Discussion Paper No. 40

DISADVANTAGEOUS SYNDICATES

by

Andrew Postlewaite

and

Robert W. Rosenthal

Revised October 10, 1973

Northwestern University, Evanston, Illinois 60201

1. INTRODUCTION.

By organization of economic agents, we refer to formation of a single entity (e.g. corporation, union) from several individual agents. From the real world as well as economic theory, one is familiar with situations in which it is economically advantageous for individuals to organize. In fact, except for the direct costs of forming and operating an organization, one can hardly imagine a situation in which there are not some advantages to be gained. In our minds, however, the notion of the universal advantage of organization has been called into some question by the examples contained in Aumann [2]. We shall argue in this paper that under some circumstances the organization of economic agents may confer only harmful effects on all of those who organize; and we shall attempt to describe circumstances under which this may occur.

For any game an organization, or to follow [3] a syndicate, is a set of players who are bound by the rules to operate as a single individual; i.e., no coalition may form which contains some, but not all, of the members of the syndicate. We wish to examine the question : in an economically motivated setting can all the members of a coalition be in some sense worse off if they form a syndicate than if they don't

In section 2, we discuss Aumann's examples. In section 3, we present some further examples in an attempt to convince the reader that there are circumstances in which a syndicate may be meaningfully described as disadvantageous. In section 4 we show that if the unorganized agents of the main example are a set of individually small agents relative to the market, then Aumann's phenomenon disappears.

2. AUMANN'S EXAMPLES.

Aumann's three examples involve the game-theoretic solution concept, core. Briefly, a possible outcome for a game is dominated if some nonempty coalition can guarantee that some other outcome of the game, preferred by each of its members, will result instead. The core is the set of undominated outcomes. In an exchange economy, a coalition can guarantee its members whatever can be achieved by trades within the coalition. It has been shown that in exchange economies every competitive allocation is in the core. Furthermore, if, in an exchange economy, no individual owns a nonnegligible portion of any good and if only coalitions holding a nonnegligible portion of some good are permitted to dominate, then under mild assumptions on individual preferences, the set of competitive allocations coincides with the core. (See [1], [5], [7] for different versions of the "equivalence theorem".)

In each of Aumann's examples there are two goods and three types of traders. For each type there is a continuum of traders possessing identical preferences and endowments. The traders of type 1 hold only good 1 initially. The traders of types 2 and 3 hold only good 2 initially. Types 2 and 3 differ only in their preferences. The three examples differ only in the form of the utility functions for each type. Each example has a unique competitive allocation, the only core point in the case of no syndicates as prescribed by the equivalence theorem.

In each example Aumann compares the core in the unorganized case with the core when all the traders of type 1 form a syndicate.

In example A a continuum of core points results, some of which are preferred by every member of the syndicate to the competitive allocation and some of which are less preferred. In example B, the continuum of added core points are all less preferred than the competitive allocation by all the players of the syndicate. In example C, a single point is added to the core, less preferred than the competitive allocation by the syndicate.

The syndicate in each of Aumann's games is disadvantageous in the sense that : the core of the game with the syndicate contains points less preferred by the syndicate to every point in the core of the same game without the syndicate. In examples B and C, this is accompanied by the phenomenon of no additional core points in the syndicate game which are preferred to any core points in the same game without the syndicate. Under such circumstances, there seems to be some disincentive for the syndicate to form (or incentive for the syndicate to disband, if formed). This conclusion is so strange in an economic setting, that Aumann evidently regards it as sufficient evidence for calling into question the use of the core. He further suggests that a Shapley-value approach might shed some light on the situation.

3. MORE DISADVANTAGEOUS SYNDICATES.

In this section we shall argue that Aumann's examples point out a legitimate economic phenomenon. To do this we first examine some examples which are finite-player, side-payment, characteristic-function games. In [6], it is shown that every game of this class which is totally balanced (i.e., every subgame has a nonempty core) is equivalent, in a natural way, to a market of a certain type. Thus, it is natural to ask whether totally balanced side-payment games can exhibit Aumann's phenomena.

Before coming to that question, however, let us look at some phenomena which can be readily demonstrated with side-payment games. The first example shows disadvantageous syndicates can be readily found in side-payment games which do not correspond to markets.

Example 1 : $v(\{i\}) = 0 \qquad i = 1,\ldots,4.$

$v(\{1,2\}) = v(\{1,3\}) = v(\{2,3\}) = 1.$

$v(S) = 1 \quad \text{if } |S| = 3.$

$v(\{1,2,3,4\}) = 2.$

In example 1, it is easily seen that at no core point can the sum of the payoffs to players 1, 2, and 3 be less than 3/2. If $\{1,2,3\}$ forms a syndicate, however, the game becomes a bargaining game between $\{1,2,3\}$ and 4 for a unit of payoff. Thus the core of the syndicate game contains points at which $\{1,2,3\}$ receives a total of 1.

In example 1, the disadvantageous nature of the syndicate is closely tied in with the fact that the subgame played by the syndicate itself has an empty core. It is the instability of this coalition which leads to its ability to control a total payoff of 3/2 in the 4-player no-syndicate game, even though its characteristic-function value is only 1. This phenomenon cannot be present in a totally balanced game.

The second example indicates that Shapley value is not necessarily a good indicator of disadvantageous syndicates.

Example 2 : $v(S) = 0 \quad$ for $|S| \leq 5$.

$v(\{1,2,3,4,5,6\}) = 1$.

It is easy to verify that this game is totally balanced. In one associated market, each player is endowed with one unit of a personal good. Each player's utility for any feasible commodity bundle is just the minimum of the amounts he holds of each of the goods. From symmetry, the Shapley value is (1/6, 1/6, 1/6, 1/6, 1/6, 1/6). If players 1 through 5 form a syndicate, however, the game becomes a two-player bargaining game in which the Shapley value for the syndicate is 1/2, less than the total payment to that coalition at the Shapley value of the no-syndicate game. One hardly expects that any syndicate is in any real sense disadvantaged in this game, however. Rather, what we observe in example 2 is simply the peculiar implications of the symmetry requirement in Shapley value.

We now turn to the case of a disadvantageous syndicate (in Aumann's sense) in a totally balanced game.

Example 3 : $v(\{i\}) = 0$ for $i = 1, \ldots, 4$.

$v(\{1,3\}) = v(\{2,3\}) = 1$.

$v(\{1,2\}) = v(\{i,4\}) = 0$ for $i = 1, \ldots, 3$.

$v(S) = 1$ for $|S| = 3$.

$v(\{1,2,3,4\}) = 3/2$.

In example 3, the unique core point is (1/2, 1/2, 1/2, 0). Now suppose that {1,2} forms a syndicate. Then among other new core points is the point (1/4, 1/4, 1/2, 1/2), worse for both players 1 and 2 than the unique point in the core of the no-syndicate game. Of course, there are added points in the core of the syndicate game which are better for both 1 and 2, but we are concerned first with the disadvantageous possibility.

Let us consider the following interpretation of example 3. 1 and 2 are laborers. 3 and 4 are firms which use labor input. Firm 3's profits per day equal the total number of man-days employed or one, whichever is smaller (i.e., firm 3 has a labor capacity of 1 man-day per day). Firm 4's profits per day equal the man-days of laborer 1 or the man-days of laborer 2, whichever is smaller (i.e., firm 4 needs the complementary skills of both laborers in equal amounts). The maximum total profits are achieved when each firm uses each laborer 1/2 time. The only stable distribution of the profits, however, is (1/2, 1/2, 1/2, 0); since for any other non-negative distribution of 3/2, either {1,2,4} receives less than 1 unit or one of the combinations, {1,3} or {2,3}, receives less than 1 unit. This interesting situation arises from a combination of factors.

a) There is an inherent labor shortage.

b) Firm 3's position which allows it to play one laborer off against the other in effect draws all the resources away from firm 4. This is similar to the phenomenon of example 1, where the instability of the subgame played by {1,2,3} resulted in the favorable position of that coalition.

c) Firm 3's ability to play one laborer off against the other is limited by the presence of alternative employment opportunities for both.

d) Firm 4, in having to divide profits with two laborers, cannot compete effectively against firm 3, which has the same potential profit to divide with only one laborer.

If {1,2} forms a union, 3 loses the ability to play one off against the other. In the absence of firm 4, this would result in a bargaining game between 3 and the coalition {1,2}, a situation the laborers would prefer to the ununionized situation. But, in the presence of firm 4, this is no longer the case; for 3's ability to play 1 off against 2 was actually working to the benefit of {1,2}. The union can now play one firm off against the other. If v(N) were 1, this would result in all the profits going to labor. The extra 1/2 unit of capacity in the system, however, lessens the union's capability in this direction and results in core points which range from the union receiving only 1/2 to the union receiving 3/2. Thus the effect of the unionization is to add much stability to a situation which was precariously balanced in a fairly advantageous position for labor. Of the resulting stable outcomes, some are worse for labor and some are better.

To mimic the core structure of Aumann's example B, it suffices to change v({3,4}) from 0 to 1/2 in example 3. This eliminates all of the points in the core of the syndicate game which are preferred by {1,2} to (1/2, 1/2, 1/2, 0). The game remains totally balanced, but our interpretation becomes somewhat more complicated by a profit capability for the firms together but without labor input. (The discrete nature of the syndicate core in Aumann's example C cannot be mimicked in side-payment games, since the core of any side-payment game must be a convex set.)

While we recognize that the core must be used with some care in applications (for example, it does not seem realistic to expect firm 4 to receive a zero profit in this situation), it is our opinion that the disadvantage of the syndicate in this example does not arise solely because of a peculiarity in the notion of core. Rather, there is a legitimate economic phenomenon being observed. The coalitions {1,3} and {2,3} lose their leverage in a real sense when {1,2} forms a syndicate. The point (1/4, 1/4, 1/2, 1/2) can only be dominated by {1,3} or {2,3}, but player 4 will not believe such domination threats if he knows that the syndicate must work for the benefit of both 1 and 2. (Of course, if 4 were unaware of the syndicate structure this could not be argued; but such informational considerations would seem to be beyond the scope of the characteristic-function form). In a sense, the syndicate is too large for the circumstances. In becoming unbreakable, it loses the ability to make certain decentralized agreements (harmful to one of its members) the possibilities of which, however, work to the members' overall advantage.

This discussion suggests that, although in most cases it may be only advantageous for any group of economic agents to form a syndicate, there exist situations in which organization beyond a certain point may confer potential disadvantages as well, and perhaps no advantages at all.

4. A RELATED ECONOMY.

In this section we reconsider example 3, but under the assumption that players 3 and 4 are replaced by sets of individually small agents. Let $(T,\mathcal{T},\mu)$ be a measure space with T the disjoint union of sets T_1, T_2, T_3, T_4 where $\mu(T_i) = 1$ $(i = 1,\ldots, 4)$, T_1 and T_2 are atoms, and T_3 and T_4 contain no atoms. We consider the class of games with player set T and allowable coalitions $\mathcal{T}$, the characteristic functions of which agree with example 3 on the coalitions which consist of unions of sets T_i (replacing the four players, respectively, in example 3). From this class of games it is not difficult to select some which have the disadvantageous syndicate of example 3. (For example, let all other coalitions S have characteristic-function values equal to that of $\bigcup_{T_i \subseteq S} T_i$.)

We consider a class of exchange economies on the same measure space. There are four types of traders, the i^{th} type represented by a concave nondecreasing utility function u^i and initial endowment e_i - a nonnegative vector in the finite dimensional commodity space. For any set $S = S_1 \cup \ldots \cup S_4$ where S_i is a measurable subset of T_i, $i = 1,\ldots, 4$, we define

$$v(S) = \max \sum_{i=1}^{4} \int_{S_i} u^i(x(t))d\mu(t)$$

$$\text{subject to } \sum_{i=1}^{4} \int_{S_i} x(t)d\mu(t) = \sum_{i=1}^{4} e_i \; \mu(S_i).$$

This is the characteristic-function derivation for monetary markets used in [5] and a natural extension of the derivation for finite-player markets in [6].

Theorem : If concave functions $u^1, \ldots, u^4$ give rise to a characteristic function which satisfies :

$$v(T_i) = 0 \qquad i = 1, \ldots, 4;$$
$$v(T_1 \cup T_3) = v(T_2 \cup T_3) = 1;$$
$$v(T_1 \cup T_2) = v(T_i \cup T_4) = 0 \quad \text{for} \quad i = 1, \ldots, 3;$$
$$v(T_i \cup T_j \cup T_k) = 1 \quad \text{for } i \neq j \neq k;$$
$$v(T) = 3/2;$$

then the syndicate $(T_1 \cup T_2)$ is not disadvantageous[1].

We need one intermediate result.

Lemma : Let S_4 be any subset of T_4 with measure 1/2. Then if the game arises from an economy with concave utilities, $v(T_1 \cup T_2 \cup T_3 \cup S_4) = 3/2$.

Proof : Let x and (e_1+e_3-x) be a distribution of the endowments of $(T_1 \cup T_3)$ at which $v(T_1 \cup T_3)$ is achieved; i.e.,

$$u^1(x) + u^3(e_1+e_3-x) = 1.$$

Expressing similarly points at which $v(T_2 \cup T_3)$ and $v(T_1 \cup T_2 \cup T_4)$ are achieved, we have

$$u^2(y) + u^3(e_2+e_3-y) = 1$$
$$u^1(z) + u^3(w) = 1-u^4(e_1+e_2+e_4-z-w).$$

Adding the three equations, dividing by two and applying the definition of concavity yields

$$u^1(\frac{1}{2}(x+z)) + u^2(\frac{1}{2}(y+w)) + u^3(\frac{1}{2}(e_1+e_2+2e_3-x-y))$$
$$\geq \frac{3}{2} - \frac{1}{2}\, u^4(e_1+e_2+e_4-z-w).$$

Thus, from the commodity vector $\frac{1}{2}(e_1+e_2+2e_3+z+w)$, $(T_1 \cup T_2 \cup T_3)$ can generate total utility of at least $(\frac{3}{2} - \frac{1}{2}\, u^4(e_1+e_2+e_4-z-w))$. Subtracting that commodity vector from the total of the endowments $(e_1+e_2+e_3+\frac{1}{2}e_4)$ of $(T_1 \cup T_2 \cup T_3 \cup S_4)$ and distributing the remainder to S_4 (the amounts being doubled by virtue of the measure of S_4) we see that S_4 can add total utility of $\frac{1}{2}\, u^4(e_1+e_2+e_4-z-w)$ to the coalition. Hence $v(T_1 \cup T_2 \cup T_3 \cup S_4) \geq 3/2$ with equality holding from the superadditivity of v.||

Proof of theorem :

Since $v(T_1 \cup T_2 \cup T_3) = 1$, no core point can assign T_4 total utility more that 1/2. From the lemma, it is immediate then that T_4 can receive no more than zero at any core point. Since $v(T_1 \cup T_2 \cup T_4) = 1$, $(T_1 \cup T_2)$ receives at least one at any core point; and the syndicate is not disadvantageous[2].||

It is somewhat curious that the disadvantageous feature is lost in this example when the unsyndicated traders are treated as atomless, since Aumann's examples do the same. Nevertheless, Samet's main theorem [5] indicates that in side-payment market situations (with differentiable utilities) the continuum will generally be treated no better at any core point than at the competitive outcome. For this reason it seems to us that the subject of disadvantageous syndicates generally and the evidently special structure of side-payment markets deserve further study.

Acknowledgement : The authors gratefully acknowledge enlightening discussions with Robert Aumann on the subject matter of this paper.

Footnote

1. This result was originally conjectured by Robert Aumann. A somewhat weaker version (assuming differentiable utility functions) can be obtained directly from the main theorem in [5].

2. Note that the assumption that T_3 and T_4 are atomless is stronger than necessary. All that is needed for the proof is that there is a subset of T_4 of measure 1/2.

REFERENCES

1. R. AUMANN, Markets with a continuum of traders, Econometrica 32 (1964), 39-50.

2. R. AUMANN, Disadvantageous monopolies, J. Econ. Theory 6 (1973), 1-11.

3. J. GABSZEWICZ and J. DREZE, Syndicates of traders in an exchange economy, in Differential Games and Related Topics (H. Kuhn and G. Szego, eds.), North-Holland, 1971, 399-414.

4. W. HILDENBRAND, The core of an economy with a measure space of economic agents, Rev. Econ. Studies 35 (1968), 443-452.

5. D. SAMET, Monetary market with a continuum and atoms (in Hebrew), M.S. thesis, Department of Mathematics, Hebrew University of Jerusalem, 1971.

6. L. SHAPLEY and M. SHUBIK, On market games, J. Econ. Theory 1 (1969), 9-25.

7. K. VIND, Edgeworth allocations in an exchange economy with many traders, Int. Econ. Review 5 (1964), 165-177.