Discussion Paper No. 415

An Experimental Study of a Stochastic Model of Committee Voting with Exogenous Costs*

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February 1980

+Support from National Science Foundation Grant SOC 790-7366

*The authors are indebted to David Grether and Forrest Nelson for their helpful comments. Michael Hennel and Natalene Olson helped run the experiments.

AN EXPERIMENTAL STUDY OF THE EFFECT OF EXOGENOUS VOTING COSTS ON THE DECISIONS OF MAJORITY RULE COMMITTEES

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ABSTRACT

This paper examines experimentally the effect of exogenous voting vosts on the decisions of 3-person majority rule committees with induced preferences making decisions under strict Roberts Rules of Order. Based on previous work by the cuthors the committees were predicted to choose an equilibrium outcome with probability one without costs and another distribution of outcomes with costs. The experimental outcomes are consistent with these predicted distributions and the dynamics of the amendment process are generally consistent with the dynamics predicted by the myopic stochastic solution concept, despite several attempts at nonmyopic strategic voting.

INTRODUCTION

Recent theoretical and experimental research has criticized the core/equilibrium as a solution concept for majority rule games. First, the core generally fails to exist (Rubinstein, 1979), with existence requiring strict symmetry conditions on the distribution of preferences (Plott, 1967; Sloss, 1973; McKelvey and Wendell, 1976). Second, even when a core does exist, it is not always chosen as the outcome of a majority rule voting process (Hoffman and Plott, 1979; Isaac and Plott, 1978; McKelvey and Ordesbook, 1979; Plott and Levine, 1978; Plott and Rogerson, 1979).

These difficulties with the core have led to the development of a variety of nonequilibrium solution concepts, each predicting a set or distribution of points when the core does not exist (Simpson, 1969; Ferejohn, Fiorina, and Weisberg, 1977; McKelvey, Ordeshook, and Winer, 1978; Ferejohn, Fiorina, and Packel, 1980; Packel, 1979).

In a recent paper (Hoffman and Packel, 1979), we extended one particular nonequilibrium solution concept (Ferejohn, Fiorina, and Packel, 1980) to a more general model which allows for the possibility that the core will not be chosen when it exists. The Ferejohn, Fiorina, and Packel (1980) solution concept, called the stochastic solution, defines on the set of alternatives a probability measure which is derived from a Markov chain model of the voting process. While this stochastic model might be questioned for its assumption of myopia, it does seem to be consistent with the experimental results of Fiorina and Plott (1978) for committee decisions in the absence of a core. A more direct test by Plott and Rogerson (1979) gave only qualified support, however. In our model (Hoffman and Packel, 1979) we developed an exogenous decision-theoretic stopping rule for the Markov process and we explored the effect of exogenous decision costs on

the predictions of the stochastic solution. In particular, we found that while the stochastic solution predicts that the core will be chosen with probability one when it exists, our model predicts that the probability of choosing the core can be less than one and that the overall probability measure is dependent upon the decision cost functions.

In this paper we discuss an experimental test of our model, providing data relevant to a variety of hypotheses about individual and group voting behavior. We conclude that the model predicts reasonably well the actual distribution of alternatives chosen, but the stepwise process by which intermediate outcomes are chosen does not fully conform to the myopic dominance relation predicted by the Markov model. On average, committees seem to have acted "as if" voters were myopic. However, the use of one particular and highly nonmyopic search process by some of the committees resulted in intermediate and final alternatives less in accord with the assumptions and predictions of the model.

THE STOCHASTIC MODEL WITH EXOGENOUS COSTS

This section provides a summary of the model developed in Hoffman and Packel (1979) which in turn extends the stochastic solution of Ferejohn, Fiorina, and Packel (1980).

$X = \{1,2,\ldots,j,\ldots,J\}$ is the set of alternatives.

$V = \{1,2,\ldots,i,\ldots,N\}$ is the set of voters.

$k = 1,2,\ldots$ denotes the number of amendments to the original status quo passed under the given voting rule (majority rule in this experiment).

c_i^k = total cost incurred by voter i after k amendments have passed.

$\dot{c}_i^k = c_i^k - c_i^{k-1}$ = marginal cost to voter i as the k^{th} amendment is passed.

U_{ij} = payoff to voter i if the group chooses alternative j (ignoring costs).

$U_{ij}^k = U_{ij} - c_i^k$ = payoff to voter i if the group chooses alternative j after k amendments have passed.

M = the collection of minimal winning coalitions (two voter subsets of V in this experiment).

$A_{jh}^k = |\{m \in M | U_{ij}^{k-k} < U_{ih}^k \ \forall\ i \in m\}|$ = (the number of minimal winning coalitions preferring h after k amendments are passed to j after k-1 amendments are passed $(h,j \in X)$).

$$p_{jh}^k = \begin{cases} A_{jh}^k / \sum_{\ell=1}^{J} A_{j\ell}^k & \text{if } \sum_{\ell=1}^{J} A_{j\ell}^k > 0 \\ \delta_{jh} & \text{otherwise,} \end{cases}$$

where $\delta_{jh} = \begin{cases} 1 & \text{if } j = h \\ 0 & \text{otherwise .} \end{cases}$

$P_k = [P^k_{jh}]$ = stochastic matrix whose entries are the transition probabilities for the passage of the k^{th} amendment.

$P^{(T)} = \prod_{k=1}^{T} P_k$ = transition matrix through the passage of T amendments.

$P^{(\infty)} = \lim_{T \to \infty} P^{(T)}$ = the "final" transition probability matrix.

Given that $Q(0) = (q_1, q_2, \ldots, q_J)$ is a vector of starting probabilities over the various alternatives, the limiting probability distribution over outcomes is then given by

$$O = Q(0)P^{(\infty)}.$$

A crucial aspect of the model is that Q will exist under reasonable assumptions on the costs c^k_i and that the core, when it exists, will not necessarily be assigned a probability of one.

EXPERIMENTAL DESIGN

A. The Experimental Model

The specific decision procedures investigated were 3 person committees (N=3) choosing among 8 alternatives (J=8) using absolute majority rule. The study consisted of 5 experimental committees without decision costs and 36 committees with decision costs imposed. The no-cost sessions were intended as a control to test the hypothesis that, for the voting procedures employed, the core would always be chosen in the absence of decision costs. The payoff functions for the no-cost and cost experimental sessions are given in Tables I and II.

TABLE I

PAYOFFS TO PARTICIPANTS IN "NO-COST" EXPERIMENTAL SESSIONS

Motion Adopted	Voter No. 1	Value to Voter No.2	Voter No. 3
A	$ 4.00	$ 4.00	$ 14.00
B	10.00	14.00	6.00
C	2.00	16.00	4.00
D	8.00	10.00	12.00
E	6.00	2.00	16.00
F	14.00	12.00	8.00
G	16.00	8.00	2.00
H	12.00	6.00	10.00

TABLE II

PAYOFFS TO PARTICIPANTS IN "COST" EXPERIMENTAL SESSIONS

Motion Adopted	Voter No.1	Value to Voter No.2	Voter No.3
A	$10.50	$10.50	$ 18.00
B	15.00	18.00	12.00
C	9.00	19.50	10.50
D	13.50	15.00	16.50
E	12.00	9.00	19.50
F	18.00	16.50	13.50
G	19.50	13.50	9.00
H	16.50	12.00	15.00

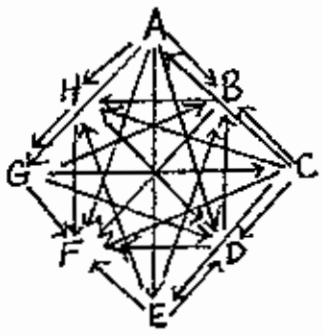
No. Amendments Passed	Total Cost	Marginal Cost
0	$ 0.00	--
1	.50	$ .50
2	1.00	.50
3	3.00	2.00
4	5.50	2.50
5	9.00	3.50
6 or more	9.00	--

Payment to voter = Value of motion adopted - cost of No. of Amendments passed

Induced preference theory says that the participants will rank alternatives according to the rankings of the associated payoffs. Note that the preference rankings correspond in the no-cost and cost sessions. Figure 1 shows the dominance relations in the absence of costs. Alternative F is the strong core in this no-cost situation.

Figure 1

Dominance Relation for No-cost Experimental Sessions



The stochastic solution when costs are present is constructed as follows. First, stochastic transition matrices P_k are constructed for each step $k = 1, 2, \ldots$ where k is the number of amendments that have been passed. Recall that entry (h, j) denotes the probability, according to the model, of moving to alternative j given that the current status quo is alternative h. In the situation described by Table II, we obtain:

$P_1 = P_2 =$

	A	B	C	D	E	F	G	H
A	0	1/6	0	1/6	1/6	1/6	1/6	1/6
B	0	0	0	0	0	1/2	0	1/2
C	1/6	1/6	0	1/6	1/6	1/6	0	1/6
D	0	1/2	0	0	0	1/2	0	0
E	0	1/5	0	1/5	0	1/5	1/5	1/5
F	0	0	0	0	0	1	0	0
G	0	1/4	1/4	1/4	0	1/4	0	0
H	0	0	0	1/3	0	1/3	1/3	0

$P_3 = P_4 =$

0	1/2	0	0	0	1/2	0	0
0	1	0	0	0	0	0	0
0	0	0	1/2	0	0	0	1/2
0	0	0	1	0	0	0	0
0	1/4	0	0	0	1/4	1/4	1/4
0	0	0	0	0	1	0	0
0	1/2	0	0	0	1/2	0	0
0	0	0	0	0	0	0	1

$$P_5 = \begin{bmatrix} 0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

P_k = Identity matrix for $k \geq 6$.*

Note that F is no longer a strong core after 2 amendments have been passed. Indeed F no longer dominates B, D, and H, which also become undominated and hence (weak) cores for $k \geq 2$. After 4 amendments have been passed, G also becomes a core.

Given our handling of P_k for $k \geq 6$, the limiting matrix $P^{(\infty)}$ is given by $P^{(\infty)} = P_1P_2P_3P_4P_5 = P_1^2P_2^2P_5$. Since all experiments took alternative A the starting status quo, Q(0) = (1,0,0,0,0,0,0). The stochastic solution is then obtained as

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*This assumes that the number of amendments passed will never exceed 5. This could have been assured by letting marginal costs for $k \geq 6$ exceed $6.00. This was not done since it could, in theory, have led to negative payoffs to subjects. In fact, none of the experimental committees passed more than 5 amendments anyway.

$$Q = Q(0)P^{(\infty)} = (0,.205,0,\ .141,\ 0,.521,.003,.130)$$

This vector provides the predicted probability distribution over alternatives A through H respectively.

B. Experimental Test of the Model

Testing the predictions of the model as summarized in the Q vector required that several experimental criteria be met. First, the procedures used by the subjects had to approximate as closely as possible the assumptions implicit in the stochastic solution model. Since a "step" in this model is defined as the passage of an amendment, subjects could avoid costs by simply discussing all possible alternatives before voting on any of them. This problem was handled (with only partial success as we shall discuss later) by allowing committees to discuss only two alternatives at a time. Also, side payments and other deals were forbidden to ensure that preference orderings were maintained. Finally, since many experimental sessions were to be run, it was felt that some blinding of experimenters would be advisable as the distribution of outcomes unfolded.

1. Procedures

The procedures used in both the no-cost and the cost experimental sessions were basically the status quo and amendment procedures of

Roberts Rules of Order (1970) already tested in Fiorina and Plott (1978), Hoffman and Plott (1979), and Plott and Rogerson (1979). The main difference between these procedures and ours is that we severely limited the amount of information subjects could exchange during the discussion of each possible alternative to the status quo. Both the no-cost and cost instructions are given in the Appendix.

In our procedures, the voting process began with letter A as the motion on the floor. Subjects could propose and discuss any alternative letter as a possible amendment to the current motion on the floor, but they were forbidden from discussing more than two letters at any one time. Violations of this rule were cause for terminating the experiment immediately with minimum payoffs. Each time an alternative letter passed or failed in a formal majority vote, one other letter could be brought up for consideration as a possible motion on the floor. Subjects were free to repropose previously defeated letters as long as they obeyed the two-at-a-time rule. Formal closure rules for ending debate were also imposed each time a vote was requested. A session was terminated when a majority (2 or more) subjects voted to adopt a current motion on the floor as the committee decision. Payoffs were then awarded in accordance with the payoff schemes described in Table I (no costs) and Table II (costs).

To ensure that committee members could not make side payments or other deals, the mention of specific monetary amounts was not allowed. Violations of this rule were also grounds for terminating a session with minimum payoffs.

2. Statistical Testing

Before beginning our experiments we decided that a χ^2 test for goodness

of fit would be the appropriate test for the probability distribution predicted by the model. Since a rule of thumb for the χ^2 test requires that the predicted number of outcomes in each cell be at least 5, we chose to group letter alternatives with the lowest predicted probabilities. Recall that the predictions of the cost model were

$$\begin{matrix} & A & B & C & D & E & F & G & H \\ Q = & (0, & .205, & 0, & .141, & 0, & .521, & .003, & .130) \end{matrix}$$

Combining alternatives A,C, E, G, and H led us to run 36 experimental sessions with costs to guarantee that the expected frequency cells in Table III all exceeded or came close to 5.

Table III

Predicated Distribution of Cost Experiment Outcomes

	ACEGH	B	D	F
Theoretical Probability	.133	.205	.141	.521
Expected Frequency (36 trials)	4.79	7.38	5.08	18.75

3. Blinding of Experimenters

The experiments were run single blind. There were four experimenters, each of whom knew the purpose of the experiment. Consequently, letters were assigned to payoffs according to 5 different permutations so that experimenters would have less awareness during a session of how events that transpired related to the theoretical model. This also served as safeguard against the leakage of information to subjects about payoffs and results of prior sessions. Alternative A was the starting alternative

for all the permutations so that no information was revealed by where things started.

C. Subjects

Subjects were male and female undergraduates from Lake Forest College and Northwestern University with no prior experience in the experiments conducted They were recruited from classes and promised "more money than they were likely to earn in their next best alternative employment." Extras were recruited in case of no shows and paid $1.00 for showing up. Participants in the no-cost experiments were paid $2.00 in addition to their earnings from the committee. Subjects in sessions which were terminated for rule violations were paid $2.00. Generally, four 3-person committees were conducted at a time and assignment to committees was done randomly.

When they were recruited, the subjects were told only that they would participate in a committee voting experiment. After each experiment they were strongly instructed not to reveal anything about the amount of money they made, the structure of monetary payoffs, or the specific nature of the experiment. Conversations with later subjects indicated that previous subjects did strictly guard that information.

RESULTS AND ANALYSIS

A. The Final Outcome Predictions of the Model

Table IV summarizes our experimental results and a χ^2 test of whether the distribution of cost experiment outcomes fits that predicted by our exogenous cost model. The fact that all no-cost experimental sessions chose the core (F) as predicted while 24 of 35 cost sessions chose the core clearly shows that costs make a significant difference in whether the

core will be reached. The fact that the null hypothesis provided by the predictions of our cost-based model cannot be rejected at the .10 level appears to provide support for the model, but we hasten to point out a serious difficuty which is partially buried in our grouping of alternatives. The occurrence of alternative A despite its zero predicted probability requires discussion and reevaluation, which we provide in the next several paragraphs.

Table IV

No-Cost and Cost Experimental Results

Possible Outcomes	Number of Each Outcome*			
	No-Cost		Cost	
	Actual	Predicted	Actual	Predicted
A	0	0	1	0
B	0	0	3	7.17
C	0	0	0	0
D	0	0	4	4.94
E	0	0	0	0
F	5	5	24	18.24
G	0	0	1	.10
H	0	0	2	4.55
Totals	5	5	35	35

Null Hypothesis: Outcomes will occur with distribution predicted by the stochastic solution with costs.

$\chi^2 = 4.51$ (Note that A,C,E,G,H were grouped together to compute this statistic)

$\chi^2_{.10}(3) = 6.25$

Result: Null Hypothesis cannot be rejected at the .10 level.

*One cost experiment had to be terminated for repeated rules violations. A no-cost experiment was also terminated, but the core was the status quo at termination.

A harsh view of even a simple occurrence of a theoretically impossible event would call for a direct rejection of the theory. Since the stochastic solution with costs model is clearly superior to its only established competitor, that the core must occur with probability one, and since the overall results do seem encouraging, we shall consider the matter more sympathetically.

It is reasonable to suppose, and later discussion will bear this out, that there is a certain degree of "noise" in our experimental procedures. Such noise, be it in the form of improper understanding of the rules by subjects, imperfect experimental design, or whatever, could lead subjects to choose any final outcome, regardless of its theoretical probability. Let us assume a small background probability ε of choosing each of the 8 outcomes with the remaining $1-8\varepsilon$ of probability allocated in proportion to the probabilistic predictions of the stochastic solution with costs. The multinomial distribution can then be used to compute the probability, as a function of ε, of obtaining the outcome frequencies observed in 35 trials. This probability will be proportional to:

$$L(\varepsilon) = \prod_{i=1}^{8} (\varepsilon + (1-8\varepsilon)P_i)^{N_i}$$

where P_i and N_i $(i = 1,2,\ldots,8)$ give the respective probabilities and observed frequencies for outcomes A through H. The ε value maximizing $L(\varepsilon)$ (or equivalently $\log(L(\varepsilon))$) turns out to be $\varepsilon = .010$ (this was obtained by computer "combing" for ε in increments of .001). Repeating the χ^2 calculations with $\varepsilon = .01$ in the "noise" model with outcomes grouped as before, $\chi^2 = 5.80$. The predictions of this noise augmented model cannot be rejected at the .10 level. This provides one plausible way to account for difficulties with "impossible" events.

Having introduced the possibility of noise in our analysis, we now consider the hypothesis that noise alone might explain the occurrence of noncore outcomes. Consider the core model that predicts a core probability (P_6 in our design) of one with all other probabilities equal to zero. Using ϵ to allow for noise and defining $L(\epsilon)$ as above, we obtain for this simpler situation

$$L(\epsilon) = \epsilon^{11}(\epsilon + (1-8\epsilon))^{24} = \epsilon^{11}(1-7\epsilon)^{24}$$

It is easy to check that L is maximized by $\epsilon = \frac{11}{7\cdot35} \approx .045$, giving a core probability of $\frac{24}{35} \approx .686$ and noncore probabilities of .045. (This turns out to be equivalent to a noise-free model in which the core recurs with precisely the probability that matches the experimental results with the remaining probability equally distributed among the 7 noncore outcomes. Grouping outcomes for a χ^2 test as done earlier, we obtain a χ^2 statistic of 6.95. Thus the core theory with noise (or the noncore equal distribution theory) would be narrowly rejected at the .10 level and hence does not do as well as our stochastic theory.

B. Dynamics of the Experimental Decision Process

Table V summarizes the sequence of amendments considered and then passed or rejected by each experimental committee. An asterisk indicates that the vote violated the predicted dominance relation. That is, an amendment passed despite the fact that it made at least two voters worse off, given their costs; or it failed even though it would have made at least two voters better off.

Key: → Amendment Passed

↛ Amendment Failed

* Vote Violated Dominance Relation

** Session Terminated Prematurely Because of Rule Violations

TABLE V

Sequence of Amendments Considered by Experimental Committees

No Cost

1. A ↛ C, A*↛ G, A → F ↛ B, F ↛ E
2. A → D ↛ E, D ↛ C, D → B ↛ D, B → H → F ↛ A, F ↛ C
3. A ↛ C, A → E → B → H → F ↛ E, F ↛ D **
4. A → H ↛ B, H → D ↛ G, D ↛ C, D → F
5. A → B ↛ E, B ↛ C, B ↛ D, B *↛ H, B → F ↛ H

Cost

1. A → E → F
2. A *↛ G, A → B ↛ E
3. A → F ↛ E, F ↛ C, F ↛ G, F *→ B *→ H, H ↛ B, H → F
4. A → B ↛ C
5. A → G ↛ E, G → F ↛ C
6. A ↛ C, A → F
7. A *↛ H, A *↛ B, A ↛ C, A *↛ D, A *↛ E, A *↛ F, A *↛ G, A → D *↛ B, D *↛ F, D ↛ H, D *↛ F, D ↛ B, D → F
8. A → G ↛ C, G → F
9. A → H *↛ F, H ↛ E, H *↛ G, H *↛ D, H ↛ F, H ↛ B, H *↛ D, H ↛ B, H ↛ E, H *↛ G, H *↛ F
10. A
11. A → E → B ↛ A, B *→ F ↛ B, F *→ E ↛ C, E → G
12. A → E → F
13. A *↛ H, A → D
14. A → G → B ↛ A, B ↛ C, B ↛ F, B ↛ E, B ↛ D
15. A *↛ F, A *↛ H, A *↛ B, A *↛ D, A *↛ G, A *↛ B, A ↛ C, A → F
16. A ↛ C, A *↛ G A *↛ B, A *↛ F, A *↛ E, A *↛ H, A*↛ D, A*↛ B, A *↛ F, A *↛ H, A → F
17. A ↛ C, A *↛ G, A *↛ D, A *↛ F, A *↛ H, A *↛ E, A *↛ B, A*↛ F, A* ↛ D, A* ↛ H, A → F
18. A *↛ E, A *↛ G, A *↛ E, A *↛ H, A → B ↛ D, B ↛ G, B → F
19. A *↛ B, A ↛ C, A → E → F ↛ B, F ↛ D, F ↛ G, F ↛ E, F ↛ C, F ↛ E
20. A *↛ G, A → E → D

21. $A \rightarrow B \not\rightarrow E,\ B \not\rightarrow G,\ B \not\rightarrow D,\ B \not\rightarrow A,\ B \rightarrow F$

22. $A \ast\!\!\not\rightarrow G,\ A \ast\!\!\not\rightarrow D,\ A \rightarrow F \not\rightarrow E,\ F \not\rightarrow G,\ F \not\rightarrow C,\ F \not\rightarrow H$

23. $A \not\rightarrow C,\ A \ast\!\!\not\rightarrow E,\ A \rightarrow B \not\rightarrow D,\ B \rightarrow F \not\rightarrow D,\ F \not\rightarrow E$

24. $A \ast\!\!\not\rightarrow F,\ A \rightarrow G \rightarrow B \not\rightarrow E,\ B \not\rightarrow F,\ B \ast\!\!\rightarrow F \not\rightarrow C$

25. $A \ast\!\!\not\rightarrow H,\ A \ast\!\!\not\rightarrow E,\ A \not\rightarrow C,\ A \ast\!\!\not\rightarrow F, A \ast\!\!\not\rightarrow D, A \ast\!\!\not\rightarrow B,\ A \ast\!\!\not\rightarrow G,$
$A \ast\!\!\not\rightarrow D,\ A \ast\!\!\not\rightarrow B,\ A \ast\!\!\not\rightarrow F,\ A \ast\!\!\not\rightarrow H,\ A \rightarrow F \not\rightarrow D,\ F \not\rightarrow B,\ F \not\rightarrow H,$
$F \not\rightarrow G$

26. $A \not\rightarrow C, A \ast\!\!\not\rightarrow E, A \ast\!\!\not\rightarrow D, A \ast\!\!\not\rightarrow F, A \ast\!\!\not\rightarrow G, A \ast\!\!\not\rightarrow H,\ A \ast\!\!\not\rightarrow B,\ A \rightarrow F$

27. $A \ast\!\!\not\rightarrow G,\ A \ast\!\!\not\rightarrow D,\ A \ast\!\!\not\rightarrow H,\ A \ast\!\!\not\rightarrow C,\ A \ast\!\!\not\rightarrow F,\ A \ast\!\!\not\rightarrow B,\ A \not\rightarrow C,\ A \ast\!\!\not\rightarrow F,\ A \rightarrow D$

28. $A \ast\!\!\not\rightarrow G,\ A \ast\!\!\not\rightarrow B,\ A \ast\!\!\not\rightarrow E,\ A \not\rightarrow C,\ A \rightarrow F \not\rightarrow D,\ F \ast\!\!\rightarrow H$

29. $A \not\rightarrow C,\ A \ast\!\!\not\rightarrow G,\ A \not\rightarrow E,\ A \rightarrow F \not\rightarrow E$

30. $A \ast\!\!\not\rightarrow E,\ A \ast\!\!\not\rightarrow G,\ A \ast\!\!\not\rightarrow F,\ A \not\rightarrow C,\ A \ast\!\!\not\rightarrow B,\ A \ast\!\!\not\rightarrow D,\ A \ast\!\!\not\rightarrow H$ **

31. $A \ast\!\!\not\rightarrow E,\ A \not\rightarrow C,\ A \ast\!\!\not\rightarrow D,\ A \ast\!\!\not\rightarrow B,\ A \ast\!\!\not\rightarrow G,\ A \ast\!\!\not\rightarrow F,\ A \ast\!\!\not\rightarrow H,\ A \rightarrow D$

32. $A \ast\!\!\not\rightarrow H,\ A \ast\!\!\not\rightarrow B,\ A \not\rightarrow C,\ A \ast\!\!\not\rightarrow D,\ A \ast\!\!\not\rightarrow E,\ A \ast\!\!\not\rightarrow F,\ A \ast\!\!\not\rightarrow G,\ A \ast\!\!\not\rightarrow H,$
$A \ast\!\!\not\rightarrow B,\ A \ast\!\!\not\rightarrow D,\ A \ast\!\!\not\rightarrow F,\ A \rightarrow F$

33. $A \rightarrow E \not\rightarrow C,\ E \rightarrow B \not\rightarrow D,\ B \not\rightarrow F,\ B \not\rightarrow H,\ B \not\rightarrow G,\ B \ast\!\!\rightarrow F$

34. $A \rightarrow D \not\rightarrow G,\ D \rightarrow F \not\rightarrow E$

35. $A \ast\!\!\not\rightarrow G,\ A \not\rightarrow C,\ A \rightarrow F$

36. $A \not\rightarrow C,\ A \rightarrow F$

Table VI summarizes the frequency of deviations from the dominance relation predicted by myopic voter computation of costs and payoffs. Notice that, in general, the dynamics of the no-cost experiments conform to the predictions of the stochastic solution. No amendment contrary to the predicted dynamics was passed and only 9% of the defeated amendments violated the dominance relation. These results are in strong contrast to those of Plott and Rogerson (1979), whose experiments were done without imposing costs. They found, in a situation where alternatives were admittedly much more equally balanced in their voter appeal, that 62% of

the amendments passed violated the predicted dominance relation.

Table VI

Deviations From the Predicted Dominance Relation

	No Cost	Cost	"Nonstrategic" Cost
Total Amendments Passed	14	58	44
No. Violating Dominance Relation (%)	0 (%)	7 (12%)	6 (14%)
Total Amendments Defeated	22	177	73
No. Violating Dominance Relation	2(9%)	101 (57%)	17 (23%)

On the other hand, the dynamics of our cost experiments only partly conform to the dynamics predicted by the stochastic solution. Looking first at the amendments passed, we see that only 12% violated the dominance relation. While this is significantly higher than the 0% for the no-cost experiments, it is still much below the 62% found by Plott and Rogerson (1979). The substantial deviation from the predicted dynamics comes when we consider the amendments which were defeated. Indeed, 57% violated the dominance relation. If we look at Table V, we can see that this 57% is not evenly distributed across cost experiments, however. 85 of the 101 deviations come from 12 of the 36 experiments. (session numbers 7, 15, 16, 17, 18, 25, 26, 27, 28, 30, 31, 32). In each of these sessions a particular strategy for avoiding decision costs was articulated and adopted by the participants. Subjects in these sessions decided in advance to consider each letter alternative in turn, gathering information about it and then defeating it to avoid costs. Then, being careful not to violate the rules against considering more than 2 letters at a time (one group failed

in this attempt and had their session terminated), these groups tried to adopt some mutually acceptable new status quo. This clever strategy was somewhat successful at undermining experimental procedures designed to enforce the binary consideration of alternatives assumed in the stochastic model and can be considered one form of experimental noise as referred to earlier. The third column of Table VI shows that elimination of these 12 overly "strategic" sessions lowers the 57% to a much more reasonable 23% for the percentage of defeated amendments violating the predicted dominance relation.

We also note that 15 of the 24 "nonstrategic" cost sessions (62%) chose the core, giving a percentage much closer to the 52% predicted by our cost model. A core/no core binomial proportion test at the .05 level narrowly avoids rejecting the null hypothesis of $p = .52$ for core occurrence in our cost model with the full sample size of 35. Elimination of "strategic" cost sessions leads to a failure to reject at well above the .10 level.

CONCLUSIONS

The experimental results summarized in this paper provide moderate but by no means unqualified support for the stochastic solution with an endogenous stopping mechanism presented in Hoffman and Packel (1979). In addition to substantiating the model's conclusion that the core may not occur with probability one, the distribution of experimental outcomes and the sequence of amendments passed generally reflect the predictions of the model.

One explanation of differences between the model and the experimental results may be attributed to the fact that committees in some sessions developed a decision strategy to circumvent the rules. In doing so, these committees followed voting patterns which tended to violate the dominance relation and to choose more core outcomes than predicted. In general, however, this tendency may reflect significantly on the stochastic solution model as a predictor of real world voting under Roberts Rules of Order (1970) only to the extent that the agenda manipulation implied by that particular strategy were allowed by parliamentary procedures.

One possible shortcoming of a stochastic model (with and without costs) is the admittedly myopic assumption behind the step-by-step selection procedure. The particular committee strategy referred to above is one example of nonmyopic behavior while, at the other extreme, the committee decision (session No. 10) to avoid all decision costs by choosing the starting alternative is another.

A more sophisticated and farsighted model of committee behavior would certainly be worth considering. There is some evidence, however, that nonmyopic behavior tends to occur in the early steps of a decision process (Plott and Rogerson, 1979), and then only with inexperienced subjects.

Thus, simple and concise myopic models of the sort we have proposed and tested would seem to be an important starting point. The particular cost-based model we have considered not only receives reasonable experimental support, but appears to be the only current model explaining the nonoccurrence of the core.

Costs

INSTRUCTIONS

General

You are about to participate in a committee process experiment in which one of several competing alternatives will be chosen by majority rule. The purpose of the experiment is to gain insight into certain features of complex political processes. The instructions are simple. If you follow them carefully and make good decisions, you might earn a considerable amount of money. You will be paid in cash at the end of the experiment.

Instructions to Committee Members

The alternative choices are represented by the letters A,B,C,D,E,F,G,H. The committee will adopt as the committee decision one and only one letter. Your compensation depends on the particular letter chosen by the committee and the number of amendments which have been passed. The value of each letter to you and the cost to you of passing any given number of amendments are shown on the attached payoff sheet. For example, if $2000 is next to point A on your payoff sheet and your cost for passing 2 amendments is $500, you will be paid $1500 if the committee adopts A by majority rule after passing 2 amendments. Your payoff sheet is your own private information. Do not show it to any other participant.

The amount of compensation associated with each letter and the cost associated with passing each additional amendment may differ among individuals. This means that at each step in the decision process the point which would result in the highest payoff to you may not result in the highest payoff to someone else. You should decide what decision you want the committee to make and do whatever you wish within the confines of the rules to get things to go your way. The experimenters, however, are not primarily concerned with

whether or how you participate, as long as you stay within the confines of the rules. Under no circumstances may you do anything to indicate how much you will be paid if any particular letter is chosen. That means you may not mention either monetary amounts or goods which would cost as much as your payoff. At any point you may discuss only two letters, the letter which is the motion on the floor and the letter proposed as an amendment to the motion on the floor. If you mention any other letter, the experiment may be terminated with minimum payoffs awarded. Under no circumstances may you mention anything about activities which might involve you or other committee members after the experiment (no deals to split up the compensation and no physical threats).

Voting Rules

The decision process begins with the letter A. Thus, A will be the beginning motion on the floor. You are free to propose any other letter as an amendment to this motion. Suppose, for example, E is the motion on the floor at some time during the meeting and 3 amendments have been passed. If you now want the group to consider B, simply raise your hand and when you are recognized by the chair, say "I move to amend the motion to B". After an amendment has been suggested the committee may discuss it as long as discussion stays within the rules. During the discussion a committee member may request an immediate vote to decide whether the amendment under discussion will become the new motion on the floor, replacing the old one. Voting on the amendment will be a two-step procedure. First, the chair will ask whether anyone objects to an immediate vote with no further discussion. If there are no objections a vote on the amendment will be taken. If the amendment passes by receiving 2 or more votes (henceforth called a majority) it becomes the new motion on the floor and is subject itself to amendments. In our example, B would become the new motion on

on the floor and four amendments would have been passed. If the amendment to replace E by B did not pass, letter E would remain the motion on the floor and other letters could be proposed as possible amendments. An amendment which fails or a motion on the floor which has been replaced may be reproposed at any time.

If there are objections to a vote on the amendment, a majority must consent to close discussion on the amendment. If a majority consents to end discussion, an immediate vote on the amendment will be taken. If a majority does not consent to close discussion, discussion on letter B, for example, continues. Note that there is no cost if an amendment fails.

Whenever a proposed amendment has been passed or rejected by a majority, a motion to terminate the meeting and adopt the current motion on the floor may be made. In the example above, if you wished the meeting to terminate with B as the committee decision after four amendments had been passed, you could make a motion to terminate the meeting at that time. The committee members may then discuss whether they wish to terminate the meeting and adopt the letter which is the current motion on the floor, but they may not discuss any other letters. The voting procedure on a motion to terminate the meeting will be the same as the voting procedure on a letter amendment. If someone asks for a vote, the chair will ask if there are any objections to ending the discussion on the motion to terminate the meeting and voting.

If there are no objections, the committee will vote on the motion to terminate the meeting and adopt the current motion on the floor. If the motion to terminate the meeting passes by a majority vote, the meeting ends and each individual is payed the amount on his or her payoff sheet associated with the current motion on the floor minus the total cost incurred from passing amendments. In our

example, if the motion to terminate and adopt B passed, each person would be paid the value to him or her of B minus the cost for passing four amendments.

If an individual objects to taking a vote on the motion to terminate the meeting, a majority must consent to taking a vote. If a majority consents to taking a vote the committee will immediately vote on the motion to terminate the meeting and adopt the current motion on the floor. If a majority does not consent, discussion on the motion to terminate continues. Note that there is no cost to taking a vote to terminate the meeting.

In summary, the initial motion on the floor will be letter A. You are free to amend this motion or move to terminate the meeting as you wish. The meeting will not end until a majority consents to terminate the meeting and adopt some motion. Your compensation will be determined by the motion on the floor finally adopted by the majority and the number of amendments passed.

Are there any questions?

We would like you to answer the questions on the attached page. Answering them should help you understand the instructions.

QUESTIONS

1. Letter ____ makes me the most possible money. If this letter is adopted after two amendments are passed, the amount I would receive is _____.

2. Letter ____ makes me the least possible money. If this letter is adopted after one amendment is passed, the amount I would receive is _____.

3. Suppose B is the motion on the floor and an amendment to C is proposed and there is no objection to a vote. If C passes by a majority vote, the new motion on the floor is _____. If C fails by a majority vote, the new motion on the floor is _____.

4. Suppose an amendment to move to D is the third and final amendment to pass. If the motion on the floor is adopted by the majority, my compensation is _____.

5. Suppose an amendment to move from A to F is passed, making F the motion on the floor, and someone proposes letter B as an amendment. You may at this point mention in discussion only letters _____ and _____.

6. Suppose 2 amendments have passed and 3 others were considered but defeated. The meeting then terminates with C as the final motion on the floor. My compensation is _____.

No Costs

INSTRUCTIONS

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You are about to participate in a committee process experiment in which one of several competing alternatives will be chosen by majority rule. The purpose of the experiment is to gain insight into certain features of complex political processes. The instructions are simple. If you follow them carefully and make good decisions, you might earn a considerable amount of money. You will be paid in cash at the end of the experiment.

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The alternative choices are represented by the letters A,B,C,D,E,F,G,H. The committee will adopt as the committee decision one and only one letter. Your compensation depends on the particular letter chosen by the committee. The value of each letter to you is shown on the attached payoff sheet. For example, if $2000 is next to point A on your payoff sheet, you will be paid $2000 if the committee adopts point A by majority rule. Your payoff sheet is your own private information. Do not show it to any other participant.

The amount of compensation associated with each letter may differ among individuals. This means that the point which would result in the highest payoff to you may not result in the highest payoff to someone else. You should decide what decision you want the committee to make and do whatever you wish within the confines of the rules to get things to go your way. The experimenters, however, are not primarily concerned with whether or how you participate, as long as you stay within the confines of the rules. Under no circumstances may you do anything to indicate how much you will be paid if any particular letter is chosen. That means you may not mention either monetary

amounts or goods which would cost as much as your payoff. At any point you may discuss only two letters, the letter which is the motion on the floor and the letter proposed as an amendment to the motion on the floor. If you mention any other letter, the experiment may be terminated with minimum payoffs awarded. Under no circumstances may you mention anything about activities which might involve you or other committee members after the experiment (no deals to split up the compensation afterward and no physical threats).

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If there are no objections a vote on the amendment will be taken. If the amendment passes by receiving 2 or more votes (henceforth called a majority) it becomes the new motion on the floor and is subject itself to amendments. In our example, B would become the new motion on the floor. If the amendment to replace E by B did not pass, letter E would remain the motion on the floor and other letters could be proposed as amendments. An amendment or a motion on the floor which has been replaced may be reproposed at any time.

If there are objections to a vote on the amendment a majority must consent to close discussion on the amendment. If a majority consents to end discussion an immediate vote on the amendment will be taken. If a majority does not consent to close discussion, discussion on letter B, for example, continues.

Whenever a proposed amendment has been passed or rejected by a majority, a motion to terminate the meeting and adopt the current motion on the floor may be made. In the example above, if you wished the committee to make B its final decision, you could make a motion to terminate the meeting as soon as B became a new motion on the floor. The committee members may then discuss whether they wish to terminate the meeting and adopt the letter which is the current motion on the floor, but they may not discuss any other letters. The voting procedure on a motion to terminate the meeting will be the same as the voting procedure on a letter amendment. If someone asks for a vote, the chair will ask if there are any objections to ending discussion on the motion to terminate the meeting and voting.

If there are no objections, the committee will vote on the motion to terminate the meeting and adopt the current motion on the floor. If the motion to terminate the meeting passes by a majority vote, the meeting ends and each individual is payed the amount on his or her payoff sheet associated with the current motion on the floor. In our example, if the motion to terminate the meeting and adopt B passed, each person would be payed the value of B listed on his or her payoff chart.

If an individual objects to taking a vote on the motion to terminate the meeting, a majority must consent to taking a vote. If a majority consents to taking a vote, the committee will immediately vote on the motion to terminate the meeting and adopt the current motion on the floor. If a majority does not consent, discussion on the motion to terminate continues.

In summary, the initial motion on the floor will be letter A. You are free to amend this motion or move to terminate the meeting as you wish. The meeting will not end until a majority consents to terminate the meeting and adopt some motion. Your compensation will be determined by the motion on the floor finally adopted by the majority.

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We would like you to answer the questions on the attached page. Answering them should help you understand the instructions.

QUESTIONS

1. Letter ____ makes me the most possible money. The amount I would receive is _____.

2. Letter ____ makes me the least possible money. The amount I would receive is _____.

3. Suppose B is the motion on the floor and an amendment to C is proposed and there is no objection to a vote. If C passes by a majority vote, the motion on the floor is _____.
   If C fails by a majority vote, the new motion on the floor is _____.

4. Suppose an amendment to move to D passes and no further amendments pass. If the motion on the floor is adopted by the majority, my compensation is _____.

5. Suppose an amendment to move from A to F is passed, making F the motion on the floor, and someone proposes letter B as an amendment. You may at this point mention in discussion only letters ____ and ____.

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