Discussion Paper 390

INFLATION AND GROWTH IN THE LONG RUN: A CRITICAL NOTE ON MODELS OF KARNI AND MUNDELL

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1. Introduction

The earlier writings of Robert Mundell on inflation and real interest (1963; 1971, chapter 2) examine the effects of inflationary expectations on the real interest rate and the level of real money balances in a short-run model in which the stock of capital is treated as a constant. The basic conclusions of the model are that an increase in inflation causes the nominal interest rate to rise by less than the increase in inflation, resulting in a lower stock of real money balances and a higher level of saving and investment.

Long-run extensions of that model are presented by Karni (1972) and Mundell (1971, chapter 5) to take into consideration the effects over time of changes in the capital stock. Karni's basic model is formulated with an assumption of all-inclusive investment. It concludes that the economy converges to a stationary state and that the real interest rate and level of real wealth occurring in the stationary state are independent of the rate of inflation. The effects of inflation are to reduce the level of real money balances and increase the stock of capital occurring in the stationary state such that the level of real wealth remains unchanged.

Mundell's long-run model (1971, chapter 5) differs from Karni's model mainly in that it includes money as an argument of the production function. In this case inflation leads to a lower real interest rate in the stationary state since the decline in the ratio of real money balances to capital lowers the marginal product of capital. However, the other conclusions are qualitatively the same as Karni's: inflation reduces the real money stock and increases the capital stock occurring in the stationary state.

The purpose of this note is to argue that the basic conclusions of the long-run models of Karni and Mundell flow from certain critical but untenable assumptions concerning the saving function, and that a more realistic saving assumption yields radically different results. Karni and Mundell have made saving a function of wealth but not income.[1] This assumption is reasonable within the framework of Mundell's short-run model (1963; 1971, chapter 2) since income is fixed in that model. However, when income is a variable, as is the case in the Karni and Mundell long-run models, then a reasonable saving function should include income as an argument. They have essentially assumed that the marginal propensity to consume out of income (for a given level of real wealth) is unity. This article will examine modified versions of these models in which it is assumed that the fraction of income saved is a function of the ratio of wealth to income. (A discussion of the rationale for this saving assumption can be found in such works as Ando and Modigliani (1963) and Friedman (1976).) This modification, along with other reasonable restrictions to be specified below, leads to the conclusion that an increase in the rate of inflation induces 1) a permanently lower real interest rate, 2) a permanently greater rate of growth in the capital stock with no tendency for the economy to converge to a stationary state, and 3) a temporary decline in real money balances followed by a permanently more rapid rate of growth in real balances, so that in the longer run, real money balances increase with inflation.

In the subsequent discussion it will be assumed that the anticipated rate of inflation is always equal to the actual rate, so that the terms "inflation rate" and "rate of inflationary expectations" can be regarded as synonymous.

2. The Karni Model and a Modified Version

Short-run equilibrium in the Karni model is determined by the system of equations

$$I(\rho - r) = S(\omega)$$

$$m = L(i, \omega)$$

$$\omega \equiv m + \frac{cY}{r}$$

where Y is real income, r is the real interest rate, i is the nominal interest rate, I is real investment, S is real saving, m is real money balances, L is money demand, ω is real wealth, and c is the fraction of income going to capital (assumed to be a constant.) It is assumed that $I' > 0$, $S' \gtrless 0$, $L_i < 0$, and $L_\omega > 0$ (where a subscript denotes a partial derivative.) ρ is the marginal efficiency of capital. Since this is a model of a one-commodity economy ρ equals the marginal product of capital if there is no depreciation of capital. If it were assumed that capital depreciates at a constant proportional rate then ρ would equal the marginal product of capital minus the depreciation rate. Henceforth we shall simply refer to ρ as the marginal product of capital, with the understanding that if capital depreciates, this refers to the marginal product of capital net of depreciation.

Long-run equilibrium is characterized by the conditions that saving and net investment equal zero.

Karni works with an all-inclusive concept of capital and hence the marginal product of capital is constant. In figure 1 the marginal product of capital is represented by the height of the horizontal MPC line. This is the locus of points at which investment demand equals zero. (Karni calls this curve the I=0 curve.)

For a given capital stock the S=0 curve represents the locus of points at which saving equals zero, and the IS curve represents the locus of points at which saving equals investment demand. For a given capital stock and rate of inflationary expectations, the LM curve represents the locus of points at which the money market is in equilibrium. A short-run equilibrium is determined by the intersection of the IS and LM curves. A long-run equilibrium or stationary state occurs when the intersection of the IS and LM curves coincides with the intersection of the S=0 and MPC curves.

Let us consider an economy which is initially at the stationary state and review Karni's analysis of how the economy responds to an increase in inflationary expectations. The initial position of the economy is at E_o (figure 1) where the LM_o, IS_o, $(S=0)_o$, and MPC curves all intersect. There is no saving or capital accumulation at this point. Now if there is an increase in inflationary expectations the LM curve will shift downward by the amount of the increase in inflationary expectations -- say to LM_1. The new short-run equilibrium occurs at E_1, where the IS_o and the LM_1 curves intersect. The real interest rate has fallen, the nominal interest rate has risen, and the stock of real money balances has fallen. Also, net saving and capital accumulation begin. As the capital stock increases, the LM curve moves to the right. Under Karni's saving and investment functions the IS and S=0 curves shift to the left. This process continues until a new stationary state occurs at E_2, where the MPC, $(S=0)_2$, LM_2, and IS_2 curves all intersect. Here real wealth and the real interest rate are restored to the levels that existed prior to the inflation. However, the capital stock is higher and the stock of real money balances is lower than in the initial stationary state.

Now consider a modified version of Karni's model in which it is assumed that the fraction of income saved is a function of the ratio of wealth to income and that the fraction of income invested is a function of the difference between the marginal product of capital and the real interest rate. Hence

$$s\left(\frac{m+\frac{cY}{r}}{Y}\right) = v(\rho - r) \tag{1}$$

defines the IS curve, and

$$s\left(\frac{m+\frac{cY}{r}}{Y}\right) = 0 \tag{2}$$

defines the S=0 curve, where s is the fraction of income saved and v is the fraction of income invested.

The essential difference between this model and the Karni model is how the IS and S=0 curves shift in response to a change in the capital stock. Under the Karni assumptions, the IS and S=0 curves shift to the left with an increase in the capital stock. However, if (1) and (2) are assumed, the IS and S=0 curves shift to the right in proportion to the increase in income; the reasoning is simple: for a given real interest rate, an increase in real income requires a proportional increase in real money balances for the same wealth to income ratio to be maintained. Hence for a given real interest rate, a proportional increase in income and real balances maintains equalities (1) and (2).

If it is furthermore assumed that the money demand function

$$L(\omega, Y, i)$$

is linearly homogeneous in wealth and income, then an increase in the capital stock also induces a rightward shift in the LM curve which is exactly proportional to the increase in income. Since an increase in the capital stock induces proportional shifts in the IS, LM, and S=0 curves, the IS-LM equilibrium point never

catches up with the S=0 curve. There is no tendency for the economy to converge to a stationary state. As the capital stock increases, income, wealth, and real balances all increase in the same proportion and the real and nominal interest rates remain constant.

In this model the decline in the real interest rate which accompanies an increase in inflation is not temporary but permanent. Furthermore, the decline in real money balances that accompanies an increase in inflationary expectations is only temporary; eventually the stock of real money balances becomes greater than it otherwise would have been.

3. The Long-Run Mundell Model and a Modified Version

There are two basic differences between the Karni model and the long-run Mundell model (1971, chapter 5). First, in the Karni model the marginal product of capital is constant as a consequence of the assumption that investment is all-inclusive. The Mundell model, on the other hand, has a production function containing real money balances and capital as arguments:

$$\text{output} = X(m,K)$$

where K is "capital". (This model could be interpreted as representing an economy in which investment augments material capital and human capital but not monetary capital, where K is an aggregate of material and human capital.) In this model the MPC function (in figure 2) slopes upward since an increase in real balances increases the marginal product of capital.

Secondly, there is a sense in which Mundell's long-run model (1971, chapter 5) differs not only from Karni's model, but also from Mundell's own models in earlier chapters of the same book. Karni assumes there is a relation defining a flow demand for investment as a function of the difference between the marginal product

of capital and the real interest rate. The Mundell (1971, chapter 5) model, on the other hand, assumes that the real interest rate is always equal to the marginal product of capital. There is no function defining a flow demand for investment[2] and hence there is no IS curve.[3] Short-run equilibrium in the Mundell (1971, chapter 5) model is, within the geometrical framework of figure 2, determined by the intersection of the LM curve and the MPC curve. (The MPC curve could, however, be viewed as a limiting case of an IS curve as marginal adjustment costs of capital expansion approach zero.)

The Karni and Mundell (1971, chapter 5) models do have the common feature that saving is made a function of wealth but not income. We shall, as with the Karni model, compare Mundell's model to a revised version in which the saving function is modified.

In order to make clear the similarities and differences between the Karni and Mundell models we shall examine the Mundell model within the same geometrical framework used by Karni. We shall suppose that the economy is initially at a stationary state and then consider how the economy adjusts over time after the rate of inflationary expectations moves to some higher level and remains at that level. This is a slightly different dynamic exercise than the one performed by Mundell, who treats the rate of monetary expansion as the exogenous variable while inflation is endogenous; since the inflation rate depends not only on the rate of monetary expansion but also on the rate of growth of output, Mundell's dynamic exercise allows the inflation rate to vary over time even as the exogenous variable -- the rate of monetary expansion -- remains constant. The approach we adopt will be, instead, to suppose that the monetary authority targets a certain rate of inflation and successfully maintains that rate, so that inflation

is the exogenous variable. The two approaches are really quite similar. This change is necessary in order to enable us to construct Mundell's model within the geometrical framework used by Karni.

Suppose that the economy is initially at a stationary state. This is represented by point E_o in figure 2 where the $(S=0)_o$, MPC_o, and LM_o curves all intersect. If there were an increase in inflationary expectations, the LM curve would shift downward and the new equilibrium would be at E_1 where the LM_1 and MPC_o curves intersect. Since the short-run equilibrium is now to the left of the S=0 curve, net saving and capital accumulation occur. As this happens the MPC curve and the LM curve each move to the right. Since Mundell assumes that the production function is linearly homogeneous in money and capital and that money demand is linearly homogeneous in capital, the shifts of the LM and MPC curves are each in proportion to the increase in the capital stock.

Under Mundell's saving assumption the S=0 curve moves to the left as the capital stock increases. Hence eventually the economy converges to a stationary state like E_2 where the LM_2, MPC_2, and $(S=0)_2$ curves all intersect.[4]

Now consider a modified version of the model in which the fraction of income saved is assumed to be a function of the wealth to income ratio. In this case an increase in the capital stock causes the S=0 curve to shift to the right in proportion to the increase in the capital stock. As with our modified version of the Karni model, the short-run equilibrium point never catches up with the S=0 curve; real balances, capital, and income increase proportionally forever and there is no tendency for the economy to approach a stationary state.[5] Clearly the decline in real balances resulting from an increase in inflationary expectations is only temporary and is offset over time by the greater rate of growth in real balances.

4. Conclusions

It has been shown that many of the results of the Karni (1972) and Mundell (1971, chapter 5) models flow from the critical but untenable assumption that saving is a function of wealth but not income. The Karni and Mundell models both lead to the result that the stationary state is dynamically stable and that inflation causes the stock of real balances occurring in the stationary state to become lower. In addition, the Karni model leads to the result that the real interest rate occurring in the stationary state is independent of the rate of inflation. The preceding sections have demonstrated that if these models are modified so that the fraction of income saved is made a function of the ratio of wealth to income, then the basic conclusions of the Karni and Mundell models no longer hold. In our modified versions of both the Karni and Mundell models, there is no tendency for the economy to approach a stationary state. An increase in inflationary expectations leads to a permanent rather than a temporary increase in growth. Furthermore the decline in real money which accompanies an increase in inflationary expectations is only temporary; this is followed by a permanently greater rate of growth in the stock of real balances, so that in the longer run an increase in inflation causes the stock of real money balances to become greater than it otherwise would have been. Also, unlike the Karni model, the decline in the real interest rate induced by an increase in inflationary expectations in not temporary, but permanent.

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FOOTNOTES

1/ In Karni's article this assumption is stated explicitly. In Mundell's (1971, chapter 5) model it is implicit; Mundell's conclusion that inflation reduces the level of real balances but not the level of wealth occurring in the stationary state (pages 43 and 49) follows from this assumption.

2/ In this type of model the market-clearing mechanism must be viewed as a system of differential equations in which prices and interest change at a rate necessary to induce firms to absorb the capital supplied by manufacturers. For other macroeconomic models based on an assumption of a continuous adjustment of the stock of capital to the firms' optimal level, see Floyd-Hynes (1978), Foley-Sidrauski (1970a, 1970b), and Thompson (1977).

3/ In chapter 5 of his book, Mundell calls the LM curve the ii schedule and he calls the MPC curve the rr schedule. It is tempting to interpret the rr schedule in Mundell's book (page 44) as an IS schedule since it has the same slope that an IS schedule would have and because an IS curve was used in earlier chapters of the same book. However, this is not how Mundell defines the rr schedule. More importantly, the rr schedule in Mundell's model does not behave the way an IS schedule would behave. Within the geometrical framework used by Mundell on pages 47-49 (which is not the same geometrical device that will be applied in this article), an increase in the capital stock would, under Mundell's saving assumption, have two effects if the rr curve were to be interpreted as an IS curve: 1) The rr schedule would shift to the left, and 2) The $\lambda\lambda$ schedule would

move closer to the rr schedule. If the rr schedule were interpreted as a MPC schedule, then an increase in the capital stock would have only the second effect but not the first; this is what Mundell assumes.

4/ One of the differences between the dynamic exercise we just performed and the one performed by Mundell is that under Mundell's approach, during the "inter-run" period the rate of inflationary expectations is less than that which eventually occurs in the stationary state; consequently Mundell's approach yields the result that the real interest rate is higher in the inter-run than in the stationary state.

5/ The same result can be demonstrated with the geometrical device used by Mundell (pages 47-49); it merely needs to be observed that if the fraction of income saved is a function of the wealth to income ratio, then an increase in the capital stock would not alter any of the curves in Mundell's figure 5-3 (page 47).

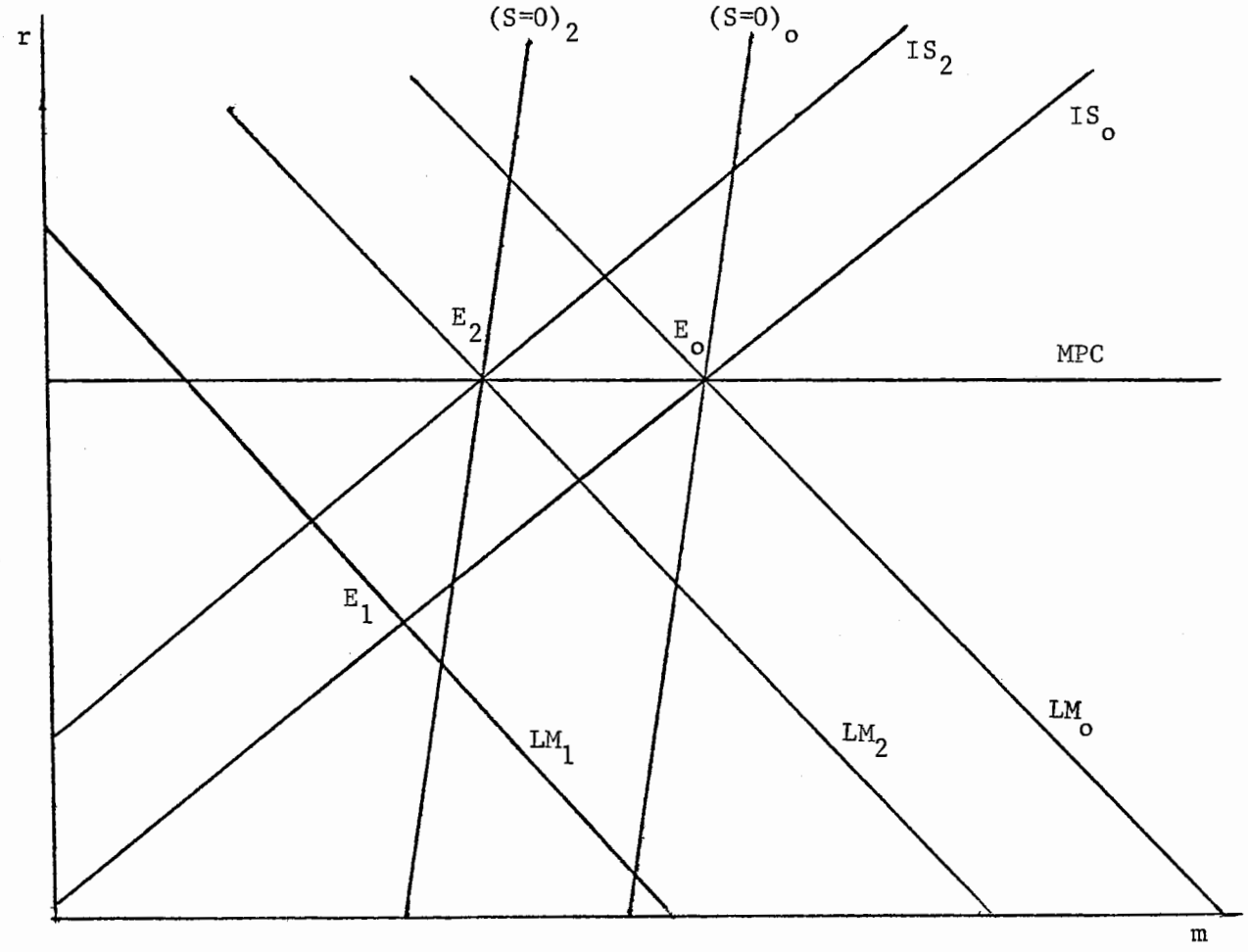

Figure 1

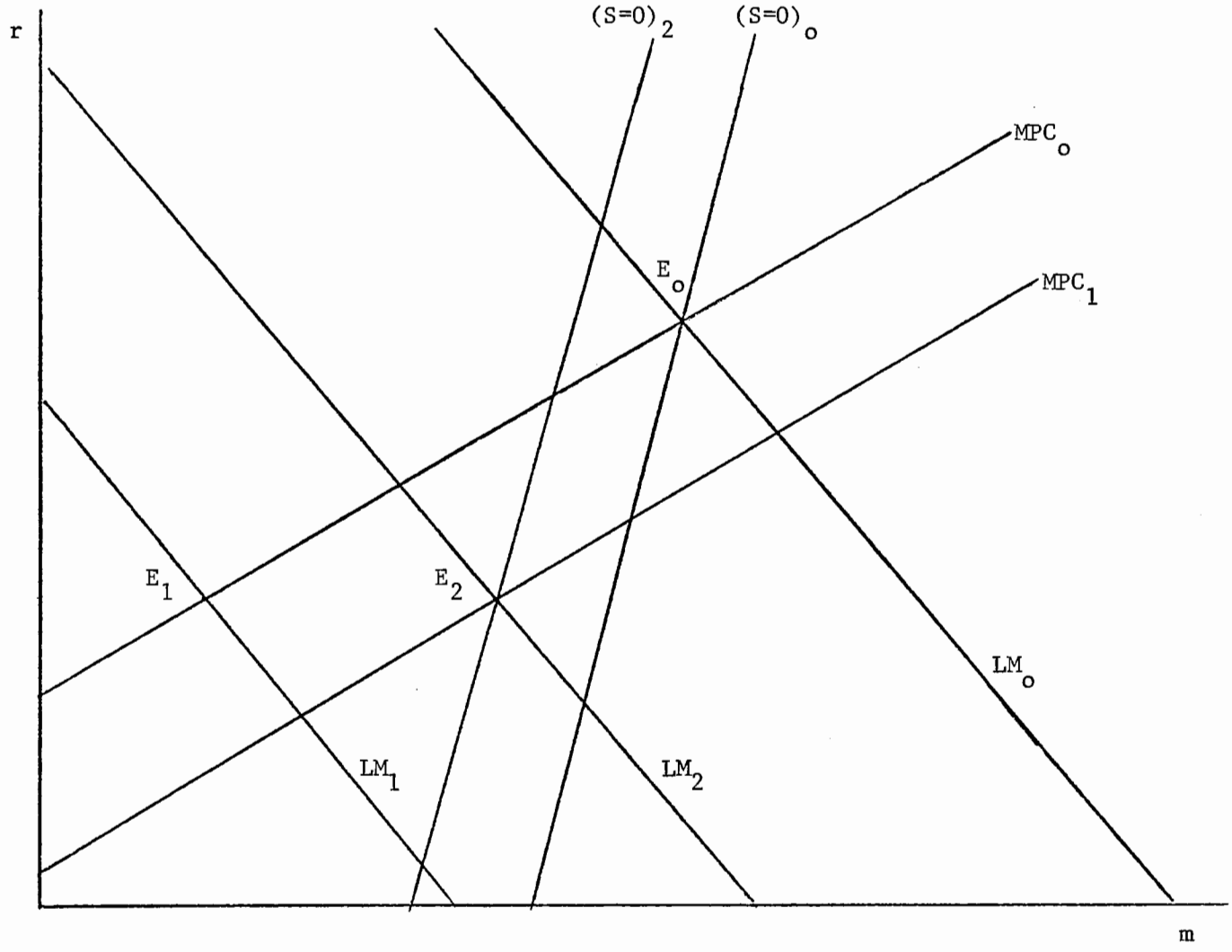

Figure 2