DISCUSSION PAPER NO. 437

ON THE FORMATION OF PRICE EXPECTATIONS:
An Analysis of Business Test Data
by Log-Linear Probability Models

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August 1980
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* The research on which this paper is based was carried out under a program
financed jointly by the National Science Foundation (USA), Grant SOC 74-21194,
by the Deutsche Forschungsgemeinschaft, Grant 219/10, and with the generous
assistance of the Institut National de la Statistique et des Etudes Économiques,
the Direction de la Prévision of the Ministère de l’Economie, and a grant from
the Centre National de la Recherche Scientifique under a joint cooperative
agreement between France and the United States for 1979. None of our benefac-
tors are responsible for the opinions herein expressed.

Footnote continued next page.
We wish to thank G. Flaig, S. Kawasaki and K. Zimmermann of the University of Mannheim; John Link and Quang Vuong of Northwestern University; and J.P. Grand-jean and Benoit Ottenwaelter of INSEE for able research assistance. A. Renard of INSEE constructed the data sets from the INSEE Enquête sur l'Industrie used in our work. We are indebted to W. Strigel, of the Ifo Institut für Weltwirtschaftsforschung, Munich, for making the data from the Ifo Konjunktur Test available to us. We thank him and Karl Kuhlo (Ifo) for helpful comments on research related to this paper and Edmond Malinvaud, Georges de Menil, and other participants of the Econometrics Seminar, CNRS, Paris, for comments on a preliminary version of part of it.

We also thank the participants of the International Seminar on Macroeconomics, June 23-24, 1980, Oxford, especially our discussants Laurits Christensen and Hugh Wills, for useful suggestions. We are most grateful to Jean Waelbroek and Robert J. Gordon for their suggestions and help in revising and shortening our paper, as presented to the ISOM, for publication in its present form.
1. Introduction

There are a number of studies on the formation of price expectations which make use of aggregates over time based on survey data. In addition, there have been some attempts to incorporate other types of time-series aggregates of intentions or attitudes of consumers or firms drawn from surveys. With but one exception (Theil, 1966, pp. 417-24), however, there are no studies which make use of the microdata themselves to study reported expectations and plans directly and thus to get inside the "black box" of the firm to test various models of the formation and revision processes. In most macroeconometric models and other studies involving aggregate time-series data, it is necessary to make a series of assumptions about how firms form expectations and plans and how they revise them on the basis of new information. These assumptions can be tested only indirectly within the context of the behavioral model assumed. Survey data on actual expectations and plans, aggregate or micro, seldom play any role at all.

For many years and in a large number of countries data have been collected from individual firms on expectations, plans, appraisals, and past realizations for a variety of variables (Strigel, 1977). The oldest and most famous of these surveys is that done by the Ifo-Institut, Munich, every month since November 1949, for the Federal Republic of Germany. Other surveys such as that conducted by the
"Service de la Conjoncture" of INSEE, beginning in the late 1960's for France, are administered less frequently but often contain data on a greater number of variables. These business tests, as they are called, offer a unique source of data for the analysis of how expectations and plans are formed and revised at the level of the individual firm.

Unfortunately, the Ifo data are not available in machine-readable form. Our resources enabled us only to put data in this form beginning in January 1977. As of time of writing, our data were complete only to July 1978. Although the INSEE data are in machine-readable form, tapes are available to us only beginning in 1974 and most of our analyses had to be carried out with the 1977 and 1978 tapes, initially made available to us.

While the business-test data present formidable problems of analysis and are available to us only over a relatively short period of time, we believe that they offer a challenging opportunity for the investigation of how firms form and revise plans and expectations. In this paper, we concentrate on prices. Elsewhere (König, Nejover, Oudiz 1979a, 1979b), we have dealt with production plans and subsequent realizations; König (1980) has also considered prices for a relatively small group of firms in the German textile industry over the period January 1975 - July 1977. Here we use data on a much larger number of firms for France, as well
as for Germany, to formulate and test a number of different models of price
effectuation/plan formation and revision, including some models involving the joint
determination of production plans and price expectations/plans.

Our principal finding is that there exists a strong, and relatively stable
for the French data, relationship between price expectations/plans and subsequent
revisions. We also find changes in price expectations/plans strongly positively
associated with deviations of previous expectations/plans from the realizations
to which they refer. We find this also to be the case for production plans,
and that, for the German case, revisions of production plans are significantly
positively associated with revisions of price expectations/plans. In addition,
expectations with respect to future demand conditions appear to be positively re-
lated to price expectations/plans for the German data but not for the French
data.

Nature of the Data: Problems and Advantages

Our analyses are based, for Germany, on monthly data for approximately 4500
individual establishments over the period January 1977 - December 1978 and, for France, on data collected in March, June and November from approximately 1600 firms for the period March 1974 - November 1977.

Almost all data are categorical, indeed most are trichotomous. The categori-
cal data can be classified into three groups:

i) variables that reflect plans and/or expectations (ex ante data)
ii) variables referring to realizations (ex post data)
iii) variables indicating appraisals.

In the Ifo business-test data, the realizations describe monthly changes of
variables; data on plans and expectations are given in terms of changes over the
next three-month period. An exception is expected changes in business conditions,
which refer to six-month period in the future. This difference in the unit period
of observation creates serious problems in the analysis of the relation between
expectations/plans and realizations. The approach used here aggregates the data
on realizations for three consecutive months into a new variable having a unit
period of three months. ¹

The INSEE data are collected every three or four months (March, June and
November); the periods to which expectations and plans refer are always compar-
able to the realizations reported on the following survey despite the variation
in periods.
With respect to prices, the questions in the Ifo Business Test are:

- "Our domestic prices (net), considering changing conditions, have risen, not changed, fallen over the preceding month."

- "Our domestic prices (net) will, considering changing conditions, rise, remain the same, fall in the course of the next three months."²

The corresponding questions in the INSEE questionnaire are as follows:

- "Would you indicate the variation of your sales prices (net of tax) for the period [since the last survey]."

- "What will be the probable variation of your sales prices (net of tax) for the period [until the date of the next survey]."

The first of each of these pairs of questions is ambiguous; the sources of ambiguity are whether respondents do or do not take into account seasonal variations or changes in the general price level. The latter are perhaps not so important in the countries and periods under consideration, but would certainly be in countries experiencing high rates of inflation. In any case, in almost every period, a substantial number of firms report declining prices or price expectations/plans despite the increasing general level of industrial prices.

The question with respect to expectations/plans is more difficult to interpret. It is not clear from the way in which the question is asked in both the German and French surveys whether the responses represent primarily forecasts or
primarily planned prices, i.e., whether the majority of firms are price takers or price setters. Although it is possible to obtain some indirect evidence on this point, we remark here that over periods as short as three months, it is perhaps not extremely important, since set prices may respond to market conditions over longer periods of time. As more data are accumulated, it will be possible to formulate models to distinguish between these alternatives more definitively.

The short periods of time over which we have been able to obtain data on individual firms presents another difficulty: The general trend of industrial prices has been upward in both countries; although there have been fluctuations in the rates of change, neither economy experienced a recession during the period. This severely limits our ability to test models in which economy-wide variables affect individual behavior and may also influence the results we obtain using the microdata. Nonetheless, there is substantial variation in behavior at the micro-level, with many firms reporting or anticipating price decreases. Such variation in individual behavior offers a considerable advantage over the use of aggregate time series alone in modeling the formation of price expectations/plans.
The principal difficulty encountered in the analysis of business-test data at the micro-level is that the data are almost exclusively categorical. Moreover, we would like to examine a number of simultaneous inter-relations among several such variables. Methodological problems of the analysis of cross-classified data have been the subject of considerable current interest (Reynolds, 1977; Fienberg, 1977; Upton, 1978). We also have dealt elsewhere at some length with a particular methodological approach especially useful in the analysis of categorical data of the kind we encounter in the Ifo and INSEE Business Tests (Nerlove and Press, 1976, 1978; König and Nerlove, 1979; König, Nerlove and Oudiz, 1979a, 1979b). The approach is based on a parameterization of the probabilities characterizing large multidimensional contingency tables developed by Birch (1963), Hosteller (1968), Bishop (1969), and in many recent papers by Goodman (1978). The approach also lends itself to a suitable generalization of measures of ordinal bivariate association to analyses involving more than two categorical variables. 2a
While these methods of analysis do not permit structural estimation in the traditional econometric sense, as developed for the analysis of continuous data, the methods we use do permit formulation of conditional probability models, with relatively few parameters, which correspond under certain assumptions to reduced form equations. Estimation of conditional probability models permits us to make inferences about the direction and strength of association, although, as is typically the case in the analysis of cross-section data, inferences about the direction of causality are hazardous, since it is hardly ever appropriate to assume that post hoc implies propter hoc. Nonetheless, throughout the remainder of the paper, we do generally assume that timing determines the direction of causality, i.e., that prior events, plans and expectations reported by the firm influence current expectations and plans and not vice versa.

Plan of the Remainder of the Paper

The remainder of our paper is organized as follows: In Section 2, we discuss previous studies, some of which have been based on survey data, and the various models of price expectation/plan formation we estimate and test, including models of the joint determination of production plans and price expectations/plans. Section 3 briefly describes the log-linear probability model, which is the statistical
tool used in our analyses, and also considers measures of partial bivariate association among ordinal variables. Section 4 takes up the question of the stability and significance of the association between current price expectations/plans and subsequent realizations. Although stability and significance of this relationship does not imply corresponding stability and significance of the underlying process of price expectation/plan formation, it provides both a useful introduction to the method of analysis and a justification for studying the formation process, since a strong and stable relation between price expectations/plans and subsequent realizations, noted above, suggests the utility of the former in forecasting the latter, a subject to which we will return in a subsequent paper. Section 5 discusses the estimates of a number of conditional probability models for price expectations/plans. Section 6 discusses estimates of joint models of production plans and price expectations/plans. We conclude with some remarks on the implications of our results for further directions of research.

2. Models of Price Expectation/Plan Formation

Previous Studies

Previous studies on the formation of price expectations are based upon the aggregates of business test data or similar survey data. G. de Neenil and S. S. Bhalla (1975) extend the concept of balances, discussed below, assuming a normal distri-
bution of response of the SCF data on price expectations in order to construct an appropriate time series for testing the Phillips-curve concept. In a study by J. A. Carlson and N. Parkin (1975), answers in categorical form from the Gallup Poll for the United Kingdom are assumed to be normally distributed and, under the additional assumption of constancy of the variance during the sample period, can be converted into a time series of inflationary expectations. These are, in turn, related to actual price changes by an adaptive expectation mechanism.

Similarly, A. Knübl (1974) uses the same approach for analyzing the price expectation formation process in Germany. He also shows that price expectations are strongly related to current and past actual price changes as well as to variations in the demand pressure.

In most previous work, so-called balances have been used to aggregate the categorical responses obtained from surveys of the business test type. In this procedure the number or percent of the total number of respondents reporting a negative response (−) is subtracted from the number or percent reporting a positive response (+). As noted by Carlson and Parkin (1975), inter alia, the aggregate balances neglect the information afforded by the no change (±) category or those who report that they don't know. Carlson and Parkin have devised an ingenious procedure for using the information in a manner that nonetheless permits a single
aggregate time series to be created. Their method is based on some rather stringent distributional assumptions and certain arbitrary assumptions about the thresholds at which changes in response occur. These are normality of the frequency distribution of responses, constancy over time, and a choice of numerical scaling of a threshold for answers of the type "prices will remain the same" (supposed to be independent of the rate of inflation and to be constant over time). The normality of the distribution function has been questioned in another study by J. A. Carlson (1975), indicating strong evidence for skewness; other problems are discussed extensively by J. Forster and M. Gregory (1977).

A second problem that arises in the use of aggregate balances as a statistic in the analysis of business test data derives from the fact that the value of a balance must always lie between two limits (e.g., -1 and +1 or -100% and +100%). While such limits are of no importance when the balance is treated as an independent or explanatory variable, use of the balance as a dependent variable causes certain well-known problems. The use of probit or logit analysis, or variants thereof, offers a partial solution, although only recently have such methods been extended to situations in which there are more than two categories. In earlier work, for example, Theil (1966, pp. 417-24), more than two categories
were aggregated by treating, say, the "u" and the "n" categories as a single category or the "m" and the "n" categories as a single category. Unfortunately, the results are likely to be sensitive to such collapsing since the proportions in the various categories may vary significantly over time in response to variations in general business conditions or other economy-wide variables affecting all firms or consumers. The methods we have used to analyze the microdata from Ifo and INSEE do not aggregate categories and treat the categories symmetrically.

A third problem arises simply because of the aggregate nature of the balances. It is easy to construct an example for two categorical variables which are in fact independent but, for which, over time, the balances show perfect positive correlation. This situation may easily arise because changing common environmental factors influence all individuals over time. It is analogous to the problem of spurious correlation among time series, but use of the micro data may enable us to establish the true nature of the relationship (in the example, independence) even though we may not be able to observe or specify the common environmental factors responsible for the spurious relationship among the aggregate variables.

Price Expectations/Plans and Subsequent Realizations

As indicated in Section 1, price expectations/plans for the German data refer to a three-month period in the future, whereas realizations are reported at the
time of the monthly survey for the preceding month. Aggregates are formed in the manner described in footnote 1. This problem does not arise for the French data where realizations reported at the survey date refer to the period since the last survey, whether it be three or four months. In our notation the subscript always refers to the date of the survey, and we do not explicitly note the fact that the German price realizations are constructed from the current survey and two previous monthly surveys. Since the period between the French surveys is variable, we denote the current survey date by \( t \), the immediately preceding survey date by \( t - q \). An asterisk will denote an expectation/plan. Thus \( P_t^* \) is the price expectation/plan for the coming period and \( P_{t+q}^* \) is the subsequent realization reported for that period in the survey taken on date \( t+q \).

In section 4 below, we examine the strength and stability of the association between \( P_t^* \) and \( P_{t+q}^* \).

Models for the Determination of Price Expectations/Plans

All variables reflect changes rather than levels. Some of the models formulated and estimated here, however, refer to changes in responses from one period to the next or to differences between expectations/plans and the realizations to which they refer, i.e., surprises. Since all of the data with which we deal in this paper are categorical, generally trichotomous and ordinal, it is necessary
to define what we mean by a change of a variable between two points in time (survey dates) and by a surprise. In each case, we do so by converting two trichotomous variables at different points in time into a third trichotomous variable. A backward difference of the variable $X_t$ is defined by the table

$$
\begin{array}{ccc}
\Delta X_t & X_t & X_{t-q} \\
+ & - & + \\
+ & - & + \\
+ & - & - \\
\end{array}
$$

The subscript $t$ to $\Delta X$ refers to the date of the survey on which the realization or anticipation is reported; thus, for realizations, $\Delta X_t$ refers to the change between events two periods ago and the period preceding the date of the survey, whereas for anticipations it refers to the change between the currently reported anticipation and the one previously reported on the preceding survey date. For example, the Northwest corner of the table shows a positive change $\Delta X_t$: $X_{t-q}$ was negative, but $X_t$ is positive. Similarly, the Southwest corner shows a negative change. The diagonal elements are cases in which $X_{t-q}$ and $X_t$ are both the same.
A surprise is defined by the table

\[
\begin{array}{c|c|c|c}
+ & = & + & + \\
- & = & - & - \\
- & - & - & - \\
\end{array}
\]

Thus, the subscript \( t \) in \( EX_t \), referring to the date of the current survey, points to realizations since the last survey date, \( t-q \). The anticipations, \( X^*_t \), are for the period forward to the date of the current survey.\(^4\) The interpretation of the surprise variable is similar to that of the change variable. For example, the Northeast corner of the table indicates that although \( X_t \) was expected to decrease at time \( t-q \), in fact, in the interval to \( t \), it increased; thus, a positive surprise occurred.
With these distinctions in mind, we henceforth denote models by enclosing the variables which they include in parentheses; causality is assumed to run in the direction past to present, so that we think of current values of variables as conditioned on past values of variables included in the model.

MODEL I: Adaptive Expectations. \( (\hat{P}_t^\hat{o} \mid \hat{P}_{t-\tau}^o, P_t^o) \)

In its original early formulation, the adaptive expectations model related the change in expected normal prices to the difference between last period's realized price and last period's expectation (Nerlove, 1956, 1958). The variables \( \hat{P}_t^\hat{o} \) and \( P_t^o \) already represent changes in levels, but we can consider changes in \( \hat{P}_t^\hat{o} \) in relation to surprises as defined above. Below we call this the "error-learning" model of expectation formation. In the notation used in this paper, we would write the error learning model as \( (\Delta \hat{P}_t^\hat{o} \mid E\hat{P}_t) \). Clearly this model represents a special restricted case of the adaptive expectations model that places no quantitative restrictions on the relation between \( \hat{P}_t^\hat{o} \) and the previous expectation/plan and realization, \( \hat{P}_{t-\tau}^o \) and \( P_t^o \), respectively. We view the more general form as more appropriate in the case of quantitative data since, this form allows the strength of the association between current expectations/ plans and immediately preceding ones to differ from the strength of the association of the former with realizations. Thus, if we write, as we would for contin-
uos variables

\[ P_t^s = \theta P_t + (1-\theta) P_{t-q}^s \]

it suggests that the stronger the association between \( P_t^s \) and \( P_t \) relative to the association between \( P_t^s \) and \( P_{t-q}^s \), the larger the coefficient of expectation, \( \theta \).

The formulation implies that the probability that \( P_t^s \) takes on a particular categorical value is conditional on the expectation/plan of the previous period and subsequent price behavior, but does not rule out association between \( P_{t-q}^s \) and \( P_t \). Indeed, this association is generally strong and stable. Moreover, for reasons explained elsewhere (e.g., Kawasaki, 1979), it is generally more efficient computationally to estimate conditional log-linear probability models by estimating the joint probability model from which the conditional is derived.

MODEL II. Error-Learning. \( (\Delta_p^s | EP_t) \)

Because the adaptive expectations model places no restrictions on the relationship among the three variables: \( P_t^s, P_{t-q}^s \) and \( P_t \), it is possible that the strong relationship between \( P_{t-q}^s \) and \( P_t \) so dominates any relationship among the three variables that only weak and unstable relationships between \( P_t^s \) and/or \( P_{t-q}^s \) and \( P_t \) remain. Model II represents the simple adaptive expectations model in restrictive form as a relation between \( \Delta P_t^s \) and \( EP_t \).
MODEL III. Extrapolative Expectations. (P_t^* | P_t, P_{t-d})

An alternative to adaptive expectations or the special case of this model, the error-learning model, is a model of purely extrapolative expectations. In this model current price expectations/plans are related to the two preceding realizations. Estimation of this model requires complete data on each firm included in the sample for a five-month period in the German case and for the preceeding survey in the French case.

We also formulate a model of price expectation/plan formation which contains a variable related to expected future business conditions (German data) or expected future demand (French data). The relevant survey questions are

German data:

\( G_t^* \): "Our business conditions for product XY are expected to be in the next 6 months, corrected for seasonal variation -- improved, about the same, deteriorated."

French data:

\( D_t^* \): "Change in demand (domestic and foreign). Probable trend in the period until the next survey - increasing, stable, decreasing."

We have argued elsewhere (König, Nerlove and Oudis, 1979a) that, although, the Ifo-survey question with respect to business conditions is highly ambiguous, the responses probably represent expectations with respect to future changes in demand and that, therefore, \( G_t^* \) is comparable to \( D_t^* \).
MODEL IV. Adaptive Expectations with Expected Future Demand or Business Conditions.
\[ (P^*_t \mid \mathbb{I}_{t-1}, P^*_t, Q^*_t \text{ or } D^*_t) \]

This model simply adds \( Q^*_t \) or \( D^*_t \) to the list of conditioning variables in Model I above. We have not estimated Models II or III with this addition.

Joint Models of Production Plans and Price Expectations/Plans

To the extent that production plans and price expectations/plans are influenced by common variables, we would expect them to be jointly determined. Clearly, it would be desirable to introduce such variables in the model directly if they are observable. At this stage of our work, however, we have only considered very simple models involving previous realizations of both variables. Curiously, it turns out that the association between production plans and price expectations/plans is strong for the German data but weak for the French data, when the association with past realizations is taken into account.

The relevant additional questions with respect to realized production, \( Q_t \), and production plans, \( Q^*_t \), are

German data:

- \( Q_t \): "Our production with respect to product XY was, in relation to the preceding month- augmented, unchanged, diminished."

- \( Q^*_t \): "Our production with respect to product XY is planned to be in the course of the next three months, corrected for seasonal variations- increased, remain the same, decreased."
French data:

\( Q^e_t \): "Change in your production. Trend in the past period - increase, stability, decrease."

\( Q^n_t \): "Change in your production. Probable trend in the next period - increase, stability, decrease."

The joint models considered are:

MODEL V. Future Production Plans and Past Realizations

\[ (p^n_t, Q^n_t \mid P^e_t, Q^e_t) \]

In Model V, we add future production plans, \( Q^n_t \), and the immediately preceding realizations, \( Q^e_t \), to a simple extrapolative model of price expectations

\[ (p^n_t \mid P^e_t) \].

MODEL VI. Changes in Price Expectations/Plans and Production Plans in Relation to Surprises.

\[ (\Delta P^n_t, \Delta Q^n_t \mid EP_t, EQ_t) \]

Thus, Model VI adds an error-learning component in production plans to the simple error-learning model of price expectations/plans.

Results for these two models are reported in Section 6 below.

3. Method of Analysis: The Log-Linear Probability Model

The ANOVA Decomposition

The log-linear probability model for the analysis of large multidimensional contingency tables represents a parameterization of the probabilities underlying the
table, which are generally assumed to be strictly positive. The parameterization may be chosen in several different ways to facilitate the interpretation of the data. In particular, the model represents the logarithm of the probabilities in a form, analogous to the analysis of variance (ANOVA), involving main effects, bivariate interaction effects, and so on. Restriction of the number and order of interaction configurations in the model permits us to characterize the probabilities in terms of a relatively small number of parameters. Moreover, we can choose these parameters (corresponding to a particular choice of basis for the vector space of the logarithm of the probabilities) in several different ways in order to facilitate the interpretation of relationships among variables. Finally, the conditional probabilities associated with the joint probabilities of a log-linear probability model are also log-linear involving a reduced set of main and interaction configurations.

Let \( Q = \{ A_1, \ldots, A_q \} \) be a set of categorical random variables, which may take on, respectively, \( I_1, \ldots, I_q \) possible values. If we have a sample of \( N \) observations on the \( q \) categorical random variables, we might arrange these in an \( I_1 \times I_2 \times \cdots \times I_q \) table of correspondings to a similar arrangement of the probabilities

\[
\begin{align*}
P_1, \ldots, P_q, i_1 = 1, \ldots, I_1, i_2 = 1, \ldots, I_2, \ldots, i_q = 1, \ldots, I_q
\end{align*}
\]

Alternatively, one can order the logarithms of the

\[
Q = \sum_{k=1}^{q} I_k
\]
probabilities into a Q×1 vector by some principle, e.g., lexicographically,

\[
\log p = \begin{bmatrix}
\log p_1 & \ldots & 1 \\
\vdots & \ddots & \vdots \\
\log p_{11} & \ldots & 1_q
\end{bmatrix}
\]

The vectors \log p may be thought of as points in a vector space. The problem is then to choose the basis in a convenient manner (Nerlove and Press, 1978). Convenience may be defined both in terms of ease of interpretation and ease of reduction in the number of parameters characterizing the probabilities, which amounts to reduction in the dimensionality of the space in which \log p is represented, that is, a reduction in the number of basis vectors spanning the space.

There are clearly many possible choices of a basis. One of the most interesting and useful of these is the choice, which in the case of full dimensionality, allows us to represent the logarithms of the probabilities in a traditional analysis of variance format

\[
\log p_{i_1, \ldots , i_q} = \mu + \alpha_{i_1}(i_1) + \ldots + \alpha_q(i_q) + \beta_{12}(i_{11},i_{12}) + \ldots + \gamma_{q-1,q}(i_{q-1},i_q) + \ldots \\
+ \alpha_1, \ldots , \gamma_q(i_1, \ldots , i_q)
\]
The representation (3) still leaves scope for choice, however, without further restriction, since, for \( i_1 \neq 1, \ldots, i_1, \ldots, i_q \neq 1, \ldots, i_q \), the right-hand side of (3) contains many more parameters than the left-hand side.

By including a vector consisting entirely of ones in the basis, the parameter \( \mu \) may be chosen so as to normalize to one the sum of the probabilities defined in (3). As noted (Nerlove and Press, 1976, pp. 7, 14-18), this normalization leads to a representation of the probabilities in a multivariate generalization of the well-known logistic form. Further, restrictions, however, are required to reduce the dimensionality of the right-hand side of (3) to the dimensionality of the left-hand side, or less. When the two are exactly equal, the model is called saturated. One convenient choice imposes the traditional analysis of variance summation constraints on the parameters of the configurations

\[ \alpha_1(\cdot), \ldots, \alpha_i(\cdot), \ldots, \alpha_q(\cdot) \]:

\[ \begin{align*}
\alpha_1(\cdot) - \alpha_2(\cdot) - \ldots - \alpha_q(\cdot) &= 0, \\
\beta_{12}(i_1, \ldots, i_2) &= 0, \ldots, \beta_{q-1,q}(i_1, \ldots, i_q) = 0 \\
\omega_1, \ldots, \omega_q(\cdot) &= 0, \ldots, \omega_1, \ldots, \omega_q(\cdot, i_2, \ldots, i_q) = 0.
\end{align*} \]

The dot used in place of an index denotes summation over that index. The parameters \( \alpha_i(i_1), \ldots, \omega_1, \ldots, \omega_q(i_1, \ldots, i_q) \) have the usual ANOVA interpretation: \( \mu \)
denotes an overall effect; \( \alpha_k(l_1) \) denotes an effect due to \( A_k \) (at "level" \( l_1 \));
\( \beta_{12}(l_1, l_2) \) denotes a bivariate interaction effect between \( A_1 \) and \( A_2 \) (at "levels\( l_1 \) and \( l_2 \), respectively); and \( \omega_1, \ldots, \omega_q(l_1, \ldots, l_q) \) denotes a \( q \)-order interaction
among \( A_1, \ldots, A_q \) (at "levels" \( l_1, \ldots, l_q \), respectively). The basis corresponding to the restrictions (4) is called the deviation-contrast basis.\(^6\)

Hierarchical Models. Deviation-Contrast versus Score Parametrization

In all of the models, for which estimates are presented in Sections 4-6 below, higher-order configurations than bivariate are suppressed, leading to a parsimonious representation of the probabilities of the model. Corresponding main effects are, however, always included; thus our models are members of a special class of log-linear probability models called hierarchical.\(^7\) Kawasaki (1979, p. 154) has shown that, for hierarchical models, the joint probability for any number of
categorical variables may be decomposed into a product of component probabilities,
each summing to one and the product of which is equal to the joint probability,
each of which depends only on one interaction configuration. Thus we may interpret a bivariate interaction configuration as contributing a component to the probability depending only on a pair of variables, after other interactions and pro-
portional variations represented by main effects are accounted for.
For example, in the trichotomous case the nine deviation-contrast parameters characterizing any bivariate interaction configuration may be arranged in the form of a table

<table>
<thead>
<tr>
<th></th>
<th>$g(1,1)$</th>
<th>$g(1,2)$</th>
<th>$g(1,3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Only the four parameters in the upper left corner are estimated, the remainder being derived from the restrictions (4). The parameters of this table are proportional to the component probabilities of the configuration in the Kawasaki decomposition. As formulated, the deviation-contrast parameterization makes no use of any ordering among the variables; however, one can see how the estimates might reflect ordering and, therefore, association as follows: Suppose that $g(1,1) - g(2,1) = g(2,1) = g(2,2) = 0$; the table shows that then

$g(1,1) - g(3,1) = -g(1,3) = -g(3,1)$, indicating a strong positive or negative association between the two variables of the configuration depending on the sign of $g(1,1)$. Similarly, a large positive value of $g(2,2)$ indicates a concentration in the no change categories; whereas a negative value indicates its absence. The presence or absence of significant concentration in the no change categories is particularly important in econometric applications. Large positive or negative values for the off-diagonal elements, $g(1,2)$ and $g(2,1)$, in relation
to the diagonal, reflect variation in the degree of concentration of one variable with respect to changes in the other and tend, given the corner values and the center, to lower the degree of bivariate association, positive or negative.

An alternative parameterization in terms of "scores" has been suggested by Haberman (1974) and developed in some detail in several unpublished papers by Quang Vuong (1979a, 1979b). This alternative may be useful in interpreting directions and other characteristics of association among ordinal variables. (A somewhat different approach is that of Goodman, 1979). Both Haberman and Vuong suggest equispaced scores corresponding to the ordering of the variables.

In the trichotomous case, the bases for both are identical. We norm all basis vectors, except that corresponding to the over-all effect to $\sqrt{2}$, which corresponds to the norms of these vectors in the deviation-contrast basis. 9

For example, in the case of a trichotomous bivariate interaction configuration, we have the following relationship:

$$
\sigma_{11} = \beta(1,1) + \frac{1}{2} \left[ \beta(1,2) + \beta(2,1) \right] + \frac{1}{4} \beta(2,2)
$$

$$
\sigma_{12} = -\sqrt{\frac{3}{2}} \left[ \beta(1,2) + \frac{1}{2} \beta(2,2) \right] = \sqrt{\frac{3}{4}} \left[ -\beta(1,2) + \beta(3,2) \right]
$$

$$
\sigma_{21} = -\sqrt{\frac{3}{2}} \left[ \beta(2,1) + \frac{1}{2} \beta(2,2) \right] = \sqrt{\frac{3}{4}} \left[ -\beta(2,1) + \beta(2,3) \right]
$$

$$
\sigma_{22} = \frac{1}{4} \beta(2,2).
$$
where $\alpha_{11}$, $\alpha_{12}$, $\alpha_{21}$, and $\alpha_{22}$ are the particular score parameters chosen for the analyses reported below. Equations (5) permit an easy interpretation of the score parameters in terms of our previous discussion: $\alpha_{22}$ provides an unambiguous measure of clumping in the center categories of no change, since $\beta(1,1)$ provides a measure of direction of association when $\beta(1,2) = \beta(2,1) = \beta(2,2) = 0$, then so must $\alpha_{11}$. The parameters $\alpha_{12}$ and $\alpha_{21}$ depend only on the skewness parameters $\beta(1,2)$ and $\beta(2,1)$ of the deviation-contrast representation, holding $\beta(2,2)$ constant.

### Measures of Association and Partial Association

One of the most important topics in the analysis of categorical data is the measurement of association among ordinal variables, especially partial association, controlling for the influence of additional variables when more than two variables are considered at the same time. This is analogous to the estimation of regression coefficients or partial correlations in multiple regression analysis, if attention is restricted to conditional log-linear probability models. As shown in Nerlove and Press (1976) and elsewhere, one can always interpret joint probabilities in terms of a series of conditional probabilities and vice versa. Thus, the analogy may be carried over to situations in which joint dependence among several categorical variables, some of which may be ordinal, is of interest. Besides Haberman (1974), extensive discussions of measures of
ate association between two ordinal categorical variables are contained in Wilson (1974), Reynolds (1977, Chapter 3), Upton (1978, pp. 34-38), and Goodman (1979).

An important and frequently used measure is the Goodman-Kruskal \( \gamma \)-coefficient, developed in a series of four papers, reprinted as Goodman and Kruskal (1979).

This measure has been generalized by Kawasaki (1979, Chapter 6), in the context of multivariate log-linear probability models, to a so-called component gamma coefficient, which is a measure of partial bivariate association based on the bivariate-interaction parameter estimates from a joint or a conditional log-linear probability model. Davis (1967) extends the Goodman-Kruskal coefficient to the multivariate case in a manner based directly on the observed contingency table and without reference to the log-linear model representation of the contingency table probabilities. As we have seen, the first parameter, \( \gamma_{11} \), of the bivariate interaction configuration also provides a measure of partial association.

The Goodman-Kruskal gamma coefficient is defined for two-way tables; to generalize it to a measure of partial bivariate association in the multivariate case, Kawasaki (1979, Chapter 6) makes use of the multiplicative decomposition of the joint probabilities discussed above: Neglecting trivariate and higher-order interaction configurations, a \( \gamma \)-coefficient defined for a particular bivariate compon-
ent probability represents the bivariate association after main effects and other bivariate interactions have been taken into account. Asymptotic variances of this component gamma coefficient may be obtained from the values of, and the variance-covariance matrices for, the underlying parameters of the configuration by the delta method (Kawasaki, 1979, pp. 161-163).

When trivariate and higher-order interactions are included in the model, definition and measurement of partial association becomes more difficult. Perhaps the simplest way to proceed is to regard the measure of bivariate association between two variables, say A and B, as a function of the level of a third variable C, or of a third and a fourth variable, etc. In the case of business-test data, however, it is rarely possible to estimate interactions of order greater than two.

4. Stability and Significance of the Relation between $P_1$ and $P_{t-q}$

The association between price expectations/plans and subsequent realizations is positive, significant, and one of the strongest found in our models. Such an association may arise either because price expectations are very good forecasts (rational expectations) or because firms are setting prices. In this section, we examine this relationship in more detail without attempting to resolve the issue of whether the association arises because price expectations are rational or be-
cause, over short periods at least, prices are simply set. Note that stability of the relation between price expectations/plans and subsequent realizations does not imply, nor is it implied by, stability of the underlying process generating the price expectations/plans.

Table 4.1 shows the score parameters, gamma coefficient and $\chi^2$ test against independence for the Ifo data for the saturated model ($P_t^0, P_t^{eq}$). The periods are April 1977, July 1977, October 1977, January 1978, April 1978 and July 1978. The number of observations in each sample, $N$, is given in the last line. The results show gamma coefficients ranging from 0.85 to 0.93, all highly significant.

The first score parameter is quite stable across periods, ranging from 1.21 to 1.66 and also highly significant. The values of the fourth score parameter, $\gamma_{22}$, show that there is a definite tendency towards clumping in the no change categories; the values of $\gamma_{21}$ are positive, significant, and stable over time. The remaining score parameters are insignificant and unstable. Thus, the very high values of gamma are due in part to clumping in the no change categories, but largely to the pure positive association indicated by the first score parameter.

Corresponding results for the INSEK data, June 1977, November 1977, and March 1978 are presented in Table 4.2. Although the associations are slightly less strong than in the German data, identical conclusions emerge.
Table 4.1: Score Parameters for the Bivariate Interaction Configuration for the Model \( (P_t^*, P_{t+q}) \), Gamma Coefficient and Chi-Square Test against Independence. Ifo Data, 1977-1978.
(t-statistics in parentheses.)

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_{11} )</td>
<td>1.5115 (5.8352)</td>
<td>1.2088 (8.7119)</td>
<td>1.2864 (10.2600)</td>
<td>1.2808 (11.5310)</td>
<td>1.6645 (6.3911)</td>
<td>1.5647 (5.9253)</td>
</tr>
<tr>
<td>( \alpha_{12} )</td>
<td>-0.2578 (1.3836)</td>
<td>-0.2787 (2.6242)</td>
<td>-0.1521 (1.6561)</td>
<td>-0.5461 (4.9806)</td>
<td>0.0190 (0.1167)</td>
<td>0.0682 (0.4064)</td>
</tr>
<tr>
<td>( \alpha_{21} )</td>
<td>-0.1058 (0.6904)</td>
<td>0.1583 (1.7855)</td>
<td>-0.1209 (1.3465)</td>
<td>0.0125 (0.1723)</td>
<td>-0.2529 (1.6376)</td>
<td>-0.1521 (1.5468)</td>
</tr>
<tr>
<td>( \alpha_{22} )</td>
<td>0.5587 (5.2941)</td>
<td>0.6001 (10.3491)</td>
<td>0.6002 (7.0867)</td>
<td>0.5856 (8.7754)</td>
<td>0.3956 (4.1050)</td>
<td>0.5305 (5.3486)</td>
</tr>
<tr>
<td>( \gamma_{AB} )</td>
<td>0.9190 (23.5311)</td>
<td>0.8542 (21.2804)</td>
<td>0.8624 (29.1773)</td>
<td>0.8692 (31.2050)</td>
<td>0.9324 (32.9739)</td>
<td>0.9254 (27.2937)</td>
</tr>
</tbody>
</table>

\( X^2 \) against independence
\( DF=4 \)

| 526. | 497. | 475. | 697. | 647. | 531. |

\( N = \) number of observations

| 2362 | 2210 | 2154 | 2373 | 2533 | 2548 |
Table 4.2: Score Parameters for the Bivariate Interaction Configuration for the Model \( (P_t, P_{t+q}) \), Gamma Coefficient, and Chi-Square test against independence. INSEE Data, June 1977, November 1977 and March 1978. (t-statistics in parentheses.)

<table>
<thead>
<tr>
<th>Bivariate interaction</th>
<th>June 1977</th>
<th>November 1977</th>
<th>March 1978</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_t \times P_{t+q} )</td>
<td>( \alpha_{11} )</td>
<td>.1466 (8.7531)</td>
<td>.9743 (7.2246)</td>
</tr>
<tr>
<td>( \alpha_{12} )</td>
<td>-.2345 (-1.9970)</td>
<td>-.2979 (-2.5005)</td>
<td>-.2948 (-1.2266)</td>
</tr>
<tr>
<td>( \alpha_{21} )</td>
<td>.0104 (.1044)</td>
<td>-.0712 (-.6984)</td>
<td>-.3161 (-1.5343)</td>
</tr>
<tr>
<td>( \alpha_{22} )</td>
<td>.4273 (5.3070)</td>
<td>.3359 (4.1310)</td>
<td>.5345 (3.4562)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>.8225 (19.666)</td>
<td>.7477 (13.142)</td>
<td>.8957 (16.117)</td>
</tr>
</tbody>
</table>

\( \chi^2 \) against dependent DF=4

| \( \chi^2 \) | 286 | 144 | 217 |
| \( N \) | 1043 | 964 | 1008 |
In order to test whether the relation between price expectations/plans and subsequent price realizations is significantly stable over time (insignificantly different), we fit the joint model \( (P_t^o, P_{t+q}^o, P_t^b, P_t) \) and test the hypothesis that the bivariate interaction configurations \( P_t^o \times P_{t+q} \) and \( P_{t-q}^0 \times P_t^b \) are identical between adjacent periods. This test is carried out using the deviation-contrast parameterization and an approximate Chi-square test based on Theil’s suggestion for testing linear restrictions in regression analysis (Theil, 1971, pp. 238, 285). The tests are made for the INSEE data for six pairs of dates between June 1974 and November 1977 and for the Ifo data for five pairs of periods January 1977 – April 1977.

The idea of the test is very simple. In the ANOVA representation each bivariate interaction configuration is characterized by 4 parameters, the remaining 5 are determined by the ANOVA restrictions. Let variable 1 be \( P_t^o \) and let variable 2 be \( P_t \), both as of a given date; we consider two periods and the unconstrained model (\( P_{t-q}^0 \), \( P_t^o \), \( P_t^b \), \( P_{t-q}^0 \)). Let the first period be denoted by a superscript 0 and the second by a superscript 1; then the restriction that the relation \( (P_t^o, P_{t+q}) \) is stable over time may be made by testing the linear restrictions

\[
\delta_{12}^{0} (i_1, i_2) = \delta_{12}^{1} (i_1, i_2), \quad i_1, i_2 = 1, 2.
\]
Table 4.3 presents the Chi-square test statistics and the associated probabilities for the upper tail of the Chi-square distribution for four degrees of freedom for selected pairs of months between June 1974 and November 1977 for the INSEE data and between January 1977 and October 1978 for the Ifo data.

In the INSEE case for the most part, except the last period, the values of Chi-square are low resulting in large probabilities for accepting the null hypothesis of stability in consecutive periods in the relation between price expectations/plans and subsequent realizations: The first probability is 0.77 which means we can accept the null hypothesis with a high degree of confidence, whereas the last probability is only 0.084 which means we would reject the null hypothesis at a 10 percent significance level but accept it at a 5 percent level.

The Ifo data show a substantially greater degree of instability in the relation between $P_t^+ x P_{t+q}$ and $P_{t-q}^+ x P_t^+$. In only two cases out of five can the null hypothesis that the interaction parameters are the same be accepted at a five percent level. If thus appears that the high degree of association between price expectations/plans and subsequent realizations found for both the German and the French data, masks considerable differences both among periods and between behavior in the two economies.
Table 4.3. Tests of the Stability of the Relationship between $P^*_t$ and $P^*_t \text{eq}$, Chi-Square Value Constrained vs. Unconstrained Model, Associated Probability and Degrees of Freedom, Sample Size. Various Periods INSEE and Ifo Data.

<table>
<thead>
<tr>
<th>Period</th>
<th>$\chi^2$</th>
<th>df</th>
<th>Probability</th>
<th>Sample Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>INSEE</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>June 1974 - November 1974</td>
<td>1.836</td>
<td>4</td>
<td>0.7660</td>
<td>654</td>
</tr>
<tr>
<td>November 1974 - March 1975</td>
<td>3.120</td>
<td>4</td>
<td>0.5379</td>
<td>704</td>
</tr>
<tr>
<td>March 1975 - June 1975</td>
<td>3.537</td>
<td>4</td>
<td>0.4722</td>
<td>836</td>
</tr>
<tr>
<td>June 1975 - November 1975</td>
<td>7.752</td>
<td>4</td>
<td>0.1011</td>
<td>899</td>
</tr>
<tr>
<td>March 1977 - June 1977</td>
<td>6.798</td>
<td>4</td>
<td>0.1469</td>
<td>880</td>
</tr>
<tr>
<td>June 1977 - November 1977</td>
<td>8.218</td>
<td>4</td>
<td>0.0839</td>
<td>784</td>
</tr>
<tr>
<td>Ifo</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>January 1977 - April 1977</td>
<td>10.32</td>
<td>4</td>
<td>0.0353</td>
<td>2519</td>
</tr>
<tr>
<td>July 1977 - October 1977</td>
<td>25.50</td>
<td>4</td>
<td>0.0000</td>
<td>2622</td>
</tr>
<tr>
<td>October 1977 - January 1978</td>
<td>8.899</td>
<td>4</td>
<td>0.0637</td>
<td>2933</td>
</tr>
<tr>
<td>January 1978 - April 1978</td>
<td>15.74</td>
<td>4</td>
<td>0.0034</td>
<td>3044</td>
</tr>
<tr>
<td>April 1978 - July 1978</td>
<td>8.754</td>
<td>4</td>
<td>0.0675</td>
<td>3069</td>
</tr>
</tbody>
</table>
5. **Conditional Models for $P_t^*$**

In Section 2, we describe the models of price expectation/plan formation based on past price expectations and realizations alone. In this section some results for those models are given in abbreviated form for chosen periods in which all relevant bivariate interaction configurations were fully estimable. Although more comprehensive statistics are available, only the component gammas and first score parameters of the bivariate configurations are given here.

**MODEL 1. Adaptive Expectations**


For both the German and the French data, the strongest and most stable association appears to be between $P_t$ and $P_{t-q}^*$; this is not a consequence of the adaptive expectations hypothesis, but it persists in the results for all models containing these two variables individually, regardless of what others are included. This result suggests that the $P^*$'s are more of the nature of plans than of expectations. The lack of significant positive association between production plans, noted in Section 6, or expectations of future demand and price
<table>
<thead>
<tr>
<th>Data Set, Period, Chi-square Test against Independence</th>
<th>Bivariate Interaction Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_t^* \times P_{t-q}^*$</td>
</tr>
<tr>
<td><strong>Ifo</strong></td>
<td></td>
</tr>
<tr>
<td>April 1977. $\chi^2 = 987$</td>
<td>0.50 (4.19)</td>
</tr>
<tr>
<td>$\eta_{11}$</td>
<td>0.61 (3.26)</td>
</tr>
<tr>
<td>July 1977. $\chi^2 = 878$</td>
<td>0.38 (2.62)</td>
</tr>
<tr>
<td>$\eta_{11}$</td>
<td>0.42 (2.30)</td>
</tr>
<tr>
<td>October 1977. $\chi^2 = 794$</td>
<td>0.66 (7.03)</td>
</tr>
<tr>
<td>$\eta_{11}$</td>
<td>0.87 (4.26)</td>
</tr>
<tr>
<td>January 1978. $\chi^2 = 902$</td>
<td>0.56 (4.65)</td>
</tr>
<tr>
<td>$\eta_{11}$</td>
<td>0.68 (3.32)</td>
</tr>
<tr>
<td>April 1978. $\chi^2 = 1041$</td>
<td>0.28 (5.67)</td>
</tr>
<tr>
<td>$\eta_{11}$</td>
<td>0.72 (4.02)</td>
</tr>
<tr>
<td>July 1978. $\chi^2 = 974$</td>
<td>0.69 (6.80)</td>
</tr>
<tr>
<td>$\eta_{11}$</td>
<td>0.90 (3.96)</td>
</tr>
<tr>
<td><strong>INSEE</strong></td>
<td></td>
</tr>
<tr>
<td>June 1977. $\chi^2 = 362$</td>
<td>0.35 (2.25)</td>
</tr>
<tr>
<td>$\eta_{11}$</td>
<td>0.36 (2.07)</td>
</tr>
<tr>
<td>November 1977. $\chi^2 = 227$</td>
<td>0.62 (3.81)</td>
</tr>
<tr>
<td>$\eta_{11}$</td>
<td>0.74 (2.67)</td>
</tr>
<tr>
<td>March 1977. $\chi^2 = 289$</td>
<td>0.41 (1.23)</td>
</tr>
<tr>
<td>$\eta_{11}$</td>
<td>0.65 (1.20)</td>
</tr>
<tr>
<td>June 1978. $\chi^2 = 393$</td>
<td>0.34 (1.86)</td>
</tr>
<tr>
<td>$\eta_{11}$</td>
<td>0.36 (1.70)</td>
</tr>
</tbody>
</table>
expectations/plans for the French data, noted below, also supports this hypothesis, but the evidence from the German data does not, since in most cases, a strong and fairly stable relation between expected future business conditions (probably an indicator of demand; see König, Nerlove and Owidz, 1979a) and price expectations/plans is found; there is also a positive association between production plans and price expectations/plans in the German data, noted in Section 6.

On the whole, the adaptive expectations hypothesis is not well supported by the French data, although it is somewhat better supported by the German data. The two crucial interactions \( P_t^0 \times P_{t-4}^0 \) and \( P_t^0 \times P_t \) are relatively strong and stable in the German data, but the results are quite mixed for the French data.

**MODEL II. Error-Learning**

The results for the error-learning model for prices are presented in Table 5.2. The model is estimable for the German data in all periods selected for analysis except April 1977. Unfortunately, the Model II is estimable for the INSEE data for 1977-78 only for one period, November 1977; we have also estimated the model for the period 1974-76, although these results are not presented here. For the periods for which the model is fully estimable, the results show
<table>
<thead>
<tr>
<th>Data Set</th>
<th>Period</th>
<th>Chi-Square Test against Independence</th>
<th>Bivariate interaction Configuration $\Delta P_x \times \Delta P_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ifo</td>
<td>July 1977. $\chi^2 = 451$</td>
<td>$\gamma$</td>
<td>$\sigma_{11}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.89 (21.50)</td>
<td>1.55 (5.75)</td>
</tr>
<tr>
<td></td>
<td>October 1977. $\chi^2 = 284$</td>
<td>$\gamma$</td>
<td>$\sigma_{11}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.78 (16.85)</td>
<td>1.05 (7.32)</td>
</tr>
<tr>
<td></td>
<td>January 1978. $\chi^2 = 218$</td>
<td>$\gamma$</td>
<td>$\sigma_{11}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.83 (16.49)</td>
<td>1.55 (3.99)</td>
</tr>
<tr>
<td></td>
<td>April 1978. $\chi^2 = 297$</td>
<td>$\gamma$</td>
<td>$\sigma_{11}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.88 (24.53)</td>
<td>1.79 (6.85)</td>
</tr>
<tr>
<td></td>
<td>July 1978. $\chi^2 = 447$</td>
<td>$\gamma$</td>
<td>$\sigma_{11}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.91 (21.82)</td>
<td>1.78 (6.77)</td>
</tr>
<tr>
<td></td>
<td>November 1977. $\chi^2 = 103$</td>
<td>$\gamma$</td>
<td>$\sigma_{11}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.82 (16.66)</td>
<td>1.45 (6.51)</td>
</tr>
</tbody>
</table>
a strong, significant, and positive association. It is interesting to note, in
the fuller detail available for this model, but not presented here, that $\alpha_{12}$ of
the bivariate interaction in the score parameterization is negative and signifi-
cant in two cases and that the French result yields an $\alpha_{22}$ many times that for the
German data; that is, there is an absence of clumping in the no change/no change
category. In fact, in these cases the reverse is true; the no change/no change
category contains fewer observations than the other cells despite the strong
positive association between $\Delta P_t$ and $EP_t$ in all cases. An examination of the
main effects for the INSEE estimate also reveals large, negative, and highly
significant main effects $\alpha_{20}$ and $\alpha_{02}$, i.e., for the second category of both vari-
able. (Results for the INSEE data 1974-76 confirm these conclusions.)

Since the joint model $(\alpha P_t, EP_t)$ is fully saturated the Chi-square test
against independence provides a useful measure of goodness of fit, open to fewer
objections than is the case for unsaturated models. Although the component gamma
coefficient and first score parameter are highly stable and significant, we note
that the Chi-square statistic, although always highly significant for 6 degrees
of freedom, varies considerably.

On the whole one may conclude that Model II is strongly supported by the data
at our disposal. A note of caution, however, is in order: although constructed by
the method outlined above and, therefore, not directly comparable to a quantita-
tive difference or error, \( \Delta \hat{Q}_t \) "contains" \( P_{t-q} \) and \( \hat{Q}_t \) "contains" \( P_t \), both with the
same "sign." Moreover, the positive association encountered here, in the case
of Model II, is similar in magnitude and significance to that encountered for
\( P_{t-q} \) and \( P_t \) in the estimates for Model I; thus, the encouraging results present-
ed for Model II may not be entirely what they seem. A careful examination of the
results over a longer period of time could not alter this conclusion unless, we
were to find evidence against Model II, i.e., instability over time or lack of
significant association.

MODEL III. Extrapolative

The model \( (P_t^* \mid P_t, P_{t-q}) \) is estimable from the Ifo data only for July 1977,
January 1977, April 1978 and July 1978; from the INSEE surveys we can obtain
estimates for all four periods in 1977 and 1978.

The estimates, presented in Table 5.3., show significant and reasonably
stable positive association between realized prices in consecutive periods
\( P_{t-q} \times P_t \) in France but only a weak association for the German data. For both
countries there exists a positive association between immediately past price
realizations and current price expectations/plans \( P_t \times P_t^* \) comparable in magnitude,
significance and stability to what we found for the Model II, the error-learning
<table>
<thead>
<tr>
<th>Data Set, Period, Chi-Square Test Against Independence</th>
<th>Bivariate Interaction Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \beta \times \beta_L )</td>
</tr>
<tr>
<td>Ifo</td>
<td></td>
</tr>
<tr>
<td>July 1977. ( \chi^2 = 528 )</td>
<td>0.36 ( 5.80)</td>
</tr>
<tr>
<td></td>
<td>0.37 ( 4.96)</td>
</tr>
<tr>
<td>January 1978. ( \chi^2 = 496 )</td>
<td>0.41 ( 6.55)</td>
</tr>
<tr>
<td></td>
<td>0.42 ( 5.43)</td>
</tr>
<tr>
<td>April 1978. ( \chi^2 = 501 )</td>
<td>0.44 ( 7.90)</td>
</tr>
<tr>
<td></td>
<td>0.47 ( 6.09)</td>
</tr>
<tr>
<td>July 1978. ( \chi^2 = 385 )</td>
<td>0.33 ( 5.28)</td>
</tr>
<tr>
<td></td>
<td>0.35 ( 4.73)</td>
</tr>
<tr>
<td>INSEE</td>
<td></td>
</tr>
<tr>
<td>June 1977. ( \chi^2 = 144 )</td>
<td>0.80 (16.10)</td>
</tr>
<tr>
<td></td>
<td>1.18 ( 7.12)</td>
</tr>
<tr>
<td>November 1977. ( \chi^2 = 222 )</td>
<td>0.78 (14.15)</td>
</tr>
<tr>
<td></td>
<td>1.25 ( 6.46)</td>
</tr>
<tr>
<td>March 1978. ( \chi^2 = 211 )</td>
<td>0.51 ( 3.77)</td>
</tr>
<tr>
<td></td>
<td>0.58 ( 3.88)</td>
</tr>
<tr>
<td>June 1978. ( \chi^2 = 91 )</td>
<td>0.71 ( 6.15)</td>
</tr>
<tr>
<td></td>
<td>0.97 ( 3.81)</td>
</tr>
</tbody>
</table>
model. The association between price realizations two periods ago and current price expectations/plans $P_{t-q}^* \times P_t^*$ is weak, unstable, and frequently insignificant for the French data, although for the Ifo data, the association is even stronger than for $P_t^* \times P_t^*$. In this case we do not have the problem, as we did in the case of Model II, of confounding the strong current association of price expectations/plans and subsequent realizations with the adequacy of the underlying model of expectation formation.

MODEL IV. Adaptive with Expectations of Future Demand

Model IV adds expected future changes in demand (INSEE data), or in business conditions (Ifo data), to the adaptive-expectations model in the form discussed above. Our results are presented in Table 5.4.

In all cases, the association for the interaction $P_t^* \times G_t^*$ is positive and significant for the Ifo data; in no case is the association for the interaction $P_t^* \times D_t^*$ significant for the INSEE data, although it is always positive. Only in June 1978 does the magnitude of the association approach the magnitudes found for the Ifo data. The results for the other interaction configurations remain about the same as we found earlier for Model I, that is, the addition of $G_t^*$ or $D_t^*$ does not appear to affect the relation among $P_t^*$, $P_{t-q}^*$, and $P_t^*$. 
### Table 5.4: Model IV: Measures of Association. Chi-Square Test against Independence. Ifo and INSEE Data. Various Periods. (t-statistics in parentheses.)

<table>
<thead>
<tr>
<th>Data Set, Period</th>
<th>Chi-square Test for Independence</th>
<th>Bivariate Interaction Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\chi^2$</td>
<td>$P_{\chi^2}$</td>
</tr>
<tr>
<td>Ifo</td>
<td></td>
<td>$P_{\chi^2}$</td>
</tr>
<tr>
<td>April 1977</td>
<td>$\chi^2 = 1093$</td>
<td>0.55 (3.67)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.68 (2.57)</td>
</tr>
<tr>
<td>July 1977</td>
<td>$\chi^2 = 1045$</td>
<td>0.54 (5.10)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.63 (3.73)</td>
</tr>
<tr>
<td>October 1977</td>
<td>$\chi^2 = 864$</td>
<td>0.39 (3.51)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.43 (3.03)</td>
</tr>
<tr>
<td>January 1978</td>
<td>$\chi^2 = 1006$</td>
<td>0.32 (2.36)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.34 (2.05)</td>
</tr>
<tr>
<td>April 1978</td>
<td>$\chi^2 = 1103$</td>
<td>0.58 (4.27)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.74 (2.81)</td>
</tr>
<tr>
<td>July 1978</td>
<td>$\chi^2 = 1112$</td>
<td>0.56 (5.39)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.66 (3.90)</td>
</tr>
<tr>
<td>INSEE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>June 1977</td>
<td>$\chi^2 = 393$</td>
<td>0.14 (3.79)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.16 (0.76)</td>
</tr>
<tr>
<td>November 1977</td>
<td>$\chi^2 = 386$</td>
<td>0.654 (0.36)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.047 (0.31)</td>
</tr>
<tr>
<td>March 1978</td>
<td>$\chi^2 = 236$</td>
<td>0.21 (0.78)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.26 (0.75)</td>
</tr>
<tr>
<td>June 1978</td>
<td>$\chi^2 = 290$</td>
<td>0.40 (1.80)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.48 (1.57)</td>
</tr>
</tbody>
</table>
That we find a significant effect of expected future business conditions for the Ifo data and that we do not find a significant effect of expected future demand for the INSEE has no obvious interpretation. In earlier work (Könnig, Nerlove and Oudiz, 1979a and 1979b), we found a strong positive relationship between expected business conditions or expected future demand and production plans. Moreover, this relationship was stable over time and the deviation-contrast parameters were nearly the same numerically. This suggested to us that $G_t^*$ was in fact being interpreted by the Ifo respondents as a question about future demand. That $G_t^*$ is positively associated with $P_t^*$ for the Ifo data is consistent with either the hypothesis that the Ifo responses represent forecasts or the hypothesis that they represent plans, but the lack of association of $G_t^*$ and $P_t^*$ for the INSEE respondents suggests that their responses may be more of the nature of plans than forecasts.

6. Joint Models of Production Plans and Price Expectations/Plans

Due to data limitations we were not able to estimate joint adaptive expectations models of production plans and price expectations/plans. However, we did estimate a joint extrapolative model (Model V) and a joint error-learning model (Model VI). The results are presented in this section.
Table 6.1 presents a summary of our results for the model \( \left( P_{t}^{*}, Q_{t}^{*} \mid P_{t}, Q_{t} \right) \). Last price realizations and current price expectations/plans are strongly positively and significantly associated for both the Ifo and the INSEE data, so are past production realizations and future production plans (as noted in our previous work). However, it is interesting to note the remarkable difference between the Ifo and the INSEE data with respect to the bivariate configurations \( P_{t}^{*} \times Q_{t}^{*} \) and \( P_{t}^{*} \times Q_{t} \). For the Ifo data both show positive and significant association, although this association is by no means as great as for the configuration \( P_{t}^{*} \times P_{t} \) and \( Q_{t}^{*} \times Q_{t} \). On the other hand, the association between production plans and price expectations/plans is never significant, although always weakly positive, for the INSEE data; the association between past realizations is always positive but significant only two times out of four.

We remark above that the positive association of future business conditions for the Ifo data is consistent with either the hypothesis that the reported price expectations/plans are really plans or the hypothesis that they are forecasts. The results obtained for Model V do not unambiguously resolve this uncertainty, since both production plans and price expectations/plans may be the result of the firm's perception of future demand conditions in the German case: A firm would normally
<table>
<thead>
<tr>
<th>Data Set, Period</th>
<th>Chi-Square Test against Independence</th>
<th>Bivariate Interaction Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>IFO</td>
<td></td>
<td>$P_\gamma \times Q_\gamma$</td>
</tr>
<tr>
<td>July 1977, $\chi^2 = 765$</td>
<td>$\gamma$</td>
<td>0.48 (4.60)</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{11}$</td>
<td>0.57 (3.68)</td>
</tr>
<tr>
<td>October 1977, $\chi^2 = 704$</td>
<td>$\gamma$</td>
<td>0.40 (2.03)</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{11}$</td>
<td>0.46 (1.67)</td>
</tr>
<tr>
<td>January 1978, $\chi^2 = 631$</td>
<td>$\gamma$</td>
<td>0.38 (2.89)</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{11}$</td>
<td>0.42 (2.47)</td>
</tr>
<tr>
<td>April 1978, $\chi^2 = 712$</td>
<td>$\gamma$</td>
<td>0.47 (3.53)</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{11}$</td>
<td>0.53 (2.97)</td>
</tr>
<tr>
<td>July 1978, $\chi^2 = 734$</td>
<td>$\gamma$</td>
<td>0.57 (4.77)</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{11}$</td>
<td>0.70 (3.39)</td>
</tr>
<tr>
<td>INSEE</td>
<td></td>
<td>$\gamma$</td>
</tr>
<tr>
<td>June 1977, $\chi^2 = 197$</td>
<td>$\gamma$</td>
<td>0.17 (1.01)</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{11}$</td>
<td>0.20 (0.96)</td>
</tr>
<tr>
<td>November 1977, $\chi^2 = 188$</td>
<td>$\gamma$</td>
<td>0.11 (0.75)</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{11}$</td>
<td>0.12 (0.70)</td>
</tr>
<tr>
<td>March 1978, $\chi^2 = 179$</td>
<td>$\gamma$</td>
<td>0.30 (1.28)</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{11}$</td>
<td>0.35 (1.17)</td>
</tr>
<tr>
<td>June 1978, $\chi^2 = 143$</td>
<td>$\gamma$</td>
<td>0.23 (1.10)</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{11}$</td>
<td>0.28 (1.00)</td>
</tr>
</tbody>
</table>
plan to raise both production and prices if it believed that the market for its product would be stronger in future, irrespective of whether market forces primarily determined prices or whether the firm simply set them. The distinction, however, between a price setter and a price taker is somewhat blurred under such circumstances. On the other hand, the lack of significant positive association between expected future demand or planned production and price expectations/plans for the INSEE is strongly suggestive that, at least over the short periods analysed, the group of French firms being studied is not highly responsive to market conditions in setting prices.

**MODEL II. Joint Error-Learning**

In the previous section, we considered the relationship between surprises or errors in price expectations/plans and revisions of the same (Model II). We consider here a joint model of production plans and price expectations/plans, which corresponds to the error-learning model for prices alone. Although the results are not highly stable, there are a number of significant interactions which are suggestive of more complicated interrelationships among the variables than a simple error-learning model applied to each variable separately. The results are summarized in Table 6.2.
<table>
<thead>
<tr>
<th>Data Set</th>
<th>Period, Chi-Square Test against Independence</th>
<th>Bivariate Interaction Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ifo</td>
<td></td>
<td>( \Delta \hat{\sigma}_t^2 \times \Delta \hat{\sigma}_c^2 )</td>
</tr>
<tr>
<td></td>
<td>July 1977, ( \chi^2 = 782 )</td>
<td>0.24 (2.72)</td>
</tr>
<tr>
<td></td>
<td>( \hat{\sigma}_{11} )</td>
<td>0.26 (2.58)</td>
</tr>
<tr>
<td></td>
<td>October 1977, ( \chi^2 = 690 )</td>
<td>0.17 (1.72)</td>
</tr>
<tr>
<td></td>
<td>( \hat{\sigma}_{11} )</td>
<td>0.18 (1.67)</td>
</tr>
<tr>
<td></td>
<td>January 1978, ( \chi^2 = 478 )</td>
<td>0.20 (3.10)</td>
</tr>
<tr>
<td></td>
<td>( \hat{\sigma}_{11} )</td>
<td>0.22 (3.01)</td>
</tr>
<tr>
<td></td>
<td>April 1978, ( \chi^2 = 743 )</td>
<td>0.23 (3.71)</td>
</tr>
<tr>
<td></td>
<td>( \hat{\sigma}_{11} )</td>
<td>0.25 (3.57)</td>
</tr>
<tr>
<td></td>
<td>July 1976, ( \chi^2 = 793 )</td>
<td>0.18 (2.16)</td>
</tr>
<tr>
<td></td>
<td>( \hat{\sigma}_{11} )</td>
<td>0.19 (2.10)</td>
</tr>
<tr>
<td>INSEE</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>November 1977, ( \chi^2 = 386 )</td>
<td>0.83 (16.20)</td>
</tr>
<tr>
<td></td>
<td>( \hat{\sigma}_{11} )</td>
<td>1.16 (6.90)</td>
</tr>
</tbody>
</table>
The Ifo data show strong positive and significant associations between $EP_t$ and $\Delta P^*_t$ and between $EO_t$ and $\Delta Q^*_t$. Weaker associations, but still significant with the exception of July 1977, are found between $EP_t$ and $EO_t$ and between $\Delta P^*_t$ and $\Delta Q^*_t$.

A few scattered interaction configurations such as $\Delta P^*_t \times EO_t$ also show significant, but weak, associations (not reported here). Curiously, the INSEE data show a negative, albeit insignificant, association between $EP_t$ and $\Delta P^*_t$ when the positive associations between $EO_t$ and $\Delta P^*_t$ are taken into account.

The relationships over time between $EP_t$ and $\Delta P^*_t$ and between $EO_t$ and $\Delta Q^*_t$ are susceptible to the same source of bias that we noted earlier in connection with the error-learning model for prices alone, namely, that an expectation or plan occurs in the surprise variable and the corresponding realization occurs in the change variable. Since expectations or plans are strongly positively related to subsequent realizations for both prices and production, it is possible that the relation between surprises and changes in a spurious one. However, not only does this not appear to be the case for the INSEE data, but the significant associations between $\Delta Q^*_t$ and $\Delta P^*_t$ for the Ifo data confirm the existence of more complicated dependencies. This model does not shed further light on the possible difference between French and German price behavior, which we note above, however, since the
associations observed are consistent with both the hypothesis that the $P^*_t$ are expectations and the hypothesis that they are plans.

7. Conclusions. Directions for Further Research

Business-test data of the type collected by Ifo and INSEE offer a wealth of information on the expectations and plans of individual firms, subsequent realizations, and appraisals of current variables. While the present paper is limited to analyses of price expectations/plans and past and future realizations, and in relation to production plans, the surveys contain data on many more variables of interest, such as incoming orders, demand or business conditions, backlogs of orders, inventories, appraisals of inventory levels and order backlogs. The surveys contain some quantitative information and, in the French case, may be linked to surveys of investment intentions and realizations. We believe that our preliminary work on models of price expectation/plan formation demonstrates the utility of these data for exploring processes at the level of the individual firm in the determination of many different variables in addition to price expectations or plans. In earlier work we dealt with production plans and demonstrated that the results we anticipated on the basis of a very simple theory did emerge. We concluded that the business test data were thus suited to more
elaborate analyses of relationships not so easily anticipated \textit{a priori}.

The results of the present paper demonstrate, first, that there is a very strong positive association between price expectations/plans and subsequent realizations for both the French and German data. If this association is found to persist over longer periods and for more widely separated points in time, the utility for forecasting realizations from expectations/plans should be further enhanced. However, the strong positive association found for consecutive periods does not imply stability of the association over time, nor does stability over time imply stability of the underlying process of expectation/plan formation. We find that while the bivariate association is relatively stable for the French data, it is not for the German data; moreover, it is apparent that the main effects (proportional variations across categories) for both variables are changing significantly over time due to variations in macroeconomic factors affecting all firms simultaneously. It is clearly important to investigate the nature of these factors and their relationship to the changing main effects and also why the French and German bivariate relationships appear to be different with respect to their stability over time. To do so may involve disaggregating German data by broad industry group and examining the bivariate relationships over longer periods of time than we can with data presently at our disposal.
We have also shown that a restricted form of the adaptive expectations model, the error-learning model, yields the best results for both the French and German data and for joint models involving production plans as well as price expectation plans. Our assessment rests largely on sign, strength and stability over time of key bivariate associations. Clearly, more clear-cut goodness-of-fit criteria are needed to compare models and the stability over time needs to be tested in a more formal way. Both matters are of concern in our on-going investigation.

We found in earlier work that expectations of future demand or of future business conditions were the most important variables associated with production plans (since the variables are simultaneously reported one cannot infer direction of causality). Such an association, but not so strong or stable, also emerges for price expectations/plans in the German data but not in the French data. This suggests that French firms may be less responsive to market forces in setting or forecasting prices than are German firms, but further work is necessary before reaching any firm conclusions on this matter. It is particularly necessary to examine price expectations/plans over longer periods of time and to introduce economy-wide variables which may be affecting all firms simultaneously.

Some additional warnings with respect to the tentative character of our substantive conclusions seem necessary. First, in neither country do the data cover
a whole business cycle; prices in both countries have been increasing almost steadily during the observation period in general (although not for each firm and/or product). Thus, for the individual firm, changes in its relative position determine changes in sales, an aspect we have not considered here. Second, the approach used here rests upon the implicit assumption of homogenous price formation behavior by industries and firms and for all products. Thus, the effects of different market structure, important for the distinction between price-setting and price-taking behavior, have also been neglected, as has the impact of various government policies on pricing behavior and/or expectation formation.

An analysis, for example, for two-digit industry groups reduces the number of observations per group considerably exacerbating the problem of estimability due to empty cells in the margins. It is therefore difficult to resolve these issues by disaggregation.

The traditional static theory of the firm does not provide much guidance in studying the intrinsically dynamic process of price expectation or plan formation under conditions of differing market structure. Nor do recent formulations of optimal dynamic behavior under certainty or uncertainty provide a framework with which to analyse data for individual firms such as those used in this study.

A modest beginning has been made here in the specification and testing of several models, which may in turn provide a firmer empirical basis for the formulation
of a microeconomic theory of price expectation formation and behavior at the level of the individual firm.

Above all, we believe we have demonstrated the utility and flexibility of methods of analysis based on the log-linear probability model for formulating and testing simple hypotheses about the kind of categorical variables encountered in business tests of the type carried out by Ifo and INSEE. It is true that our conditional probability models are not structural in the usual econometric sense; we can do no more than examine partial associations with other variables accounted for. Nonetheless, the vast amount of information contained in the contingency tables for any sizable number of variables is reduced to manageable proportions in terms of parameters which may be readily interpreted; relatively complex hypotheses can be tested easily when the data are summarized in convenient parametric form. Although models with more structure can be developed, e.g., in terms of latent variables or by superimposing log-linear models with a priori zero cells, we believe that the relatively unstructured approach taken here is more suitable for preliminary work with these relatively unfamiliar data.
In the Ifo Business-Test firms report at the end of each month $t$ if they expect their selling prices to increase, remain unchanged or decrease in the course of the next three months, i.e., for months $t+1$, $t+2$ and $t+3$. Actual price changes, however, refer to the preceding month, i.e., to the change in the reporting month $t$ compared to month $t-1$. Similar problems arise with respect to production plans and expected business conditions.

In order to have the same unit period of observation for both expectations and realizations, firms for the sample are selected according to the following rules:

1. If sign $P_{t-1}$ is identical for all $i=0,1,2$ firms are included in the sample with the reported sign.
2. If responses differ in sign for each $i$, firms are deleted.
3. If sign $P_{t-1}$ is equal for two periods, firms are included if response for the third period is not opposite in sign. For example, if responses are $(+,-,+,-)$ or $(+,-,+,-)$ etc. firms are included, but firms are deleted with responses are $(+,-,-)$ or $(-,+,+)$. Sample sizes were reduced by less than 10% by this procedure. We denote the backward aggregate formed in this way simply by $P_t$, omitting any indication that it is an aggregate and using the subscript $t$ to indicate the current survey date to which the first of the previous months' realizations is referred. Comparable aggregates were formed for realizations in production plans, $Q_t$.

The phrase, "considering changing conditions," is generally taken to refer changes in the overall level of industrial prices, but there is a substantial element of ambiguity present.
The classic papers on this subject are collected in Goodman and Kruskal (1979); Wilson (1974) gives a good general discussion; an early attempt at generalization is Davis (1962); our approach is basically that of Kawasaki (1973) supplemented by an extension of the work of Haberman (1974).
Suppose we observe the following series of contingency tables for the variables $A$ and $B$ at the times $t=1,2,3$:

![Contingency tables](image)

The aggregate balances for these data show a perfect positive correlation.

$B_A$ at $t=1$: $75\%$; $B_B$ at $t=1$: $75\%$.

$B_A$ at $t=2$: $0\%$; $B_B$ at $t=2$: $0\%$.

$B_A$ at $t=3$: $-75\%$; $B_B$ at $t=3$: $-75\%$.

Yet, as is readily verified by computing the marginal frequencies and observing that the joint frequencies are products of the corresponding marginal frequencies, it is clear that the variables $A$ and $B$ are in fact independent.

As one of our discussants, Hugh Willa, at the ISCM conference pointed out, restricting $\Delta P_t$ and $E_t$ to be trichotomous introduces a certain incompatibility with the definitions of the original variables. In particular, if "scores" are assigned to the original variables on the basis of their ordering the distances will not in general be preserved in the ordering of the new variables $I_X$ and $E_X$.

An exception is the case of incomplete tables which contain a priori zero cells. We do not deal with such tables in the work discussed in this paper.

To illustrate: The basis for the $2 \times 2$ case consists of the columns of the matrix

$$U = \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1
\end{bmatrix}.$$
which yields the representation for \( \log p = (\log p_{11}, \log p_{12}, \log p_{21}, \log p_{22})' \), as in (3) above, as

\[
\log p = \Psi \\
\begin{bmatrix}
\mu \\
\alpha_2(1) \\
\alpha_2(2) \\
\beta_{12}(1,1) \\
\beta_{12}(1,2) \\
\beta_{12}(2,1) \\
\beta_{12}(2,2)
\end{bmatrix}
\]

where \( \mu \) corresponds to the overall effect, \( \alpha_2 \) to the main effect for the first variable, \( \beta_{12} \) to the main effect for the second variable, and \( \beta_{12} \) to the bivariate interaction effect. The values of the parameters for other combinations of indices are recovered from the ANOVA restrictions (4).

In the 3x3 case, the deviation-contrast basis consists of the columns of the matrix

\[
\Psi = \begin{bmatrix}
1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & -1 & -1 & -1 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & -1 & -1 & 0 & 0 & 1 \\
1 & 1 & 0 & -1 & -1 & 0 & -1 & 0 \\
1 & 1 & 0 & 1 & 1 & 0 & -1 & 0 \\
1 & 1 & 0 & -1 & -1 & 1 & 1 & 1
\end{bmatrix}
\]

and \( \log p = \Psi h \), where \( h \) is the vector

\[
\Psi = \begin{bmatrix}
\mu \\
\alpha_2(1) \\
\alpha_2(2) \\
\alpha_2(3) \\
\beta_{12}(1,1) \\
\beta_{12}(1,2) \\
\beta_{12}(2,1) \\
\beta_{12}(2,2)
\end{bmatrix}
\]
Kawasaki (1979, Chapter 2) shows how the basis for a general multivariate log-linear probability model may be generated from so-called elementary bases for univariate models by direct (Kronecker) product operations.

A model is hierarchical if the inclusion of any interaction configuration implies the inclusion of all lower-order interaction configurations involving only the variables in the higher-order configuration. Equivalently, exclusion of any configuration implies exclusion of all higher-order configurations that include all of the variables included in the lower-order configuration. Hierarchical models are generally more plausible in the log-linear probability context than nonhierarchical models. In particular, models that omit certain main effects, but include interactions, are non-hierarchical; omission of a main effect suppresses common factors affecting all individuals proportionally across categories of one variable. A similar argument can be made for the implausibility of models that include higher-order interactions but suppress lower-order interactions containing subgroups of the variables.

Under the usual assumptions about how the sample is generated, it can be shown that the likelihood depends on the distances among scores; in principle, therefore, these distances can be estimated simultaneously with the associated parameters.

In the $3 \times 3$ case, the basis consists of the columns of the matrix

$$
V = \begin{bmatrix}
1 & 1 & 1/3 & 1 & 1/3 & 1 & 1/3 & 1/3 \\
1 & 1 & 1/3 & 0 & -2/3 & 0 & -2/3 & 0 \\
1 & 1 & 1/3 & -1 & 1/3 & -1 & 1/3 & 1/3 \\
1 & 0 & -2/3 & 1 & 1/3 & 0 & 0 & -2/3 \\
1 & 0 & -2/3 & 0 & -2/3 & 0 & 0 & 4/3 \\
1 & 0 & -2/3 & -1 & 1/3 & 0 & 0 & 2/3 \\
1 & -1 & 1/3 & 1 & 1/3 & -1 & -1/3 & 1/3 \\
1 & -1 & 1/3 & 0 & -2/3 & 0 & 2/3 & 0 \\
1 & -1 & 1/3 & -1 & 1/3 & 1 & -1/3 & -1/3
\end{bmatrix}
$$
We write

\[
\log p = V
\]

\[
\begin{align*}
\mu \\
\sigma_{10}(1) \\
\sigma_{10}(2) \\
\sigma_{20}(1) \\
\sigma_{20}(2) \\
\sigma_{12}(1,1) \\
\sigma_{12}(1,2) \\
\sigma_{12}(2,1) \\
\sigma_{12}(2,2)
\end{align*}
\]

For convenience, even in multivariate cases, we often refer to the bivariate score parameters, which correspond to the last 4 columns of V above, simply, in subscript notations, as \( \alpha_{11}, \alpha_{12}, \alpha_{21}, \alpha_{22} \). The score parameters are linearly related to the deviation-contrast parameters (since a simple change of basis is involved).

Estimation, estimability and partial estimability is a complex subject which we cannot discuss here because of space limitations (but see Nerlove and Press, 1980, Section 5). An important result, however, which bears on the present discussion is that, if a maximum-likelihood estimate exists for a hierarchical model, then any marginal table corresponding to a configuration contained in the model has no empty cells. Since the contingency tables which characterize business-test data typically have a great many sampling zeros, three way and higher-order marginal tables frequently contain one or more empty cells. Such cells preclude the estimation of any hierarchical log-linear probability model containing all the parameters of the configuration corresponding to that model.
These restrictions and revised estimates can be cast in very simple form. Let \( \theta \) be the vector of deviation-contrast parameters, and let \( R \) be a 4x24 vector of zeros, ones and minus ones such that

\[
R \theta = 0
\]

represents the restrictions on the bivariate interaction parameters of the text. Let \( \hat{\theta} \) be the unconstrained maximum-likelihood estimate of \( \theta \) and let \( \hat{V} \) be its estimated variance-covariance matrix. The unconstrained maximum-likelihood estimate \( \hat{\theta} \) is asymptotically normally distributed with mean \( \theta \) and variance-covariance matrix \( V \). The maximum-likelihood estimate \( \hat{\theta} \) of \( \theta \) under the constraints (i) is:

\[
\hat{\theta} = \hat{\theta} - \hat{V} R (R \hat{V} R')^{-1} R \theta.
\]

Under the null hypotheses, \( \hat{\theta} \) is also asymptotically normally distributed with mean \( \theta \) and variance-covariance matrix \( V^* \), where \( V^* \) is consistently estimated by

\[
\hat{V}^* = \hat{V} - \hat{V} R (R \hat{V} R')^{-1} R \hat{V}.
\]

An asymptotic test of the null hypothesis that the bivariate interactions are the same for \( X \) and \( X + q \) and \( X - q \) may be made by computing the statistic

\[
(\hat{\theta} - \hat{\theta}^*)' \hat{V}^{-1} (\hat{\theta} - \hat{\theta}^*),
\]

which asymptotically is distributed as Chi-square with 4 degrees of freedom (the number of restrictions imposed).

We also tested whether both the bivariate interactions and main effects could be considered stable in consecutive periods; this hypothesis was decisively rejected for both the Ifo and INSEE data for every period except March 1975 - June 1975 (INSEE). These results are consistent with the hypothesis that the main effects vary over time in a manner reflecting economy-wide conditions, but that the bivariate association between price expectations/plans and subsequent
realizations may or may not be stable in consecutive periods depending on the nature of the expectation formation process in the two countries.

To test the stability of the extrapolative expectations model over time, we have computed a Chi-square test statistic as described in footnote 11 for the stability of the interactions $P_t \times P_{t-1}$ and $P_t \times P_{t-2}$ in consecutive periods INSEE (June 1974 - March 1978) and Ifo (April 1977 - October 1978). In all cases but two (Ifo: January 1978 - April 1978; INSEE: November 1974 - March 1975), we reject the null hypothesis of stability at very high levels of significance. We have not been able to carry out a similar test for Model II, but the results presented in Table 5.2, suggest a far greater degree of stability, at least for Germany. This finding thus lends some additional support to the error-learning model.
REFERENCES


Bishop, Y.M.M., 1969. "Full contingency tables, logits and split continu -


