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ON THE NON-ROBUSTNESS OF BARRO'S NEUTRALITY RESULT ON SOCIAL SECURITY

by

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ABSTRACT

In a recent paper Barro [1974] argues that bequests can vitiate the effect of intergenerational tax-transfer policies. The utility function he uses has as an argument the utility of the household's direct heirs, leading to a marginal propensity to bequeath (MPB) out of transfers of unity. It is shown here that Barro's result is not robust: the case MPB=1 corresponds to an unstable equilibrium. For MPB's near unity any tax-transfer policy triggers explosive changes in the capital-labor ratio. For sufficiently small MPB's the system is stable, but in the stable region a larger MPB leads to a larger long-run effect on the capital-labor ratio.

Do Bequests Offset Social Security?

I. Introduction

In a recent paper Barro [1974] argues that the existence of a bequest motive, with bequests operative, vitiates the effect of intergenerational transfer policies. Briefly, he claims that government-initiated transfer policies will be exactly offset by changes in bequests, so that neither the consumption of any household nor the path of capital accumulation is affected. It is shown here that Barro's result is not robust.

The utility function Barro uses has as its arguments the household's own consumption and the lifetime utility of its direct heirs, leading to a marginal propensity to bequeath (MPB) out of transfers of unity. Clearly, Barro's conclusion need not hold if the MPB out of transfers is less than unity. However, one might guess that MPB's closer to unity would reduce the impact of a given transfer policy, since a larger proportion of the transfer would be offset by a change in bequests. I.e., one might suppose that the long-run effect depends continuously on the MPB.

It is shown below that this reasoning is incorrect. The case MPB=1 corresponds to an unstable equilibrium. Consequently, for MPB's slightly less than unity any intergenerational transfer policy--no matter how small--leads to explosive changes in the capital-labor ratio.

For smaller values of the MPB the system is stable. However, in the stable region a larger MPB, although it slows down the speed of adjustment, leads to a larger long-run impact on the capital-labor ratio.

II. A Model of Social Security

In this section the impact of balanced-budget, unfunded social security programs is examined using a simple overlapping generations model. The purpose is to determine how the short-run and long-run impact of such a program depends on the value of the MPB.

Each household lives for two periods, and is characterized by the period when it is old. The household supplies (inelastically) one unit of labor when young. At the end of the period it receives a wage and pays any tax levied on the young. It also receives any bequests left by the preceding generation. For simplicity assume that the young do not consume.[1]

When old the household supplies all of its wealth to be used as capital in production. At the end of the period it receives the principal plus returns, and in addition receives any transfers paid to the elderly. The household then allocates its total end-of-period assets between its own consumption and its bequests to the next generation. For simplicity assume that the MPB is constant, so that the distribution of assets within each generation can be ignored.

Homogeneous output is produced using capital and labor. The production function is standard: it is continuous and differentiable, there are constant returns to scale, and Harrod-neutral technical change occurs at the constant rate g.

$$F(K,L,t) = (1+g)^{t}L \cdot f((1+g)^{-t}K/L)$$

Assume that the population grows at the constant rate n, and let:

$$(1+v) \equiv (1+g)(1+n)$$

Assume that cohort t=1 supplies one unit of labor (during period t=0), L(0)=1. Letting k(t) denote the ratio of capital to effective labor in period t,

$$k(t) = K(t)/[(1+g)^t(1+n)^t L(0)] = (1+v)^{-t}K(t), \qquad (1)$$

where K(t) is the total capital stock in period t.

Assume that the economy is initially in a steady state with capital-labor ratio $\bar{k}$, and let r and w denote the steady-state rate of return to capital and wage rate respectively:

$$r \equiv f'(\bar{k})$$

$$w = f(\bar{k}) - r\bar{k} \quad .$$

Attention will be limited to balanced-budget programs that transfer from the young to the old, and for which the per capita tax and transfer grow at the constant rate g. Any such policy can be represented by a single parameter s > 0. The total tax paid by households in cohort t (when young) is $(1+v)^t s$, and the total transfer received by them (when old) is $(1+v)^{t+1} s$. If v > r, any such program increases the net wealth of every cohort. In order to rule out these "free lunches," assume that v < r.

In the absence of any transfer program the total capital stock in period t, K(t), and the total bequest left by cohort t, B(t), both grow at the rate v.

$$K(t) = (1+v)^t K_0$$

$$B(t) = (1+v)^t B_0 \qquad (2)$$

$$t = 0,1,\ldots \quad .$$

Suppose that in period t=0 a transfer policy described by $s > 0$ is initiated. For s sufficiently small the induced change in the capital stock--and hence in the rate of return to capital and in the wage rate--will be negligible, at least in the short run. Hence the perturbation to the paths for capital and bequests are described by:

$$\Delta K(t) = -(1+v)^t s + \Delta B(t-1) \quad , \qquad (3a)$$

$$\Delta B(t) = \beta \cdot [(1+r)\Delta K(t) + (1+v)^{t+1} s] \quad , \qquad (3b)$$

$$t = 1,2,\ldots$$

$$\Delta K(0) = 0 \quad , \qquad (3c)$$

$$\Delta B(0) = (1+v)\beta s \quad , \qquad (3d)$$

where β is the MPB. Equation (3a) says that savings by cohort t falls by the amount of the tax paid and increases by the induced increment in its inheritance. Equation (3b) says that bequests by cohort t change in proportion to the net change in its savings (with interest) plus the amount of the transfer received. Equations (3c) and (3d) are initial conditions for the period when the program is initiated.

It is easiest to consider the perturbation in the path of bequests per capita. Let,

$$\Delta b(t) = (1+v)^{-t} \Delta B(t), \qquad t=0,1,2,\ldots \tag{4}$$

Using (3)-(4),

$$\begin{aligned} \Delta b(t) &= \frac{1+r}{1+v} \beta \Delta b(t-1) - (r-v)\beta s, \quad t=1,2,\ldots \\ \Delta b(0) &= (1+v)\beta s \end{aligned} \tag{5}$$

The behavior of $\Delta b(t)$ depends on the value of β. There are four possibilities, illustrated in Figure 1.

If $\beta = 0$ there is no change in the bequest of any cohort, as shown in Figure 1a.

If $0 < \beta < (1+v)/(1+r)$, the change in the per capita bequest is initially positive. However, as shown in Figure 1b, it declines over time, asymptotically approaching the (negative) limit:

$$\Delta b^E = - \frac{(r-v)(1+v)\beta s}{(1+v) - (1+r)\beta} .$$

Note that within the stable parameter range a larger MPB, although it slows down the speed of adjustment, increases the long-run impact of the policy.

If $(1+v)/(1+r) \leq \beta < 1$ the initial change in bequests is positive. However, as shown in Figure 1c the increment falls over time, diverging to $-\infty$.

If $\beta = 1$, the case argued by Barro, the change in per capita bequest is $(1+v)s$ in every period, exactly the amount of the transfer to the elderly. However, as can be seen in Figure 1d this equilibrium is unstable, and any perturbation would result in explosive changes in bequests.

The perturbation to the capital-labor ratio, which using (1)-(3) is given by:

$$\begin{aligned} \Delta k(t) &= \frac{1+r}{1+v}\beta\, \Delta k(t-1) - (1-\beta)s \\ \Delta k(0) &= 0 \end{aligned} \tag{6}$$

displays the same types of behavior, as shown in Figure 2. The equilibrium value of the perturbation is:

$$\Delta k^E = \frac{-(1-\beta)(1+v)s}{(1+v)-(1+r)\beta} .$$

In the stable region of the parameter space Δk^E is negative, and is larger in absolute value for larger values of β (Figure 2b).

III. Conclusions

As shown above, Barro's "neutrality" result corresponds to an unstable equilibrium. A slight perturbation of the MPB away from unity will cause explosive changes in bequests and in the capital-labor ratio.

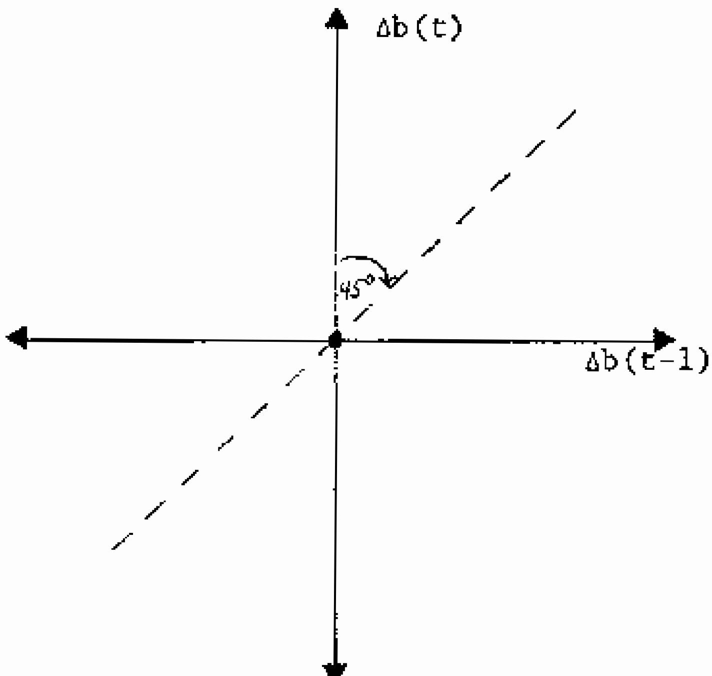
Will feedback mechanisms set in to preserve approximate neutrality for MPB's near unity? The inital decrease in the capital-labor ratio (see Figure 2c) will raise the marginal product of capital, making the system even more unstable (cf. equation (6)). Hence changes in factor payments do not appear to be a stabilizing influence.

However, as real income falls the MPB can be expected to fall, and this can stabilize the system. However, as noted above, for stable parameter values a <u>larger</u> MPB leads to a <u>larger</u> long-run impact on the capital-labor ratio.

What light does this shed on Barro's conclusion? For a multitude of reasons (variation in family size, distributional effects, etc.) the MPB out of social security payments is not going to be exactly equal to one. The model above suggests that Barro's conclusion must be viewed with skepticism, since it does not appear to be robust.

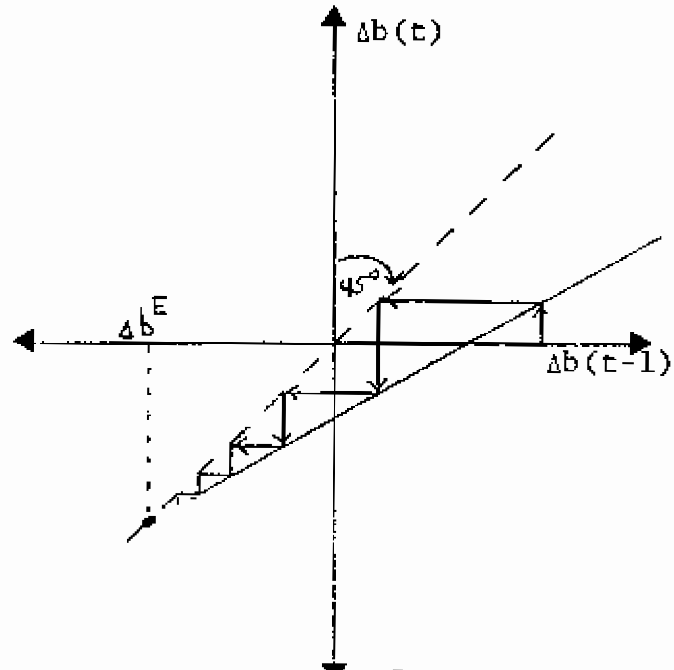
Footnotes

[1] A model with continuously overlapping generations and with consumption by all age cohorts is examined in Stokey (1979). The conclusions for that model are the same as the conclusions for the simpler model presented here.



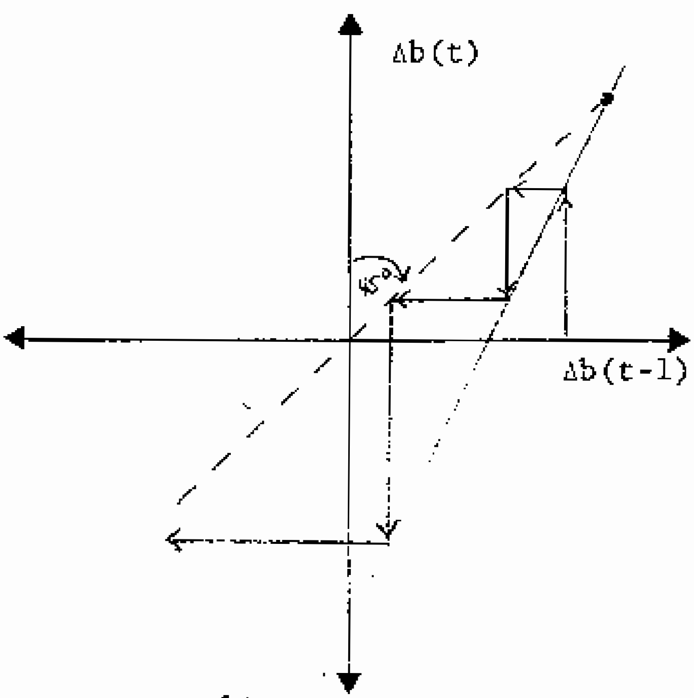
$\beta = 0$

Figure 1a



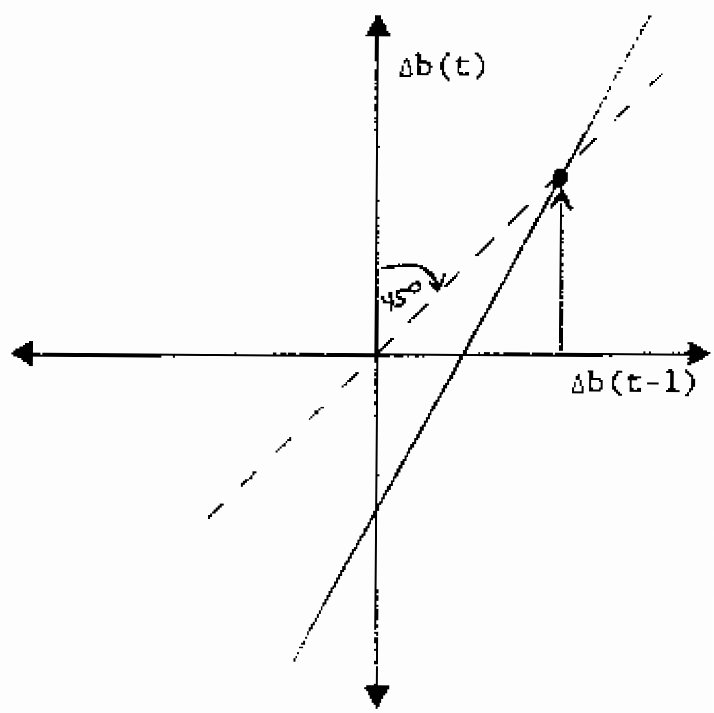
$0 < \beta < \frac{1+v}{1+r}$

Figure 1b



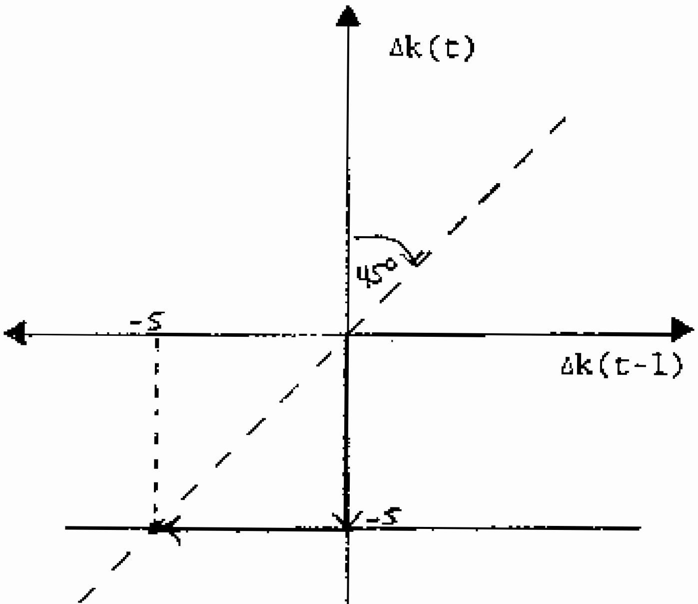
$\frac{1+v}{1+r} \leq \beta < 1$

Figure 1c



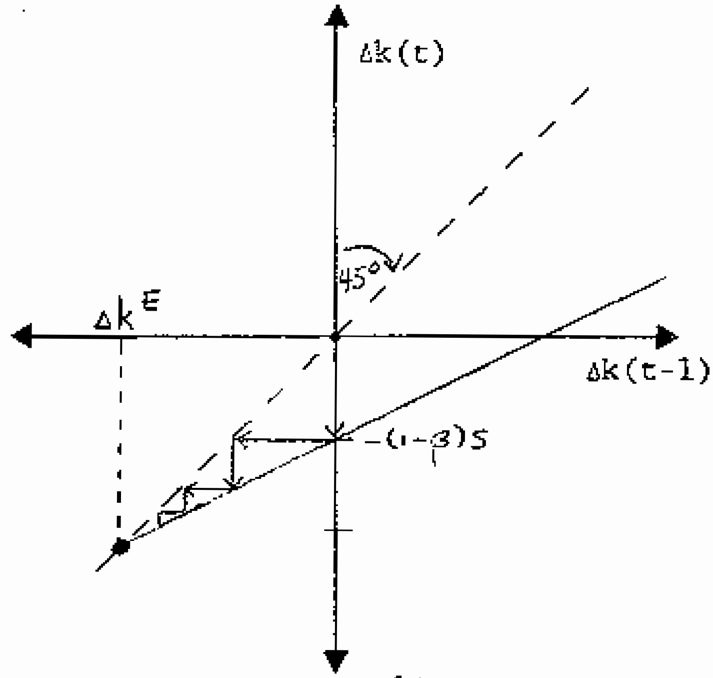
$\beta = 1$

Figure 1d



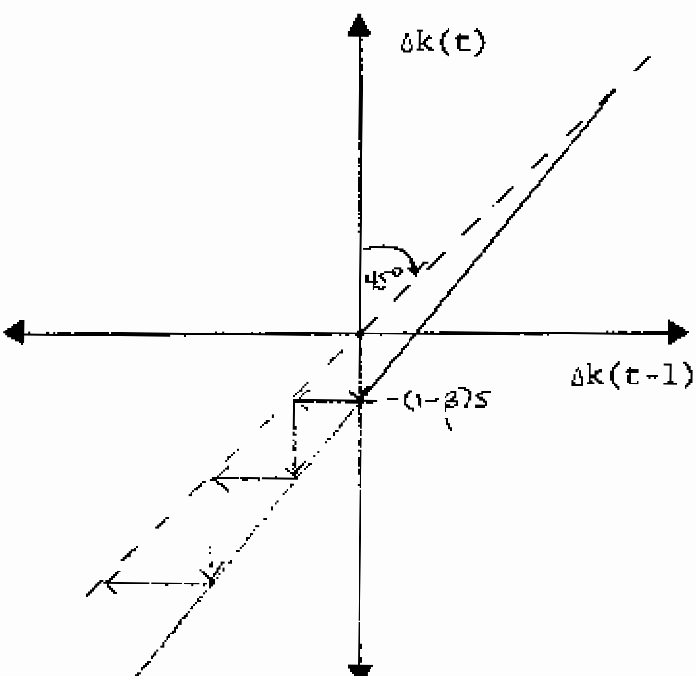
$\beta = 0$

Figure 2a



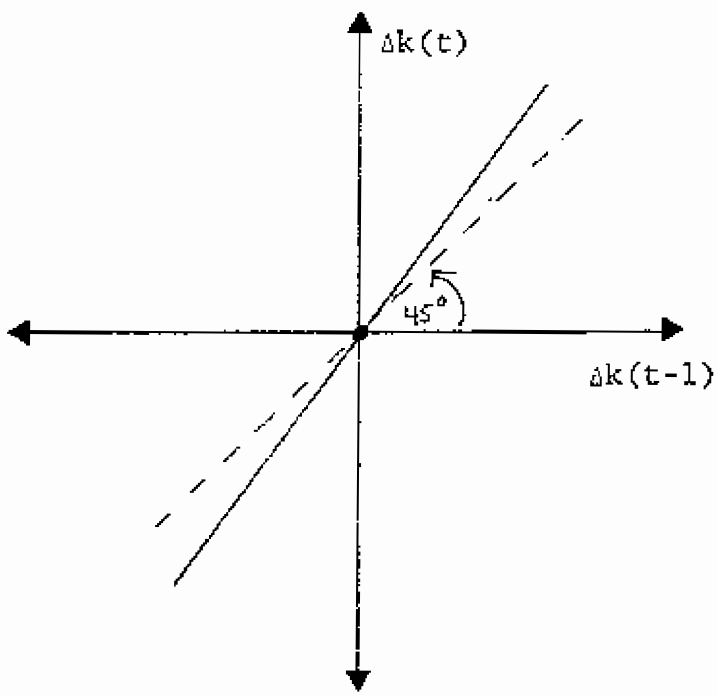
$0 < \beta < \frac{1+v}{1+r}$

Figure 2b



$\frac{1+v}{1+r} \leq \beta < 1$

Figure 2c



$\beta = 1$

Figure 2d

References

Barro, Robert J., "Are Government Bonds Net Wealth?" Journal of Political Economy, Vol. 82, No. 6 (Nov./Dec. 1974), 1095-1117.

Stokey, Nancy L., "Do Bequests Offset Social Security?" , Discussion Paper No. 376, Center for Mathematical Studies in Economics and Management Science, Northwestern University, April, 1979.