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EQUILIBRIUM LIMIT PRICING DOESN'T LIMIT ENTRY

by

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ABSTRACT

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We consider a model with one established firm and one
potential entrant. Each is modeled as a rational, strategic
agent. Since each is assumed to be uncertain about the other's
costs, we treat the situation as a game of incomplete
information. Limit pricing emerges endogenously from the
monopolist's attempt to persuade the entrant that its costs
(which are a determinant of post-entry profits for both) are
lower than they actually are. However, in equilibrium the
entrant infers the monopolist's costs correctly. Thus the same
entry occurs as would if there were no limit pricing and the
entrant responded optimally.

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Equilibrium Limit Pricing Doesn't Limit Entry[*]

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I. INTRODUCTION

The idea that a firm faced by the threat of entry would
alter its price-quantity decisions so as to prevent, slow, or
reduce the probability of entry can be traced back through the
work of J. Bain [1] and J. M. Clark [2] in the 1940's at least to
a paper by N. Kaldor [6], and it has received continuing
attention ever since.[1] In particular there has been a
significant volume of work appearing on this subject over the
last decade, beginning with the contributions by Gaskins [4],
Kamien and Schwartz [7,8], and Pyatt [11]. (See Scherer [14] and
S. C. Salop [13] for further references).

The models in this broad literature offer a wide variety of
alternative specifications, but certain features are common to

most of them. In particular, they concentrate on the decision problem of the established firm, taking as given the hypothesis that a lower pre-entry price will—for largely unexplained reasons—act to deter or limit entry. In this context, the typical conclusion is that an optimal price-output policy in the face of threatened entry will involve prices which are below the short-run monopoly level.

Such results would seem to raise potentially important welfare questions for public policy. If limit pricing results in lower pre-entry prices, then ceteris paribus, it would appear to be desirable from an efficiency point of view. But these lower prices typically are not as low as might be expected after entry. Thus, if entry is slowed or prevented by the limit pricing, a trade-off appears. Without limit pricing, society faces a period of high prices then enjoys the lower post-entry prices. With limit pricing, the prices are initially lower than in its absence, (but they are not as low as they would be after entry), and the period of such inefficiently high prices is extended, possibly indefinitely. Further, if one believes that an industry with more active participants is to be preferred on the grounds of increasing technological change, reducing X-inefficiency, limiting the concentration of power, or whatever, then the apparent trade-off becomes even more complicated.

In this paper we re-examine limit pricing in the context of an equilibrium model where both the established firm and the potential entrant are modeled as rational, strategic agents. This approach is in contrast with that employed the literature

cited above. There only the established firm is modeled as a maximizing decision-maker, while entrants' behavior is simply given via some mechanical specification of the rate, probability, or timing of entry as a function of the current price or quantity set by the established firm.

In our equilibrium model, the functional relationship between the established firm's actions and those of the entrant, which was simply assumed in the earlier literature, emerges endogenously. Specifically, observing the pre-entry price or quantity allows the entrant to make inferences regarding the established firm's costs, which are a determinant of post-entry prices and thus of the profitability of entry. Thus, the monopolist has an incentive to cut pre-entry prices and raise quantity so as to alter the inferences made about costs and thereby influence the entry decision. This incentive is effective, since the established firm will, in fact, practice limit pricing in equilibrium. However, a previously unsuspected feature of limit pricing also emerges: entry will occur under this equilibrium limit pricing in precisely the same cir-cumstances as if the established firm had been blissfully ignorant of the threat of entry and had consequently practiced no limit pricing, and the potential entrant had been aware of all this. In a sense, then, equilibrium limit pricing does not limit entry.

This of course means that the discussion of policy issues and trade-offs raised above is a complete red herring. Limit pricing unambiguously contributes to efficiency of resource

allocation, yielding lower prices before entry while not imposing any costs in terms of delayed or forestalled entry.

The framework of analysis we use is that of games of incomplete information played Bayesian players, as introduced by Harsanyi [5]. Our paper is organized in four sections, including the present one. In the next section we present our model and examine its equilibria. The third section contains a specific example worked out in detail. The subjects of the final section are some concluding remarks and suggestions for possible extensions of the model which would lead to further work.

We should note that there have been a number of other treatments of the issues raised in this paper in the last couple of years to which we have not yet explicitly referred. A survey of these is given by S. C. Salop [13]. Of particular interest are studies by J. Friedman [3], who presents a very complete if somewhat analytically formidable model of entry prevention, and by R. J. Reynolds and S. C. Salop (reported in Salop [13]). These papers employ game-theoretic equilibrium techniques in the context of imperfect information. However, the failure of limit pricing to limit entry was not noted explicitly in this work. We would also particularly like to draw attention to the remarkable dissertation by A. Ortega-Reichert [9] on bidding under incomplete information. His solution of the problem of bidding repeatedly against a rival suggested to us the nature of the solution to the limit pricing problem addressed here.

II. THE MODEL

We consider a situation with an established firm (the monopolist or firm 1) and a potential entrant (firm 2). Each is aware of the other's existence. There is a single, homogeneous product, having an inverse demand function $P = D(Q)$, where Q is the total output. We assume that D is known to both firms. Each firm has constant marginal costs C_i, so that the total variable costs for firm i in producing Q_i units of output are $C_i Q_i$, $i = 1,2$. Each firm i knows the value of its own marginal cost, C_i, but is unsure about the value of C_j, $j \neq i$. Each is assumed to have beliefs about C_j which, conditional on the value of C_i, are given by a distribution $H_i(\quad | \ C_i)$ whose support is an interval $[\underline{C}_j, \overline{C}_j]$. Firm 2 faces entry costs in the amount K in present value terms. These costs may be incorporation or license fees; the cost of some necessary level of advertising, market research, product development or non-resaleable capital goods; the amount of requisite bribes; or whatever. Each firm i attaches a present value of δ_i per dollar to earnings which accrue after the entry decision is made.

Initially, firm 1 must pick a production level Q_1. The optimal level of this output will depend typically on the costs C_1. Then, observing Q_1 and knowing its own costs C_2, firm 2 must decide whether or not to enter. Firm 1 then receives a payoff which consists of its immediate first-period profits, $\Pi^O_{C_1}(Q_1) \equiv (D(Q_1) - C_1) Q_1$, plus either $\delta_1 \Pi^M_1 (C_1)$, the the present value of the future monopoly profits (if firm 2 did not enter) or $\delta_1 \Pi^C_1(C_1, C_2)$, the present value of the Cournot

profits (if 2 entered). The payoff to firm 2 if it enters
is $\delta_2 \pi_2^C(C_1, C_2) - K$, the present value of the Cournot profits less
the entry cost, while if it does not enter its payoff is zero.

Assume that $\partial \pi_2^C / \partial C_1 > 0$ and that $\partial \pi_2^C / \partial C_2 < 0$.
Then the inequality $\delta_2 \pi_2^C(C_1, C_2) - K > 0$ can be solved for C_2 and
written in the form $C_2 < \gamma(C_1)$,
where $\gamma' = -(\partial \pi_2^C / \partial C_1) / (\partial \pi_2^C / \partial C_2) > 0$. The interpretation of γ is
that if the entrant were to know C_1, it would enter if and only
if C_2 is less than $\gamma(C_1)$.

If there are multiple Cournot equilibria after entry,
then $\bar{\pi}_1^C$ and $\bar{\pi}_2^C$ might be interpreted as the expected values of
profits over these equilibria under some expectations over the
likelihood of each of these actually obtaining. An alternative
specification would have the π_i^C given by some collusive sharing
of the monopoly profits. This would be appropriate if one
expected joint profit maximization after entry. Any of these
formulations of the payoffs correspond to a set-up with two
periods, where the Q_1 and entry decisions are made in the first
period in ignorance of the opponent's costs, while in second
period the firms make quantity decisions knowing each other's
costs and, of course, whether entry has occurred. The assumption
that costs become known after entry simplifies the analysis and
may not be too unrealistic. The restriction to one potential
entrant and the particular specification of costs are obvious
limitations of the model, as would seem to be the implicit
requirement of there being only one date at which entry can take
place. Yet, in the stationary set-up used here, this latter

assumption turns out in fact to be innocuous, since adding more
periods with the one potential entrant and the same demand and
cost conditions in each would not affect the solution. This will
become clear in working through the analysis. Allowing non-
constant unit costs definitely would make a difference in the
analysis, although, as we will argue in Section IV, the main
point of limit pricing not deterring entry should carry
through. Finally, allowing multiple potential entrants, many
time periods and the possibility of predation after entry raises
a host of issues not explored here.[2]

If the costs were known to each firm, the problem described
above would correspond to a simple game of complete information
in extensive form. The tree for this game would first have firm
1 pick Q_1, a non-negative real number, then 2 would decide
whether or not to enter after seeing Q_1, and the payoffs would
accrue accordingly.[3] But with C_i unknown to firm j, the firms no
longer know which game they are playing, since each is uncertain
about the values of both its own and its opponent's payoffs
corresponding to any particular decisions. Analysis of this
situation of incomplete information would easily appear to fall
into the miasma of infinite regress, with each firm conjecturing
about its opponent's true costs, about what its opponent believes
the original firm's costs to be, about what its opponent believes
the original firm believes the opponent's costs to be, and so
on. However, Harsanyi [5] has presented an approach to solving
such games which avoids this pitfall.

Specifically, in Harsanyi's approach one considers a game

with complete (but imperfect) information in which there is a
third player, "nature," which is indifferent over all possible
outcomes in the game. Nature moves first, and its move is to
select C_1 and C_2 at random from $[\underline{C}_1, \overline{C}_1] \times [\underline{C}_2, \overline{C}_2]$ according to
some distribution. (It is natural that the $H_i(\cdot|\cdot)$ should be the
conditional distributions corresponding to this original
distribution, and we will assume that this is, in fact, the
case.[4] We will also assume that $H_i(\cdot|C_i)$ has a density $h_i(\cdot|C_i)$
for each C_i. These densities are positive over $[\underline{C}_j, \overline{C}_j])$. Each
firm i is informed about the value of C_i resulting from nature's
move, but not about the C_j value. Then for each realization
of C_1 and C_2, the game tree unfolds as described above. In
this set up, then, there is imperfect information, in that the
players are not fully informed about the outcome of nature's
move, but there is not incomplete information, since everyone
knows fully the game being played (i.e., they know the game tree
and what the payoffs are at the end of each branch of the
tree). Harsanyi then argues that for equilibrium analysis, the
Nash equilibria of this second, imperfect information game should
be taken to be the equilibria of the original, incomplete
information game. It is this approach we adopt here.

In games in extensive form, such as we are now considering,
a strategy is a specification of what action the agent will take
in each possible information set. Here, an information set for
firm 1 is given by a particular value for his costs, while for
firm 2, an information set is given by a particular realization
of C_2 and the particular level of output, Q_1, selected by firm

1. Since firm 1's actions consist of possible levels for Q_1, a strategy for firm 1 is a function $s : [\underline{C}_1, \overline{C}_1] \to R_+$, where $Q_1 = s(C_1)$ is the output choice made by the firm if its costs are C_1. For firm 2, a strategy is a map $t : [\underline{C}_2, \overline{C}_2] \times R_+ \to \{0,1\}$, where we interpret $t(C_2, Q) = 0$ to mean that 2 does not enter if its costs are C_2 and the established firm produces Q, and $t(C_2, Q) = 1$ to mean that it does enter.[5]

We assume that each firm seeks to maximize the expected present value of profits. With these criteria, we define an equilibrium (in pure strategies) to be a pair (s,t) of strategies such that for each $C_1 \in [\underline{C}_1, \overline{C}_1]$, s maximizes the expected present value of firm 1's profits, given that firm 2's strategy is t, and for each $C_2 \in [\underline{C}_2, \overline{C}_2]$, t maximizes the expected present value of 2's profits, given that firm 1's strategy is s. In each case, the expectation is with respect to the maximizing agent's conditional distribution over the other firm's costs, given its own cost level.

Letting $R(C_1, C_2) \equiv \Pi_1^M(C_1) - \Pi_1^C(C_1, C_2)$ denote the reward accruing to firm 1 in the future if it prevents entry, we can write the equilibrium conditions for our game explicitly as: (s,t) is an equilibrium if: for any $C_1 \in [\underline{C}_1, \overline{C}_1]$ and any $s* : [\underline{C}_1, \overline{C}_1] \to R_+$,

$$\Pi_{C_1}^o(s(C_1)) + \delta_1 E\{R(C_1, C_2)[1 - t(C_2, s(C_1))] | C_1\}$$

$$> \Pi_{C_1}^o(s*(C_1)) + \delta_1 E\{R(C_1, C_2)[1 - t(C_2, s*(C_1))] | C_1\};$$

and, for any $C_2 \varepsilon [\underline{C}_2, \overline{C}_2]$ and any $t^*:[\underline{C}_2, \overline{C}_2] \times R_+ \rightarrow \{0,1\}$,

$$E\{[\delta_2 \Pi_1^C(C_1,C_2) - K]t(C_2,s(C_1)) \mid C_2\}$$

$$\geq E\{[\delta_2 \Pi_2^C(C_2,C_2) - K]t^*(C_2,s(C_1))\mid C_2\}.$$

Note that writing the second argument of t as $s(C_1)$ rather than Q is acceptable _in equilibium._ This does not, however, mean that t is a function of s. Rather, it represents part of the usual Nash, "best reply" condition in the present context.

To interpret the equilibrium conditions, recall that $t(C_2,s(C_1)) = 1$ means that 2 enters when its costs are C_2 and 1 produces $s(C_1)$, while $t(C_2, s(C_1)) = 0$ means that 2 stays out. Then the term $R(C_1,C_2)[1-t(C_2,s(C_1))]$ is zero if 2 enters, in which case 1 collects only the Cournot profit, and it is $R(C_1,C_2)$ if 2 does not enter, in which case 1 collects the Cournot profit plus the bonus $R(C_1,C_2)$, which is the difference between the monopoly and Cournot profits. Similarly, 2's payoff will be $[\delta_2 \Pi_2^C(C_1,C_2) - K]$ when $t(C_2,s(C_1)) = 1$, and zero when $t(C_2,s(C_1)) = 0$.

The general question of existence of Bayes equilibria in pure strategies appears to be an open one, and we do no attempt to provide any general treatment of the existence question here. Further, to simplify our search for equilibria, we will concentrate on a particular subclass of possible strategies. (We will later justify this aproach by giving sufficient conditions for existence of equilibrium strategies of the particular type we will be considering). One should definitely note, however, that although we concentrate on equilibrium strategies in a special

class, they are in fact equilibria when tested against all possible alternative strategies s and t.

Specifically, suppose that firm 1's equilibrium strategy, s, is a strictly deceasing function of C_1. It is on this class of monotonic strategies that we will concentrate. Some motivation for considering such strategies is that the simple monopoly solution is a strictly decreasing function of C_1 as long as D is differentiable.

If firm 2 conjectures that firm 1 has adopted the decreasing strategy $\bar{s}$, then, to respond optimally, it must behave as if C_1 were equal to $\bar{s}^{-1}(Q)$. It will then enter only if its costs are low enough to make entry profitable when 1's costs are $\bar{s}^{-1}(Q)$. Let $g(Q) \equiv Y(\bar{s}^{-1}(Q))$ be the level of C_2 at which, under 2's conjecture, entry will be just barely worthwhile. Then 2's optimal response must satisfy

$$t(C_2, Q) = \begin{cases} 1 & \text{if } C_2 \leqslant g(Q) \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

Now suppose firm 1 conjectures that 2 has adopted a strategy of the general form (1). Specifically, 1 conjectures that 2 will enter if $C_2 \leqslant \bar{g}(Q)$ for some function $\bar{g}$. Then for each C_1, an optimal quantity $Q = s(C_1)$ must maximize

$$\Pi^o_{C_1}(Q) + \delta_1 \int_{\bar{g}(Q)}^{\vec{C}_2} h_1(x|C_1) R(C_1,x) \, dx.$$

For each C_1, the first order condition for a maximum is

$$0 = \frac{d\Pi^0_{C_1}}{dQ} - \delta_1 R(C_1, \bar{g}(Q)) \, h_1(\bar{g}(Q)|C_1) \, \bar{g}'(Q).$$

In equilibrium, each firm's strategy must be optimal against the other's actual strategy, not merely against arbitary conjectures. Thus, in addition to firm 1's first order condition and condition (1), we require that $s \equiv \bar{s}$ and $g \equiv \bar{g}$. It is convenient to view these requirements as rational expectations conditions. Each firm forms expectations ($\bar{s}$ and $\bar{g}$) concerning how its competitor will respond in various situations, and in equilibrium these expectations are required to be consistent with the competitor's actual optimal decision rule.

Given these rational expectations conditions, we have that $g(Q) \equiv \gamma(s^{-1}(Q))$. Substituting this and the identity $C_1 \equiv s^{-1}(Q)$ into firm 1's first order condition leads to the equation

$$0 = \frac{d\Pi^0_{C_1}}{dQ} - [\delta_1 R(C_1, \gamma(C_1)) \, h_1(\gamma(C_1)|C_1) \, \gamma'(C_1) \, / \, s'(C_1)], \qquad (2)$$

where primes denote derivatives. In Theorem 1, we provide a condition which guarantees the sufficiency of this first order condition for the optimality of s.

Notice that condition (2) defines a differential equation for s. The equation has a family of solutions, indexed by a boundary condition. The equilibrium we seek involves firm 1 playing one such s and firm 2 playing the corresponding t described by (1).

If $s' < 0$ (as we have assumed) and if the function R^*

defined by $R^*(C_1, C_2) = R(C_1, C_2) h_1(C_2|C_1)$ is strictly positive
at $(C_1, \gamma(C_1))$, then the first order condition (2) implies that
$d\Pi^o_{C_1}/dQ$ is strictly negative. Thus, limit pricing does emerge:
the established firm has an incentive to increase quantity and
decrease price from their monopoly levels. The origin of this
incentive is that given a strategy for firm 1, a higher value of
Q_1 is perceived as indicating a lower value of C_1, which in turn
leads to a reduced probability of entry. Thus, any equilibrium
which meets (2), the first order necessary condition under our
assumptions, must involve limit pricing.

In particular, the simple monopoly solution does not meet
these conditions. Examining this monopoly outcome to show that
it is not an equilibrium may give some feel for the incentive to
adopt a limit pricing strategy. The simple monopoly solution is
defined for each C_1 by $d\Pi^o_{C_1}/dQ = 0$. Let $m(\cdot)$ denote the
corresponding strategy, which we assume is defined and
differentiable for all $C_1 \in [\underline{C}_1, \overline{C}_1]$, and recall that m is a
strictly decreasing function. Now, suppose that firm 2 believes
that 1 is playing m (as would be the case if m were an
equilibrium strategy). Then, if firm 1 were to choose an output
of $m(C_1) + \epsilon$ instead of $m(C_1)$, firm 2 would (incorrectly) infer
that 1's costs were $C_1^* \equiv m^{-1}(m(C_1) + \epsilon)$, which is strictly less
than $m^{-1}(m(C_1)) = C_1$ for all positive ϵ. From 1's point of view,
this has the effect of eliminating entrants with costs between

$\gamma(C_1)$ and $\gamma(C_1^*)$, the effect of which on expected second period profits is of the order of ε if R^* is positive. The cost of this action in terms of current profits is, however, at most of the order of ε^2, since $d\bar{\pi}_{C_1}^O/dQ$ is, by definition, zero at $m(C_1)$. Thus, there is an incentive to deviate from the simple monopoly solution, which consequently cannot be an equilibrium strategy.

Note that if the entry costs, K, are zero, and the Cournot solution will obtain after entry, then equilibrium will involve no limit pricing, since $R(C_1, \gamma(C_1))$ will be zero. The logic is the following: if K is positive, the marginal entrant (with costs of $\gamma(C_1)$) must enter with strictly positive output in order to generate enough operating revenues to recover K. This positive output has a marked effect on firm 1's profits, so it is worthwhile to attempt to reduce the probability of entry by raising Q_1. But if $K = 0$, the entry will occur as long as an infinitesimal excess of revenues over operating cost can be realized, i.e., whenever the Cournot price, as determined by C_1 and C_2, exceeds C_2. Thus, the marginal entrant has $P = C_2$, so from the usual Cournot equilibrium condition, $Q_2 \ \partial P/\partial Q$ is zero for the marginal entrant. With a downward sloping demand, this means that Q_2 is zero, and, in turn, $\pi_1^M(C_1)$ and $\pi_1^C(C_1, \gamma(C_1))$ are equal, so that $R(C_1, \gamma(C_1))$ is zero. Thus, there is no incentive to deter the marginal entrant. This point will be illustrated in the example in the next section.

However, given that $R^*(C_1, \gamma(C_1))$ is positive and given the assumptions made above on s and π_2^C, limit pricing does emerge.

Yet, we have assumed that s was strictly decreasing and, thus, invertible. Thus, when firm 2 makes its entry decision in equilibrium, it correctly infers firm 1's costs. The limit pricing thus has no impact in equilibrium in terms of disguising the established firm's costs, and exactly the same entry will occur as if no limit pricing were being practiced and the potential entrant were to be aware of this.

This result may seem counterintuitive, and indeed we find it striking. One might ask why the monopolist foregoes present profits by limit pricing in this context. The answer is that in equilibrium if firm 1 were to produce $Q < s(C_1)$, firm 2 would infer that 1's costs are $s^{-1}(Q) > C_1$. The additional expected entry this induces yields lower expected future profits, and this more than offsets the immediate gains. On the other side, as we argued above, if limit pricing is not being practiced (and is known not to be), there is an incentive to attempt to misrepresent one's costs by expanding output.

Actually, this result perhaps ought not to be surprising if one is acquainted with the signalling literature. In our model, Q_1 serves as a signal regarding C_1, and in this context there is no reason to expect that the resulting choices will be Pareto optimal. The literature on revelation of preferences suggests similar results, and Ortega-Reichert's [9] solution of the repeated bidding problem involves a parallel phenomenon to that discussed here. However, to allay any suspicions that the differential equation defined by the first order condition (2) which we have been considering is not, in fact, a solution to

firm 1's problem, we present in the following theorem a
sufficient condition for the first order necessary conditions to
define an equilibrium.

Theorem 1. Suppose that s is continuous and strictly decreasing
and that it satisfies the differential equation (2) on an
interval $(0, \bar{\bar{C}})$ with $\bar{\bar{C}} \geq \bar{C}_1$ and $s(\bar{\bar{C}}) \equiv m(\bar{C}_1)$. Let t be defined by
(1) for Q in the range of s and by

$$t(C_2, Q) = \begin{cases} t(C_2, s(\underline{C}_1)) & \text{for } Q > s(\underline{C}_1) \\ t(C_2, m(\bar{C}_1)) & \text{for } Q < m(\bar{C}_1) \end{cases} \qquad (1')$$

Then, if the marginal revenue function is decreasing and if

$$s'(Z) < \inf_{C \in [0, \bar{\bar{C}})} \delta_1 \gamma'(Z) \frac{R*(C, \gamma(Z)) - R*(Z, \gamma(Z))}{Z - C}$$

for all Z, (s,t) is an equilibrium pair.

Proof.

We have argued earlier that t is an optimal response to s.
We must show that, with t fixed, s is optimal.

Suppose firm 1's costs are C_1. Firm 1 will not wish to
produce less than $m(C_1)$, the monopoly solution, since to do so
results in both lower current profits and lower expected future
profits than does $m(C_1)$. Similarly, firm 1 will not choose to
produce more than $s(0)$, since to do so would reduce current
profits without increasing expected future profits. Note that
$s(0) > s(C_1) > m(C_1)$ so $s(C_1)$ satisfies these necessary
conditions. We show below that 1's optimization problem is
quasi-concave on $[s(\bar{\bar{C}}), s(0)]$, so that the first order conditiion

(2) is sufficient on this range. Since $s(\bar{\bar{C}}) < m(C_1)$, the first-order condition then guarantees the global optimality of the quantity $s(C_1)$.

Let $G(Z,Q)$ denote firm 1's expected payoff when its costs are Z and its quantity is Q, and let $G_2(Z,Q)$ denote $\partial G/\partial Q$ evaluated at (Z,Q). Then the differential equation (2) becomes $G_2(Z,s(Z)) \equiv 0$. Let $Q \in (s(\bar{\bar{C}}),s(0)]$ and let $Z = s^{-1}(Q)$. Then

$$G_2(C_1,Q) = G_2(C_1,Q) - G_2(Z,Q)$$

$$= (Z-C_1) - \delta_1 \gamma'(Z)[R*(C_1,\gamma(Z)) - R*(Z,\gamma(Z))]/s'(Z)$$

which is non-positive for $Q > s(C_1)$ (i.e., for $Z < C_1$) and non-negative for $Q < s(C_1)$ (i.e., $Z > C_1$) if

$$s'(Z) < \delta_1 \gamma'(Z) \frac{R*(C_1,\gamma(Z)) - R*(Z,\gamma(Z))}{Z-C_1}$$

Our assumption guarantees that this holds for all Z and C_1. This proves that $G(C_1,Q)$ is quasi-concave with an optimum at $Q = s(C_1)$.   Q.E.D.

It is instructive to describe the results so far obtained in the language of the signalling literature. Accordingly, we make the following identifications between the elements of our model and the concepts employed by Spence [17] in his job market signalling model. Let the ability of the established firm be given by $-C_1$, so that lower costs correspond to greater ability. Let the quantity Q be the signal and let $\Pi^o_{C_1}(m(C_1)) - \Pi^o_{C_1}(Q)$ be the cost of signalling at level Q (to a

firm of ability $-C_1$). Finally, let
$\delta_1 E\{R(C_1,C_2)[1 - t(C_2,Q)]|C_1\}$ be the benefit of signalling at
level Q.

In Spence's analysis, any benefit from signalling comes in
the form of higher wages, and the relative value of various wage
levels is independent of ability. This corresponds in our model
to the case $\partial R*/\partial C_1 = 0$. Signalling equilibria arise in Spence's
analysis when the marginal cost of signalling (e.g., acquiring
education) at any level decreases with ability. When the
equilibrium strategy s is strictly decreasing (and hence
invertible) the equilibrium is called a separating equilibrium.

In the limit-pricing model, the benefit of signalling is not
generally independent of the firm's ability, so the usual
conditions on the cost of signalling (which are satisfied in our
model) do not guarantee the existence of a separating
equilibrium. The inequality condition in Theorem 1 generalizes
the assumption that $\partial R*/\partial C_1 = 0$, which, as mentioned earlier, is
the relevant benefit-independence assumption.

Signalling models generally have many equilibria, some of
which (the pooling equilibria) are not separating. One may then
naturally wonder whether such equilibria exist in the limit
pricing context considered here. The following lemma and theorem
explore this question for an interesting special case.

<u>Lemma.</u> Suppose that (i) - (iv) hold.

 (i) C_1 and C_2 are independently distributed.

 (ii) $\partial R/\partial C_1 < 0$

 (iii) $R(C_1,C_2) > 0$ for all (C_1,C_2)

(iv)   the marginal revenue function, MR,

            is decreasing.

If (s,t) is an equilibrium pair with $s \geq m$, then s is non-

increasing.

Proof. From the independence of C_1 and C_2 and the

assumption that Π_2^C is monotone decreasing in C_2 one may infer

that there exists a function g such that for any Q in the range

of s,

$$t(C_2, Q) = \begin{cases} 1 \text{ if } C_2 < g(Q) \\ 0 \text{ otherwise.} \end{cases}$$

Given (i), let $h_1(C_j | C_i) = h_i(C_j)$ for all C_i, and define

$$A(C, Q) = \delta_1 \int_{g(Q)}^{\overline{C}_2} R(C, C_2) h_1(C_2) dC_2.$$

$A(C,Q)$ is the expected present value of the bonus from preventing

entry when $C_1 = C$ and $Q_1 = Q$.

Suppose that, contrary to the lemma, there exists C' and C"

with C' < C" such that $Q' = s(C') < s(C") = Q"$. By the

optimality of Q" for C", we have

$$[D(Q") - C"]Q" + A(C", Q") \geq [D(Q') - C"]Q' + A(C", Q')$$

So,

$$\delta_1 \int_{g(Q')}^{g(Q")} R(C", C_2) h_1(C_2) dC_2$$

$$= A(C", Q') - A(C", Q")$$

$$\leq [D(Q") - C"]Q" - [D(Q') - C"]Q'$$

$$= \int_{Q'}^{Q''} (MR(Q) - C'') \, dQ$$

$$\leqslant \int_{Q'}^{Q''} (MR(Q) - C') \, dQ \leqslant 0$$

The last inequality follows from the assumptions that MR is decreasing and that $Q' = s(C') > m(C')$, and from the fact that, at the monopoly solution, marginal revenue equals marginal cost: $MR(m(C')) = C'$. It then follows that $g(Q'') \leqslant g(Q')$: the probability of entry at Q' exceeds that at Q''. Note that this is intuitively sensible. The high cost firm C'' has deviated further from the simple monopoly outcome. To compensate and thus to prevent it from reducing output, it must enjoy less threat of entry at Q'' than at Q'. Then since $\partial R / \partial C_1 \leqslant 0$,

$$A(C',Q'') - A(C',Q')$$

$$= \delta_1 \int_{g(Q'')}^{g(Q')} R(C',C_2) h_1(C_2) \, dC_2 \geqslant \delta_1 \int_{g(Q'')}^{g(Q')} R(C'',C_2) h_1(C_2) \, dC_2$$

$$= A(C'',Q'') - A(C'',Q').$$

Hence,

$$[D(Q'') - C']Q'' + A(C',Q'') - A(C',Q')$$

$$\geqslant [D(Q'') - C'']Q'' + (C'' - C')Q'' + A(C'',Q'') - A(C'',Q').$$

By the optimality of Q'' for C'', this latter expression is greater than or equal to

$$[D(Q') - C"]Q' + (C" - C')Q"$$

$$= [D(Q') - C']Q' + (C" - C') (Q" - Q')$$

$$> [D(Q') - C']Q'.$$

This contradicts the optimality of Q' for C' and the result then follows.

Q. E. D.

Thus, under conditions (i) - (iv), so long as the equilibrium s never involves lower output and higher prices than under simple monopoly, s cannot increase anywhere.

Theorem 2. Suppose the conditions (i)-(iv) of the lemma hold and that, in addition,

(v) $\gamma(\underline{C}_1) > \underline{C}_2$ and $\gamma(\overline{C}_1) < \overline{C}_2$

If (s,t) is an equilibrium pair with s > m then one of the following three possibilities must hold:

    a) s is strictly decreasing.

    b) s is discontinuous and non-increasing; or

    c) s is a constant function, i.e., $s(C_1) \equiv s(\underline{C}_1)$.

Thus, all continuous non-constant solutions are strictly decreasing.

Proof. In view of the lemma, it suffices to show that if s is continuous but not constant, then s is strictly decreasing. Suppose to the contrary that for some $\hat{Q}$, $s^{-1}(\hat{Q}) = [C',C"]$ where

$C' < C''$ and is continuous . By assumption, either $C' > \underline{C}_1$
or $C'' < \overline{C}_1$. Consider first the case $C' > \underline{C}_1$.

Using the notation developed in the proofs of Theorem 1 and
the lemma, firm 1's conditional payoff function is given
(for $Q \in [s(\overline{C}_1), s(\underline{C}_1)]$) by

$$G(C,Q) = (D(Q) - C)Q + \int_{g(Q)}^{\overline{C}_2} R(C,C_2)h_1(C_2)dC_2.$$

Since $\partial \Pi_2^C / \partial C_1 > 0$, one can check that

$$\int_{C'}^{C''} \Pi_2^C(C_1, \Upsilon(C'))h_2(C_1)dC_1 / (\overline{H}_2(C'') - \overline{H}_2(C')) - X > 0$$

where $H_2(\cdot) = H_2(\cdot | C_2)$ for all C_2. Then using the continuity of
$\Pi_2^C(C_1, C_2)$ in C_2, one can infer that $g(\hat{Q}) > \Upsilon(C')$:   expected
entry is greater when Q is observed than would take place if C_1
were known to equal C'. One can also check that
for $Q > \hat{Q}$, $g(Q) < \Upsilon(C')$. Hence, for each C, $\lim_{Q \downarrow \hat{Q}} G(C,Q)$
$> G(C,\hat{Q})$, from which it follows that $\hat{Q}$ cannot be
optimal for any cost level.

For the case $C'' < \overline{C}_1$, a similar argument shows that for some
$\varepsilon > 0$ and all $C \in [C'', \overline{C}_1]$, if $Q \in (\hat{Q} - \varepsilon, \hat{Q})$ then
$G(C, \hat{Q}) > G(C,Q)$. But by continuity, there is some
$C \in [C'', \overline{C}_1]$ such that $s(C) = \hat{Q} + 1/2\varepsilon$. It follows that $s(C)$ is
not optimal for C, contrary to the equilibrium assumption.

<div align="right">Q.E.D.</div>

Theorem 2 offers a wide menu of possible equilibria, but a
closer look reveals that the equilibria of types b) and c) are
tenuous ones. If s is not strictly decreasing, then there is
some $\hat{Q}$ such that $s^{-1}(\hat{Q})$ is a non-degenerate interval,
say $s^{-1}(\hat{Q}) = [C',C'']$. Why, then does a firm with costs C' not
increase output a bit to distinguish itself from the higher cost
producers? In the preceding proof, it was shown that such a
maneuver would benefit the firm if s were continuous and $\hat{Q}$ were
not the maximum value of s. To understand what goes wrong in
cases b) and c), let us regard t as representing 1's conjecture
about how 2 will behave. If s is fixed and t is an optimal
response to s, then $t(C_2,Q)$ can be determined for each C_2 and
each Q in the range of s. But for Q not in the range of s, any
specification of $t(C_2,Q)$ is consistent with the optimality of
t. Thus, if $\hat{Q} + \epsilon$ is not in the range of s, firm 1 may
conjecture that an output level of $\hat{Q} + \epsilon$ would lead to as much or
even more entry than an output level of $\hat{Q}$. Essentially this
conjecture involves 1 believing that 2 ignores the fact that
producing $\hat{Q} + \epsilon$ rather than $\hat{Q}$ is less costly for a low cost
firm than for a high cost one. It is precisely this rather odd
sort of hypothesis about 2's behavior that is required to support
an equilibrium (s,t) where s is not strictly decreasing.
Further, as we show below in Theorem 3, under a slight
strengthening of the conditions already assumed there always
exists at least one equilibrium with s strictly decreasing.
Together these points suggest that concentrating on the
invertible equilibria is appropriate. These are, of course, the

ones where equilibrium limit pricing does not limit entry in the
sense made precise above.

Theorem 3. Suppose that, in addition to conditions (i)
through (v) of Theorem 2, h_1 is continuously differentiable
and π_2^C is twice continuously differentiable. Then there exists
an equilibrium pair (s,t) with s strictly decreasing and t
defined as in Theorem 1.

Proof. Consider the differential equation $\sigma' = f(Q,\sigma)$ where

$$f(Q,\sigma) = \frac{MR(Q) - \sigma}{\delta_1 \gamma'(\sigma) R^*(\sigma, \gamma(\sigma))} . \quad (3)$$

We will the establish the existence of a solution $C_1 = \sigma(Q)$ to
this equation, and then show that σ is invertible. Theorem 1
then implies that this inverse, s, and the corresponding t
function are an equilibrium pair.

In order to show existence of a solution to the differential
equation, we need f to be continuous in Q and differentiable in
σ for $\sigma \in [\underline{C}_1, \overline{C}_1]$ and $Q \in R_+$. For this, it is sufficient that
marginal revenue is continuous, that h_1 is positive and
continuously differentiable on $[\underline{C}_1, \overline{C}_1]$, that π_2^C is twice
continuously differentiable and that R is strictly positive.
Then, by a theorem in differential equations (see Pontryagin
[10], pp. 20-22), there exists a maximal Q* such that
$\sigma(m(\overline{C}_1)) = \overline{C}_1$, a boundary condition, and such that σ satisfies
(3) on $[m(\overline{C}_1), Q^*)$. Two possibilities then arise, according to
whether Q* is finite or infinite.

If Q* is finite, then $\sigma(Q) \to -\infty$ as $Q \to Q^*$, and in

particular, $\sigma(Q)$ includes all of $[\underline{C}_1, \overline{C}_1]$ in its range. That σ is unbounded below in this case can be seen by noting that σ could be undefined for Q greater than Q^* only if it becomes unbounded as Q^* is approached from below, and that the slope of σ is negative or zero, which insures that it goes to minus infinity in this case. The claim regarding the slope of σ is a result of the fact that $\sigma(Q) \geq MR(Q)$ with equality if $Q = m(\overline{C}_1)$. To see this, note that the boundary condition $\sigma(m(\overline{C}_1)) = \overline{C}_1$ means that $\sigma'(m(\overline{C}_1)) = 0$, while $dMR/dQ < 0$ by assumption. Thus, σ exceeds MR on some interval $(m\overline{C}_1), m(\overline{C}_1) + \varepsilon)$. Moreover, $\sigma(Q) - MR(Q)$ can never decline to zero, because if $\sigma(Q) = MR(Q)$, then $\sigma'(Q) = 0 > MR'(Q)$. It follows that σ exceeds MR for all $Q > m(\overline{C}_1)$.

If Q^* is infinite, we must verify that $\underline{C}_1 > \displaystyle\inf_{Q \geq m(\overline{C}_1)} \sigma(Q)$ in order for the range to include $\underline{C}_1$. But if this did not hold, then whenever $MR(Q) < \underline{C}_1$, i.e., $Q > m(\underline{C}_1)$, we would have

$$\sigma'(Q) < \frac{MR(Q) - \underline{C}_1}{\displaystyle\sup_{Z \,\varepsilon\, [\underline{C}_1, \overline{C}_1]} \delta_1 Y'(Z) R^*(Z, Y(Z))}.$$ Then, provided that

$MR(Q) < \underline{C}_1$ for large enough Q, which will be the case if the simple monopoly solution exists, there would exist B such that $\sigma'(Q) < B < 0$, and so $\displaystyle\lim_{Q \to \infty} \sigma(Q) = -\infty$.

Thus, the range of σ includes $[\underline{C}_1, \overline{C}_1]$ and, as we have already argued, σ' is negative except at $m(\overline{C}_1)$. Thus, $s \equiv \sigma^{-1}$ exists, and is easily seen to be a solution to the differential equation (2). With condition (i),

$\partial R/\partial C_1 < 0$ implies the condition in Theorem 1, so that (s,t) is an equilibrium.

<div align="right">Q.E.D.</div>

The preceding results point to the existence of an equilibrium in which the monopolist's strategy is a continuous, decreasing function of its costs, in which case it will be a solution of (2). However, typically there will be a whole family of solutions to this differential equation, with the solutions parameterized by a boundary condition. Further, under our conditions, a whole range of these would be equilibria. This non-uniqueness is something of a problem (as is the non-uniqueness of Nash equilibrium in any model), since the theory does not provide a complete guide to behavior: 2 must actually know which solution 1 is using and 1 must know (and use) the solution that 2 believes 1 is using. While we would insist that this is not a problem that is particular to our model, but rather is one that arises in any sort of "rational expectations" framework where inferences are being made, it does remain a difficulty.

However, there is one solution which is most appealing on economic grounds in our model. This is the one given by the boundary condition $s(\overline{C}_1) = m(\overline{C}_1)$. In it, if the monopolist's actual costs are the highest level conceivable, it does not engage in limit pricing. (In all other solutions, $m(\overline{C}_1) < s(\overline{C}_1)$). The appeal of this solution is that at $\overline{C}_1$, there is no need to lower price and forego present profits to

distinguish one's self from higher cost firms, since none could
exist. In any of the invertible equilibria, a firm with costs
of $\overline{C}_1$ will stand revealed as a weakling: why then should it give
up profits when it will gain nothing? In fact, the only way that
a solution to (2) with $m(\overline{C}_1) < s(\overline{C}_1)$ can be an equilibrium is by
specifying t in such a way that an observed value of Q_1 which is
less than $s(\overline{C}_1)$ brings on additional entry. This corresponds to
the entrant's believing that such a Q_1 value was produced by a
firm with costs higher than the highest level conceivable. If t
is not specified in this way, a monopolist with costs $\overline{C}_1$ will
always depart from the strategy given by (2) and produce $m(\overline{C}_1)$.

The importance of this is, of course, that by the uniqueness
theorem of differential equations, there is a unique solution to
(2) with $m(\overline{C}_1) = s(\overline{C}_1)$. It is this equilibrium we examine in the
next section.

III. An Example

In this section we consider an example with linear demand
and particular distributions describing the agents' beliefs about
costs. We are able then to compute an equilibrium explicitly and
analyze it.

Specifically, suppose $P = D(Q) = a - bQ$, $\underline{C}_i = 0$ and
$\overline{C}_i < a/2$, $i = 1,2$. (These latter conditions guarantee that the
first order conditions define the Cournot equilibrium.) Then we
have the following explicit forms:

$$\Pi^o_{C_1}(Q) = (a-C_1)Q - bQ^2$$

$$\Pi^M_1(C_1) = (a-C_1)^2/4b$$

$$\pi_1^C(C_1,C_2) = (a-2C_1+C_2)^2/9b$$

$$\pi_2^C(C_1,C_2) = (a+C_1-2C_2)^2/9b$$

$$Y(C_1) = (a+C_1-3\sqrt{bK})/2$$

$$R(C_1,Y(C_1)) = [2(a-C_1)\sqrt{bK} - bK]/4b$$

Note that if $K=0$, then $R(C_1,Y(C_1)) = 0$ for all C_1. Then, as we argued earlier, there will be no limit pricing in equilibrium. Thus, we will henceforth assume that K is strictly positive. In addition, we will assume that $Y(\bar{C}_1) < \bar{C}_2$, so that even a high cost established firm has some incentive to attempt to misrepresent its cost. Finally, we will assume that $Y(0) > 0$, or equivalently, that $a > 3\sqrt{bK}$, so that even zero-cost firm face some threat of entry. We will relax this latter assumption later.

With this particular specification, the first order condition (2) yields the following differential equation

$$0 = (a-C_1) s'(C_1) - 2bs'(C_1)s(C_1)$$

$$- \frac{\delta_1}{8b} h_1(Y(C_1)|C_1)[2(a-C_1)\sqrt{bK} - bK]$$

or, substituting for Y, and rearranging terms.

$$s'(C_1) = \frac{\delta_1 h_1(\frac{a+C_1 - 3\sqrt{bK}}{2}|C_1)[2(a - C_1)\sqrt{bK} - bK]}{8b[a - C_1 - 2bs(C_1)]}$$

We still have to specify the density function describing beliefs. To meet the first order conditions, we need in fact specify it only at points $(C_1, Y(C_1))$, $C_1 \in [0,\bar{C}_1]$. One simple formulation which allows an explicit solution for s is to let

h_1 be any member of the class of densities parameterized by ρ with

$$h_1(\gamma(C_1)|C_1) \equiv h_1(\frac{a + C_1 - 3\sqrt{bK}}{2}) = 8b\rho/(2(a-C_1)\sqrt{bK} - bK).$$

This expression remains bounded and non-negative as long as K and the parameter ρ are both strictly positive and $a > \sqrt{bK}$, which we have already assumed. The relevant differential equation then is $s'(C_1) = \delta_1 p/(a-C_1-2b\ s(C_1))$.

The relevant part of the family of solutions to the differential equation is graphed in Figure 1. Members of the family below the lowest one shown are not defined on all of $[0,\bar{C}_1]$ since, for the lowest one graphed, $s'(C_1)$ goes to infinity as $C_1 \rightarrow \bar{C}_1$. It is this strategy whose existence is the subject of Theorem 3 and which is the unique one with $s(\bar{C}_1) = m(\bar{C}_1)$. Those solutions that lie too high in the (C_1,Q) - plane fail the non-negativity of profits test.

All the solutions are monotonic, decreasing functions, as posited, and involved limit pricing. However, as suggested in the last section, we will concentrate on the one in which a firm with $C_1 = \bar{C}_1$ adopts the monopoly price and quantity. We call this principal solution.

Since the principal solution has an infinite derivative at $\bar{C}_1$, integrating directly is a problem. However, the differential equation for the inverse function, which is given by

$$\frac{I}{C_1'(Q)} = \frac{\delta_1 \rho}{a - C_1(Q) - 2bQ}$$

is well-behaved. Rewrite this as

$$C_1'(Q) + \frac{C_1(Q)}{\delta_1 \rho} = \frac{a - 2bQ}{\delta_1 \rho},$$

multiply both sides by $\exp(Q/\delta_1\rho)$, and note that the resulting
expression on the left hand side is the derivative of
$C_1(Q) \exp(Q/\delta_1\rho)$ with respect to Q. Then integrating yields

$$C_1(Q) = a - 2bQ + 2b\delta_1\rho + k \exp(-Q/\delta_1\rho)$$

where k is a constant of integration. Thus, s, the solution to
the original equation,[6] satisfies

$$C_1 = a - 2bs(C_1) + 2b\delta_1\rho - k \exp[(-s(C_1)/\delta_1\rho].$$

Using the boundary condition $s(\bar{C}_1) = n(\bar{C}_1) = (a - \bar{C}_1)/2b$,
we then obtain

$$0 = \frac{n(\bar{C}_1) - s(C_1)}{\delta_1 \rho} + 1 - \exp \frac{n(\bar{C}_1) - s(C_1)}{\delta_1 \rho} \qquad (4)$$

By Theorem 1, this solution with the appropriate
specification of t constitute an equilibrium.

It is relatively straightforward to obtain comparative
statics results from (4), recognizing that n and s depend on a
and b as well as on C_1 and that, in addition s depends on
ρ, δ_1 and $\bar{C}_1$. First, letting $A = [n(\bar{C}_1) - s(C_1)]/\delta_1\rho$, one
obtains

$$\partial s/\partial\rho = \delta_1 \frac{1 + (A-1)\ \exp(A)}{1 - \exp(A)}$$

$$\partial s/\partial\delta_1 = \frac{\rho\ [1 + (1-A)\ \exp(A)]}{1 - \exp(A)}$$

and

$$\partial s/\partial\bar{C}_1 = \frac{\exp(A)}{2b[1- \exp(A]}$$

Since $1 - \exp(A) = [s(C_1) - m(C_1)]/\delta_1\rho$ from (4) and since
$s(C_1) > m(C_1)$, the denominators in each of these expressions is
positive for $C_1 < \bar{C}_1$. Thus, one immediately sees that $s(C_1)$
increases as $\bar{C}_1$, the highest conceivable cost level, increases
(assuming tht h_1 is modified appropriately) . The intuition is
that the possiblity of cost levels higher than
$\bar{C}_1$ causes a firm at $\bar{C}_1$ to begin limit pricing in order to
distinguish itself from higher cost firms, and then, again to
distinguish themselves, all lower cost firms must further
increase their outputs. As well, since
$1 - \exp(A) > 0$, we have $A < 0$. Thus, $1 + (1-A)\ \exp(A) > 0$ and so
both $\partial s/\partial\rho$ and $\partial s/\partial\delta_1$ are positive: as the probability of
marginal entry or the value of second period profits increase,
limit pricing increases.

To develop a tractable example, we selected a density
function h_1 which is defined in terms of a, b, and K. For this
reason, the usual sort of comparative statics analysis on a, b,

and K does not isolate the effect of variations in the demand
parameters and entry costs. The effect of the particular density
we have chosen is to make the bonus to deterring entry at the
margin a constant: $R^*(C_1, \gamma(C)) \equiv \delta_1 \rho$. Thus, comparative
statics analysis on a and b isolate the effects of variations in
the <u>first period</u> demand parameters only.

In considering the impact of changes in a and b on limit
pricing, one must recognize that these parameters affect the
monopoly solution m as well as the limit pricing solution s. It
is appropriate, therefore, to consider the effect of these
variation on $s(C_1) - m(C_1)$. The results are easily obtained:

$$\frac{\partial s}{\partial a} = \frac{1}{2b} = \frac{\partial m}{\partial a}$$

and

$$\frac{\partial s}{\partial b} < \frac{-(a-C_1)}{2b^2} = \frac{\partial m}{\partial b} \ .$$

Thus an increase in a simply shifts m and s upward by equal
amounts while an increase in b narrows the gap between m and
s. Both of these affects can be understood in terms of the
impact of the parameter variations on the marginal revenue
function. The marginal cost of signalling at level Q for a firm
with costs C_1 is $C_1 - MR(Q)$. The marginal benefit at the
optimal level is, as noted earlier, $\delta_1 \rho$. At equilibrium,

$$\delta_1 \rho = C_1 - MR(s(C_1)) = MR(m(C_1)) - MR(s(C_1)).$$

Variations in a do not alter the slope of MR, and so these variations do not affect $s(C_1) - m(C_1)$. Increases in b make the MR line steeper ($dMR/dQ = -2b$), and so tend to reduce $s(C_1) - m(C_1)$. In effect, increases in b raise the marginal opportunity cost of signalling, and this accounts for the reduction in signalling.

To analyze the role of entry costs K, let us abandon the specific density used in our example. Instead, let h be any positive density on $[\underline{C}_2, \bar{C}_2]$ where $0 < \underline{C}_2 < \bar{C}_2 < a/2$. If there are no barriers to entry ($K = 0$), then as observed earlier, there is no bonus earned from deterring the marginal entrant: $R(C_1, \Upsilon(C_1)) \equiv 0$. Moreover, in this case, even the highest cost competitors can enter profitably against low cost established firms, because $\Upsilon(C_1) > a/2$ for all $C_1 > 0$.

Consequently, entry deterrence is hopeless and no limit pricing takes place. Similarly, if barriers to entry are very high ($K > a^2/4b$), then no competitor could ever enter profitably, and no limit pricing takes place. (In terms of analytics, both high and low values of K lead to $h(\Upsilon(C_1)) \equiv 0$. Then equation (2) implies that $s \equiv m$). Only intermediate values of K (i.e., moderate barriers to entry) lead to limit pricing.

It may be useful to be able to visualize our equilibrium solution graphically, as in Figure 2. In the first quadrant we show the principal equilibrium strategy s and the simple monopoly outcome m. The line in the second quadrant gives the identity map, while that in the third quadrant is the graph of

γ for one value of K. Using these parts of the diagram, one can construct the strategy for the entrant which is optimal, given any strategy assumed for the established firm.

In particular, suppose the entrant observes Q^*. Then, given s, this corresponds to costs of C_1^*. Tracing through the second and third quadrants, this yields $\gamma(C_1^*)$ as the marginal cost at which the entrant just recovers the entry costs K. Then if $C_2 < \gamma(C_1^*)$, firm 2 should enter, and if its costs exceed this level, it should stay out. Repeating this process for each value of C_1 in $[0,\bar{C}_1]$ yields the solid line in the fourth quadrant for $Q \in [s(\bar{C}_1), s(0)]$ Then, for values of Q in this range, the optimal strategy is defined in terms of entering or not depending on whether (C_2,Q) is to the right or left of the line in the fourth quadrant.

One can use this diagram to help see that the simple monopoly solution does not correspond to an equilibrium. The argument, however, is basically that given earlier. The dotted line in the fourth quadrant is constructed from m using the above procedure, and thus is the best response to m. Now we must see if m is, in turn, a best response to it. But this can be seen not to be the case. For example, if the established firm has costs $C_1' < \bar{C}_1$, it faces entry by firms with costs less than C_2'. By increasing output slightly to $m(C_1'')$, which has an effect on profits of $(C_1' - C_1'')^2/4b$, it causes itself to be seen as a firm with costs C_1''. This eliminates entry by firms in the region from $\gamma(C_1')$ to $\gamma(C_1'')$. The length of this region is of the order of $(C_1'' - C_1')$. Thus, the effect on first period profits is of the

order of $-\epsilon^2$, where ϵ is the increase in production, while if $R^* > 0$, the effect on second period expected profits is of the order of ϵ. Thus, at least a small deviation from m is worthwile, and m cannot be an equilibrim strategy for the established firm.

This graphical technique in fact does not generate a full strategy for firm 2, since it determines only the entry decision for Q values in the range of s, while 2's strategy must be defined for all $Q \in R_+$. As noted earlier, for other values of Q, a variety of arbitrary specifications of t will do, since in the equilibrium play of the game values of Q outside this range will not arise. In particular, as long as t is specified so that an observed $Q > s(\underline{C}_1)$ leads to no less entry than does $s(\underline{C}_1)$ and an observed $Q < s(\bar{C}_1)$ leads to as much entry as does $s(\bar{C}_1)$, then we have a full equilibrium which also meets the requirements of Selten's "perfect equilibria"[16].

Let us now consider the possibility that entry costs are so high as to preclude successful entry against low-cost established firms but not against high-cost firms. This is the case where $a^2/4b > K > a^2/9b$. In this case, the γ function as defined above is negative for low values of C_1, and an established firm with low costs faces no threat of entry. Yet, somewhat surprisingly, such firms may still practice limit pricing.

To see this let $\gamma(C_1^*) = 0$, i.e., $C_1^* = 3\sqrt{bK} - a > 0$. For $C_1 \in [C_1^*, \bar{C}_1]$, the differential equation (2) still specifies the established firm's strategy which will be given by the function $s(\cdot)$ given above. For $C_1 \in [0, C_1^*)$, no potential entrants could successfully enter, and thus there is no direct reason to adopt a

limit pricing strategy. However, if $m(C_1) \epsilon [s(C_1^*), s(\bar{C}_1)]$, a firm which produces the simple monopoly quanity is producing the same quantity as some fim with costs of $s^{-1}(m(C_1))$, which is greater than C_1^*. Against such a non-invertible strategy, the optimal response for the potential entrant is to enter after observing a quantity Q if the conditional expected profit exceeds K, where the expectation is with respect to firm 2's beliefs about C_1, conditioned on its own costs C_2 and on the event $Q = s(C_1)$. Then, if C_1 is less than but close to C_1^*, the established firm will in fact be threatened by entry. (Of course, if such entry occurs, the entrant will not recover the full amount K). This threat provides the firm with an incentive to increase its output from the monopoly level. Thus, these firms practice limit pricing, even though they face no threat of successful entry.

In particular, if firms in the range $[0,C_1^*)$ set their output equal to max $\{m(C_1),s(C_1^*)\}$, they can prevent all entry. Under certain specifications of $H_1(\cdot|\cdot)$, this simple rule with the corresponding best reply by firm 2 constitutes an equilibrium. (This situation is graphed in Figure 3).

It is clear that in the range $[0,C_1^*)$, there is no incentive to produce more than max $\{m(C_1),s(C_1^*)\}$, since this involves foregoing first period profits and has no payoff in terms of preventing entry. Also, there is no reason to produce less than $m(C_1)$. The issue is then whether a firm with costs $C_1 < C_1^*$ will find it advantageous to produce less than $s(C_1^*)$. It is easy, however, to verify that the conditions to the lemma in the

previous section are met on $[0, C_1^*]$ if we assume that C_1 and C_2 are independently distributed. (Note that we must respecify h_1 in any case, since for $C_1 < C_1^*$, $Y(C_1)$ as defined above is outside $[0, \bar{C}_2]$). The lemma then implies that s is non-increasing so that $s(C_1) \geqslant s(C_1^*)$ for $C_1 < C_1^*$. The conclusion then follows.

IV. Conclusions and Possible Extensions

By modeling the decision making of both the established firm and the potential entrant via a game of incomplete information, we have not only obtained the familiar result that it is in the interest of the established firm to lower prices in an attempt to limit entry, but also the surprising result that in equilibrium this limit pricing does not limit entry. This result has very much the flavor of rational expectations: in equilibrium, potential entrants cannot be consistently fooled, or more precisely, their beliefs cannot be systematically biased. Of course, it should by now be clear that if the monopolist were to change the quantity it was producing from its equilibrium value, there would be an impact on entry: indeed, this effect is the reason that limit pricing arises in our model. Rather, the comparison is between the entry which occurs in our equilibrium with limit pricing and that which would occur if no limit pricing were to take place and the entrant were to be aware of this.

Presumably, what would happen in the sort of situation we are considering is that established firms would not want to get involved in playing the losing game we have studied here. Two possibilities then would seem open.

First, the price output behavior adopted by the established

firm might not be what we have assumed. Although the strategies in our model are elements of function spaces, the equilibrium we have examined is a simple Nash equilibrium. As such, it is subject to the usual objections to Nash equilibria. These are familiar to economists from the Cournot model. One might, for example, then conjecture that some analogue of the Stackelberg model would describe behavior better.

In this context, a Stackelberg equilibrium would involve the established firm predicting that for any strategy which it selects, the entrant will choose a best response to its strategy. The established firm would take account of this predicted response in selecting its strategy. It appears that the simple monopoly solution would result.

The main problem with this Stackelberg solution--and, indeed, with any non-Nash solution--is that it is likely to be quite unstable. Unlike the Cournot model, the limit pricing model involves strategies which cannot be fully observed. For this reason, firms in our model can deviate from equilibrium behavior with impunity whenever a deviation is profitable. Only Nash equilibrium strategies have the stability property that neither firm ever finds a unilateral deviation to be profitable.

A second possibility for the established firm would be to attempt to separate its current price and quantity choices from the competitor's entry decision. The industrial organization literature suggests one possible approach to this: develop other methods of detering entry such as excess capacity, advertising, etc. (See e.g., Spence [18] and Salop [13]). The difficulty

with these techniques is that they would not seem to remove completely the link between current costs and output. Thus, they would not eliminate the incentive to engage in limit pricing, even though it is ultimately a fruitless strategy. An alternative approach which would be effective in the context of our model is to develop an independent, credible signal for C_1. Then, if the established firm's costs were know to the potential entrant via this signal, the established firm would be free to practice short-term profit maximization. This suggests a previously unrecognized function for the publication of financial statements which have been audited by a public accounting firm. By certifying as to the accuracy of the cost figures in the report, the CPA firm is providing credibility for such an independent signal and thereby is promoting monopoly pricing!

If one takes this suggestion seriously, the apparent reluctance of firms to provide financial data by product line must be attributed either to their perceiving no gain in deterring entry by publication of this information or to the presence of other benefits to maintaining secrecy. Possible bases for the latter alternative would seem to lead us far from the present model, but the former reason for secrecy might be explained not only by a hypothesis of ignorance but also in terms of some of the possible extensions of the model we consider now.

First one might want to investigate situations where the established firm has a more general cost structure, with two or more unknown parameters being necessary to identify the cost function. In such situations, the single variable, output, would

not be a sufficient signal for cost, and full invertibility would
not be possible. In a related vein, if first period demand is
not know exactly by the entrant (but is by firm 1) or if
Q_1 (or P) is not observed precisely, our analysis is again not
directly applicable. As well, if one models the second period
more fully to allow for such phenomena as predatory pricing by
the established firm, then a second characteristic of the
established firm, its ability or willingness to carry out
predation successfully, becomes important in the entry
decision. Again price or quantity cannot signal all the relevant
information. We plan to examine in future research such
situations in which there is more than one relevant
characteristic which is uncertain.

However, even if we admit the possibilty of noisy signals
which do not allow the exact inferences which are possible in the
present model, the basic characteristic of the rational
expectations, Bayes equilibrium--that beliefs cannot be
consistently biased in equilibrium--should not be altered. It
may be, however, that the equilibruim may influence how
informative the monopolist's behavior is to the entrant. In the
model we have analyzed, both m and s are invertible, so both are
equally informative. This is why both lead to entry in exactly
the same circumstances. In more general formulations, three
other possiblities arise: m may yield strictly finer information
than s (as already occurs in our model when there is a pooling
equilibrium); s may yield finer information than m; or they may
be informationally non-comparable.

Suppose first that limit pricing blurs information. If
entry would only rarely be profitable under full information,
limit pricing may deter entry by increasing the risk faced by the
entrant. But if entry is usually profitable, limit pricing which
deprives the entrant of information may encourage entry in
situations where the entrant with better information would stay
out. Similarly, if limit pricing it provides better cost
information than does naive monopoly behavior, limit pricing may
either discourage or encourage entry in particular situations
relative to the low information situation. Finally, it is clear
that in the non-comparable case, entry might again be either more
or less likely under limit pricing than under naive behavior.
The basic message of this discussion, and, indeed, that of our
whole paper, thus reappears: There is no basis for an immediate
presumption that limit pricing has any impact in deterring entry.

FOOTNOTES

1. F.M. Scherer ([14], p. 219) gives references for 12 papers which he considers to be among the more important contributions to the area written before 1970.

2. This set-up allows the possibility of the established firm acting very aggressively against early entrants in order to develop a reputation which will deter later potential entrants. This sort of behavior cannot easily be explained in a complete information context (Selten [15], Rosenthal [12], but would seem to constitute a potentially important phenomenon. In fact, both we and D. Kreps and R. Wilson are currently studying this issue.

3. It is clear that no limit pricing would arise in this game.

4. Harsanyi discusses conditions for given distributions H_1 and H_2 to be the marginal distributions for some distribution and argues that this "consistent case" is the appropriate one for modeling purposes. However, much of what follows here does not depend on this consistency assumption.

5. It may seem odd that, even though firm i knows its true cost level, a strategy must specify what it would do if its costs were something other than they actually are. This construction is, however, crucial to avoiding the infinite regress noted above. It may help to think of the strategy for i as giving what j conjectures that i would do for any of the cost levels that i might have. In equilibrium, this must be a correct conjecture.

6. Note that we obtain two branches for s in this fashion, corresponding to the increasing and decreasing parts of $C_1(Q)$. Only the branch with $s > m$ relevant.

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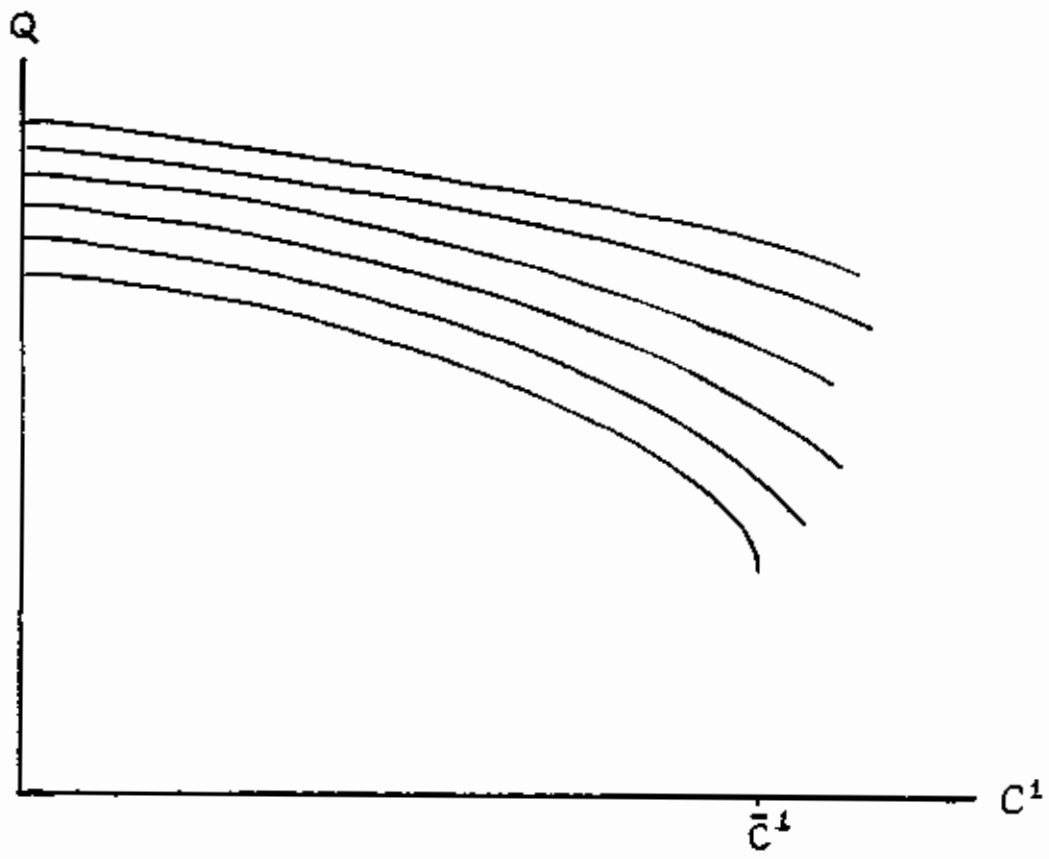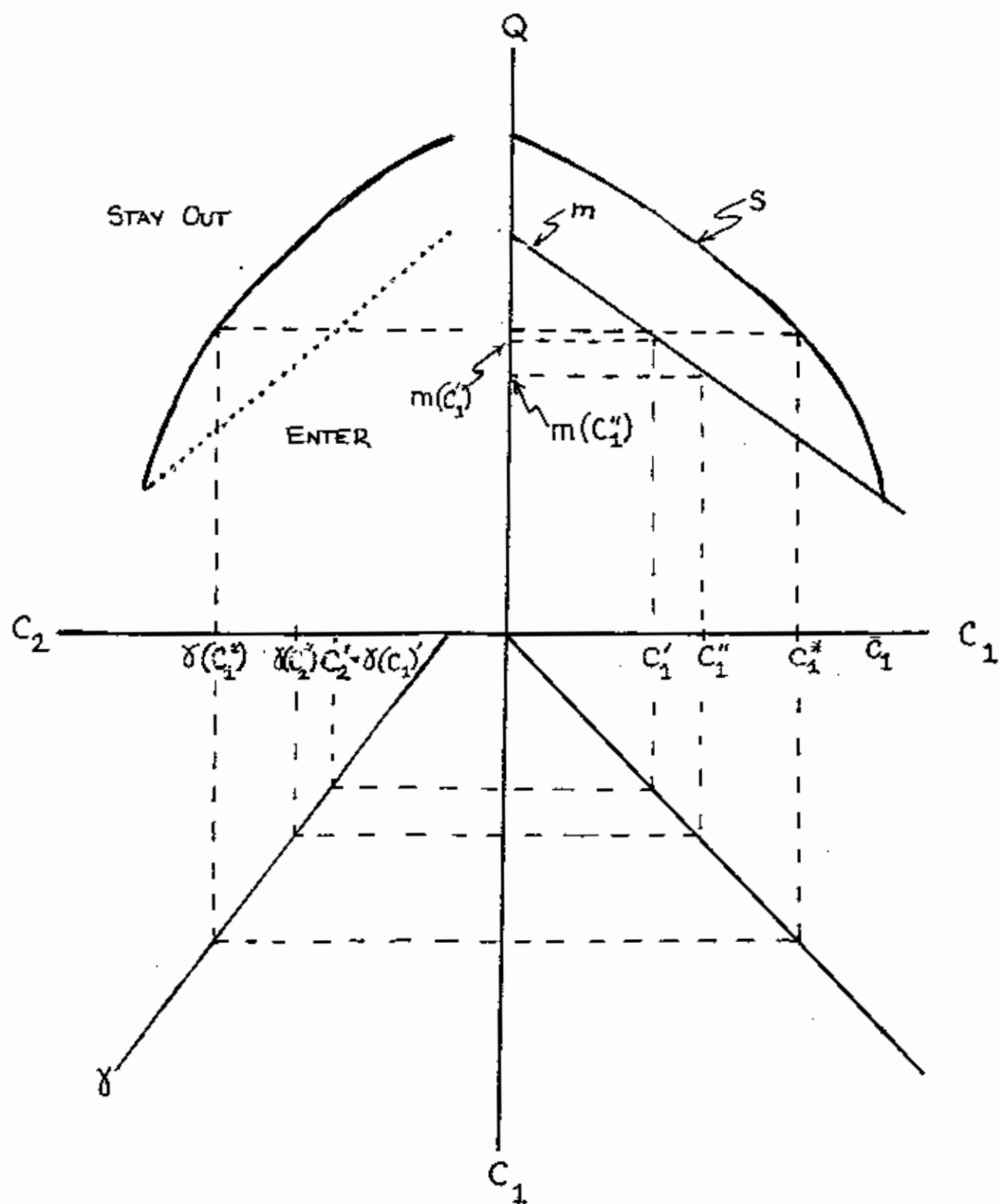   (Autumn, 1977), pp.534-544.

FIGURE 1

FIGURE 2

FIGURE 3