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CONJECTURAL VARIATIONS

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A feature common to the perfectly competitive model and the monopoly model is each firm's disregard of other firms' reactions to its price or quantity decisions. In the former case, each firm regards itself as too small to influence the market price and therefore to attract the attention of rivals; in the latter case, the monopolist regards itself as having no rivals. (In some dynamic monopoly models, the incumbent firm does take into account the effect of its actions on potential entrants.) In many markets the assumptions necessary for application of either of these two polar models are not satisfied. The firms serving a market are neither so numerous that each contributes only a negligible fraction of the total output, nor does just one firm serve the market. Instead there are relatively few firms, each of whom has some influence over the total quantity supplied and the market price. In these circumstances each firm can anticipate that its price or quantity decisions may call forth a response from rivals. All this, as we know, was recognized by Cournot almost one-hundred-and-fifty years ago. In the model of oligopoly that he formulated, firms do not take reactions into account but maximize myopically. That model has become widely employed to explain competition among the few.

One of the most appealing features of the Cournot model of oligopoly is that it yields the monopoly solution when there is just one firm and yields the perfectly competitive solution when the number of firms increases indefinitely, assuming average production costs are nondecreasing; see also Ruffin. Thus, in terms of the traditional industrial organization characterization of a market by its structure (number of firms), conduct

(their response to each others' actions), and performance (proximity of the actual market equilibrium to the one that would prevail under perfect competition), the Cournot model provides a direct link between structure and performance. This link, however, rests on Cournot's assumption that each firm regards the current output of its rivals as fixed in deciding its profit maximizing level of output. That is, each firm believes that its rivals will not alter their levels of output in response to change in its own choice of output. The firm's belief about the rivals' response to its own action was named the conjectural variation by Frisch. The Cournot assumption is that the conjectural variation is zero.

The assumption of zero conjectural variation has been recognized as naive, especially when the reactions of rivals are thought to be sequential, as in Cournot. For rivals will in fact alter their outputs in response to each others' choices for any configuration of outputs other than the equilibrium one. Indeed if one firm recognizes that its rivals hold this naive assumption, the Stackelberg solution results.

In the Cournot model, the decision variable of each firm is assumed to be its output. Thus the conjectural variation in this model is posed in terms of output responses by rivals. But it is contended that in many real situations, price is the actual choice variable. The formulation of oligopolistic interdependence with price rather than quantity as the choice variable can, as Bertrand pointed out, lead to substantially different conclusions regarding the final market equilibrium.

In this paper we examine three issues relating to the role of conjectural variations in oligopolistic models. First, we explore the role of nonzero conjectural variations in the link between market structure and performance.

Fama and Laffer and later Kamien and Anderson noted that an industry of any fixed number (greater than one) of firms may produce the monopoly output or the competitive output or any intermediate output, depending on the conjectural variations. Reinganum proved an analogous result in a context of a dynamic differential game of research and development under rivalry.

It follows immediately that a measure or index of industry structure should reflect more than the number or size distribution of firms if it is to capture elements impinging upon market performance. Cowling and Waterson, Hause, Dickson, and Dansby and Willig have addressed the question of industry structure or performance indices, taking account of possible nonzero conjectural variations. In this paper we first show in a particularly simple format how the industry and firm output depend on the conjectural variations of the industry members. We also present some other industry structure or performance indices within this format.

Second, we observe that the Cournot assumption is usually wrong. We then ask whether any assumption about rival reaction can be correct even in the most favorable circumstance of symmetric equilibrium. The very narrow class of demand curves is displayed for which there exists an interior conjectural variation that is correct in symmetric equilibrium. Also it is shown that the limiting conjectural variation of -1 can be correct for duopolists and only duopolists with any demand function in symmetric equilibrium; under this belief, they together produce the competitive output.

Third, conjectural variations may be with regard to prices or quantities, depending on whether price or quantity is perceived as the choice variable. Sonnenschein and Bergstrom each considered the two cases as dual problems, arising in related but differing circumstances. On the other hand, Levitan

and Shubik considered a differentiated market and explored the various solutions that result, depending on whether price or output is the choice variable and on the suppositions made about the conjectural variations. In this paper, we show the link between conjectural variations in prices and in quantities in symmetric equilibrium for a differentiated product.

Homogeneous Product--Firm and Industry Output

Let the industry inverse demand function be $p(Q)$, where $Q = \Sigma_{i=1}^{n} q_i$ is industry output and q_i is the output of the i^{th} firm in the n firm industry. The demand function is twice continuously differentiable, downward sloping, and has a downward sloping associated marginal revenue function. The unit cost of production is c, constant. The profit function of firm i, to be maximized by choice of q_i, is

$$(1)\quad V(q_i) = p(Q)q_i - cq_i .$$

Before discussing the profit maximization, we take up conjectural variations and their properties. Let

$$(2)\quad w_i = \partial Q/\partial q_i = 1 + \sum_{\substack{j=1 \\ j\neq i}}^{n} \partial q_j/\partial q_i \qquad i = 1,\ldots,n$$

be firm i's belief of the rate at which industry output will change with increase in its own production. The term $\partial q_j/\partial q_i$ is the conjectural variation, the rate of change in firm j's output anticipated by firm i in response to its own change. All the pertinent information about the conjectural variations of firm i is summarized in w_i. We assume that an increase in one's own output is expected to raise industry output. Further, other firms are expected to expand their own output <u>at most</u> at the same rate as does firm i in response to i's increase. That is, each w_i satisfies $0 < w_i \leq n$. The conjectural variations are assumed to be constants throughout.

Define W by

$$(3)\quad 1/W = \sum_{i=1}^{n} (1/w_i) .$$

W is $1/n^{th}$ of the harmonic mean of the w_i's and will be called the harmonic sum. Since $0 < w_i \leq n$, we have $\Sigma_{i=i}^{n}(1/w_i) \geq n/n$ so that $0 < W \leq 1$. The harmonic mean (sum) is always smaller than the arithmetic mean (sum) unless the components are identical. Thus for a fixed arithmetic sum, the harmonic mean is largest (and equals the arithmetic mean) when the firms hold identical beliefs: $w_i = w$ for all i. The harmonic sum W tends to zero if <u>any single</u> component w_i tends to zero. And the harmonic sum $W = 1$ only if <u>all</u> components $w_i = n$.

An optimal positive finite output q_i (that we assume to exist) satisfies

$$(4)\quad V'(q_i) = q_i w_i p'(Q) + p(Q) - c = 0$$

$$(5)\quad V''(q_i) = w_i[w_i q_i p''(Q) + 2p'(Q)] \leq 0$$

for i = 1,...,n. Since (4) holds for all firms, the product $q_i w_i$ must be the same for each firm. Hence $q_i = w_1 q_1 / w_i$ so that

$$Q = \sum_{i=1}^{n} q_i = w_1 q_1 \sum_{i=1}^{n} (1/w_i) = w_1 q_1 / W .$$

Therefore

$$(6)\quad q_i/Q = W/w_i \qquad\qquad i=1,\ldots,n.$$

Equation (6) tells us that the market share of each firm depends only on the beliefs of all the firms. Specifically, a firm's share of total output equals its share in the harmonic sum of beliefs. Differing beliefs generate differing market shares. A firm's market share varies inversely with its own conjectural variation and directly with the conjectural variation of its rivals.

From (6), it follows that $w_i q_i = WQ$ so that (4) can be written as

(7) $QWp'(Q) + p(Q) - c = 0.$

Industry output depends on the harmonic sum W as well as on the unit cost and the demand function. It does not depend on the number of firms as such. Of course n affects the harmonic sum W . It is clear from (7) that the entire impact of market structure and of market conduct (conjectural variations) upon market performance--industry output--is captured in the harmonic sum W.

Dansby and Willig have introduced an industry performance gradient index defined by

(8) $$\phi = [\sum_{1}^{n} ((p_i - c_i'(q_i))/p_i)^2]^{1/2}.$$

In view of (7), this index becomes

(9) $$\phi = n^{1/2} W/\varepsilon$$

where

(10) $$\varepsilon = -p/p'(Q)Q$$

is the elasticity of demand. While the industry output does not depend on the number of firms as such, recall (7), the Dansby-Willig performance index (9) does.

Other indices of market structure include the Lerner index

$$L = (p-c)/p = W/\varepsilon$$

and the Herfindal index

$$H = \sum_{i=1}^{n} (q_i/Q)^2 = W^2 \sum_{i=1}^{n} (1/w_i^2)$$

Both depend on the conjectural variations. The former depends on the demand function while the latter does not. Note that if all firms have identical beliefs, $w_i = w$, then $H = 1/n$, independent of that belief.

The conjectural variations of each firm can be inferred from market data. From (6), $w_i = W(q_i/Q)$. The w_i are proportional to market shares, where the constant of proportionality W is found from (7) to be

$$W = \varepsilon(p-c)/p.$$

Essentially this observation was employed by Iwata in his empirical estimation.

Differentiating (7) implicitly, we find that

$$(11) \quad \partial Q/\partial W = -QP'(Q)/[WQP''+(1+W)P'].$$

To sign the denominator in the right side of (11), note that

$$(12) \quad WQP''+(1+W)P' = W[QP''+(1+W)P'/W] \leq W(QP''+2P') < 0.$$

The first inequality holds since $W \leq 1$ implies that $(1+W)/W \geq 2$ and the second inequality holds since the marginal revenue function is downward sloping. Therefore, $\partial Q/\partial W < 0$; industry output varies inversely with the harmonic sum W. We noted above that for any given arithmetic sum of conjectural variations, the harmonic sum W will be largest when the firms hold identical beliefs. Thus the more homogeneous the firms' beliefs, ceteris paribus, the smaller the industry output.

Industry output Q is maximized if W=0; that occurs if $w_i=0$ for any i. Thus if any firm believes that its output will have no impact on industry output, then the industry produces the competitive output: $p(Q) = c$. (Review (7)). This result was noted by Fama and Laffer. The competitive assumption is usually stated as the firm believes it will have no impact on industry price, but so long as the industry demand is downward sloping, this is equivalent to the belief that it will have no impact on industry output.

At the other extreme, Q is minimized if W=1, which occurs if $w_i=n$ for all $i=1,\ldots,n$. The monopolistic output results if <u>each</u> firm expects its output changes to be matched by each firm in the industry.

The standard Cournot output results if $W=1/n$; one of many ways for this to happen is that $w_i=1$ for $i=1,\ldots,n$ so each firm expects others do not respond to its output changes.

Thus the oligopoly can produce the monopoly output, the competitive output, or any intermediate output, depending on the conjectural variations. The more responsive the industry is thought to be to one's own action and the more similar are the firms' beliefs about the others' responsiveness, the smaller the industry output and the larger the industry profit will be.

Homogeneous Product — Consistent Conjectural Variations

The Cournot assumption of zero conjectural variation is naive and experience usually shows it to be inappropriate. To illustrate, consider a simple two firm example with a linear industry demand $p = a-b(q_1+q_2)$. If firm 1 selects its output to maximize its own profit, taking q_2 as given, it chooses $q_1 = (a-c)/2b - q_2/2$. This implies that a change in firm 2's output will lead to a change in firm 1's output in the opposite direction and half as large; i.e. the slope of firm 1's reaction curve is $\partial q_1/\partial q_2 = -1/2$. Thus with a linear demand curve, a firm that makes the Cournot assumption will always respond to its rival's change in output and so will not itself satisfy the Cournot assumption.

Can some other conjectural variation lead to consistent expectations in symmetric equilibrium if demand is linear? If firm 1 of the previous paragraph believed that the industry output would change with its own at rate w_1, then it would select its profit maximizing output to be $q_1 = (a-c-bq_2)/(1+w_1)b$. This means firm 1's output changes with firm 2's output at

rate $\partial q_1/\partial q_2 = -1/(1+w_1)$. In symmetric equilibrium with consistent expectations, the actual rate of change of firm 1's output with firm 2's output should equal the rate conjectured by firm 2 so $1 + \partial q_1/\partial q_2 = w_2$ with $w_1 = w_2 = w$ by the assumed symmetry. This condition can be satisfied by $w = 0$ only. Only if both firms act as though they are in a perfectly competitive situation will their expectations about each other be fulfilled in symmetric equilibrium!

The discussion above rests on the supposition that demand is linear. Is there any family of demand functions and any conjectural variation for which expectations will be fulfilled in symmetric equilibrium? We will show that the answer is "yes" for a small family of demand functions and for the limiting case of w=0 in duopoly, and "no" otherwise. Specifically, we will show that if the conjectural variations are to lead to consistent expectations in an n-firm symmetric equilibrium with $w_i = w > 0$ for all i, the demand function must be

$$(13)\quad p = a + bQ^{1-r}$$

where

$$(14)\quad r = n(n+w^2-2)/w(n+w-2), \qquad 0 < w < 2, \qquad a < c, \qquad b > 0.$$

Note that the demand function depends on w parametrically. The bound on w assures that the output selected (see appendix)

$$(15)\quad Q = [b(n-1)w(2-w)/n(c-a)(n+w-2)]^{1/(r-1)}$$

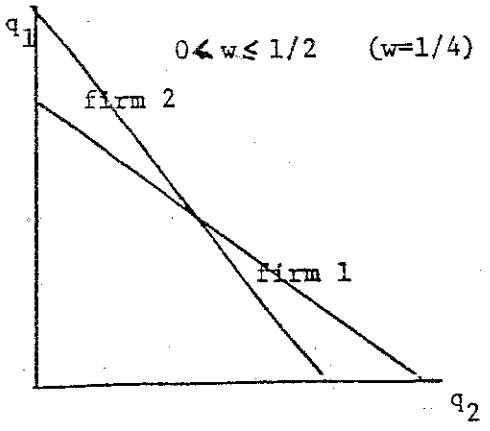
will be positive. Since the sum of a firm's conjectural variations is therefore bounded above by unity, the firms must believe that others will collectively change their output by less than the firm changes its output. The bound on the parameter a eliminates infinite profits. The parameter $r > 1$ (see appendix) so the quantity demanded varies inversely with price. The

second order conditions for a profit maximum will be satisfied under the stated conditions (see appendix).

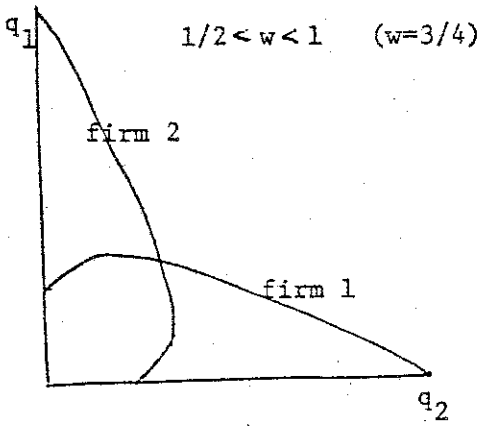
If n = 2, then r = 2, independent of w for duopoly. Thus, for a duopoly facing a unitary elastic demand $p(Q)=b/Q$, any conjectural variation less than one can be consistent in equilibrium. Firm 1's reaction function is

$$q_1 = [b(1-w)-2cq_2+(b^2(1-w)^2+4bcwq_2)^{1/2}]/2c$$

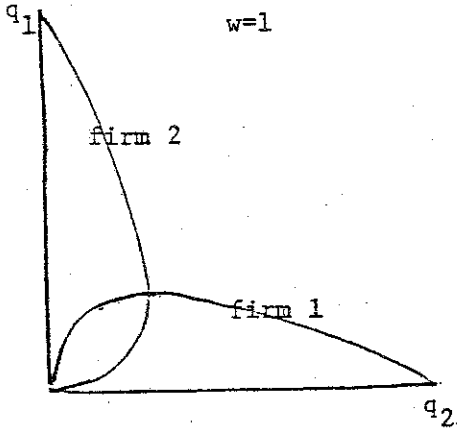
and firm 2's reaction function is analogous. These reaction functions are sketched for various values of w (with b=1, c=1/2).



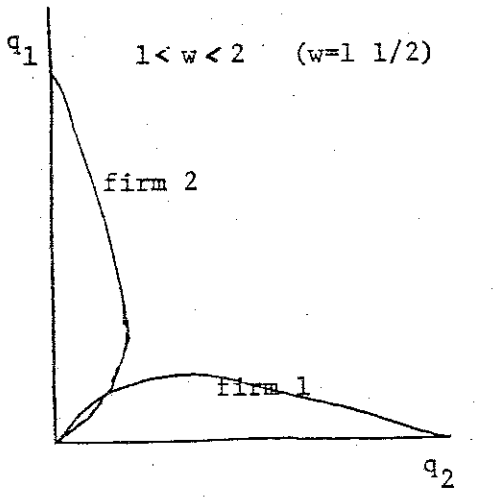
curves downward sloping



curves intersect after peak



curves intersect at peak



curves intersect before peak

If the conjectural variations are zero so that $w = 1$, then $r = n$. This (demand function (13) with r replaced by n) is the only case that the Cournot assumption will be appropriate in symmetric equilibrium.

To validate our claims in case $w > 0$, recall that under consistent expectations, a firm's conjectural variation (of the anticipated response of a rival to a change in its own output) equals the slope of the rival's reaction function (the actual response of the rival to a change in the given firm's output.) Firm i chooses its profit maximizing output q_i to satisfy

$$q_i w_i p'(q_1+\ldots+q_n) + p(q_1+\ldots+q_n) - c = 0.$$

Implicit differentiation shows that the rate of response of q_i to change in firm 1's output, i.e. the slope of firm i's reaction function, is

$$\partial q_i/\partial q_1 = -(p' + q_i w_i p")/((1+w_i)p' + q_i w_i p").$$

Therefore the rate at which industry output will actually change with firm 1's output is

$$(16)\quad 1 + \sum_{i=2}^{n} \partial q_i/\partial q_1 = 1 - \sum_{i=2}^{n} (p'+q_i w_i p")/((1+w_i)p' + q_i w_i p").$$

On the other hand, the rate at which firm 1 expects industry output to change with its own output is w_1. If expectations are to be fulfilled in symmetric equilibrium, w_1 equals expression (16). Thus with $w_i = w$ and $q_i = q$ for $i=1,\ldots,n$, we require

$$(17)\quad w = 1 - (n-1)(p'+qwp")/((1+w)p'+qwp").$$

Rearranging (17) and setting $nq = Q$ gives

$$(18)\quad Qp"/p' = -r \quad \text{where} \quad r = n(n+w^2-2)/w(n+w-2).$$

This is a differential equation for $p(Q)$. Separating variables gives $dp'/p' = -r\, dQ/Q$ that may be integrated to $p' = b_o Q^{-r}$, where $b_o < 0$ for economic sense. Integrating again produces $p(Q) = a + bQ^{1-r}$, as claimed (where $b = b_o/(1-r) > 0$ since $b_o < 0$ and $r > 1$). This shows that even in equilibrium,

conjectural variations will rarely be correct. It further suggests that a theory of oligopoly in which mutual interdependence is recognized should be dynamic rather than static, since our static theory is typically incorrect even under the most favorable circumstance of symmetric equilibrium.

We now consider the limiting case that $w=0$. With $w=0$, (17) becomes $0=2-n$. This is satisfied if and only if $n=2$. Therefore if duopolists have a conjectural variation of -1, it will be consistent for any demand function. With this belief, the industry produces the competitive output. To see why such a belief is consistent, note that in this case, the first order condition reduces to $p(q_1+q_2)=c$. Firm 1's output is therefore $q_1=p^{-1}(c)-q_2$, the competitive output minus firm 2's output. The slope of this reaction function is -1, in agreement with the conjectural variation of -1.

Differentiated Product

Suppose there are n symmetric firms, each selling a single good that is an imperfect substitute for the goods of the n-1 other firms. For instance, one may think of the products differing in color or flavor. The price each firm receives for its product can be written as a function of the quantity made available by each firm. For the first firm

$$(19) \quad p_1 = P(q_1,q_2,\ldots,q_n).$$

The price received by the i^{th} firm, $i=2,3,\ldots,n$ is written similarly, except that the positions of q_1 and q_i are interchanged on the right. Letting subscript j indicate the partial derivative with respect to the j^{th} argument, we assume the functions

$$(20) \quad P_j = P_2 \quad \text{for} \quad j = 2,\ldots,n$$

(symmetry) and

(21) $P_1 < P_2 < 0$.

An increase in the quantity supplied by any firm will depress the price received by firm 1. Further, an increase in his own output will depress his price more than will an equal increase in the output of a rival. (If the goods were perfect substitutes, in the limit, the impacts would be equal.)

As examples of the general model, one may think of the linear form

(22) $p_i = A - Bq_i - D\sum_{j=1}^{n} q_j$

or the isoelastic (loglinear) form

(23) $p_i = Aq_i^{-B}(q_1,\ldots,q_n)^{-D}$,

where $B > 0$, $D > 0$. It is readily checked that (20) and (21) are satisfied in each case.

Each firm has a constant unit cost of c. Firm 1 chooses his output q_1 to maximize his profit:

$$\max_{q_1} \; [P(q_1,q_2,\ldots,q_n)-c]q_1$$

A positive finite symmetric solution in which all n-1 rivals behave identically satisfies

(24) $P - c + (P_1+(n-1)P_2k)q_1 = 0$

where

(25) $k = \partial q_j/\partial q_1, \qquad j = 2,\ldots,n$

is firm 1's conjectural variation in quantities, the assumed rate at which each rival will adjust output in response to its own output change. The second order condition is assumed to be satisfied.

In symmetric equilibrium, each firm behaves identically, $q_j = q$ for $j=1,\ldots,n$ and (24)-(25) hold for each firm, i.e. with q_1 replaced by q in (24) and by q_i in (25). The same price is received by each firm.

Alternatively, the firms may perceive the problem as one of choosing prices. The system of n price equations represented by (19) can be inverted to express the quantities that can be sold by each firm as a function of the prices charged by each. For firm 1, the demand function is

$$(26) \quad q_1 = Q(p_1,p_2,\ldots,p_n).$$

The demand functions for the other firms are similar; in the demand function for firm i, p_1 and p_i are interchanged on the right. The properties of these demand functions are obtained using the price equations from which they have been derived, together with the implicit function theorem. To determine the partial derivatives of (26), differentiate totally the n equations represented by (19), yielding in symmetric equilibrium ($q_j=q$, $j=1,\ldots,n$) the system

$$(27) \quad dp_j = P_1 dq_j + \sum_{\substack{i=1 \\ i\neq j}}^{n} P_2 dq_i , \qquad j = 1,\ldots,n.$$

Solving this system by Cramer's rule (see appendix) gives

$$(28) \quad \partial q_i/\partial p_i = (P_1+(n-2)P_2)/(P_1+(n-1)P_2)(P_1-P_2) < 0, \quad i=1,\ldots,n$$

$$(29) \quad \partial q_i/\partial p_j = -P_2/(P_1+(n-1)P_2)(P_1-P_2) > 0, \quad i,j=1,\ldots,n;\ i\neq j$$

where the signs follow from (21). As expected, the quantity demanded from firm i will increase if either firm i reduces its price or any rival raises price.

If firm 1 perceives the problem as one of selecting price, it chooses p_1 to maximize its profit

$$\max_{p_1} \ [(p_1-c)Q(p_1,p_2,\ldots,p_n)]$$

The first order condition satisfied by a positive finite price in symmetric equilibrium is

(30) $Q + (p_1-c)[Q_1 + (n-1)Q_2m] = 0$

where Q_1 and Q_2 denote the partial derivatives of Q with respect to its first and second arguments respectively (and $Q_2 = Q_j$, $j = 2,\ldots,n$ by the assumption of symmetric equilibrium) and where

(31) $m = \partial p_j/\partial p_1$, $\quad j = 2,\ldots,n$

is firm 1's conjectural variation in prices, the rate at which it believes any rival will adjust price in response to a change in its own. By symmetry, equations (30)-(31) hold for each firm; p_1 may be replaced by p in (30) and by p_i with $i=1,\ldots,n$; $i\neq j$, in (31).

If the conjectural variation in quantities is zero, so each firm believes that its rivals will not change their quantities sold in response to an increase in its own sales, it then implicitly assumes that the rivals will lower price to maintain sales in the face of its own price reduction (that must accompany a planned increase in sales). Any conjectural variation in quantities or in prices implicitly implies a corresponding equivalent conjectural variation in prices or in quantities respectively that yields the same symmetric equilibrium price and quantity. To see more generally how the conjectural variations in prices and in quantities are related, we observe that (24) and (30) can give identical price-output prescriptions only if

(32) $-(p-c)/q = P_1+(n-1)kP_2 = 1/[Q_1+(n-1)mQ_2]$.

This was obtained by arranging both (24) and (30) to give expressions for $-(p-c)/q$ and equating the expressions.

Using (28)-(29) as expressions for Q_1 and Q_2 respectively,

(33) $Q_1+(n-1)mQ_2 = [P_1+(n-2)P_2-(n-1)mP_2]/[P_1+(n-1)P_2](P_1-P_2)$.

Substitute (33) into (32) and collect terms. Rearranging the result gives

(34) $1-m = (1-k)(1-Z)/[1+(n-1)kZ]$

where

$$(35) \quad Z = P_2/P_1, \qquad 0 < Z < 1$$

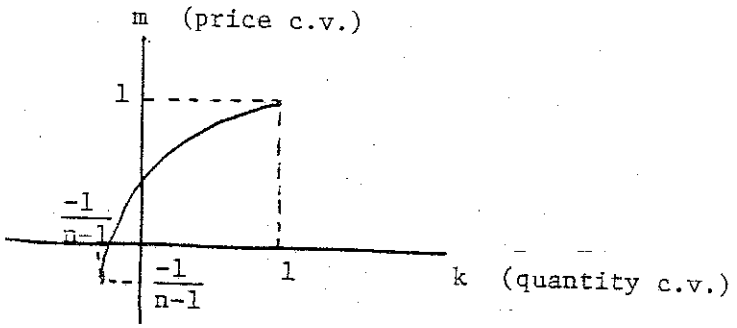
reflects the extent of perceived product differentiation. The bounds on Z follow from (21). The limiting case of Z=0 indicates independent products while the limiting case of Z=1 indicates perfect substitutes. For both the linear (22) and loglinear (23) forms of inverse demand function, Z = D/(B+D), a constant.

Equation (34) is our objective, relating the conjectural variation in prices m to the equivalent conjectural variation in quantities k. In our simple model, that relationship depends parametrically on the number of rivals and on the extent of perceived product differentiation. In the limit, if k=1, then m=1; if the firms expect their changes in output to be followed exactly, then they implicitly expect their price changes to be followed exactly, and vice versa. At the other extreme, if k = -1/(n-1), then m = -1/(n-1). If each firm believes that its increase in quantity will be just offset by reductions in quantity by each of its rivals so industry output in unchanged, then it implicitly expects that its price increase will be offset by price reductions by each of its rivals to maintain industry sales.

We can now check our assertions about the meaning of zero conjectural variations. If k=0, then m=Z. If the firm expects rivals to keep their quantities fixed despite an increase in its own output, it is implicitly expecting others to change their price in the same direction as it changes its own price, but by a lesser amount (since 0 < Z < 1). On the other hand, if m = 0, then k = -Z/(1+(n-2)Z) < 0. To understand this, suppose we raise our price and thereby reduce sales. If rivals are expected to maintain their prices unchanged (m = 0), then they must increase their output (to accommodate

the customers we lose by raising our price). Thus their output is expected to move in the opposite direction that ours does. That is, the conjectural variation in quantities is negative if the conjectural variation in prices is zero.

Having discussed (34) at several interesting points, we next look at the relationship between m and k, for given values of n and Z. It is an increasing, concave relationship from $(-1/(n-1),-1/(n-1))$ to $(1,1)$.



Thus an increase in one conjectural variation implies an increase in the other. Nonetheless, the conjectural variations need not always have the same sign, as we saw earlier. There is an interval in which a positive conjectural variation in prices implies a negative conjectural variation in quantities.

Comparative statics analysis applied to the first order condition (24) or (30) indicates, as expected, that an increase in the conjectural variation (either k or m) will lead to a reduction in the equilibrium quantity, an increase in the equilibrium price, and an increase in profits. The maximum profit is achieved by perfect coordination: $m = k = 1$.

Summary

We have discussed several facts about conjectural variations in static symmetric equilibrium. We showed that a harmonic sum of conjectural variations in a homogeneous market contains all the relevant information about market structure (numbers) and conduct (beliefs or coordination) for determining market performance (industry output). The role of similarity of beliefs in market performance was noted. We also wrote several established structure and performance indices in our format using the harmonic sum of conjectural variations.

It was shown that even in symmetric equilibrium, firms' conjectural variations are unlikely to be correct since there is only a very narrow class of demand functions and associated interior conjectural variations for which this is possible. This class was derived. It was also shown that the limiting conjectural variation that leads to the competitive output can be consistent for duopolists and any demand function, but not for any other finite number of producers.

Finally, we have demonstrated the relationship between conjectural variations in prices and conjectural variations in quantities that are equivalent in the sense of yielding the same price and output within the context of a differentiated product market in symmetric equilibrium.

Appendix

1. We show that $r > 1$, i.e. that $1 < n(n+w^2-2)/w(n+w-2)$. Since $w < n$ and $2 \leq n$, it follows that $w(n-2) < n(n-2)$ and that $w^2 < nw^2$. Adding inequalities yields $w(n+w-2) < n(n+w^2-2)$ which implies $r > 1$ as was to be shown.

To find Q in (15), we substitute (13) into (7) and solve. The second order condition (5) implies $Q^{-r}(1-r)b(2-wr/n) \leq 0$ for demand function (13). Since $1-r < 0$ while $b > 0$, this implies that $2-wr/n \geq 0$. Substituting for r and rearranging the inequality leads to $n-1 > (w-1)^2$. Since the left side is at least 1 (since $n \geq 2$) and the right side is bounded above by 1 (since $0 < w < 2$), this second order condition is satisfied.

2. We will show how to solve the system of linear equations in (27) using Cramer's rule. To evaluate the determinant of an n x n matrix with a on the main diagonal and b elsewhere, subtract the first column from each of the other columns. Then add each of the last n-1 rows to the first row. Finally expand by the first row:

(A1)

$$\begin{vmatrix} a & b & b & \cdots & b \\ b & a & b & \cdots & b \\ & & & & \\ b & b & b & \cdots & a \end{vmatrix} = \begin{vmatrix} a & b-a & b-a & \cdots & b-a \\ b & a-b & 0 & \cdots & 0 \\ & & & & \\ b & 0 & 0 & \cdots & a-b \end{vmatrix} = \begin{vmatrix} a+(n-1)b & 0 & 0 & \cdots & 0 \\ b & a-b & 0 & \cdots & 0 \\ & & & & \\ b & 0 & 0 & \cdots & a-b \end{vmatrix}$$

$$= [a+(n+1)b](a-b)^{n-1}.$$

If the first column is replaced with a vector with 1 in the first row and zero elsewhere, the determinant is readily evaluated by expanding by the first column and then applying the above result:

$$(A2) \qquad \begin{vmatrix} 1 & b & b & \dots & b \\ 0 & a & b & \dots & b \\ & \dots & & & \\ 0 & b & b & \dots & a \end{vmatrix} = [a+(n-2)b](a-b)^{n-2}$$

If the first column of the original matrix is replaced by a vector with 1 in the second row and zeros elsewhere, the determinant may be evaluated by expanding by the first column, subtracting the first column of the reduced determinant from each of the other columns, and then expanding by the first row:

(A3)

$$\begin{vmatrix} 0 & b & b & \dots & b \\ 1 & a & b & \dots & b \\ & \dots & & & \\ 0 & b & b & \dots & a \end{vmatrix} = - \begin{vmatrix} b & b & b & \dots & b \\ b & a & b & \dots & b \\ & \dots & & & \\ b & b & b & \dots & a \end{vmatrix} = - \begin{vmatrix} b & 0 & 0 & \dots & 0 \\ b & a-b & 0 & \dots & 0 \\ & \dots & & & \\ b & 0 & 0 & \dots & a-b \end{vmatrix}$$

$(n \times n)$ $\qquad$ $((n-1)x(n-1))$ $\qquad$ $((n-1)x(n-1))$

$$= -b(a-b)^{n-2}.$$

Let $a = P_1$ and $b = P_2$. The determinant of the coefficient matrix on the right in (27) is given by (A1). Using Cramer's rule, (28) is the ratio of (A2) divided by (A1) and (29) is the ratio of (A3) divided by (A1).

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