Discussion Paper No. 436

GROWTH AS A PUBLIC GOOD *

by

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August 1980

* The research reported here was done with the support of National Science Foundation Grant No. SOC-79-06672.

1) Introduction

My starting point is the idea that an economy faces the same difficulties in allocating resources between investment and consumption as it does in allocating resources betwen public and private uses. As described by Samuelson (1954), the public goods problem is to determine a Pareto optimal allocation of resources to public goods. He emphasizes that there is no question that optimal allocations exist. The problem is to find one. The difficulty arises because there can be no markets in which individuals reveal their demands for public goods.

Similarly in capital theory, it is possible to prove that Pareto optimal allocations exist. It is also possible to associate prices with the allocations, these prices being called Malinvaud prices (Malinvaud, 1953). But these prices are not market prices in the usual sense. They are present value prices at the initial period for contracts for future delivery. They are really Arrow-Debreu prices for forward claims. But as Arrow has stressed, in reality there cannot exist a complete set of forward markets (Arrow, 1974a, 1974b). Since the forward markets do not exist, the intertemporal allocation of resources cannot be market determined. This suggests that perhaps capital or growth theory should be approached by trying to find appropriate resource allocation mechanisms of the type defined by Hurwicz (1973). Groves and Ledyard (1977) and others have successfully treated the public goods problem in this way.

One might argue that the objective function in capital theory is that of a planner, so that markets are not needed to determine the optimal allocation.

My response would be, who cares about the desires of a planner? And how could he possible know the constraints of his maximization problem? These constraints must be revealed to him through markets or some other mechanism. In a previous paper (1979), I have shown how one can formulate capital theory in terms of a general equilibrium model with many consumers and no planner.

The point of view I take may be found, by and large, in Arrow's presidential address (1974b). He pointed out the importance to intertemporal economics of the absence of complete markets for contingent claims. He stressed the need to understand exactly why these markets do not exist. He observed that as soon as any markets open in the future, then people must form expectations about prices on those markets. He expressed the view that in a changing environment, agents have little basis for forming expectations rationally. All these ideas are themes of this paper.

What I do is try to carry Arrow's ideas a little further by analyzing a simple model. The model is designed to concentrate on society's problem in extracting from entrepreneurs information about production possibilities. The model is such that no contingent claims markets are incentive compatible. Nevertheless, there does exist an incentive compatible solution (or near solution) to the allocation problem. The solution involves arranging state contingent prices and trades in advance, but paying for them at the date of delivery. This solution seems utterly impractical. In fact, it makes sense only if forward transactions can be arranged without cost, and in reality contracts are indeed expensive to arrange. I do not carry the analysis further than this. The conclusion is that a sensible mechanism for dealing with intertemporal allocations must take into account the costs of making agreements. It does not seem possible to separate capital theory from larger perplexing issues having to do with how an economy should be organized.

I wish to emphasize that I reject any notion of rational expectations, for it would be inappropriate in the contexts I have in mind. Rational expectations can make sense in the context of repeated fluctuations. More precisely, rational expectations can make sense if there is no change or if change is periodic or if it is generated by a stationary stochastic process. Rational expectations make sense even when the process is not stationary, provided individuals understand the mechanism generating the fluctuations. But I have in mind the context of growth theory, in which changes are novel.

2) The Model

The model is of a world with one kind of produced commodity and with many independent producers, each of whom uses his own labor and the produced good in order to produce more of the produced good. Labor causes disutility. No producer can use anyone else's labor as an input. The produced good is not storable, so that capital should be thought of as variable capital.

It takes one period to turn input into output. The production function of each producer depends on a random variable, which he alone observes. At the beginning of each period, the producer knows his production function for that period, though he does not necessarily know what it will be in the following period.

Each producer is also a consumer. There are no other consumers in the economy. The consumer-producers are referred to as agents.

The time horizon is finite. All agents come to life in period zero. and disappear at the end of period T. Production starts in period one.

The agents should be thought of as entrepreneurs. There are no other consumers or laborers in the model simply because I want to concentrate on entrepreneurial behavior.

I now describe the model formally. There are I agents, where I is a positive integer. In period t, agent i observes the random variable s_{it}, where $t = 1,\ldots,T$. s_{it} may take on the values 0 or 1. No other agent observes s_{it}. Agent i knows the distribution of the stochastic process $\{s_{it}\}_{t=1}^{T}$. The variables s_{it}, $i = 1,\ldots,I$ are <u>not</u> assumed to be mutually independent. In my opinion, such an assumption would be inappropriate.

I use the following notation in dealing with the s_{it}. s_t denotes the random variable $(s_{1t},\ldots,s_{It})$, and S denotes the set of all possible vectors s_t. Σ denotes $\overset{T}{\underset{t=1}{X}} S$, where "X" denotes the cartesian product. $\underset{\sim}{s} = (s_1,\ldots,s_T)$ denotes an element of Σ. s_{it} may be considered to be a random variable on Σ. That is, $s_{it} : \Sigma \rightarrow \{0,1\}$ in the function which gives the $(i,t)^{th}$ component of points in Σ. $\mathscr{S}_t$ is the field on Σ generated by the random variables s_{in}, for $i = 1,\ldots,I$ and $n \leq t$. $\mathscr{S}_t$ represents the information in the economy at time t. Of course, no one agents knows all this information. P denotes the probability on Σ. $P(\underset{\sim}{s})$ is the probability of the point $\underset{\sim}{s} \in \Sigma$.

The <u>production function</u> of agent i at time t is f_{it}: $[0,1] \times [0,\infty) \times \{0,1\} \rightarrow [0,\infty)$, where $t = 1,\ldots,T-1$. (There is no production at time T.) If agent i uses a units of his own labor and b units of the produced good as inputs during period t, then his output in period $t+1$ is $f_{it}(a,b,s_{it})$.

The <u>utility function</u> of agent i for one period is u_i: $[0,1] \times [0,\infty) \rightarrow (-\infty,\infty)$. His utility in one period is $u_i(a,x)$, where a is his labor input and x is his consumption of the produced good. Utility is additively separable with respect to time and uncertainty.

Agent i is <u>endowed</u> in period zero with c_{i0} units of the produced good. This is his portion of the initial capital stock of the economy.

A <u>program</u> for an agent is of the form $(\underset{\sim}{a},\underset{\sim}{b},\underset{\sim}{c},\underset{\sim}{x}) = ((a_1,b_1,c_1,d_1),\ldots,(a_T,b_T,c_T,d_T))$, where (a_t,b_t,c_t,x_t): $\Sigma \rightarrow [0,1] \times [0,\infty) \times [0,\infty) \times [0,\infty)$ and is measurable with respect to $\mathscr{S}_t$. At time t, $a_t(\underset{\sim}{s})$ is the agent's input of labor, $b_t(\underset{\sim}{s})$ is his input of produced goods, and

$x_t(\tilde{s})$ is his consumption of the produced good. $c_t(\tilde{s})$ is his output in period $t + 1$. The program is feasible for agent i if $c_t(\tilde{s}) = f_{it}(a_t(\tilde{s}), b_t(\tilde{s}), s_{it})$, for all $\tilde{s}$ and t, and if $a_T = b_T = c_T = 0$. Notice that a program for one agent may depend on the random variables observed by other agents, even though he does not observe them himself.

Agent i's expected utility from the program $(\tilde{a}, \tilde{b}, \tilde{c}, \tilde{x})$ is $E \sum_{t=1}^{T} u_i(a_t, x_t)$, where E denotes expected value. The expected value is from the point of view of period zero.

An allocation for the economy is of the form $((\tilde{a}_i, \tilde{b}_i, \tilde{c}_i, \tilde{x}_i))_{i=1}^{I}$. It is feasible if $(\tilde{a}_i, \tilde{b}_i, \tilde{c}_i, \tilde{x}_i)$ is feasible for agent i, for all i and if $\sum_{i=1}^{I} (b_{it}(\tilde{s}) + x_{it}(\tilde{s}) - c_{i,t-1}(\tilde{s})) \leq 0$, for all $\tilde{s}$ and t, where $c_{i0}(\tilde{s})$ is simply the initial endowment, c_{i0}.

3) Assumptions

I list the assumptions I make. They will be maintained throughout the paper.

3.1) $I \geq 2$ and $T \geq 2$.

3.2) $P(\tilde{s}) > 0$ for all $\tilde{s}$.

3.3) $f_{it}(\cdot,\cdot,s)\colon [0,1] \times [0,\infty) \to [0,\infty)$ is continuously differentiable, non-decreasing and strictly concave, for all i, t and $\tilde{s}$.

3.4) $\dfrac{\partial f_{it}(a,b,s)}{\partial b} > 0$, for all i,t,a,b and s and

$\dfrac{\partial f_{it}(c,b,s)}{\partial a} > 0$, for all i,t, a and s if $b > 0$.

3.5) $f_{it}(a,b,1) > f_{it}(a,b,0)$, for all i, t and a if $b > 0$.

3.6) For all i, u_i is continuously differentiable and strictly concave. Also, $\dfrac{\partial u_i(a,x)}{\partial a} < 0$ and $\dfrac{\partial u_i(a,x)}{\partial x} > 0$, for all a and x.

3.7) $c_{i0} > 0$, for all i.

Finally, I recall what each agent observes.

3.8) Agent i and only agent i knows the functions f_{it} and u_i, observes the random variables $\{s_{it}\}_{t=1}^{T}$, and observes his own inputs and output.

4) Arrow-Debreu Equilibrium

I describe formally an Arrow-Debrew equilibrium in the model just defined. (The idea of having forward markets for contingent claims is due to Arrow (1953) and Debreu (Chapter 7 of 1959).)

A price system is a non-zero vector of the form $\underset{\sim}{p} = (p_1,\ldots,p_T)$, where $p_t = \Sigma \to [0,\infty)$ is $\mathcal{S}_t$-measurable. $p_t(\underset{\sim}{s})$ is the price of the produced good in state $\underset{\sim}{s}$ at time t. More precisely, $p_t(\hat{\underset{\sim}{s}})$ is the price for delivery at date t and in the event $[s_1 = \hat{s}_1,\ldots,s_t = \hat{s}_t]$.

The budget set of agent i is $\beta_i(\underset{\sim}{p}) = \{(\underset{\sim}{a},\underset{\sim}{b},\underset{\sim}{c},\underset{\sim}{x}) \mid (\underset{\sim}{a},\underset{\sim}{b},\underset{\sim}{c},\underset{\sim}{x})$ is a feasible program for agent i and

$$E \sum_{t=1}^{T} p_t \cdot x_t \leqq E\left[\sum_{t=1}^{T-1} (p_{t+1}c_t - p_t b_t) + p_1 c_{i0}\right]\} .$$

The response function of agent i is $\xi_i(\underset{\sim}{p})$. $\xi_i(\underset{\sim}{p})$ is the set of solutions to the problem

$$\max \{E \sum_{t=1}^{T} u_i(\underset{\sim}{a},\underset{\sim}{x}) \mid (\underset{\sim}{a},\underset{\sim}{b},\underset{\sim}{c},\underset{\sim}{x}) \in \beta_i(\underset{\sim}{p})\} .$$

An Arrow-Debreu equilibrium is a feasible allocation $((\underset{\sim}{a}_i,\underset{\sim}{b}_i,\underset{\sim}{c}_i,\underset{\sim}{x}_i))_{i=1}^{I}$ and a price system $\underset{\sim}{p}$ such that $(\underset{\sim}{a}_i,\underset{\sim}{b}_i,\underset{\sim}{c}_i,\underset{\sim}{x}_i) \in \xi_i(\underset{\sim}{p})$, for all i.

The fact that an Arrow-Debreu equilibrium exists follows from standard existence theorems of, say Arrow and Hahn (1971) or Debreu (1959). In applying their results one must identify each agent's labor as a separate commodity with its own price.

5) Incentive Compatibility of Arrow-Debreu Equilibrium

Arrow-Debreu equilibrium makes extreme demands on the honesty of agents. This should be obvious to those familiar with Radner's paper on uncertainty in equilibrium theory (Radner, 1970). Radner pointed out that a trader cannot be expected to commit himself to a contract contingent on an event he cannot observe. In spite of Radner's work, it is useful to see exactly why Arrow-Debreu equilibrium does not make sense in the model at hand. The answers shape the nature of the allocation problem in the model.

Arrow-Debreu equilibrium functions in the following way. In period zero, each agent declares to the others the probability distribution of his own random variable. Recall that by assumption 3.8, only agent i knows the probability distribution of $\{s_{it}\}_{t=1}^{T}$. Somehow, all the agents must agree on the joint distribution of the random variables s_{it}, for all i and t. Without this information, they cannot calculate the expected utility of contingent contracts.

Let us suppose that the probabilities have been revealed. The forward markets then open, and prices are determined for contingent contracts. The agents buy and sell contracts. This all happens in period zero. At later dates, there is no trade. At the beginning of each later period, each agent announces the state he observes. The contracts then specify deliveries as a function of the announcements made by all the agents.

It should be obvious that all agents would probably find it to their advantage to lie about the probabilities. Everyone would want to exaggerate the probability of being in a productive state. In this way, he would increase the price of his output contingent on the state when he could deliver more. If events were of a repetitive nature, mechanisms could be designed to overcome this problem. But I am imagining that the context is one of growth and change in which new things happen.

Let us pass over this difficulty and suppose that agents report the probabilities correctly. In later periods, the agents are free to report whichever state they please, but they must fulfill the commitments specified by their contracts. These rules define a game. I now show that an Arrow-Debreu equilibrium is unlikely to be a Nash equilibrium of this game. The idea is simply that when an agent observes 1, he can avoid work by reporting 0 and working only enough to deliver the output specified for state 0.

In order to see this problem in more detail, let $((\underset{\sim}{a}_i, \underset{\sim}{b}_i, \underset{\sim}{c}_i, \underset{\sim}{x}_i))$ and $\underset{\sim}{p}$ form an Arrow-Debreu equilibrium. Suppose that i, t and $\underset{\sim}{s}$ are such that $a_{it}(\underset{\sim}{s}) > 0$, $b_{it}(\underset{\sim}{s}) > 0$, and $s_{it} = 1$. If the Arrow-Debreu equilibrium is a Nash equilibrium in the sense just described, then we may suppose that all agents j other than i announce their true observations, s_{jt}, at time t. Given these announcements, agent i's allocation and presumed labor input depend on his announcement, s, where $s = 0$ or 1. If he announces s, he is responsible for $f_{it}(A(s),B(s),s)$ units of output, where $A(s)$ is his presumed labor input and $B(s)$ is his goods input in the Arrow-Debreu equilibrium. By assumption, $A(1) = a_{it}(\underset{\sim}{s}) > 0$, and it seems reasonable to assume that $A(0) \leq A(1)$. If he announces 0, he must deliver $f_{it}(A(0),B(0),0)$ units of output, and by assumption 3.5, $f_{it}(A(0),B(0),0) < f_{it}(A(0),B(0),1)$. By assumption 3.4, there is $A < A(0)$ such that $f_{it}(A,B(0),1) \geq f_{it}(A(0),B(0),0)$. Therefore, if agent i reports 0 when he observes 1, he may fulfill his commitments by using only A units of labor, where $A < A(0) \leq A(1)$. If he were honest he would have to use $A(1)$ units of labor. It seems unlikely that other variables in the model would depend significantly on agent i's announcements. In particular, his consumption should be nearly independent of his announcement, since the Arrow-Debreu equilibrium is supposed to provide perfect insurance. It follows that his utility should increase if he lies and works less.

In fact, it seems likely that the only Nash equilibrium would be for all agents to announce $s_{it} = 0$ at all times. Notice that in this situation, no agent could ever prove that another was lieing, even though it would be obvious to all that all were lieing.

The cheating just described would hurt the economy in two ways. Not only would agents work less than they should, but the (temporarily) more productive agents would not be favored in the allocation of the produced good.

A similar cheating problem would be the following. In period zero, the agents would know that they would always report zero. Hence, they could promise to produce more for state 1 than would be feasible, in order to be paid more in period zero for the promised output. Recall that by assumption 3.8, only the agents know their own production functions.

It might have occurred to the reader that the cheating problems just described could be overcome simply by giving an agent a sufficiently large consumption bonus every time he reported 1. Such an arrangement would destroy the Pareto optimality of the allocation, for the allocation would no longer provide perfect insurance. However, there is a deeper problem. The bonuses could not be market determined. Their determination would require fairly precise knowledge of the agents' production and utility functions. But by assumption, only the agents themselves know these functions. It would not be sufficient simply to give an agent a very large reward if he reported 1. If the advantage in reporting 1 were too great, he would have an incentive to report 1 even when he observed 0. Of course, if he did so he would have to produce in state 0 what he had promised to produce for state 1. But he could anticipate this problem in period zero and promise to deliver in state 1 only what he could produce in state 0. No one would know that he could in fact produce more.

6) The Second Best Solution

Radner (1970) suggested that the Arrow-Debreu model be modified by allowing each agent to trade only in contracts contingent on events he observed. I call equilibrium with such trade Radner equilibrium. Radner pointed out that his notion of equilibrium would be Pareto optimal in a limited sense. The equilibrium would not be dominated by any allocation which was such that each agent's allocation depended only on events he observed.

Why not be satisfied with this form of Pareto optimality? The answer is that it might be possible to achieve a Pareto superior allocation by allowing trade at times other than time zero. I suppose that many economists are aware of this fact. Radner mentioned it in an aside (1970, p. 54). Nevertheless, I give an example in order to make sure that the point is well understood.

In the model of this paper, no two agents observe the same events, so that forward contracts cannot be contingent on any events. That is, forward contracts must specify the same deliveries for all states. The idea of the example is that if agents trade when their information becomes available, then they are better able to take advantage of it.

In the example, agents trade forward at time 1, rather than at time zero as they would in a Radner equilibrium. Since agents trade forward, they do not need to form price expectations, which they would have to do if trade occurred in a succession of periods.

There is no labor in the example, since I am not concerned with incentive issues here.

<u>Example</u> There are two agents and $T = 2$. The utility function of each agent is $u(x) = \log(x+1)$. The production function of each agent in period one is $f(b,s)$, where $f(b,0) = 0$ and $f(b,1) = 2\sqrt{b}$. The initial endowment of each consumer is $c_{i0} = 3$, $i = 1,2$. The random variables s_{i1} are independent. Each one takes on the value zero with probability one-half and one with probability one-half.

The states of the world are denoted by $\underset{\sim}{s} = (s_{10}, s_{20})$. That is, $\underset{\sim}{s} = (0,1)$ means that in period zero agent 1 observes 0 and agent 2 observes 1.

The example is symmetric in the two agents. It follows that there is no trade in a Radner equilibrium. In such an equilibrium, the consumption allocation of agent 1 would be the following.

$$x_{10}(0,0) = x_{10}(0,1) = 3$$

$$x_{11}(0,0) = x_{11}(0,1) = 0$$

$$x_{10}(1,0) = x_{10}(1,1) = 0$$

$$x_{11}(1,0) = x_{11}(1,1) = 2.$$

In these formules, $x_{it}(\underset{\sim}{s})$ is the consumption of agent i in period t when the state is $\underset{\sim}{s}$. That is, agent 1 would consumer all his endowment in period zero if he observed 0. Otherwise, he would consume two units and use the other unit for production. The consumption allocation of agent 2 would be of the same nature. The expected utility of each agent in the equilibrium is log 6.

Now suppose that trade took place in period one instead of in period zero. The agents would exchange currently delivered good against good to be delivered in the following period. The resulting equilibrium depends on the state of the world $\underset{\sim}{s}$. It is described by the table below. The table should be interpreted as follows. When the state is $\underset{\sim}{s}$, prices for current and forward good are p_1 and p_2, respectively. Also, consumption of agents 1 and 2 are (x_{11},x_{12}) and (x_{21},x_{22}), respectively.

$\underset{\sim}{s} = (1,0)$: $(p_1,p_2) = (1,4/3), (x_{11},x_{12}) = (2\frac{5}{9}, 1\frac{2}{3})$

and $(x_{21},x_{22}) = (1\frac{2}{3}, 1)$.

$\underset{\sim}{s} = (0,1)$: $(p_1,p_2) = (1,4/3), (x_{11},x_{12}) = (1\frac{2}{3}, 1)$,

and $(x_{21},x_{22}) = (2\frac{5}{9}, 1\frac{2}{3})$.

$\underset{\sim}{s} = (0,0)$: $(p_1,p_2) = (1,4), (x_{11},x_{12}) = (3,0)$

and $(x_{21},x_{22}) = (3,0)$.

$\underset{\sim}{s} = (1,1)$: $(p_1,p_2) = (1,1), (x_{11},x_{12}) = (2,2)$

and $(x_{21},x_{22}) = (2,2)$.

The expected total utility of each agent in this equilibrium is $\log\ (8\sqrt{\frac{2}{3}}\ )$, which exceeds log 6, the expected utility in the Radner equilibrium.

Allowing trade at time zero does not give as good a result as does the Arrow-Debreu equilibrium. In the Arrow-Debreu equilibrium, the consumption of both agents would be identical in states (1,0) and (0,1). Equilibrium with trade at time one does not achieve full insurance. But it does make better use of information than does the Radner equilibrium.

7) Temporary Equilibrium With Forward Commitment

There is a fairly obvious way to overcome the incentive problems of Arrow-Debreu equilibrium. Imagine that agents trade every period in currently available output and in uncontingent forward contracts. Imagine also that agents agree in period zero on prices for every date and event. They also agree on the probabilities of all events. Finally, the agents must promise to announce their observations in every period. These arrangements would make it possible for agents to have rational expectations about future prices. Also, the incentive problems previously discussed would not arise, for agents would not be paid in advance for contingent delivery. I call equilibrium with these arrangements, temporary equilibrium, though it is somewhat different from the usual notion of temporary equilibrium (see Grandmont, 1977).

Temporary equilibrium does not provide perfect insurance and so does not guarantee Pareto optimality. The example of the previous section demonstrates this fact. The lack of Pareto optimality should not be of great concern, since I am concentrating on the organization of production and not on insurance problems. In the next section, I consider temporary equilibrium with self-insuring agents. With self-insurance, temporary equilibrium becomes Pareto optimal.

I now describe temporary equilibrium formally. For simplicity, I allow no trade at time zero and allow forward trade only one period in advance. A price system is a vector of the form $\underset{\sim}{p} = (p_1, \ldots, p_{T-1})$, where $p_t\colon \Sigma \to (0,\infty)$ is $\mathscr{S}_t$- measurable. Notice that prices are non-zero.

$p_t(\hat{s})$ is the number of units of good which must be delivered at date t in event $[s_1 = \hat{s}_1, \ldots, s_t = \hat{s}_t]$ in order to obtain one unit of good at date $t+1$ in the same event.

A <u>program</u> for an agent is of the form (a, b, c, x, d) where a, b, c and x are as in section 4. d is of the form $d = (d_1, \ldots, d_{T-1})$, where $d_t : \Sigma \to (-\infty, \infty)$ is $\mathscr{J}_t$-measurable. $d_t(\hat{s})$ is the quantity of good the agent buys one period forward at date t and in event $[s_1 = \hat{s}_1, \ldots, s_t = \hat{s}_t]$. The good is delivered at date $t+1$. The program is <u>feasible</u> for agent i if $c_i(s) \leqq f_{it}(a_t(s), b_t(s), s_{it})$ and $d_t(s) + c_t(s) \geqq 0$, for all t and s. The second condition says that agents cannot sell more than they plan to produce. Allowing agents to do so complicates the proof that temporary equilibria exist.

An <u>allocation</u> is of the form $((a_i, b_i, c_i, x_i, d_i))_{i=1}^{I}$, where $(a_i, b_i, c_i, x_i, d_i)$ is a program for agent i. The allocation is <u>feasible</u> if $(a_i, b_i, c_i, x_i, d_i)$ is feasible for agent i, for all i, and if

$$\sum_{i=1}^{I} (b_{it}(s) + x_{it}(s) - c_{i,t-1}(s)) = 0 \quad \text{and} \quad \sum_{i=1}^{I} d_{it}(s) = 0, \quad \text{for all } s \text{ and } t.$$

The <u>budget set</u> of agent i is

$$\beta_i(p) = \{(a,b,c,x,d) \mid (a,b,c,x,d) \text{ is a program for agent } i$$
$$\text{and, for all } s, \ c_{i0} - b_1(s) - x_1(s) = p_1(s) d_1(s),$$
$$c_{t-1}(s) + d_{t-1}(s) - b_t(s) - x_t(s) = p_t(s) d_t(s), \text{ for } t = 2, \ldots, T-1$$
$$\text{and } c_{T-1}(s) + d_{T-1}(s) - x_T(s) = 0\}.$$

The <u>response</u> of agent i, $\xi_i(p)$, is the set of solutions to the problem

$$\max \{E \sum_{t=1}^{T} u_i(a_t, x_t) \mid (\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}, \underset{\sim}{x}, \underset{\sim}{d}) \in \beta_i(\underset{\sim}{p})\} .$$

A <u>temporary equilibrium</u> is a price system $\underset{\sim}{p}$ and a feasible allocation $((\underset{\sim}{a}_i, \underset{\sim}{b}_i, \underset{\sim}{c}_i, \underset{\sim}{x}_i, \underset{\sim}{c}_i))$ such that $(\underset{\sim}{a}_i, \underset{\sim}{b}_i, \underset{\sim}{c}_i, \underset{\sim}{x}_i, \underset{\sim}{d}_i) \in \xi_i(\underset{\sim}{p})$, for all i.

I prove in an appendix that a temporary equilibrium exists. The proof is quite standard.

8) Temporary Equilibrium with Self-Insurance

In temporary equilibrium, there are no markets for insurance. But in reality, agents can insure themselves to some extent by using money or other assets. (Self-insurance has been discussed in (Yaari, 1976), (Schechtman, 1976), (Bewley, 1977) and (Grossman, Levhari and Mirman, 1979).

So far, there have been no storable assets in the model. I now go to the opposite extreme and introduce an asset so perfect that complete self-insurance is possible. Doing so has the advantage that temporary equilibria become Pareto optimal.

In reality, of course, complete self-insurance is impossible. Hence, there is a conflict between providing managers with insurance and with incentives to produce. I assume complete self-insurance simply in order to avoid this conflict.

The asset I introduce should be thought of as net credit balances. Every agent can hold an arbitrary positive or negative balance. Credit balances have no direct utility until the last period. In that period, the utility of agent i is $u_i(a,x) + B$, where B is his net balance at the end of the period. This formulation gives balances constant marginal utility over time. This constancy is startling. We may think of the credit balances as money, and one of the difficulties faced by economic planners in reality is that they cannot control or predict precisely consumers' marginal utilities of money.

In a temporary equilibrium with credit balances, it is not necessary to trade forward. Agents can use credit to finance their purchases and so do not need to sell forward.

A temporary equilibrium for the model with credit is described by a feasible allocation $((\underset{\sim}{a}_i, \underset{\sim}{b}_i, \underset{\sim}{c}_i, \underset{\sim}{x}_i))$ and a price system $\underset{\sim}{p} = (p_1, \ldots, p_T)$. The allocation and price system are just as in an Arrow-Debreu equilibrium. However, the interpretation of prices is different. $p_t(\underset{\sim}{s})$ is the price at time t and in state $\underset{\sim}{s}$ for good for current delivery. The unit of account is credit.

The credit balances of agent i are described by $\underset{\sim}{B}_i = (B_{i0}, \ldots, B_{iT})$, where $B_{it}(\underset{\sim}{s})$ is his balance at the end of period t. Agents start with no credit balance. That is, $B_{i0} = 0$. Given $(\underset{\sim}{a}_i, \underset{\sim}{b}_i, \underset{\sim}{c}_i, \underset{\sim}{x}_i)$ and $\underset{\sim}{p}$, the other balances B_{it} are defined by induction on t as follows.

$$B_{i,t+1}(\underset{\sim}{s}) = B_{it}(\underset{\sim}{s}) + p_t(\underset{\sim}{s})(c_{i,t-1}(\underset{\sim}{s}) - b_{it}(\underset{\sim}{s}) - x_{it}(\underset{\sim}{s})) .$$

A temporary equilibrium (with credit) is a feasible allocation $((\underset{\sim}{a}_i, \underset{\sim}{b}_i, \underset{\sim}{c}_i, \underset{\sim}{x}_i))$ and a price system $\underset{\sim}{p}$ such that for each i, $((\underset{\sim}{a}_i, \underset{\sim}{b}_i, \underset{\sim}{c}_i, \underset{\sim}{x}_i))$ solves the problem

$$\max \{E [\sum_{t=1}^{T} u_i(a_{it}, x_{it}) + B_{iT}] \mid (\underset{\sim}{a}_i, \underset{\sim}{b}_i, \underset{\sim}{c}_i, \underset{\sim}{x}_i) \text{ is an allocation feasible for agent } i \} ,$$

where it is understood that $B_{iT}(\underset{\sim}{s})$ depends on $(\underset{\sim}{a}_i, \underset{\sim}{b}_i, \underset{\sim}{c}_i, \underset{\sim}{x}_i)$ and $\underset{\sim}{p}$.

It is easy to see that such a temporary equilibrium exists. In fact, it is essentially unique. The feasible allocations form a compact subset of a Euclidean space. Hence, the problem

$$8.1) \quad \max\{E \sum_{i=1}^{I} \sum_{t=0}^{T} u_i(a_{it},x_{it}) \mid ((\underset{\sim}{a}_i,\underset{\sim}{b}_i,\underset{\sim}{c}_i,\underset{\sim}{x}_i)) \text{ is a feasible allocation}\}$$

has a solution. Call the solution $((\underset{\sim}{a}_i,\underset{\sim}{b}_i,\underset{\sim}{c}_i,\underset{\sim}{x}_i))$. I now define prices. The definition is by backwards on t. Let

$$p_T(\underset{\sim}{s}) = \max_i \frac{\partial u_i(0,x_{iT}(\underset{\sim}{s}))}{\partial x} .$$ Given $p_{t+1}(\underset{\sim}{s})$, let

$$p_t(\underset{\sim}{s}) = \max_i \max\left(\frac{\partial u_i(a_{it}(\underset{\sim}{s}),x_{it}(\underset{\sim}{s}))}{\partial x}, p_{t+1}(s)\frac{\partial f_{it}(a_{it}(\underset{\sim}{s}),b_{it}(\underset{\sim}{s}),s_{it})}{\partial b}\right).$$

This defines $\underset{\sim}{p}$. It is easy to check that $((\underset{\sim}{a}_i,\underset{\sim}{b}_i,\underset{\sim}{c}_i,\underset{\sim}{x}_i))$ and $\underset{\sim}{p}$ form a temporary equilibrium.

The allocation of any temporary equilibrium with credit solves problem 8.1. This is so because in equilibrium the marginal utility of credit of any agent is one in any period and any state.

9) The Incentive Compatibility of Temporary Equilibrium

If there were no costs to making forward agreements, then either form of temporary equilibrium would be incentive compatible under a reasonable assumption. The assumption needed is that no agent i can influence any price significantly in any way by misrepresenting the probability distribution of $\{s_{it}\}$ or his observations of the s_{it}. This assumption makes sense in the contexts appropriate for the model. It is clear that if the assumption were valid, then no agent could gain by misrepresenting. A rigorous statement would be an asymptotic one. Temporary equilibrium would be asymptotically incentive compatible as the number of agents went to infinity and as each agent's influcence on prices went to zero. Thus, either form of temporary equilibrium nearly solves the intertemporal allocation problem. I say "nearly" because I have not dealt adequately with the insurance problem.

But temporary equilibrium is a solution only because I have neglected the costs of forward arrangements. Such costs clearly exist, and when there are such costs, temporary equilibrium makes little sense at all. One can hardly think of temporary equilibrium as incentive compatible in this case. Suppose that all agents cooperated but one and that they determined forward prices. If making contracts required any effort, then the one agent would prefer not to cooperate and simply to trade at the announced prices. If there were many agents, his extra demand or supply would hardly be noticed.

One could try to overcome this problem by forbidding agents to trade unless they had participated in the forward planning process. But such a prohibition might be hard to enforce. Furthermore, it would not necessarily be socially advantageous to do so. The costs of arranging contracts could be enormous, for the number of dated events grows exponentially with the time horizon. Some other scheme might be better. For example, one might pay a random sample of agents for participating in forward planning. Further work in this direction would require specification of the costs involved in making contracts, in finding equilibrium prices, or in doing whatever planning requires.

10) Arrow, Coase and Hayek

The thoughts discussed in this paper are closely related to ideas of Arrow (1974a, 1974b), Coase (1932) and Hayek (1937, 1945).

Coase asked why firms exist at all. Why not have all economic interactions be arranged on markets? Part of his answer was that it is costly to interact on markets and to arrange contracts. These costs can be avoided by forming an organization which exercises at least some authority. For instance, a total market solution would lead to contracts for labor which would specify exactly what the laborer would do in every possible circumstance. If laborers did not care how their services were used, it would be cheaper to give employers authority over laborers during specified periods. (I am paraphrasing passages on pages 391 and 392 of Coase's article.)

Arrow (1974a) discusses issues very similar to those raised by Coase. Arrow takes the view that "organizations are a means of achieving the benefits of collective action in situations in which the price system fails" (1974a, p. 33). In his discussion of economic organizations, he emphasizes the costs of gathering and transmitting information.

Thus, both Coase and Arrow urge, just as I do, that one take into account the costs of planning and coordination themselves.

Hayek (1937 and especially 1945) described a version of reality which corresponds exactly to that which lies behind the model of this paper. I quote a key passes (on pages 519 and 520 of 1945).

> "The peculiar character of the problem of a rational economic order is determined precisely by the fact that the knowledge of the circumstances of which we must make use never exists in concentrated or integrated form but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess. The economic problem is thus not merely a problem of how to allocate 'given' resources -- if 'given' is taken to mean given to a single mind which deliberately solves the problem set by these 'data.'... it is a problem of the utilization of knowledge which is not given to anyone in its totality."

Hayek also emphasized the importance of change and novelty in economic life. It is this idea which lies behind my refusal to use rational expectations.

Hayek goes on to argue that the competitive system solves society's coordination problem and does so with a remarkable economy of information exchange. It gives the "man on the spot" a free hand, while giving him just the information he needs in order to take into account society's needs. I quote Hayek again (1945, pp 526,527).

> "The most significant fact about this system is the economy of knowledge with which it operates, or how little participants need to know in order to be able to take the right action . ... It is more than a metaphor to describe the price system as a kind of machinery for registering change..."

Hayek's optimism about the competitive systems seems somewhat gratuitous. The achievements of the system may strike one as impressive. But no one has considered rigorously whether alternative systems could do better in a world such as that described by Hayek. The problem is that when information is dispersed, complete Arrow-Debreu markets become impossible.

As Arrow pointed out (1974b), it is not clear that the competitive system functions efficiently when such markets are absent. The following remark of Arrow's makes an interesting contrast with Hayek's criticism of economists who advocate central planning (1974b, pp. 5,6).

> "Even as a graduate student, I was somewhat surprised at the emphasis on static allocative efficiency by market socialists, when the non-existence of markets for future goods under capitalism seemed to me a much more obvious target."

Appendix

I prove that a temporary equilibrium exists for the model of section 7.

Assumption 3.3 implies that the set of feasible outputs is bounded. Let $\bar{c} > 0$ be such that $\sum_{i=1}^{I} c_{it}(\underset{\sim}{s}) < \bar{c}$, for all i, t and $\underset{\sim}{s}$ and for any feasible allocation $((\underset{\sim}{a}_i, \underset{\sim}{b}_i, \underset{\sim}{c}_i, \underset{\sim}{x}_i, \underset{\sim}{d}_i))$.

The boundedness of production implies upper and lower bounds on prices. Let $\underline{p}_t = \min\left\{\left(\frac{\partial f_{it}(a,0,s))}{\partial b}\right)^{-1} \middle| \text{ all } i, a \text{ and } s\right\}$ and let $\bar{p}_t = \max\left\{\left(\frac{\partial f_{it}(a,\bar{c},s)}{\partial b}\right)^{-1} \middle| \text{ all } a \text{ and } s\right\}$. Assumptions 3.3 and 3.4 imply that $0 < \underline{p}_t < \bar{p}_t < \infty$. $\underline{p}_t$ and $\bar{p}_t$ are lower and upper bounds on prices, respectively.

Let $\Delta = \{\underset{\sim}{p} = (p_1, \ldots, p_{T-1}) \mid \underset{\sim}{p}$ is a price system and $\underline{p}_t \leq p_t(\underset{\sim}{s}) \leq \bar{p}_t$, for all $t\}$. Δ is a compact convex subset of a Euclidean space. I apply the Brouwer fixed point theorem to a function on Δ.

I now truncate the budget set. Let $\hat{\beta}_i(\underset{\sim}{p}) = \{(\underset{\sim}{a},\underset{\sim}{b},\underset{\sim}{c},\underset{\sim}{x},\underset{\sim}{d}) \in \beta_i(\underset{\sim}{p}) \mid$ no components of $\underset{\sim}{b},\underset{\sim}{c},\underset{\sim}{x}$, or $\underset{\sim}{d}$ exceed $\bar{c}$ in absolute value $\}$.

Let $\hat{\xi}_i(\underset{\sim}{p})$ solve the problem

$$\max \left\{E \sum_{t=1}^{T} u_i(a_t, x_t) \mid (\underset{\sim}{a},\underset{\sim}{b},\underset{\sim}{c},\underset{\sim}{x},\underset{\sim}{d}) \in \hat{\beta}_i(\underset{\sim}{p})\right\}.$$

$\hat{\xi}$ is a continuous function on Δ.

Let $z(\underset{\sim}{p})$ be defined as follows. $z(\underset{\sim}{p}) = (z_1(\underset{\sim}{p}), \ldots, z_{T-1}(\underset{\sim}{p}))$, where

$$z_t(\underset{\sim}{p}, \underset{\sim}{s}) = \sum_{i=1}^{I} (x_{it}(\underset{\sim}{s}) + b_{it}(\underset{\sim}{s}) - c_{i,t-1}(\underset{\sim}{s})) \quad \text{and}$$

$(a_i, b_i, c_i, x_i, d_i) = \hat{\xi}_i(p)$, for all i.

Finally, let $h: \Delta \to \Delta$ be defined as follows $h(\underset{\sim}{p}) = (q_1, \ldots, q_{T-1})$, where $q_t(\underset{\sim}{s}) = \min(\bar{p}_t, \max(\underline{p}_t, p_t(\underset{\sim}{s}) - z_t(\underset{\sim}{p}, \underset{\sim}{s})))$, for all $\underset{\sim}{s}$ and t. h is continuous, so that it has a fixed point, $\hat{\underset{\sim}{p}}$. Let $(\underset{\sim}{a}_i, \underset{\sim}{b}_i, \underset{\sim}{c}_i, \underset{\sim}{x}_i, \underset{\sim}{d}_i) = \hat{\xi}_i(\hat{\underset{\sim}{p}})$, for all i.

It is sufficient to show that $z(\hat{\underset{\sim}{p}}) = 0$, for this means that

A.1) $$\sum_{i=1}^{I} (b_{it}(\underset{\sim}{s}) + x_{it}(\underset{\sim}{s}) - c_{i,t-1}(\underset{\sim}{s})) = 0, \quad \text{for all } \underset{\sim}{s} \text{ and } t.$$

Also, by adding the budget constraints of the individuals one obtains

A.2) $$\sum_{i=1}^{I} (b_{it}(\underset{\sim}{s}) + x_{it}(\underset{\sim}{s}) - c_{i,t-1}(\underset{\sim}{s}) - d_{i,t-1}(\underset{\sim}{s})) = - p_t(\underset{\sim}{s}) \sum_{i=1}^{I} d_{it}(\underset{\sim}{s}), \quad \text{for all } \underset{\sim}{s} \text{ and } t,$$

where it is understood that $d_{i0}(\underset{\sim}{s}) = 0$. Equations A.1 and A.2 imply that

A.3) $$\sum_{i=1}^{I} d_{it}(\underset{\sim}{s}) = 0, \quad \text{for all } \underset{\sim}{s} \text{ and } t.$$

This equation is proved by induction on t.

Equations A.1 and A.3 state that the allocation $((\underset{\sim}{a}_i, \underset{\sim}{b}_i, \underset{\sim}{c}_i, \underset{\sim}{x}_i, \underset{\sim}{d}_i))$ is feasible. Since it is feasible its components are all less than $\bar{c}$ in absolute value. A standard argument now proves that

$(\underset{\sim}{a}_i, \underset{\sim}{b}_i, \underset{\sim}{c}_i, \underset{\sim}{x}_i, \underset{\sim}{d}_i)$ belongs to $\xi_i(\underset{\sim}{\hat{p}})$ as well as to $\hat{\xi}_i(\underset{\sim}{\hat{p}})$. This proves that $((\underset{\sim}{a}_i, \underset{\sim}{b}_i, \underset{\sim}{c}_i, \underset{\sim}{x}_i, \underset{\sim}{d}_i))$ and $\underset{\sim}{\hat{p}}$ form a temporary equilibrium.

I now prove that $z(\underset{\sim}{\hat{p}}) = 0$. Suppose that $z_{\underline{t}}(\underset{\sim}{\hat{p}}) \neq 0$ and let $\underline{t}$ be the smallest value of t such that $z_t(\underset{\sim}{\hat{p}}, \underset{\sim}{s}) \neq 0$, for some $\underset{\sim}{s}$. Then, the allocation $((\underset{\sim}{a}_i, \underset{\sim}{b}_i, \underset{\sim}{c}_i, \underset{\sim}{x}_i, \underset{\sim}{d}_i))$ is feasible in the periods before $\underline{t}$, so that $\sum_{i=1}^{I} c_{i,\underline{t}-1}(\underset{\sim}{s}) < \bar{c}$.

Suppose that $z_{\underline{t}}(\underset{\sim}{\hat{p}}, \underset{\sim}{s}) < 0$. Then, $b_{1,\underline{t}}(\underset{\sim}{s}) \leq \sum_{i=1}^{I} c_{i,\underline{t}-1}(\underset{\sim}{s}) < \bar{c}$. Also, $\hat{p}_{\underline{t}}(s) = \bar{p}_{\underline{t}}$. But then by the definition of $\bar{p}_t$, $b_{1\underline{t}}(\underset{\sim}{s}) \geq \bar{c}$. That is, the price of forward output is so high that agent 1 finds it profitable to use at least $\bar{c}$ units of input. This contradiction implies that $z_{\underline{t}}(\underset{\sim}{\hat{p}}, \underset{\sim}{s}) \geq 0$.

Suppose that $z_{\underline{t}}(\underset{\sim}{\hat{p}}, \underset{\sim}{s}) > 0$. Then, $\hat{p}_{\underline{t}}(\underset{\sim}{s}) = \underline{p}_{\underline{t}}$, so that $d_{i\underline{t}}(\underset{\sim}{s}) \geq 0$, for all i. That is, the price is so low that no agent finds it profitable to produce and none sell forward. Since the allocation $((\underset{\sim}{a}_i, \underset{\sim}{b}_i, \underset{\sim}{c}_i, \underset{\sim}{x}_i, \underset{\sim}{d}_i))$ is feasible up to time $\underline{t} - 1$, it follows that $\sum_{i=1}^{I} d_{i,\underline{t}-1}(\underset{\sim}{s}) = 0$. Hence by equation A.2, $\sum_{i=1}^{I} d_{i,\underline{t}}(s) = -\underline{p}_{\underline{t}}^{-1} z_{\underline{t}}(\underset{\sim}{\hat{p}}, \underset{\sim}{s}) < 0$. This contradiction implies that $z_{\underline{t}}(\underset{\sim}{\hat{p}}, \underset{\sim}{s}) = 0$.

This completes the proof that $\underset{\sim}{\hat{p}}$ is an equilibrium price vector.

Acknowledgement

I am very grateful to John Ledyard for helpful advice. I am also grateful to Michael Magill for having told me to read Hayek.

REFERENCES

ARROW, Kenneth (1953), " Le rôle des valeurs bourières pour la repartition la meilleure des risques," Econométrie (Centre Nationale de la Recherche Scientifique, Paris), pp. 41-48.

____________ (1974a), The Limits of Organization (Norton, New York).

____________ (1974b), "Limited knowledge and economic analysis," American Economic Review, 64, pp 1-10.

ARROW, Kenneth and Frank Hahn (1971), General Competitive Analysis (Holden-Day, San Francisco).

BEWLEY, Truman (1977), "The permanent income hypothesis: a theoretical formulation," Journal of Economic Theory, 16, pp. 252-292.

____________ (1979), "An integration of equilibrium theory and turnpike theory," Discussion Paper 405, Center for Mathematical Studies in Economics and Management Science, Northwestern University, Evanston, Illinois.

COASE, R. H. (1937), "The nature of the firm," Economica, 4, pp. 386-405.

DEBREU, Gerard (1959), Theory of Value (John Wiley, New York).

GRANDMONT, Jean Michel (1977), "Temporary general equilibrium theory," Econometrica, 45, pp. 535-572.

GROSSMAN, Sanford, David Levhari and Leonard Mirman (1979), "Consumption under uncertainty," in General Equilibrium, Growth and Trade, Essays in Honor of Lionel McKenzie, Jerry Green and José Scheinkman (eds.) (Academic Press , New York).

GROVES, Theodore and John Ledyard (1977), "Optimal allocation of public goods: a solution to the 'free rider' problem," Econometrica, 45, pp. 783-809.

HAYEK, Friedrich (1937), "Economics and knowledge," Economica, 4, pp. 35-54. Reprinted in F. Hayek, Individualism and Economic Order (Univ. of Chicago, 1948).

________________ (1945), "The use of knowledge in society," American Economic Review, 35, pp. 519-530. Also reprinted in Individualism and Economic Order.

HURWICZ, Leonid (1973), "The design of resource allocation mechanisms," American Economic Review, 58, pp 1-30.

MALINVAUD, Edmond (1953), "Capital accumulation and efficient allocation of resources," Econometrica, 21, pp. 233-267.

RADNER, Roy (1970), "Competitive equilibrium under uncertainty," Econometrica, 36, pp. 31-58.

SAMUELSON, Paul (1954), "The Pure Theory of Public Expenditure," Review of Economics and Statistics, 36, pp. 387-389.

SCHECHTMAN, Jack (1976), "An income fluctuation problem," Journal of Economic Theory, 12, pp. 218-241.

YAARI, Menaham (1976), "A law of large numbers in the theory of consumers' choices under uncertainty," Journal of Economic Theory, 12, pp. 202-217.