series is our measure of a series's variability and the correlation with real GNP our measure of a series's covariability. These measures are computed for the first and the second half of the sample as well as for the entire sample. This is a check for the stability of the measures over time.

A variable might be strongly associated with real output but lead or lag real output. Therefore, as a second measure of the strength of association with real output, the R-squared for the regression

$$(8) \quad c_{jt} = \alpha_j + \sum_{i=-2}^2 \beta_{ji} \text{GNP}_{t-i}$$

for each series j was computed.

The ratio of the explained sum of the squares for this regression to the explained sum of squares for the regression when the coefficients are not constrained to be equal in the first and the second halves of the sample is our measure of stability. It is a number between zero and one with one indicating that the best fit equation is precisely the same in the first and second halves of the sample.

We choose this measure rather than applying some F-test for two reasons. First we do not think the assumption of uncorrelated residuals is maintainable. Second, even if it were, it is very difficult to deduce the magnitude of the instability from the reported test statistic.

TABLE 2

AGGREGATE DEMAND COMPONENTS: STANDARD DEVIATIONS WITH GNP

Sample Period: 1950.1 - 1979.2

	Standard Deviations in Percents			Correlations with Real Output			Average Percent of Real of Real GNP
	Whole	First Half	Second Half	Whole	First Half	Second Half	
Real GNP	1.8	1.7	1.9	-	-	-	-
Total Consumption	1.3	1.2	1.4	.739	.503	.917	61.7
Services	.7	.7	.6	.615	.441	.781	26.8
Non-Durables	1.2	1.0	1.3	.714	.575	.808	26.5
Durables	5.6	6.1	5.0	.574	.298	.884	8.4
Total Invest. Fixed	5.1	4.2	5.9	.714	.454	.884	14.2
Residential	10.7	8.5	12.4	.436	.123	.637	4.4
Nonresidential	4.9	4.4	5.3	.684	.554	.777	9.7
Equipment	5.8	5.6	5.9	.707	.642	.760	6.0
Structures	4.5	3.8	5.1	.512	.225	.698	3.7
Total Government	4.8	6.5	2.2	.258	.353	.152	22.6
Federal	8.7	11.6	4.2	.266	.377	.125	10.8
State & Local	1.3	1.6	1.0	-.170	-.408	.131	11.8

TABLE 3

AGGREGATE DEMAND COMPONENTS
STRENGTH OF ASSOCIATION WITH GNP AND
MEASURE OF STABILITY

Sample Period: 1950.1 - 1979.2

	Correlation with Real Output Squared	R^2 for Regression $c_{jt} = \alpha_j + \sum_{i=2}^k \beta_{ji} \text{GNP}_{t-i}$	Stability Measure
Total Consumption	.546	.620	.922
Services	.378	.424	.877
Non-Durables	.510	.589	.968
Durables	.329	.415	.829
Total Investment Fixed	.509	.552	.785
Residential	.190	.441	.809
Non-residential	.468	.602	.831
Equipment	.500	.631	.908
Structures	.262	.367	.834
Total Government	.067	.119	.509
Federal	.071	.129	.482
State & Local	.029	.095	.298

Aggregate Demand Components

The first set of variables studied are the real aggregate demand components. The results are summarized in Tables 2 and 3. The series that are most stable are consumption of services, consumption of non-durables and state and local government purchases of goods and services. Each of these has standard deviation less than the 1.8 percent value for real output. The investment components including consumer durable expenditures are about three times as variable as output. Covariabilities of consumption and investment with output are much stronger than the covariability of government expenditures with output.

Factors of Production

The second set of variables considered are the factors of production and productivity which is output per hour. These results are summarized in Tables 4 and 5. There is a strong and stable positive relationship between hours and output. In addition, the variability in hours is comparable to the variability in output. The contemporaneous association between productivity and output is weak and unstable with the standard deviation of productivity being much smaller than the standard deviation of output. It is interesting to note that when lead and lag GNP's are included, the association between GNP and productivity increases dramatically with the R-squared increasing from .010 to .453.

Capital stocks both in durable goods and non-durable goods industries are less variable than real output and negatively associated with output. Inventory stocks, on the other hand, have a variability comparable to output and the correlation with output is positive. Further,

TABLE 4

FACTORS OF PRODUCTION: STANDARD
DEVIATIONS AND CORRELATIONS WITH GNP

Sample Period: 1950.1 - 1979.2

	Standard Deviations in Percents			Correlations with Real Output		
	Whole	First Half	Second Half	Whole	First Half	Second Half
Real GNP	1.8	1.7	1.9	-	-	-
Capital Stocks						
Inventory	1.7	2.0	1.4	.507	.686	.309
Capital Stock Durables	1.2	1.4	1.0	-.210	-.178	-.274
Capital Stock Non-Durables	.7	.7	.7	-.236	-.185	-.297
Hours						
Work Week	2.0	2.1	1.8	.853	.896	.824
Employees	.5	.6	.5	.820	.854	.800
Productivity	1.4	1.6	1.2	.773	.831	.732
	1.0	1.0	1.1	.100	-.231	.361

TABLE 5

FACTORS OF PRODUCTION
STRENGTH OF ASSOCIATION WITH GNP
AND MEASURE OF STABILITY

Sample Period: 1950.1 - 1979.2

	Correlation with Real Output Squared	R^2 for Regression $c_{jt} = \alpha_j + \sum_{i=-2}^2 \beta_{ji} \text{GNP}_{t+i}$	Stability Measure
Capital Stocks			
Inventory	.257	.622	.828
Capital Stock Durable	.044	.235	.782
Capital Stock Non-Durable	.056	.129	.740
Hours			
Work Week	.728	.838	.954
Employees	.672	.700	.513
	.600	.801	.935
Average Product of Labor	.010	.453	.773

the strength of association of inventories with GNP increases when lag and lead GNP's are included in the regression. This is indicated by the increase in the R-squared from .257 to .622.

Monetary Variables

Results for the final set of variables are presented in Tables 6 and 7. Correlations between nominal money, velocity, and real money with GNP are all positive. The differences in the correlations in the first and second half of the sample with the exception of nominal M1 suggests considerable instability over time in these relationships. A similar conclusion holds for the short term interest rate. The correlations of GNP with the price variables are positive in the first half of the sample and negative in the second half with the correlation for the entire period being small and negative.

TABLE 6

MONETARY AND PRICE VARIABLES
STANDARD DEVIATIONS AND CORRELATIONS
WITH GNP

Sample Period: 1950.1 - 1979.2

	Standard Deviations in Percents			Correlations with Real Output		
	Whole Period	First Half	Second Half	Whole Period	First Half	Second Half
Real GNP	1.8	1.7	1.9	-	-	-
M1						
Nominal Value	.9	.8	1.0	.661	.675	.649
Velocity	1.6	2.0	1.0	.614	.801	.415
Real Value	1.5	1.2	1.7	.565	.079	.865
M2						
Nominal	1.1	.9	1.3	.480	.175	.665
Velocity	1.9	2.4	1.2	.529	.818	.131
Real Value	1.8	1.4	2.1	.432	-.221	.828
Interest Rate						
Short	.24	.27	.19	.510	.738	.255
Long	.06	.06	.06	.193	.640	-.175
Price Indexes						
GNP Deflator	1.0	1.0	1.1	-.239	.490	-.814
CPI	1.3	1.3	1.3	-.316	.223	-.799

TABLE 7

MONEY AND PRICE VARIABLES
STRENGTH OF ASSOCIATION WITH GNP AND
MEASURE OF STABILITY

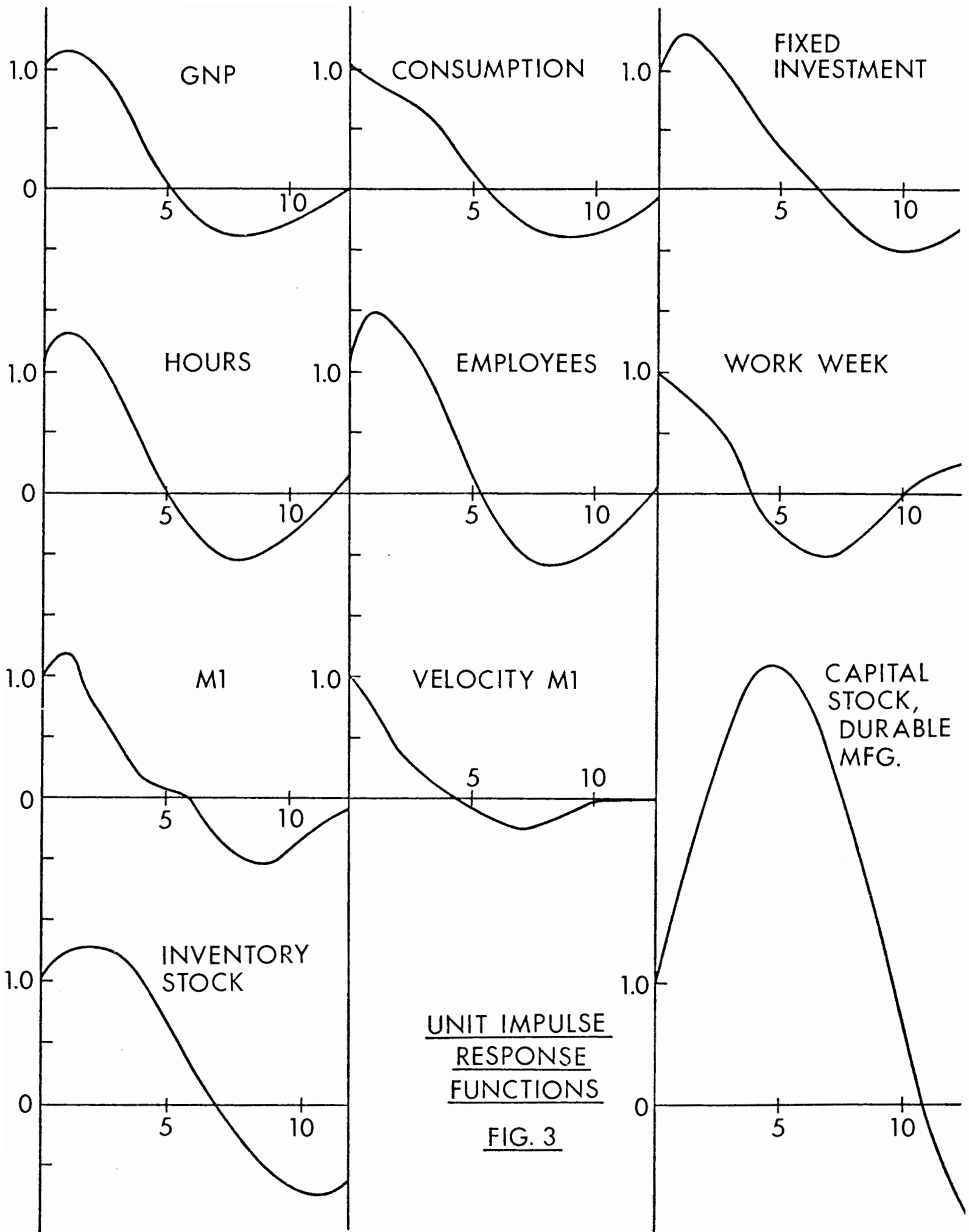
	Correlation with Real Output Squared	R^2 for Regression $c_{jt} = \alpha_j + \sum_{i=-2}^2 \beta_{ji} \text{GNP}_{t+i}$	Stability Measure
M1			
Nominal Value	.437	.445	.378
Velocity	.378	.408	.281
Real Value	.319	.495	.678
M2			
Nominal Value	.230	.371	.749
Velocity	.280	.376	.650
Real Value	.187	.428	.684
Interest Rate			
Short	.260	.506	.748
Long	.037	.381	.724
Price Index			
GNP Deflator	.057	.261	.567
CPI	.010	.330	.481

4. Serial Correlation Properties of Data Series

A sixth order autoregressive process was fit to a number of the series which displayed reasonable stable comovements with real output. In Figure 3 are plots of the unit impulse response functions for GNP and nine other series for the estimated autoregressive function.⁶ The function for GNP increases initially to a peak of 1.15 in period one and has a minimum of $-.39$ in period eight. The patterns for consumption and investment are similar except that for consumption the peak is in the initial period. The function for consumption and each of its three components (not pictured) are similar to the one for the aggregate.

The pattern for total hours and the number of employees, except for the greater amplitude, is very similar to the pattern for GNP. The average work week pattern, however, begins to decline immediately and the period of damped oscillation is shorter. The monetary variables have very different response patterns, indicating serial correlation properties very different than those of real output.

There is a dramatic difference in the response pattern for the capital stock in durable goods industries. The maximum amplitude of the response is much greater, being about 3.6, and occurs slightly over a year subsequent to the unit impulse. The pattern for the capital stock in the non-durable goods industries (not pictured) is similar though the maximum amplitude is smaller, being 2.8. For both capital stocks the peaks in the unit response function are in period five.



UNIT IMPULSE
RESPONSE
FUNCTIONS

FIG. 3

APPENDIX

All the data were obtained from the Wharton Economic Forecasting Association Quarterly Data Bank.

The short-term interest rate was the taxable three-month U.S. Treasury bill rate, and the long-term interest rate, the yield on U.S. Government long-term bonds.

FOOTNOTES

1

Lucas [1980] interprets the work of Mitchell [1913] in a similar light.

2

Examples include Litterman and Sargent [1979], Nelson and Plosser [1980], Neftci [1978], Sargent and Sims [1977], Sims [1980, a,b] and Singleton [1980].

3

We thank Paul Milgrom for bringing to our attention that the procedure we employed has been long used by actuarians.

4

This minimization has two elements, g_0 and $g_0 - g_{-1}$, which are treated as unknown parameters with diffuse priors. The Kalman smoothing technique (see Pagan [1980]) was used to efficiently compute the conditional expectations of the g_t , given the observed y_t . The posterior means of g_0 and $g_0 - g_{-1}$ are the generalized least squares estimates. The conditional expectation of the g_t for $t \geq 1$ are linear functions of these parameters and the observations.

5

See Miller [1946] for a derivation. There are certain implicit restrictions on the y_t sequence when the sample is infinite. Otherwise the g_{jt} may not exist. We require that the $\{y_t\}$ sequence belong to the space for which

$$\sum_{t=-\infty}^{\infty} .8941^{|t|} |y_{jt}| < \infty .$$

6

Letting a_t be the innovations and

$$c_t = \sum_{i=0}^{\infty} \theta_i a_{t-i},$$

be the invertible moving average representation, parameter θ_i equals the value of the unit response function in period i . One must take care in interpreting the response pattern. Two moving average processes can be observationally equivalent (same autocovariances function) yet have very different response patterns. We choose the invertible representation because it is unique. It is just one way to represent the serial correlation properties of a covariance stationary stochastic process. Others are the spectrum, the autoregressive representation and the autocovariance function.

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