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A DUOPOLY MODEL OF THE ADOPTION OF AN INNOVATION OF UNCERTAINTY PROFITABILITY

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I. Introduction

Economists have long been aware of and frequently conjectured about the importance of uncertainty, information, learning, attitudes, and rivalry on the rate of adoption and diffusion of an innovation.[1] However, since most of the work on diffusion has been empirical in nature, rather difficult measurement problems have nearly precluded examination of the effects of these factors.[2] Moreover, none of the theoretical work on diffusion has incorporated all of these factors into the analysis. The earliest work in this area by Fellner [4], Salter [19], Smith [22], and Hinomoto [8], among others looked at capital-embodied innovations whose adoption could be delayed due to the necessity of building a new plant in order to adopt the innovation. Both of these efforts, however, avoided the issues of uncertainty and rivalry. More recently, work by Flaherty [5] and Reinganum [17] has confronted the rivalry question by examining optimal adoption behavior in duopolistic industries. Both of these approaches, however, suffer from two major flaws: first, they avoid the issue of uncertainty; and second, they generate delays in adoption by essentially assuming that adoption will not increase the firm's present discounted value for some time after the new technology is first introduced, which raises a serious question as to whether or not the new technology really is an innovation when first introduced.

Recent work by this author [9] shows that it may be optimal for a firm to delay its adoption of an innovation which would increase the firm's present discounted value if adopted immediately when the firm is simply uncertain about whether or not the innovation will be profitable and this uncertainty can be reduced by waiting to gather information about the innovation. This analysis is also general enough to include both new process and new product innovations, but is lacking since it avoids the issue of rivalry. The purpose of this paper is to extend the analysis in [9] by examining the effects of rivalry, as well as uncertainty, information, learning, and attitudes, on the adoption and diffusion of an innovation. In particular, a simple duopoly model is developed in which each firm faces a sequence of decision dates at which it has the option of either adopting an innovation of uncertain profitability or waiting to obtain some information about the innovation. Rivalry enters because each firm knows that its rival's decision to adopt or wait will have an effect on the expected values of its two possible decisions.

The remainder of this paper is organized in the following fashion. Section II presents the basic model describing each firm's decision problem and some preliminary technical results. In Section III the existence of unique optimal adoption rules is demonstrated for both the case where the rival firm has not yet adopted and that where it has already adopted. This rule is used to obtain optimal adoption probabilities for the firm from which several interesting results can be derived:

(1) It may be optimal to delay the adoption of an innovation which would be profitable (increase the firm's present discounted value) if it were adopted immediately;

(2) The length of such a delay will be shorter the more optimistic the firm is that the innovation will be profitable when it first appears or the more favorable is the information received about it; and

(3) Adoption by the rival firm need not increase, and may possibly decrease, the probability of adoption by the firm still waiting at the last decision date.

In Section IV the existence of an equilibrium for each decision date at which neither firm has yet adopted is demonstrated. This equilibrium occurs when each firm's endogenous estimate of the probability the rival will adopt then (which is required to solve the firm's decision problem) is equal to the rival's optimal adoption probability. These equilibrium estimates are endogenous in that they are revised at each decision date to incorporate both new information about the innovation and the information provided by the fact that the rival firm has not yet adopted. Section V then concludes the paper.

II. The Model

Consider an industry composed of two firms which are initially operating in a certain environment earning a period profit of $r^i > 0$ $(i=1,2)$ and then are confronted with an exogenously developed innovation. Information about the innovation's existence comes from a source external to the firms which will continue to provide information about the innovation at discrete intervals thereafter. Each piece of information is an observation which can be classified as being either favorable or unfavorable to the innovation and therefore can be represented by a Bernoulli random variable Z which takes on the value 1 if the observation is considered favorable and 0 if not. At each stage of the decision process each firm has the option of either adopting (if it has not already done so) or waiting to learn more about the innovation (i.e., take an observation). A firm which has not adopted will then observe a sequence of Bernoulli random variables which are assumed to be independent and identically distributed with unknown parameter $\theta = \Pr\{Z=1\}$. It is also assumed that each duopolist believes that θ must be one of two values, θ_1 and θ_2, where $0<\theta_2<\theta_1<1$. Additionally, it is assumed that after one of the firms has adopted the waiting firm will ignore any external information and observe only the experience of its rival with the innovation.[3] Finally, no explicit cost of taking an observation will be assumed, although it is evident that waiting to take an observation may have an opportunity cost if the innovation is a good one (i.e., will increase the firm's expected discounted present value).

After the innovation appears, the revenue earned by each firm in a given period will be uncertain, depending upon its action, its rival's action, and the true state of nature, θ. In particular, letting a_1 and a_2 be the actions of firms 1 and 2, respectively, the period certain returns to firm i will be represented by $r_z^i(a_i,a_j)$, where $i,j=1,2(i\neq j)$ and $z=0,1$. Let $a_i=1$ designate adoption and $a_i=0$ designate waiting by firm i. Then, for instance, $r_1^i(1,1)$ is the period return to firm i when both firms have adopted and their experience is favorable. The expected period returns to firm i will then be one of the following:

$$\theta r_1^i(1,1) + (1-\theta)r_0^i(1,1) \tag{1a}$$

$$\theta r_1^i(1,0) + (1-\theta)r_0^i(1,0) \tag{1b}$$

$$\theta r_1^i(0,1) + (1-\theta)r_0^i(0,1) \tag{1c}$$

$$r^i \tag{1d}$$

Equation (1a) is the period return expected when both firms have adopted; (1b) is that when firm i has adopted but its rival is waiting; (1c) is that when firm i is waiting but its rival has adopted; and (1d) is, of course, that when both wait.

The impact of rivalry on the returns earned by each firm will be characterized by the following assumptions:

$$r_1^i(1,0) > r_1^i(1,1) > r^i > r_0^i(1,1) > r_0^i(1,0) \tag{2a}$$

$$r_0^i(0,1) > r^i > r_1^i(0,1) \tag{2b}$$

Inequalities (2a) show that, no matter what the rival firm does, if firm i adopts it will earn a return above the pre-innovation level in any period in which its experience is favorable and a return below the pre-innovation level in any period in which its experience is unfavorable. However, the gain when its experience is favorable will be larger if its rival has not adopted and the loss when its experience is unfavorable will be smaller if its rival also has adopted. Inequalities (2b) show that if firm i waits when its rival has adopted, then it will earn more than the pre-innovation return in any period when its rival's experience is unfavorable and less when its rival's experience is favorable. Thus, in a given period there are potential gains to be obtained from having made the right decision for that period when the rival has made the wrong decision and potential losses to be incurred from having made the wrong decision for that period when the rival has made the right decision. It is worthwhile to note that the inequalities in (2) can be derived from a simple Cournot duopoly model where demand is linear and marginal cost is constant for both the case of a process innovation (shifting the marginal cost curve either up or down and the case of a product innovation (shifting the demand curve to the left or right).

Finally, the adoption decision is assumed to be irreversible and require a fixed outlay $c^i > 0$. Let $\beta \varepsilon (0,1)$ be the discount factor and define $R_z^i(a_i, a_j) = \frac{r_z^i(a_i, a_j)}{1-\beta}$ and $R^i = \frac{r^i}{1-\beta}$. Then the expected discounted return to firm i from adoption will be

$$\theta R_1^i(1,1) + (1-\theta)R_0^i(1,1) - c^i \qquad (3a)$$

if its rival has adopted and

$$\theta R_1^i(1,0) + (1-\theta)R_0^i(1,0) - c^i \qquad (3b)$$

if its rival never adopts. Similarly, if firm i does not adopt its expected discounted return will be

$$\theta R_1^i(0,1) + (1-\theta)R_0^i(0,1) \tag{3c}$$

if its rival adopts and R^i if it also does not adopt. The following relationships among these expected discounted returns are assumed:

$$\theta_1 R_1^i(1,0)+(1-\theta_1)R_0^i(1,0)-c^i>\theta_1 R_1^i(1,1)+(1-\theta_1)R_0^i(1,1)-c^i>R^i \tag{4a}$$

$$\theta_2 R_1^i(1,0)+(1-\theta_2)R_0^i(1,0)-c^i<\theta_2 R_1^i(1,1)+(1-\theta_2)R_0^i(1,1)-c^i<R^i \tag{4b}$$

$$\theta_2 R_1^i(0,1)+(1-\theta_2)R_0^i(0,1)>R^i>\theta_1 R_1^i(0,1)+(1-\theta_1)R_0^i(0,1). \tag{4c}$$

These assumptions allow the event $\{\theta=\theta_1\}$ to be interpreted as the innovation being good and $\{\theta=\theta_2\}$ as the innovation being bad since firm i, regardless of its rival's actions, would prefer to adopt if it knew $\theta=\theta_1$ and not adopt if it knew $\theta=\theta_2$.[4] They also extend the effects of rivalry on period returns assumed in (2) to the firm's expected discounted returns since, as in (2), whether firm i decides to adopt or not, its expected discounted return will always be higher if its rival makes the wrong decision (i.e., adopts when $\theta=\theta_2$ or does not adopt when $\theta=\theta_1$) and lower if its rival makes the correct decision.

The complexity of the firm's decision problem can now be seen. There is uncertainty about whether or not the innovation will be profitable and when, if ever, the rival will adopt. It is therefore necessary for the firm to estimate both θ and the probability that the rival will adopt (if it has not already) in order to obtain estimates of the values of adopting and waiting at each stage of the process.

The estimate of θ is derived in standard Bayesian fashion. If p^i is firm i's subjective probability assessment that $\theta=\theta_1$, then the estimate of θ is given by

$$q(p^i) = p^i\theta_1 + (1-p^i)\theta_2. \tag{5}$$

It is assumed that when the innovation first appears each firm assigns a subjective probability $g^i\varepsilon[0,1]$ to the event $\{\theta=\theta_1\}$. This initial belief the innovation is good is then revised as observations are seen by the application of Bayes theorem. In particular, if p^i is the firm's current belief the innovation is good, then after a favorable observation

p^i will be adjusted upward to $h_1(p^i) = \dfrac{p^i\theta_1}{q(p^i)}$ (6a)

and after an unfavorable observation p^i will be adjusted downward to

$$h_0(p^i) = \frac{p^i(1-\theta_1)}{1-q(p^i)} \tag{6b}$$

Hence, after n observations have been seen, k of which were favorable ($k\leq n$), the firm's updated belief the innovation is good will be given by

$$p(n,k,g^i) = [1+(\frac{\theta_2}{\theta_1})^k \quad (\frac{1-\theta_2}{1-\theta_1})^{n-k} \quad (\frac{1-g^i}{g^i})]^{-1} \quad . \tag{7}$$

Because $g^i\varepsilon[0,1]$, it is clear that the firm's current belief at any stage, $p^i=p(n,k,g^i)$, may attain any value in [0,1]. The assignment of g^i, which will depend upon such things as the past experience of firm i with similar innovations and the expertise and attitudes of its decision-makers, will not be dealt with explicitly in this paper, but instead will be taken as given.

The estimate of the probability that the rival firm j will adopt at a given decision date, if it has not done so already, is less straightforward. For now it will simply be assumed that this probability does not depend upon the firm's belief the innovation is good, p^i, but is updated at each stage in the decision process so that it may be written as

$$\Pr\{a_{jt} = 1 | a_{j\tau} = 0 \text{ for all } \tau > t\} = \pi_t^i \tag{8}$$

where $\pi_t^i \varepsilon [0,1]$ for each i and t. The presentation and discussion of the method used by each firm to estimate π_t^i will be deferred until Section IV, where it will serve as the basis for demonstrating the existence of an equilibrium for this duopoly. All that is required for the analysis in this section is that this probabilistic belief the rival firm will adopt does not depend upon p^i.[5]

The stochastic process underlying the firm's decision problem can now be completely specified. The state variable for firm i can be taken to be $(p^i, a_j) \varepsilon [0,1] x \{0,1\}$, an ordered pair describing the firm's current belief the innovation is good and whether or not the rival firm has adopted. There are four states to which (p^i, a_j) can move after an observation is seen: $(h_1(p^i),1)$, $(h_0(p^i),1)$, $(h_1(p^i),0)$, and $(h_0(p_i),0)$. The transition probabilities are then determined from equations (6), (7), and (8) as follows:

$$\Pr\{(h_1(p^i),1) | (p^i,0)\} = q(p^i)\pi_t^i \tag{9a}$$

$$\Pr\{(h_0(p^i),1) | (p^i,0)\} = (1-q(p^i))\pi_t^i \tag{9b}$$

$$\Pr\{(h_1(p^i),0) | (p^i,0)\} = q(p^i)(1-\pi_t^i) \tag{9c}$$

$$\Pr\{(h_0(p^i),0) | (p^i,0)\} = (1-q(p^i))(1-\pi_t^i) \tag{9d}$$

$$\Pr\{(h_1(p^i),1) | (p^i,1)\} = q(p^i) \tag{9e}$$

$$\Pr\{(h_0(p^i),1) | (p^i,1)\} = 1-q(p^i) \tag{9f}$$

$$\Pr\{(h_1(p^i),0) | (p^i,1)\} = \Pr\{(h_0(p^i),0) | (p^i,1)\} = 0. \tag{9g}$$

Since these probabilities do not depend on the stage of the process, this stochastic process is Markovian with stationary transition probabilities and so the firm faces a Markovian decision process.

The firm's decision problem can now be formally stated as an optimal stopping problem where the stopping value is the expected return from adoption and the optimal continuation (or waiting) value is the expected value of the next observation. Following standard procedure, let the value function $V_t^i(p^i,a_j)$ be the maximum expected return to firm i when the state is (p^i,a_{j_6}) and there are $t=0,1,\ldots N$ decision dates remaining in the problem. When $t=0$, the firm has lost the opportunity to adopt and so the value function there will be, for all $p^i \epsilon [0,1]$,

$$V_0^i(p^i,1)=q(p^i)R_1^i(0,1)+(1-q(p^i))R_0^i(0,1) \tag{10a}$$

$$V_0^i(p^i,0)=R^i. \tag{10b}$$

The $V_t^i(p^i,a_j)$ for $t \geq 1$ are then defined recursively by the functional equation

$$V_t^i(p^i,a_j)=\max\{V_t^i(1,p^i,a_j),\ V_t^i(0,p^i,a_j)\} \tag{11}$$

where the expected adoption returns are

$$V_t^i(1,p^i,1)=q(p^i)R_1^i(1,1)+(1-q(p^i))R_0^i(1,1)-c^i \tag{12a}$$

$$\begin{aligned} V_t^i(1,p^i,0)=&\pi_t^i[q(p^i)R_1^i(1,1)+(1-q(p^i))R_0^i(1,1)-c^i] \\ &+(1-\pi_t^i)[q(p^i)R_1^i(1,0)+(1-q(p^i))R_0^i(1,0)-c^i] \end{aligned} \tag{12b}$$

and the expected waiting returns are

$$\begin{aligned} V_t^i(0,p^i,1)=&q(p^i)r_1^i(0,1)+(1-q(p^i))r_0^i(0,1) \\ &+\beta[q(p^i)V_{t-1}^i(h_1(p^i),1)+(1-q(p^i))V_{t-1}^i(h_0(p^i),1)] \end{aligned} \tag{13a}$$

$$\begin{aligned} V_t^i(0,p^i,0)=&\pi_t^i[q(p^i)r_1^i(0,1)+(1-q(p^i))r_0^i(0,1)]+(1-\pi_t^i)r^i \\ &+\beta\pi_t^i[q(p^i)V_{t-1}^i(h_1(p^i),1)+(1-q(p^i))V_{t-1}^i(h_0(p^i),1)] \\ &+\beta(1-\pi_t^i)[q(p^i)V_{t-1}^i(h_1(p^i),0)+(1-q(p^i))V_{t-1}^i(h_0(p^i),0)]. \end{aligned} \tag{13b}$$

for all $p^i \epsilon[0,1]$. It is significant to observe that this model of the decision problem of a firm which is confronted with an innovation of uncertain profitability and is aware of the impact of its rival's actions on its expected returns can easily lend itself to a rather broad interpretation of innovation. In particular, since each firm's expected present value will in general be different after the innovation is introduced, this approach adopts the Schumpeterian view of innovation as a change in the status quo. Moreover, the model is general enough to encompass both new process and new product innovations.

Before examining optimal behavior and its implications under this model, it will be necessary to derive some of the properties of the value function defined by (10)-(13) in order to demonstrate the existence of optimal decision rules for this problem. These properties are summarized in the following lemmas. Since the decision problem is the same for both firms, the superscript i, denoting firm i, will hereafter be deleted whenever it can be without causing any notational confusion.

Lemma 1: For all $t=1,2,\ldots,N$, $V_t(p,1)$ and $V_t(p,0)$ are continuous and convex in p for all $p\epsilon[0,1]$.

Proof: First, since $q(p)$ is continuous, it follows from (10a) that $V_t(1,p,1)$ is continuous in p for all t. And since $q(p)q(h_1(p))+(1-q(p))q(h_0(p))=q(p)$ under Bayesian learning,

$$\begin{aligned}V_1(0,p,1)&=q(p)r_1(0,1)+(1-q(p))r_0(0,1)+\beta q(p)V_0(h_1(p),1)\\&\quad+\beta(1-q(p))V_0(h_0(p),1)\\&=q(p)r_1(0,1)+(1-q(p))r_0(0,1)+\beta q(p)[q(h_1(p))R_1(0,1)+(1-q(h_1(p)))R_0(0,1)]\\&\quad+\beta(1-q(p))[q(h_0(p))R_1(0,1)+(1-q(h_0(p)))R_0(0,1)]\\&=q(p)r_1(0,1)+(1-q(p))r_0(0,1)+\beta[q(p)R_1(0,1)+(1-q(p))R_0(0,1)]\end{aligned}$$

$$=q(p)R_1(0,1)+(1-q(p))R_0(0,1), \tag{14}$$

which also is continuous in p. Hence, $V_1(p,1)$ is continuous in p. Since $h_1(p)$ and $h_0(p)$ are continuous, a simple induction on t will then give $V_t(p,1)$ continuous in p for all t.

Next, from (12b) it is obvious that $V_t(1,p,0)$ is continuous in p for all t. It can readily be verified (as with (14) above) that

$$V_1(0,p,0)=\pi_1[q(p)R_1(0,1)+(1-q(p))R_0(0,1)]+(1-\pi_1)R, \tag{15}$$

which also is continuous, so $V_1(p,0)$ is continuous. Because $h_1(p)$ and $h_0(p)$ are continuous and because $V_t(p,1)$ was shown above to be continuous, a simple induction on t will then give $V_t(p,0)$ continuous in p for all t.

Third, since q(p) is linear in p, both $V_1(1,p,1)$ and $V_1(0,p,1)$ are linear, and so convex, in p. Therefore, as the maximum of convex functions $V_1(p,1)$ is convex in p on [0,1]. Assume that $V_T(p,1)$ is convex in p on [0,1]; that is, for all $\lambda\varepsilon[0,1], V_T(\lambda,1) \le (1-\lambda)V_T(0,1)+\lambda V_T(1,1)$. Since $V_{T+1}(1,p,1)$, given by (12a), is linear in p, $V_{T+1}(p,1)$ will be convex for $p\varepsilon[0,1]$ if it can be shown that $V_{T+1}(0,p,1)$ is convex. And since $q(p)r_1(0,1)+(1-q(p))r_0(0,1)$ is linear, it can be seen from (13a), with $t=T+1$, that showing $q(p)V_T(h_1(p),1)$ and $(1-q(p))V_T(h_0(p),1)$ are convex for $p\varepsilon[0,1]$ will suffice to show that $V_{T+1}(p,1)$ is convex. Now, by the induction hypothesis and the definition of $h_1(p)$ in (6a), for all $\lambda\varepsilon[0,1]$

$$\begin{aligned} q(\lambda)V_T(h_1(\lambda),1) &\le q(\lambda)[(1-h_1(\lambda))V_T(0,1) + h_1(\lambda)V_T(1,1)] \\ &= (1-\lambda)\theta_2 V_T(0,1) + \lambda\theta_1 V_T(1,1) \\ &= (1-\lambda)q(0)V_T(h_1(0),1) + \lambda q(1)V_T(h_1(1),1), \end{aligned}$$

which shows that $q(p)V_T(h_1(p),1)$ is convex for $p\varepsilon[0,1]$. Similarly, the induction hypothesis and (6b) imply that

$$(1-q(\lambda))V_T(h_0(\lambda),1)\leq(1-q(\lambda))[(1-h_0(\lambda))V_T(0,1)+h_0(\lambda)V_T(0,1)]$$
$$=(1-\lambda)(1-\theta_2)V_T(0,1)+\lambda(1-\theta_1)V_T(1,1)$$
$$=(1-\lambda)(1-q(0))V_T(h_0(0),1)+\lambda(1-q(1))V_T(h_0(1),1)$$

for all $\lambda\varepsilon[0,1]$, which shows $(1-q(p))V_T(h_0(p),1)$ is convex for $p\varepsilon[0,1]$. Therefore, $V_t(p,1)$ is convex for all $p\varepsilon[0,1]$.

Finally, $V_1(p,0)$ is convex for $p\varepsilon[0,1]$ since it is the maximum of $V_1(1,p,0)$ and $V_1(0,p,0)$, which can be seen to be linear, and therefore convex, in p from (12b) and (15). $V_t(p,0)$ can then be shown to be convex for $p\varepsilon[0,1]$ if $V_{T+1}(p,0)$ is convex when $V_T(p,0)$ is convex. Since $V_{T+1}(1,p,0)=V_1(1,p,0)$ and $q(p)r_1(0,1)+(1-q(p))r_0(0,1)$ are linear, it can be seen from (13b), with t=T+1, and the preceding proof that showing $q(p)V_T(h_1(p),0)$ and $(1-q(p))V_T(h_0(p),0)$ are convex for $p\varepsilon[0,1]$ will suffice to show that $V_{T+1}(0,p,0)$, and thus $V_{T+1}(p,0)$, are convex. But this can be done in the same way that $q(p)V_T(h_1(p),1)$ and $(1-q(p))V_T(h_0(p),1)$ were shown to be convex above. Q.E.D.

Lemma 2: For all $t=1,2,\ldots,N$:

(i) $V_t(0,1)=\theta_2R_1(0,1)+(1-\theta_2)R_0(0,1)$

(ii) $V_t(1,1)=\theta_1R_1(1,1)+(1-\theta_1)R_0(1,1)-c$

(iii) $V_t(0,0) = R$

(iv) $V_t(1,0)=\pi_t[\theta_1R_1(1,1)+(1-\theta_1)R_0(1,1)-c]$
$+(1-\pi_t)[\theta_1R_1(1,0)+(1-\theta_1)R_0(1,0)-c]$

Proof: From the inequalities in (4b) and (4c) and equations (10a), (11), (12a), and (13a), it can be seen that showing $V_t(0,0,1)=\theta_2R_1(0,1)+(1-\theta_2)R_0(0,1)$ for all t is sufficient to prove (i). It follows from (14) and (4b,c) that $V_1(0,0,1)=\theta_2R_1(0,1)+(1-\theta_2)R_0(0,1)$. Assume that $V_T(0,0,1)=\theta_2R_1(0,1)+(1-\theta_2)R_0(0,1)$. Then from (13a),

$$V_{T+1}(0,0,1)=\theta_2r_1(0,1)+(1-\theta_2)r_0(0,1)+\beta[\theta_2V_T(h_1(0),1)+(1-\theta_2)V_T(h_0(0),1)]$$

$$=\theta_2r_1(0,1)+(1-\theta_2)r_0(0,1)+\beta[\theta_2V_T(0,1)+(1-\theta_2)V_T(0,1)]$$

$$=\theta_2r_1(0,1)+(1-\theta_2)r_0(0,1)+\beta[\theta_2R_1(0,1)+(1-\theta_2)R_0(0,1)]$$

$$=\theta_2R_1(0,1)+(1-\theta_2)R_0(0,1).$$ Q.E.D.

The proofs of (ii)-(iv) can be done in similar fashion and will therefore be omitted.

Lemma 2 simply shows that in this model it is optimal for the firm to adopt if it knows the innovation is good (p=1) and to wait if it knows the innovation is bad (p=0), regardless of its rival's actions. Of course, this behavior is precisely what one would expect a reasonable model to predict. Lemmas 1 and 2 together are enough to show that the firm's optimal adoption rule in this model can be expressed in the form of a reservation probability rule. These results, and some of their implications for optimal firm behavior, will be presented and discussed in the next section.

III. Optimal Adoption Rules and the Probability of Adoption

Consider the decision problem of either firm when both have not yet adopted, but instead are waiting to learn more about the innovation; this problem is described by equations (10b), (11), (12b), and (13b) given (2) and (5)-(7). Because each firm is uncertain about both the profitability of the innovation and whether or not the rival will adopt, the expected adoption return is a weighted average of the expected adoption returns if the rival decides to adopt and if it does not, where the weights are the probabilities the rival will or will not adopt. The expected waiting return is similarly calculated. As indicated in the previous section, it can be shown that the solution to this problem takes the form

$$V_t(p,0) = \begin{cases} V_t(1,p,0) & \text{if} \quad p \geq p^*_{t0} \\ V_t(0,p,0) & \text{if} \quad p < p^*_{t0} \end{cases}$$

for $t=1,2,\ldots,N$ where p^*_{t0} is the reservation probability when there are t decision dates remaining at which the firm can adopt and when the rival firm has not yet adopted ($a_j=0$). This result is proved in the following theorem.

Theorem 1: For each $t=1,2,\ldots,N$ there exists a unique $p^*_{t0}\epsilon(0,1)$ such that $V_t(1,p^*_{t0},0)=V_t(0,p^*_{t0},0)$ and:

$$\text{(i) } V_t(1,p,0) > V_t(0,p,0) \text{ for all } p\epsilon(p^*_{t0},1]$$

$$\text{(ii) } V_t(1,p,0) < V_t(0,p,0) \text{ for all } p\epsilon[0,p^*_{t0}).$$

Proof: It follows from Lemma 2 and equation (2) that $V_t(1,0,0) < V_t(0,0,0)$ and $V_t(1,1,0) > V_t(0,1,0)$ for all $t\geq 1$. Because $V_t(1,p,0)$ is linear in p, it follows from Lemma 1 that the function $V_t(0,p,0)-V_t(1,p,0)$ is continuous and convex in p on [0,1]. Hence, $V_t(0,p,0)-V_t(1,p,0)$ is a continuous, convex function of p on [0,1] which is positive at $p = 0$ and negative at $p = 1$. Q.E.D.

An immediate consequence of this theorem is that both the firm's optimal adoption rule and the probability the firm will adopt at any stage of the process (when its rival is also waiting) can be completely specified by p^*_{t0} for $t=1,2,\ldots,N$. In particular, Theorem 1 proves the existence of an optimal decision rule for each firm, when neither have adopted, of the form

$$\delta_t(p,0) = \begin{cases} 1 & \text{if} \quad p \geq p^*_{t0} \\ 0 & \text{if} \quad p < p^*_{t0} \end{cases} \quad . \tag{16}$$

That is, the firm's optimal adoption rule is a reservation probability rule: at any stage of the decision process when there are t decision dates remaining, the firm's current belief the innovation is good is p, and the rival has not yet adopted, the firm should adopt if $p \geq p^*_{t0}$ and otherwise wait. Moreover, the probability the firm will adopt under these circumstances can be expressed as

$$\Pi_t(p,0) = \Pr\{p \geq p^*_{t0}\}. \tag{17}$$

Before discussing the implications of these results for optimal firm behavior, it will be worthwhile to derive similar ones for the case where the rival firm has adopted so that optimal behavior in both cases can be compared and contrasted.

The firm's decision problem when its rival has already adopted is of interest for several reasons. First, industry equilibrium, however defined, need not require that both firms adopt at the same date since the duopolists may have different initial beliefs, receive different external information, or evaluate the information in a different manner. If adoption dates differ, one of the firms must face this problem after its rival has adopted. Moreover, it is conceivable that only one of the firms will have access to the external source of information, as could happen if the supplier and one of the duopolists had a special arrangement; in this event the other firm would not even learn of the innovation's existence until its rival adopted. Finally, although it was assumed that the innovation was developed externally to the industry, this decision problem could also be used to analyze the behavior of a firm whose rival developed a non-patentable innovation. Under any of these circumstances the waiting firm will face the decision problem described by equations (10a), (11), (12a), and (13a) given (2) and (5)-(7), which is somewhat simpler than the other since the rival's action is known and the only uncertainty is whether the innovation will be profitable or not. Again it can be shown that the solution to this problem takes the form

$$V_t(p,1) = \begin{cases} V_t(1,p,1) & \text{if} \quad p \geq p^*_{t1} \\ V_t(0,p,1) & \text{if} \quad p < p^*_{t1} \end{cases}$$

Theorem 2: For each $t=1,2,\ldots,N$, there exists a unique $p^*_{t1} \varepsilon (0,1)$ such that $V_t(1,p^*_{t1},1) = V_t(0,p^*_{t1},1)$ and:

(i) $V_t(1,p,1) > V_t(0,p,1)$ for all $p \varepsilon (p^*_{t1},1]$

(ii) $V_t(1,p,1) < V_t(0,p,1)$ for all $p \varepsilon [0,p^*_{t1})$.

Proof: This proof is identical to that of Theorem 1 with $V_t(\cdot,\cdot, 1)$ replacing $V_t(\cdot,\cdot,0)$ and therefore is omitted.

As with Theorem 1, this theorem proves the existence of an optimal decision rule for the firm whose rival has already adopted of the form

$$\delta_t(p,1) = \begin{cases} 1 & \text{if} \quad p \geq p^*_{t1} \\ 0 & \text{if} \quad p < p^*_{t1} \end{cases} \tag{18}$$

and, therefore, shows that the probability this firm will adopt at any date can be expressed as

$$\Pi_t(p,1) = \Pr\{p \geq p^*_{t1}\}. \tag{19}$$

Again the firm's optimal adoption rule is a reservation probability rule: at any stage when there are t decision dates remaining, the firm's current belief the innovation is good is p, and the rival has already adopted, the firm should adopt if $p \geq p^*_{t1}$, which occurs with probability $\Pi_t(p,1)$, and otherwise wait.

It is now possible to examine the implications of this model of firm behavior in terms of the probability a waiting firm will adopt at any stage of the decision process. Since $p^*_{ta_j} \varepsilon (0,1)$ for all t and $a_j=0,1$, it is evident that both firms may have initial beliefs g^i which are either high enough to imply both will adopt immediately or low enough to imply neither will adopt when the innovation is introduced. If either waits to learn about the innovation, the length of the delay until adoption occurs (if it ever does) then depends upon the nature of the information received and the reservation probabilities in addition to the firm's original belief the innovation is good. The following result shows that a firm is more likely to adopt the innovation at any stage in the decision process when its original belief is higher or the information received is more favorable to the innovation.

Proposition 1: Whether the rival firm has adopted or not, the probability of adoption by the waiting firm will be higher (lower) when the firm's original belief the innovation is good or the proportion of observations which are favorable are higher (lower).

Proof: Recalling that $p = p(n,k,g)$ as defined in equation (7), this result follows immediately from (17) and (19) since $p(n,k+1,g) > p(n,k,g)$ and $\frac{\partial p(n,k,g)}{\partial g} > 0$.

This behavior is, of course, precisely what one would expect any reasonable model to predict. The following group of results, however, are surprising when viewed in the context of the existing literature concerning the diffusion of innovations. In nearly every study of diffusion published it has been assumed, at least implicitly, that the probability a waiting firm will adopt at any time will be higher the greater the proportion of all potential users of the innovations who have already adopted. In what follows it will be shown that optimal behavior for the firm will not be consistent with this assumption under certain circumstances. In particular, it will be shown that, under certain conditions on the returns $R_z(a_i,a_j)$ and R, the probability a firm will adopt at the last decision date (t=1) when its rival has already adopted may be the same as, greater, or less than the probability it will adopt when its rival has not yet adopted. This will be done after the following preliminary result is derived.

Theorem 3: $p^*_{10} \gtreqless p^*_{11}$ if and only if

$$\frac{R_0(0,1)-R_0(1,1) + c}{R_1(1,1)-R_1(0,1)+R_0(0,1)-R_0(1,1)} \gtreqless \frac{R-R_0(1,0) + c}{R_1(1,0)-R+R-R_0(1,0)}$$

Proof: It follows from Theorem 1 that $p^*_{10} \gtreqless p^*_{11}$ if and only if $V_1(1,p^*_{11},0) \lesseqgtr V_1(0,p^*_{11},0)$. And since

$$q(p^*_{11})R_1(1,1)+(1-q(p^*_{11}))R_0(1,1)-c = q(p^*_{11})R_1(0,1)+(1-q(p^*_{11}))R_0(0,1)$$

by Theorem 2 and (14), it follows that

$$V_1(1,p^*_{11},0)-V_1(0,p^*_{11},0) = (1-\pi_1)[q(p^*_{11})R_1(1,0)+(1-q(p^*_{11}))R_0(1,0)-c-R].$$

Therefore,

$$p^*_{10} \gtreqless p^*_{11} \text{ if and only if } q(p^*_{11})R_1(1,0)+(1-q(p^*_{11}))R_0(1,0)-c \lesseqgtr R. \tag{20}$$

The theorem's statement can now be derived from (21) by solving the equation $V_1(1,p^*_{11},1) = V_1(0,p^*_{11},1)$ for $q(p^*_{11})$, substituting this into the last condition, and rearranging terms.

The result of interest here can now be obtained from Theorem 3 and equations (17) and (19); it is stated in the following corollary.

Corollary 1: $\Pi_1(p,0) \gtreqless \Pi_1(p,1)$ if and only if

$$\frac{R_0(0,1)-R_0(1,1)+c}{R_1(1,1)-R_1(0,1)+R_0(0,1)-R_0(1,1)} \gtreqless \frac{R-R_0(1,0)+c}{R_1(1,0)-R+R-R_0(1,0)}. \tag{21}$$

Furthermore:

(i) $\Pi_1(p,0)=\Pi_1(p,1)$ if $R_z(1,0)-R_z(1,1)=R-R_z(0,1)$, $z=0,1$.

(ii) $\Pi_1(p,0)<\Pi_1(p,1)$ if $R_z(1,0)-R_z(1,1)\leq R-R_z(0,1)$, $z=0,1$,

where the inequality is strict for one z.

(iii) $\Pi_1(p,0)>\Pi_1(p,1)$ if $R_z(1,0)-R_z(1,1)\geq R-R_z(0,1)$, $z=0,1$,

where the inequality is strict for one z.

Proof: The first statement (21), follows immediately from Theorem 3 and equations (17) and (19). Simple algebraic manipulations of this expression then give (i)-(iii).

The relationship defined by (21) provides conditions both necessary and sufficient for the probability of adoption by the waiting firm at its last chance to be higher, the same, or lower when the rival firm has adopted than when it has not. The sufficient conditions given by (i)-(iii), however, provide the best basis for interpretation. By the definitions of $R_z(a_i,a_j)$ and R, it is evident that the conditions in (i)-(iii) can be replaced with equivalent ones in terms of $r_z(a_i,a_j)$ and r. Hence, the interpretation of (i)-(iii) can be made in terms of these period returns. Now, if the firm adopts but its rival doesn't, then the firm will gain from its rival's error if its experience is favorable and lose from its own error if its experience is unfavorable. The gain is $r_1(1,0)-r_1(1,1)$ and the loss is $r_0(1,0)-r_0(1,1)$. And if the firm does not adopt but its rival does, then the firm will suffer a loss if its rival's experience is favorable and a gain if it is unfavorable. This loss is $r_1(0,1)-r$ and this gain is $r_0(0,1)-r$. Hence, the probability a waiting firm will adopt at its last chance will be the same whether the rival firm has adopted or not if: (1) when the experience is favorable, the gain the firm would realize if it adopts and its rival does not equals the loss it would incur if it did not adopt and its rival did; and (2) when the experience is unfavorable, the loss the firm would incur if it adopts and its rival does not equals the gain it would realize if it did not adopt and its rival did. This is a situation in which there is no bias toward adopting or waiting in the returns. That is, the gain the firm realizes by making the correct decision when its rival errs is the same for both the decision to adopt and the decision to wait; and the loss the firm incurs by making the wrong decision when its rival makes the correct one is also the same for both the adoption and waiting decisions.

Similar interpretations can be applied to (ii) and (iii) of Corollary 1. The probability a waiting firm will adopt at its last chance will be greater (smaller) when the rival firm has (has not) already adopted if: (1) when the experience is favorable, the gain the firm would realize by adopting when its rival waits is less than the loss it would incur by waiting when its rival adopts; and (2) when the experience is unfavorable, the loss it would incur by adopting when its rival waits is greater than the gain it would realize by waiting when the rival adopts. In this case there is a clear bias against adoption and in favor of waiting when the rival firm has not yet adopted, which results in the firm being less likely to adopt if the rival has not adopted than if it has. Finally, the probability a waiting firm will adopt at its last chance will be smaller (greater) when the rival firm has (has not) adopted if: (1) when the experience is favorable, the gain the firm would realize by adopting when its rival does not is greater than the loss it would incur by waiting when its rival adopts; and (2) when the experience is unfavorable, the loss it would incur by adopting when its rival does not is less than the gain it would realize by waiting when its rival adopts. In this case there is a clear bias in favor of adoption and against waiting when the rival has not yet adopted, which results in the firm being more likely to adopt if the rival has not adopted than if it has. Stated somewhat more loosely, if there is an advantage to being the first to adopt, then a firm will be less likely to adopt after its rival has preempted that advantage by adopting.

It is significant to observe that the inequality conditions (i)-(iii) of Corollary 1 may obtain under very general circumstances. In particular, all of these inequalities can be derived from a simple Cournot duopoly model where demand is linear and marginal cost is constant for both new product innovations (which shift the demand curve either to the right or left) and new process innovations (which shift the marginal cost curve either downward or upward). Hence, even though the model used here is rather specific in several ways, it is reasonable to state the following proposition based on Theorem 3 and Corollary 1.

Proposition 2: Adoption by a rival firm need not increase, and may possibly decrease, the probability that a firm waiting will adopt the innovation at the last decision date.[7]

In concluding this section it will be worthwhile to indicate that the claim of Proposition 2 holds under conditions more broadly applicable than those embodied in this model. In particular, this result does not depend critically on the assumption that a waiting firm ignores the external source of information once its rival has adopted.

If a waiting firm sees an observation from both its rival and the external source after the rival has adopted, then it can again be shown that a reservation probability strategy is optimal (if the firm estimates q(p) and learns in Bayesian fashion). Denote these reservation probabilities by $\hat{p}_{t1}$ and $\hat{p}_{t0}$ for $t=1,2,\ldots,N$. Then it can, in fact, be shown that $\hat{p}_{11}=p^*_{11}$ and $\hat{p}_{10}=p^*_{10}$ and that Theorem 3 and Corollary 1 obtain in this case also. The relevant comparison for probabilities of adoption at t=1, if the rival adopts at t=2 for instance, will be $\Pi_1(h_i(p),1)$ and $\Pi_1(p,0)$. Since $h_1(p)>p>h_0(p)$ and $p^*_{11}=p^*_{10}$ is still possible, the claim of Proposition 2 will also hold when a waiting firm observes both the external source and its rival's experience.

However, it is also possible that the probability a waiting firm will adopt will be higher after rival adoption. Assuming $p^*_{11}=p^*_{10}$ for convenience, then $\Pi_1(h_1(p),1)>\Pi_1(p,0)$ and $\Pi_1(h_0(p),1)<\Pi_1(p,0)$. That is, rival adoption will increase the probability the waiting firm will adopt if and only if its experience is favorable. Although this conclusion is not surprising, it does help to clarify the apparent contradiction between the result of Proposition 2 and the often reported empirical result that the probability a waiting firm will adopt increases as the proportion of users who have already adopted increases. As the discussion of Corollary 1 indicated, rival adoption *per se* will have a different impact on the likelihood a waiting firm will adopt when there are differences in the advantages and disadvantages of adopting before the rival or having the rival adopt first. These differences depend on such things as the nature of the innovation and the market power of each firm. If these are neutralized, by setting $p^*_{11}=p^*_{10}$, then the waiting firm will be more (less) likely to adopt after its rival has if and only if the rival adoption was successful (unsuccessful). In other words, it is not the simple information that a rival has adopted which increases the likelihood of adoption by a firm waiting, but instead the additional information favorable to the innovation which will be generated by rival adoption when the innovation is a good one. Hence, the proportion of an industry which has adopted may be significant in explaining the rate of adoption only because it is serving as a proxy for favorable information received by waiting firms about the innovation.

However, it is also possible that the probability of adoption by a waiting firm will be higher after rival adoption. Assume for expositional convenience that $p^*_{11} = p^*_{10}$. Then it follows that $\Pi_1(h_1(p)_1 1) > \Pi_1(p,0)$ and $\Pi_1(h_0(p),1 < \Pi_1(p,0)$. That is, rival adoption will increase (decrease) the probability of adoption by the waiting firm if and only if the rival's experience is favorable (unfavorable). Although this result does not seem surprising, it is especially significant in the light of the assumption, commonly made in empirical studies of diffusion, that the probability of adoption by a waiting firm will be higher the larger the proportion of firms in the industry who have adopted. It is normally argued (see, for instance, Mansfield [14,16]) that this should be the case because as the proprotion of adopters increases: (1) information and experience related to the innovation accumulate, (2) it becomes less risky to adopt, and (3) competitive pressures to adopt build up. This assumption also conveniently implies a diffusion curve for the innovation which is logistic (S-shaped, as are most of the ones empirically observed). Although these reasons certainly seem plausible, they are basically <u>ad hoc</u> justifications for testing an empirical relationship, rather than implications of optimal firm behavior toward adoption. The approach used here clearly shows that, under uncertainty, optimal behavior may imply a lower adoption probability for a waiting firm after the rival has adopted. For instance, the nature of the innovation and market structure may be such that rival adoption will reduce the adoption probability of a waiting firm (as in (iii) of Corollary 1, where $p^*_{11} > p^*_{10}$). Furthermore, in the case where there is no bias for or against adoption (as in (i) of Corollory 1, where $p^*_{11} = p^*_{10}$), rival adoption will increase the waiting firm's adoption probability if and only if the rival's experience is favorable. Certainly if a firm adopted and suffered an unfavorable experience, then no one would predict that this information would increase the adoption probability of a waiting firm.

Therefore, the analysis of optimal firm behavior under this model shows that there is no *a priori* reason to expect rival adoption *per se* to increase the probability of a waiting firm will adopt. It also shows that the reason empirical studies find a significantly positive relationship between the probability of adoption and the proportion of firms that have already adopted is that the latter variable is serving as a proxy for information favorable to the innovation. This is probably not a bad proxy since one would expect information generated by the experience of adopters with an innovation which eventually diffuses throughout the industry to be favorable, at least on average. Nevertheless, there may be other variables which are better measures of favorable information and whose use would not only provide greater explanatory power, but also avoid the fallacious assumption that a higher proportion of firms using an innovation necessarily implies a higher adoption probability for waiting firms.

IV. Industry Equilibrium

Consider the situation when neither firm has adopted at a given decision date. As a consequence of Theorem 1, once firm i has assigned a probability to the event that its rival will adopt at t (π_t^i), its optimal decision rule can be completely specified by solving the equation $V_t^i(1,p,0)=V_t^i(0,p,0)$ for p to obtain $p_{t0}^{i*}(\pi_t^i)$.[8] Moreover, given the firm's current belief the innovation is good, p^i, the probability of adoption by firm i at t implied by its optimal decision rule can also be determined. Thus, an appropriate equilibrium concept for the situation in which neither firm has yet adopted is one which requires that each firm's probabilistic belief that its rival will adopt then be equal to the probability of adoption implied by the rival firm's optimal decision rule. That is, at each decision date t when neither firm has adopted, the industry is in equilibrium if $\pi_t^1 = \Pi_t^2(p,0)$ and $\pi_t^2 = \Pi_t^1(p,0)$. This is a type of fully rational, or self-fulfilling, expectations equilibrium since it requires that each firm's belief its rival will adopt be the same as that which obtains when each firm follows its optimal decision rule. The purpose of this section is to develop a method which each firm can use to form estimates of the π_t^i which are endogenous to the model and admit the existence of such an equilibrium at each decision date when neither firm has yet adopted.[9]

It will be assumed that each firm has complete but imperfect information abouts its rival. In particular, each firm is assumed to know all relevant information about its rival except for the rival's initial prior, which it knows only in distribution. It is also assumed that each firm believes its rival's prior to be distributed over the unit interval according to the continuous cumulative distribution function F. Finally, assume that each firm sees the same sequence of observations, uses the same updating rule, (7), and faces the same finite number of decision periods, given by the positive integer N. Under these assumptions, each firm i can, for given π_t^j, determine the reservation probability of its rival j at any decision date, $p_{t0}^{j*}(\pi_t^j)$, by solving a decision problem similar to its own. It can then form an estimate of the probability of rival adoption at that date by using the distribution F. To see how this is done, consider the situation when the innovation first appears, at which time neither firm has adopted, no observations have been seen, and there are t=N decision dates remaining for both firms. Given any value of π_N^2, firm 1 can determine $p_{N0}^{2*}(\pi_N^2)$ by solving the decision problem of firm 2 (which it is assumed to know). The probability that firm 2 will adopt at t=N implied by its optimal decision rule for that π_N^2 is then $\Pr\{g^2 \geq p_{N0}^{2*}(\pi_N^2)\}$. The best estimate that firm 1 can make of this probability will be $\hat{\pi}_N^1(\pi_N^2)=1-F(p^{2*}_{N0}(\pi_N^2))$. Similarly, given any value of π_N^1, the best estimate firm 2 can make of the optimal probability that firm 1 will adopt at t=N will be $\hat{\pi}_N^2(\pi_N^1)=1-F(p_{N0}^{1*}(\pi_N^1))$. The industry will then be in equilibrium at t=N if there exists a $(\pi_N^{1*},\pi_N^{2*})\varepsilon[0,1]x[0,1]$ such that $\hat{\pi}_N^1(\pi_N^{2*})=\pi_N^{1*}$ and $\hat{\pi}_N^2(\pi_N^{1*})=\pi_N^{2*}$.

Now suppose that neither firm adopts at t=N. Then both will receive two pieces of information. The first is the information about the innovation provided by the external source, and the second is that the rival's initial prior g^j must be less than $p_{N0}^{j*}(\pi_N^{j*})$ since it did not adopt. Naturally, each firm's estimate of rival adoption at t=N-1 should be revised in accordance with this new information. Again, given any π_{N-1}^2, firm 1 can determine the reservation probability of firm 2 at t=N-1, $p_{N-1,0}^{2*}(\pi_{N-1}^2)$. The probability that firm 2, using its optimal decision rule, will adopt at t=N-1 will then be

$\Pr\{p(1,z_1,g^2)\geq p^{2*}_{N-1,0}(\pi^2_{N-1})\}$ where z_1 is either 0 or 1. It follows from the definition of $p(n,k,g)$ given by (7) that for each i and $z_1=0,1$, $p(1,z_1,g^i)\geq p^{i*}_{N-1,0}(\pi^i_{N-1})$ if and only if $g^i\geq g^*(1,z_1,p^{i*}_{N-1,0}(\pi^i_{N-1}))$,[7] so that this optimal adoption probability can also be written as $\Pr\{g^i\geq g^*(1,z_1,p^{i*}_{N-1,0}(\pi^i_{N-1}))\}$. However, since neither firm adopted at t=N, it is more accurate to write this probability as $\Pr\{g^i\geq g^*(1,z_1,p^{i*}_{N-1,0}(\pi^i_{N-1}))|g^i<p^{i*}_{N0}(\pi^{i*}_N)\}$. Now, since $p^{i*}_{N-1,0}(\pi^i_{N-1})$ may be greater than, equal to, or less than $p^{i*}_{N0}(\pi^{i*}_N)$ (depending upon the relationships between $r^i_z(1,0)-r^i_z(1,1)$ and $r^i-r^i_z(0,1)$ and the values of π^{i*}_N and π^i_{N-1}), $g^*(1,z_1,p^{i*}_{N-1,0}(\pi^i_{N-1}))$ may be greater than, equal to, or less than $p^{i*}_{N0}(\pi^{i*}_N)$ whether $z_1=0$ or $z_1=1$. The optimal adoption probability will thus be zero if $g^*(1,z_1,p^{i*}_{N-1,0}(\pi^i_{N-1}))\geq p^{i*}_{N0}$ and positive otherwise. Hence, given any π^2_{N-1}, the best estimate firm 1 can make of the probability that firm 2 will adopt at t=N-1, given that it did not adopt at t=N and that the first observation was z_1, is

$$\hat{\pi}^1_{N-1}(\pi^2_{N-1}) = \begin{cases} F(p^{2*}_{N0}(\pi^{2*}_N))-F(g^*(1,z_1,p^{2*}_{N-1,0}(\pi^2_{N-1}))) \\ \qquad \text{if } p^{2*}_{N0}(\pi^{2*}_N)>g^*(1,z_1,p^{2*}_{N-1,0}(\pi^2_{N-1})) \\ 0 \qquad \text{otherwise} \end{cases}$$

The best estimate that firm 2 can make of the probability that firm 1 will adopt at t=N-1 is defined in a similar fashion. As before, the industry will be in equilibrium at this date if there exists a $(\pi^{1*}_{N-1},\pi^{2*}_{N-1})\epsilon[0,1]x[0,1]$ such that $\hat{\pi}^1_{N-1}(\pi^{2*}_{N-1})=\pi^{1*}_{N-1}$ and $\hat{\pi}^2_{N-1}(\pi^{1*}_{N-1})=\pi^{2*}_{N-1}$.

In order to generalize this estimation procedure for any decision date at which neither firm has yet adopted it will now be helpful to present another technical result and introduce some new notation.

Lemma 3: For any triple (n,k,p_{t0}^{*}), where $t=N-n$ and $k\leq n$, there exists a unique $g^{*}(n,k,p_{t0}^{*})\varepsilon(0,1)$ such that $p(n,k,g)\geq p_{t0}^{*}$ if and only if $g\geq g^{*}(n,k,p_{t0}^{*})$; moreover,

$$g^{*}(n,k,p_{t0}^{*})=[1+(\frac{1-p_{t0}^{*}}{p_{t0}^{*}})(\frac{\theta_1}{\theta_2})^{k}(\frac{1-\theta_1}{1-\theta_2})^{n-k}]^{-1}. \tag{22}$$

Proof: The definition of $p(n,k,g)$ given by (7) can be used to solve $p(n,k,g)=p_{t0}^{*}$ explicitly for g and thereby obtain (22). That $p(n,k,g)\geq p_{t0}^{*}$ if and only if $g\geq g^{*}(n,k,p_{t0}^{*})$ follows from the fact that $\frac{\partial p(n,k,g)}{\partial g}>0$. That $g^{*}(n,k,p_{t0}^{*})$ is unique and an element of (0,1) follows from Theorem 1, where p_{t0}^{*} was shown to be unique and belong to (0,1).

This lemma allows each firm's optimal decision rule to be restated in terms of a reservation initial prior $g^{*}(n,k,p_{t0}^{i*})$ for each $t=N-n=1,\ldots,N$ and $k\leq n$. That is, at any decision date t when the reservation probability is p_{t0}^{i*} and $n=N-t$ observations have been seen, k of which were favorable, firm i will adopt if and only if $g^{i}\geq g^{*}(n,k,p_{t0}^{i*}),i=1,2$. Now let $s_n=(n,\sum_{k=1}^{n} z_k)$ for $n=1,\ldots,N-1$ and $s_0=(0,0)$, so that $g^{*}(n,k,p_{t0}^{i*})$ can be written $g^{*}(s_{N-t},p_{t0}^{i*})$ for $t=N-n=1,\ldots,N$. Finally, let

$\bar{g}^{i}(s_n)=\min\{g^{*}(s_0,p_{N0}^{i*}),\ g^{*}(s_1,p_{N-1,0}^{i*}),\ldots,g^{*}(s_n,p_{t0}^{i*})\}$ for $n=N-t-1,\ldots,N-1$.

That is, $\bar{g}^{i}(s_n)$ is the minimum reservation prior realized by firm i throughout the first n+1 decision dates (and first n observations) while both firms are waiting. Hence, at any stage of the process when there are t decision dates remaining and neither firm has yet adopted, it must be the case that $g^{i}<\bar{g}^{i}(s_{N-t-1})$ for $i=1,2$. The probability that either firm will adopt at t, calculated from the optimal decision rule, is thus

$\Pr\{g^{i}\geq g^{*}(s_{N-t},p_{t0}^{i*})|g^{i}<\bar{g}^{i}(s_{N-t-1})\}$ for $t=1,\ldots,N-1$. Hence, given any π_t^{j}, the best estimate that firm i can make of the probability firm j will adopt at t, for t=1,...,N-1, will be $\hat{\pi}_t^i(\pi_t^j)=F(\bar{g}^i(s_{N-t-1}))-F(g^*(s_{N-t},p_{t0}^{j*}(\pi_t^j)))$ if $\bar{g}^i(s_{N-t-1})>g^*(s_{N-t},p_{t0}^{j*}(\pi_t^j))$ and $\hat{\pi}_t^i(\pi_t^j))=0$ otherwise. The best estimates of the probability of rival adoption at decision date t for all t=1,...,N can now be summarized as follows: for any given π_t^j, the best estimate that firm i can make of the probability of adoption by its rival, firm j, at t is:

$$\hat{\pi}_N^i(\pi_N^j) = 1-F(p_{N0}^{j*}(\pi_N^j)) \tag{23}$$

$$\hat{\pi}_t^i(\pi_t^j) = \begin{cases} F(\bar{g}^j(s_{N-t-1}))-F(g^*(s_{N-t},p_{t0}^{j*}(\pi_t^j))) & \\ \qquad \text{if } \bar{g}^j(s_{N-t-1})>g^*(s_{N-t},p_{t0}^{j*}(\pi_t^j)) & \\ 0 \qquad \text{otherwise} \end{cases} \tag{24}$$

for t=1,..., N-1

and for i,j=1,2 where i≠j.

The existence of a self-fulfilling (rational) expectations equilibrium for every decision date t at which neither firm has yet adopted can now be demonstrated.

Theorem 4: Let $\hat{\pi}_t^1(\pi_t^2)$ and $\hat{\pi}_t^2(\pi_t^1)$ be defined as in (23) and (24). Then at any decision date t when neither firm has yet adopted and each has seen the sequence of observations summarized by s_{N-t}, there exists a $(\pi_t^{1*},\pi_t^{2*})\varepsilon[0,1]x[0,1]$ such that $\hat{\pi}_t^1(\pi_t^{2*}) = \pi_t^{1*}$ and $\hat{\pi}_t^2(\pi_t^{1*}) = \pi_t^{2*}$.

Proof: It will suffice to show that there exists a $\pi_t^{1*}\varepsilon[0,1]$ such that $\hat{\pi}_t^1(\hat{\pi}_t^2(\pi_t^{1*})) = \pi_t^{1*}$ for any t. The proof for the case where $\hat{\pi}_t^1(\pi_t^2)=\hat{\pi}_t^2(\pi_t^1)=0$ is trivial. For the remaining cases Schauder's fixed point theorem will give the result if the composite function $\hat{\pi}_t^1 \circ \hat{\pi}_t^2$, which maps [0,1] into itself, can be shown to be continuous on [0,1]. Since F was assumed to be continuous on [0,1], it therefore will suffice to show that the reservation probability mapping $p_{t0}^{i*}(\pi_t^i)$ is continuous on [0,1]. Equations (12b) and (13b) can be used to solve explicitly for π_t^i as a function of p_{t0}^{i*}. This mapping is continuous and one-to-one. Therefore its inverse, which is $p_{t0}^{i*}(\pi_t^i)$, is also continuous. Q.E.D.

There are three features of the equilibrium guaranteed by this existence theorem which are worth noting. First, it shows that at every stage of the decision process, until either one of the firms adopts or both lose the opportunity to adopt (at t=0), each firm accurately predicts the optimal probability of adoption by its rival and uses this probability to solve its own decision problem. Second, these equilibrium probabilities are endogenously determined, being revised at each decision date to take into account both the new information about the innovation from the external source and the information about the rival's initial prior gleaned from observing that the rival has not yet adopted. That is, while both firms are waiting they learn about both the innovation and their rival's prior, and adjust their expectations about the probability of rival adoption accordingly. And third, the results of the previous section are consistent with this equilibrium. For instance, Proposition 1 shows that a diffusion with adoption delayed by both firms is a possible equilibrium outcome. That is, since the firms' initial priors were not assumed to be the same, one possibility is that both firms delay their adoption of the innovation, where the length of the delay is longer for the less optimistic firm (the one with the lower prior). Moreover, Proposition 2 shows that if there is an advantage in being the first to adopt, then in the aforementioned case of a delayed-adoption diffusion the adoption by the first firm will actually reduce the probability of adoption by the other firm.

Finally, this equilibrium also has a game theoretic interpretation. The situation at a decision date when neither firm has yet adopted can be viewed as a game in which the strategies are to adopt or wait and the value of the game to each firm is given by the value function in (10b), (11), (12b), and (13b). Since each firm is assumed to know all relevant information except for the rival's prior belief, which it knows only in distribution, this is a game of complete but imperfect information played by Bayesian pleayers.[11] The Bayes equilibrium strategies for this game are then the optimal decision rules of equation (16) evaluated at the equilibrium optimal adoption probabilities (π_t^{1*}, π_t^{2*}) given by Theorem 4. And since a sequence of these games is played until either one firm adopts or t=0, this result also shows the existence of equilibrium pure strategies for the stochastic game which is comprised of Bayesian games of the type described above. This is an uncommon result for a stochastic game.[12]

V. Concluding Remarks

There are three major results of this paper. First, optimal adoption rules characterized by unique reservation probabilities (representing the minimum level of confidence in the innovation's profitability which the firm must attain to induce adoption) exist for each firm in both the case where the rival has not yet adopted and that where it has. Second, optimal adoption probabilities derived from these decision rules show that it may be optimal to delay the adoption of a good innovation and that adoption by one firm may actually decrease the probability of adoption by the firm waiting. And third, there exists an equilibrium for the game played at each decision date at which neither firm has yet adopted (so that each is uncertain about both the profitability of the innovation and its rival's action) which is characterized by both firms accurately and endogenously estimating the optimal adoption probability of its rival.

These results further strengthen the argument put forth by this author in [9] that uncertainty, information, learning, and attitudes of decision-makers are factors critical to the adoption and diffusion of an innovation. In particular, it has been shown that an equilibrium outcome in a duopoly could involve both delayed adoption and different adoption dates (i.e., a diffusion) and could be entirely explained by differences in the firm's initial beliefs that the innovation will be good. In addition, Proposition 2 and the subsequent discussion clearly show that it is not rival adoption per se, but rather additional favorable information generated by the rival's experience, which will increase the probability of adoption by a waiting firm. Admittedly, some of the assumptions made are rather restrictive in nature and, therefore, the results of this paper are not perfectly general. Nevertheless, they are sufficiently significant and interesting to conclude that further analysis of adoption and diffusion should take into account the factors emphasized both here and in [9].

Finally, one important issue which has not been considered here should be mentioned. This is the question of how market structure affects the rate of adoption of an innovation.[13] The natural approach would be to extend this model to an arbitrary number of firms and examine the relationship between the number of firms and the probability of adoption by a particular firm. This should be the focus of future research in this area.

FOOTNOTES

1. See Mansfield[14,16]for the most comprehensive discussion of these issues.

2. See Herregat [7] and Mansfield [14] for two examples of attempts to examine these factors.

3. These last two assumptions are not critical to any of the major results that follow and are made strictly for convenience. See [9] for a more thorough discussion of the implications of relaxing these assumptions.

4. Here and throughout the remainder of this paper an innovation which would increase the firm's present discounted value if adopted immediately will be referred to as "good" and one which would not will be called "bad".

5. As long as g^1 and g^2 are independent, as will be assumed here, $p(n,k,g^1)$ and $p(n,k,g^2)$ will be independent. (See, e.g., DeGroot [3]).

6. It will be assumed throughout that there are a finite number of decision dates, N. In the context of this model this assumption merely implies that there are a finite number of observations available to the firm, so that its ability to learn about the innovation by waiting is limited. Existence of the value functions $V^i(p^i,a_j) = \lim_{t\to\infty} V_t^i(p^i,a_j)$ for $a_j = 0,1$ can be shown when additional assumptions are made on the period returns $r_z^i(a_i,a_j)$ and r. However, analysis of this case (when the number of decision dates is infinite) yields no additional insights.

7. It is tempting to conjecture that this result will hold for any t. However, this cannot be demonstrated because the relative magnitudes of p_{t0}^{i*} and p_{t1}^{i*} cannot be determined except at t=1. This issue seem important enough to merit further investigation.

8. It follows from Theorem 1 that the equation $V_t^i(1,p,0) = V_t^i(0,p,0)$ implicitly defines a mapping which assigns to each $\pi_t^i \epsilon [0,1]$ a unique reservation probability $p_{t0}^{i*}(\pi_t^i)$.

9. Since the adoption decision is irreversible, once either firm has adopted the element of rivalry is gone from the remaining firm's decision problem and there is no need for an equilibrium concept.

10. See equation (22) in Lemma 3 for the precise definition of $g^*(1,z_1,p_{N-1,0}^{i*}(\pi_{N-1}^i))$.

11. As Harsanyi [6] has shown, an equilibrium for such a Bayesian game will also be an equilibrium for the equivalent game of incomplete information which would arise if each firm had no information about its rival's prior.

12. See Sobel [23,24] for a discussion of this.

13. For studies of the related issue of how market structure affects the timing of the development of innovations see Scherer [20], Kamien and Schwartz [10, 11, 12], Loury [13], Reinganum [18] and Dasgupta and Stiglitz [2].

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