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DEVIATIONS FROM OPTIMAL PRICING OF LOWER COLORADO RIVER WATER*

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INTRODUCTION

The Colorado River system is the major source of water for irrigated agriculture, industry, and direct consumption in western Colorado, Utah, western Arizona, southern Nevada, and southern California.[1] Although it drains a large geographical area, its flow is both quite small relative to the demand for it and highly variable. In the first quarter of this century competing claims for the water among those five states led to a bitter battle over both the distribution of water rights and the location and size of a possible storage and flood control dam and reservoir system. This battle culminated in approval in 1928 of the Boulder Canyon Project Act and the Colorado Compact of 1922. At that time the building of Boulder Dam and Lake Mead reservoir by the U.S. Bureau of Reclamation were sanctioned and the river flow was divided between the Upper Basin (Colorado, Utah, and extreme northern Arizona) and the Lower Basin (Nevada, California, and the rest of Arizona). The division was to be at Lee's Ferry, just north of the Grand Canyon. The agreement was that the Upper Basin would deliver 75 million acrefeet of water to the Lower Basin through Lee's Ferry every 10 year period. Later battles divided the Lower Basin water among the three states and among users within states. A U.S. treaty with Mexico forced both basins to agree to deliver together 1.5 million acrefeet per year of low-saline-content water to Mexico.

At the time the compact was drafted, the mean annual flow of the river was thought to be 15-16 million acrefeet above Lee's Ferry and 1 million acrefeet below, so that each basin was granted about half on average. Since then the estimate has been lowered to about 13-14 million acrefeet total, but the compact has not been revised to accord with this new estimate.

At the same time, the demand for water from the Colorado system is growing significantly. Because Arizona has not been able to use its full allotment until now, California has expanded its consumption nearly 1 million acrefeet per year beyond the 4.4 million acrefeet per year it is legally allowed. In the early 1980's, however, with the completion of the Central Arizona Project to supply water to Tucson and Phoenix, Arizona can legally demand that California restrict its consumption to 4.4 million acrefeet per year if California's additional consumption competes with Arizona's right to receive deliveries of 2.8 million acrefeet per year. In addition, exploitation of oil shale deposits in the Upper Basin could increase Upper Basin demand for water.

While a water shortage will not develop immediately because Lake Mead in the Lower Basin and Lake Powell in the Upper Basin are both full, some further rationing scheme will have to be employed in the near future. This paper sets out a general model for the allocation of river water and discusses the inefficiencies in the current allocation system. Then, recognizing the difficulties involved in making second-best policy prescriptions, it suggests some ways that inefficiencies might be reduced without upsetting the delicate balance of property rights and income distributions associated with the use of that water.

The paper is divided into three sections. First, an optimal pricing scheme for water is outlined and discussed. The second section identifies important ways in which current pricing schemes differ from the optimum. Finally, some policy changes are suggested.

AN OPTIMAL PRICING MODEL FOR WATER

Colorado River water can be modelled as one of a general class of storable goods subject to uncertain supply. In a given year the supply of water is a random draw from a probability distribution which is determined not by man, but by the climate of the area. In order to use river water investments have to be made in pipes, pumping stations, and diversion canals. Storage of water requires investment in dams and reservoirs. Because of the time required to build these facilities, the decisions about delivery and storage capacity size have to be made before it is known how much water will be available. Because the sizes of these facilities cannot be changed easily once they are built and the amount of river flow will be different every year, the capacity sizes will generally not be the cost minimizing sizes after the river flow is known even if they are the expected cost minimizing sizes from an ex-ante point of view. Some years river flow will be so low that neither the storage nor the delivery capacity will be full and other years it will be so high that some water will go to waste.

The demand for water is also important in determining the optimal size storage and delivery capacity. If demand as well as supply is uncertain, the ex-ante choice of optimal storage and delivery capacity is more complicated than if supply alone were uncertain. In a river system which dominates a region to the extent that the Colorado River dominates the American southwest differences in river flow from year to year may result in differences in relative prices and wages. Variations in relative prices and wages from year to year translate into variations in demand from year to year.

Once it is known how much water will be available and what the demand curve will be it has to be decided how much to consume this year and how much to store for future use. Since next year's flow and demand are also unknown, the storage decision is as complicated as the capacity size decision. If you store a lot and next year's flow is large, some water may go to waste, but if you don't store much and next year's flow is low you may wish you had consumed less and stored more. Ideally you would always have a full reservoir and full pipes with none going to waste. Since that situation will rarely be possible you have to choose between present consumption and storage for the future when future supply and demand are uncertain.

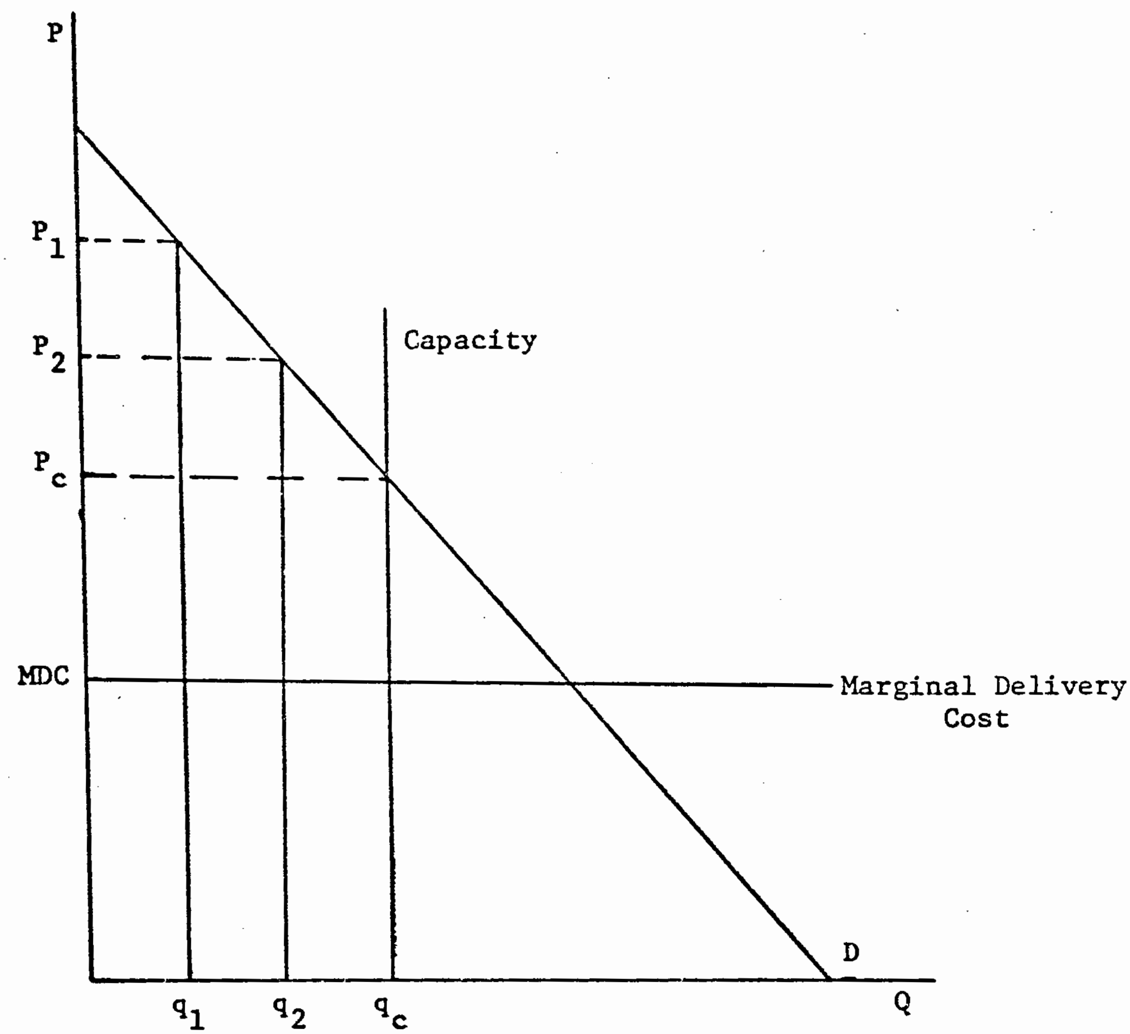
Having allocated water between present and future consumption, other distributional questions still remain. How many variable inputs will be hired to deliver the water? How much water will go to production, such as growing food, and how much will go to direct consumption, such as for drinking and washing?

The appendix sets out a mathematical formulation of the choices described above. The results can be summarized briefly. Let us begin with an optimal price for water once it has been determined how much will be delivered this year and what the demand curve will be. As long as the supply to be delivered is less than delivery capacity, the optimal price is at short run marginal delivery cost plus a competitive price for the right to deliver an acrefoot of water. If available supply exceeds delivery capacity the optimal price is at short run marginal delivery cost plus a competitive surcharge which brings demand and delivery capacity size to equilibrium.

Figure 1 illustrates how these pricing rules are determined. Let q_c be the available delivery capacity, MDC be the marginal delivery cost and D be the known demand curve for water. If $q > q_c$ the price of P_c and the surcharge will be P_c - MDC per acrefoot. At q_2 and $q_1 < q_c$, the prices will be P_2 and P_1 with P_2 - MDC and P_1 - MDC being the competitive prices for the right to deliver an acrefoot of water. This pricing rule should be applied both to water consumed directly and to water used to produce food or other products. As long as the marginal cost of delivering water to consumers is the same as the marginal cost of delivering water to firms, the price paid by each should be the same. The price of a good which uses water in its production process will then be higher by the marginal cost of whatever other inputs are required.

It is more difficult to decide what is the correct price for water when delivery and storage capacity sizes and the amount of water to store have to be chosen also. Since these choices have to be made before it is known how much water will be available, they have to be made on the basis of how much water individual consumers and firms think will be available. If an omniscient planner were deciding how much delivery and storage capacity to build and how much water to store, he or she would face the following welfare problem. Make those choices in such a way that, at those capacity sizes and amount of stored water, it would not be possible, without knowing how much water would be available, to increase the expected utility of any one consumer without lowering the expected utility of some other consumer. Given this definition, an ex-ante optimal size delivery or storage capacity or amount of storage is one which equates each consumer's discounted expected marginal rate of substitution between other goods and water with the ratio

FIGURE 1



of discounted expected long run marginal costs, including the costs of building the delivery and storage capacity and storing the water. Also from an ex-ante point of view, an optimal set of anticipated prices for water and other goods associated with each possible amount of river flow equates discounted, probability-weighted marginal rates of substitution across all consumers.

If consumers have different beliefs about how much water will be available, gains from trade exist among consumers when decisions about both delivery and storage capacity size and storage of water are being made. Consider the following simple example. Suppose Mr. A and Mr. B jointly own a river on which they wish to build a dam, storage facility, and diversion works to deliver the water to their homes. Mr. A thinks it highly probable that river flow will be high in the next two periods, but Mr. B thinks it will probably be only moderate. Consequently, Mr. A considers a large storage facility unnecessary but he does want a relatively large diversion works. In constrast, Mr. B wants to build a relatively large storage facility and a smaller diversion works. Assume that larger capacity costs more in both cases. Clearly, in the absence of transactions costs, Mr. A would be willing to bribe Mr. B to build a larger diversion works and Mr. B would be willing to bribe Mr. A to build a larger storage facility and store more water. Thus, even though water itself is a private good, delivery and storage capacity and stored water look like public goods unless each person can build his own dam and store his own water.

Now consider two different ways of financing capacity expansion and organizing storage and water deliveries. In the first case, suppose firms producing both delivery and storage capacity sell shares of those capacities to consumers in a competitive market before starting to build. Consumers who think there will be a great deal of water with high probability will buy more delivery capacity shares than consumers who think capacity should be small. Firms will then build only as much delivery capacity as they can sell in shares to consumers at competitive prices. similarly, consumers will buy storage capacity shares on the basis of their beliefs about both the supply of and the demand for water in the future. Firms will also only build as much storage capacity as they can sell in shares to consumers. Consumers can then trade delivery and storage capacity units and water rights among themselves after both facilities have been built. In addition, consumers own water rights which they can either rent to firms or use to store water for the future in the storage facilities in which they own shares. In order to deliver a certain number of acrefeet of water to their customers, firms would then have to rent an equal number of acrefeet of water rights and delivery capacity rights from consumers.

In this market, the sale of shares becomes the mechanism for allowing the group represented by Mr. A to bribe that represented by Mr. B and visa versa. When water is to be delivered, delivery capacity shares will rent for a zero price when the delivery capacity is not full and for a positive price when it is. Similarly, storage capacity shares will sell for a zero price when the reservoir is not full and a positive price when it is and rights to stored water will rent for a positive price when this year's flow is low and a zero price when it is high. The price per acrefoot of delivered water will then reflect all the scarcity and capacity charges which are positive in that year. At a competitive equilibrium in the capacity rights market the expected rate of return will depend upon the distribution of risk preferences among consumers.

In the second market, suppose each firm which delivers water builds or purchases its own capacity and stores its own water on the basis of the firm manager's beliefs about how much water will be available, discount rate on future profits, and attitude towards risk. Before observing how much water will be available a competitive expected utility maximizing firm manager will build delivery and storage capacity and store water so that the discounted expected marginal utility of the price for delivered water equals the discounted expected marginal disutility of the cost. Since firms and not consumers have evaluated the capacity and storage choices, it is unlikely that this allocation mechanism will lead to an optimal choice.

In fact, if firms own delivery and storage capacity and store water, finding an optimal price is very similar to designing an optimal tax. Before the available supply of water is known, each consumer should pay a different lump sum tax for the use of delivery and storage capacity and stored water, which takes account of his individual utility function, beliefs, and discount rate for future consumption.

As is well known, however, finding such a set of prices and then enforcing them in the face of arbitrage possibilities is close to impossible.

If the prices are not designed optimally, some consumers would still be willing to bribe the firm to build more delivery or storage capacity or store more water and others would wish the firm had built or stored less. If, however, all consumers and firms owning delivery and storage capacity and storing water have identical probability beliefs and discount rates over future consumption or profits and firms owning delivery and storage capacity maximize expected profits then these gains from trade do not exist to begin with. In that case, a competitive market mechanism which has firms instead of consumers owning delivery and storage capacity and storing water would generate an optimal price for water, optimal sized delivery and storage facilities and optimal quantities of stored water.

The above discussion of optimality conditions under the two market structures assumed that incomes and relative prices were the same regardless of how much water was available in a particular year. If we relax that assumption, each consumer's demand for water will also be uncertain. These income effects distort consumers' ex-ante evaluations of the anticipated price ratios acorss different river flows and make it difficult for a competitive mechanism to achieve an allocation of water and other goods that equates the discounted probability-weighted marginal rates of substitution across consumers. If the anticipated allocation is not optimal, then the signals consumers or firms receive when they are deciding how many delivery or storage capacity units to buy or the amount of water to store will not be correct and those decisions will not be made optimally.

There are two ways to avoid the problems created by these income effects when they are significant. One is to make all production and consumption decisions before it is known how much water will be available. In two seminal works on the competitive market under uncertainty Arrow (1964) and Debreu (1959) suggested a market in contingent commodity claims as a way of circumventing the welfare problems created by uncertainty. In their market consumers would purchase a portfolio of consumption plans, one for each possible amount of river flow, for example. Similarly, firms would contract for a set of inputs to be used in conjunction with each possible river flow. All these transactions would be carried out before any production or consumption was allowed. After the river flow was known firms would simply produce the appropriate amount of goods and distribute the output on the basis of the distribution of contingent commodity claims. No further trading would be allowed. Since all transactions would take place before the river flow was known, there would be no uncertainty about income, relative prices, or firm profits. Thus, there would be no uncertainty about demand and firms would assume no risks. This implies that regardless of which group owned delivery and storage capacity and stored water, these ex-ante choices would be made optimally.

If all the futures markets envisioned by Arrow and Debreu did not exist, then some choices would be made in spot markets after it was known how much water would be available. In that case, the following restrictions on differentiable expected utility functions would be necessary to achieve the condition that ex-ante discounted probability-weighted marginal rates of substitution would be equal across consumers. Ex-ante ratios of marginal utilities of income across all pairs of possible river flows in all time periods would have to be equal across consumers. In other words, if two possible river flows were 2 and 4 and my ex-ante marginal utilities of income were 5 and 8, respectively, then your ratio of ex-ante marginal utilities across those two possible river flows would have to be 5/8. If this condition is fulfilled, it implies that ex-ante, the marginal rates of substitution for income and therefore total consumption in different states of the world and different time periods are equal across consumers.

If consumers own delivery and storage capacity and stored water, then this condition on consumer utility functions is sufficient to ensure an ex-ante optimal choice of delivery or storage capacity or stored water. If firms own these risky assets, then a stronger sufficient condition is needed, although the condition discussed above is still necessary. This stronger condition is that consumers' ex-ante marginal utilities of income are discounted constant functions of their subjective probability beliefs. This assumption implies that after it is known how much water is available, each consumer's marginal utility of income will be the same for every amount of water available.

PRICING OF WATER IN SOUTHERN CALIFORNIA

The actual pricing of water in the Colorado River distribution system of Southern California differs from the optimal pricing structure discussed in a variety of ways. To begin with, the market for delivered water is not competitive. Instead, water is treated as a public utility and each consumer is served by only one water company. Each water company, whether municipal or private, has the exclusive right to serve a particular area. Prices are set either by municipal governments or by water district governments or, in the case of private water companies, by the California Public Utilities Commission. Although pricing rules differ, a majority of water companies use a two part or even a three part tariff. Generally property taxes levied on customers of a water district cover some or all of the price for delivered water. In addition, there is often a fixed monthly charge, at least for residential and commercial customers, which varies with the size of the establishment. This fixed charge may or may not cover some minimum number of acrefeet consumed each month. Agricultural users may not always pay a fixed charge in addition to their acreage assessments even when residential users in the same area do pay one. Finally, with some exceptions, users pay a price per acrefoot for the water they actually use in addition to whatever minimum they might be allowed with their fixed monthly payments.

On the face of it this pricing structure appears as though it might be an attempt to legislate a pricing structure which resembles an optimal structure. Customers pay lump sum taxes and charges as entrance and maintenance fees and then pay marginal charges for the water they actually use. These charges may not be optimal in the sense that 1) they probably do not equate discounted expected marginal rates of substitution and discounted expected marginal costs and 2) they are not based on consumer probability beliefs and discount rates. However, the structure itself might be second-best.

This surface similarity with an optimal pricing and taxing structure hides some important inefficiencies. Three in particular merit comment. First, the marginal charge paid by customers is generally too low from a welfare point of view. This statement can be made because there is no market for rights simply to divert the water. The marginal charges only cover pumping, treatment, transport, and delivery costs. Rights to divert the water in the first place are allocated according to what is called the appropriative water rights system. Briefly, the system works as follows. Water districts gain the right to deliver to customers a certain number of acrefeet of water per year by diverting it and putting it to "beneficial consumptive use." In a given year, the available river flow is allocated by allowing the water district which diverted water first, historically, to take all its allotment before the next more recent water district takes it and so on. In other words, "first in time means first in right." Once the water has been divided in this manner, a secondary market in which more junior water districts would pay more senior water districts for the right to divert more water in a given year is generally ruled out because the Bureau of Recalmation does allow sales or rental of water among users whose diversion works it helped finance. All the major water districts such as the Imperial Valley Irrigation District and the Metropolitan Water District of Southern California are subject to this exclusion.

This allocation mechanism is economically inefficient for two reasons. First, there is no mechanism for allocating water to its most productive uses. Each senior water district is encouraged to use water until its marginal productivity or utility is zero even though higher profits or utility could be achieved if senior water districts could rent or sell rights to junior water districts. Second, as Burness and Quirk (1978) have shown, the priority system forces junior water districts to bear all the risks. If each water district were a farm, an optimal allocation of risk bearing would have agricultural firms of equal size bearing equal risks.

The second type of inefficiency in the price and tax structure is that agriculture is subsidized at the expense of residential and commercial use. Agricultural users pay about half to two thirds the marginal charges that other users pay for the same water.. From the standpoint of economic efficiency a better way to provide relief to both farmers and consumers would be to give lump sum grants and make water and food prices equal to their long run marginal costs.

The third type of inefficiency is in the way that large scale delivery and storage capacity projects are financed. Most of them have been built with federal grants or low interest loans from the general tax revenues of the U.S. and state governments. That means of financing would not necessarily be inefficient if water districts paid the true marginal cost for the use of those capacities and the price paid by customers reflected those rental payments. Instead, however, general tax payers pay for storage and delivery capacity and the prices of water and food are lower than the prices which cover long run marginal costs.

SUGGESTED POLICY ALTERNATIVES.

I would like to propose three possible changes in public policy. Each is directed at attempting to correct one of the specific inefficiencies outlined above. First, suppose water districts could buy and sell percentage shares of their own and other water districts' priority rights. Suppose further that water districts could purchase a portfolio of shares from each other water district, one set of shares for each possible flow of the river. After the flow were known they could even trade again among themselves if they did not like the portfolio they had bought before. Such a market would be like the Arrow-Debreu contingent commodity claims market discussed above, but flexibility would be allowed if gains from trade still existed after the river flow were known.

The model outlined in the appendix shows that such a market would achieve an efficient allocation of water rights and an optimal scarcity charge for water if it were competitive. To evaluate the usefulness of that result, we need to know 1) whether such a semi-contingent claims market might develop if the ban on sales of rights were lifted and 2) if so, whether it would be competitive? The first question is somewhat easier to speculate about than the second. Currently there is little incentive for the development of such a system because all water districts have been able to get more than their full allotments every year and acreage limitations imposed by the Bureau of Recalmation preclude expansion of water use in most water districts. The expectation is the Metropolitan Water District of Southern California which could probably sell more water than it currently does if it built a larger delivery system.

Such a market might easily become more attractive after the Central Arizona Project is completed, however. The Metropolitan Water District is a relatively junior appropriator in California and will therefore be one of the first to have to reduce its diversions when California must reduce diversions to 4.4 million acrefeet annually. Further, even when California restricts itself to 4.4 million acrefeet annually, the Central Arizona Project, the most junior appropriator, may face random shortages if the Upper Basin states go several years without delivering their full commitment through Lee's Ferry. This can be anticipated because power demands on Hoover Dam[2] require that the Lake Mead reservoir mever be drained down very far. If Tucson and Phoenix were in danger of losing their supplemental water supply, the Central Arizona Project also might wish to purchase or rent rights from other water districts.

At least two water districts can be expected to want to sell or rent rights to other water districts. The Imperial Valley and Coachella Valley irrigation districts have the right to about 3.5 million acrefeet of water per year. They are second in seniority in California behind the Palo Verde Irrigation District which only has rights to about 350,000 acrefeet. The canal which brings water from the Colorado to the Salton Sea area is unlined and traverses one of the hottest deserts in the world. The districts' allotment of water is so large relative to the demand for water in the districts that water is practically free and there is therefore no incentive to invest resources to conserve water by lining and covering the canal. They must continue to divert that much water every year in order to maintain their appropriative right, however. If they do not continue to make "beneficial consumptive use" of all the water in their right, they could lose some without compensation. The availability of such a market would provide the Imperial and Coachella Valley Irrigation Districts with a more productive use for their water which is currently allowed to go to waste.

While the demand for some kind of market in water rights may exist, water districts may find a contingent claims market too difficult to administer and enforce. It is more likely that more senior firms will simply rent percentages of their rights to more junior firms in spot transactions during shortages. Such an unpredictable market would not help firms make better ex-ante capacity and storage choices because it would be difficult to anticipate quantities available and prices, but it would improve the spot allocation and pricing of water, ex-post.

The question of whether the market might be competitive is more difficult to answer. The above discussion suggests that there are at least two buyers and two sellers of rights which can be easily identified. Such a distribution of economic agents could hardly be seen as competitive. More potential buyers and sellers might be found among rights holders in the Upper Basin, but transfers of water between basins would require a renegotiation of the Colorado Compact. That drastic a change in the distribution of the water is unlikely in the near future because of long-standing animosity between the basins. Even transactions between rights holders in California and Arizona may be hampered by the legacy of feuding over water allocations between those two states. In the near future, therefore, it appears that such a market in rights would be subject to some monopoly power on both the supply side and the demand side. Even though this market would probably not be competitive, however, it could be argued that it would be better to let some market attach a scarcity value to water than to continue to have two water districts waste water that other districts and their customers might be willing to pay for.

The second policy suggestion is that agricultural and residential charges for water of equal quality be equalized gradually. A sudden equalization might correct inefficiencies, but it would not be a Pareto superior policy. Farmers and purchasers of farm products would suffer and direct consumers of water would benefit. A policy of gradually equalizing water rates would allow farmers to make necessary long run adjustments and, therefore, reduce the hardship to farmers and consumers of food.

The third suggestion is that there be a fundamental change in the way large scale water storage and delivery systems are financed in the future. An immediate reduction in the state and federal subsidy for these projects is probably neither possible nor desirable. Large sums of money have already been sunk into dams, reservoirs and diversion canals, all of which are relatively long-lived. Given that these capacity units exist and are seldom used to capacity an optimal pricing rule is to price at the expected value of short run delivery and storage costs including a scarcity for the virgin flow. Decisions on future delivery and storage capacity expansion projects might be made by selling capacity units to individuals served by the water projects. These would then be rented back to firms delivering water. While this is a radical departure from the current policy of government support, such a market might develop like a commodities market. The benefits would be that only as much capacity would be build as individuals would be willing to pay for and that prices paid for delivered water would reflect the costs of providing additional delivery and storage capacity.

Conclusion

The above theoretical and institutional remarks indicate that consumers of water districts which have senior rights to Colorado River water benefit substantially from federal policy regarding the pricing and allocation of that water. They pay no scarcity charge, they do not finance their own delivery or storage facilities, and, if they are farmers, they pay less than whatever marginal delivery costs their water districts might incur. While this observation is not new, there has not been a comprehensive statement of what an optimal pricing rule and market allocation mechanism might be. While more research is needed on such details as the potential for a market in shares of appropriative rights and the economic effects of changing to private share financing of capital projects, this paper provides an attempt at a comprehensive theoretical framework for organizing a review of federal water allocation policy.

Appendix

This appendix sets out the model used to develop the propositions set forward in the text. Proofs of the propositions follow by solving the constrained maximization problems.

I. Ex-ante Optimality Conditions

Assume that there are n consumers, each denoted i, three goods produced and traded, S random states of the world each denoted s, and 2 time periods. Good x is water directly consumed, v is food, which uses water in its production process, and y is a composite good.

In this model, there are 2 consumption periods and delivery and storage capacity are built before any consumption takes place. In that pre-consumption period, consumption plans are made contingent on the observation of the random variable in period 1, subject to the constraint that no more water can be consumed than is available. Then, for every observation of the random variable in period 1, there is an ex-ante optimal storage, an optimal change in delivery capacity, and on optimal portfolio of consumption plans contingent on the observation of the random variable in period 2. All capacity units are dismantled at the end of period 2.

The technique applied to analyze this problem will be to maximize a welfare function subject to random constraints. The results are mathematically equivalent to those obtained from a dynamic programming analysis. This technique was employed in preference to dynamic programming, however, because the results closely resemble the results obtained from other analyses of welfare conditions under uncertainty.

Now, to introduce the model, let x^1_{is} be i's consumption of water in period 1, state s and x^2_{irs} be i's consumption of water in period 2, state r, given s in period 1. v_{is}^1 is food consumed by i in period 1, state s and v_{irs}^2 is food consumed by i in period 2, state r, given s. Food is produced according to production functions $v(x_{vs}^1, L_{vs}^1)$ and $v(x_{vrs}^2, L_{vrs}^2)$ in periods 1 and 2, states s and r, given s, where x_{vs}^1 and x_{vrs}^2 are water devoted to food production and L_{vs}^1 and L_{vrs}^2 are labor devoted to food production. $\sum_{i=1}^{n} v_{is}^1 \leq v(x_{vs}^1, L_{vs}^1)$ ∀s and $\sum_{i=1}^{n} v_{irs}^2 \leq v(x_{vrs}^2, L_{vrs}^2)$ ∀r,s.

Water is delivered according to delivery functions $x(L_{xs}^1)$ and $x(L_{xrs}^2)$ in periods 1, state s and 2, state r, given s, where L_{xs}^1 and L_{xrs}^2 are labor devoted to water delivery. $\sum_{i=1}^{n} x_{is}^1 + x_{vs}^1 \leq x(L_{xs}^1)$ ∀s and $\sum_{i=1}^{n} x_{irs}^2 + x_{vrs}^2 \leq x(L_{xrs}^2)$ ∀r,s. z_s^1 and z_r^2 are the random supplies of water available in periods 1, state s and 2, state r, where z_s^1 and z_r^2 are independently and identically distributed according to i's discrete subjective probabilities α_{is} and α_{ir}. z_s^1 can either be consumed as $x(L_{xs}^1)$ or stored as X_s^1 in the storage facility, subject to storage losses of $\theta \varepsilon (0,1)$ per unit of storage. Delivery of water is through capacity produced according to functions $\hat{x}(L_{\hat{x}}^0)$ in period 0 and $\hat{x}(L_{\hat{x}s}^1)$ in period 1, state s, where $L_{\hat{x}}^0$ is labor devoted to delivery capacity in period 0 and $L_{\hat{x}s}^1$ is labor devoted to delivery capacity in period 1, state s. $x(L_{xs}^1) \leq \hat{x}(L_{\hat{x}}^0)$ ∀s and $x(L_{xrs}^2) \leq \hat{x}(L_{\hat{x}s}^1)$ ∀r,s. Storage capacity is produced in period 0 according to production function $F(L_f)$, where L_f is labor devoted to the production of storage capacity. $X_s^1 \leq F(L_f)$ ∀s and $L_f + L_{\hat{x}}^0 \leq \sum_{i=1}^{n} L_i^0$, where L_i^0 is labor supplied by i in period 0.

Now, let y_{is}^1 and y_{irs}^2 be i's consumption of a composite good in periods 1, state s and 2, state r, given s. y is produced according to production functions $y(L_{ys}^1)$ and $y(L_{yrs}^2)$, where L_{ys}^1 and L_{yrs}^2 are labor devoted to the production of y in periods 1, state s and 2, state r, given s. $L_{xs}^1 + L_{vs}^1 + L_{ys}^1 + L_{\hat{x}s}^1 \leq \sum_{i=1}^{n} L_{is}^1$ ∀s and $L_{xrs}^2 + L_{vrs}^2 +$

$L^2_{yrs} \leq \sum_{i=1}^{n} L^2_{irs}$ $\forall r,s$, where L^1_{is} and L^2_{irs} are labor supplied by i in periods 1, state s and 2, state r, given s.

Assume that no consumption takes place in period 0. Consumer i supplies labor to the production of storage and delivery capacity according to differentiable utility function $u_i(L^0_i)$. In periods 1, state s and 2, state r, given s, consumer i has strictly quasi-concave differentiable utility functions $u_i(x^1_{is}, v^1_{is}, y^1_{is}, L^1_{is})$ and $u\ (x^2_{irs}, v^2_{irs}, y^2_{irs}, L^2_{irs})$. These utility functions are assumed to have the following regularity properties:

$$\lim_{x^1_{is} \to 0} \frac{\partial u_i}{\partial x^1_{is}} = \lim_{v^1_{is} \to 0} \frac{\partial u_i}{\partial v^1_{is}} = \lim_{y^1_{is} \to 0} \frac{\partial u_1}{\partial y^1_{is}} = \lim_{x^2_{irs} \to 0} \frac{\partial u_1}{\partial x^2_{irs}} = \lim_{v^2_{irs} \to 0}$$

$$\frac{\partial u_i}{\partial v^2_{irs}} = \lim_{y^2_{irs} \to 0} \frac{\partial u_i}{\partial y^2_{irs}} = +\infty \ \forall i,r,s; \lim_{\left[L^0_i \to 24 \text{ hrs/day}\right]} \frac{\partial u_i}{\partial L^0_i} = \lim_{\left[L^1_{is} \to 24 \text{ hrs/day}\right]} \frac{\partial u_i}{\partial L^1_{is}} =$$

$$\lim_{\left[L^2_{irs} \to 24 \text{ hrs/day}\right]} \frac{\partial u_i}{\partial L^2_{irs}} = -\infty \ \forall i,r,s; \quad \lim_{x^1_{is} \to \infty} \frac{\partial u_i}{\partial x^1_{is}} > 0 \ \forall i, s; \lim_{v^1_{is} \to \infty} \frac{\partial u_i}{\partial v^1_{is}} > 0 \ \forall i, s;$$

$$\lim_{y^1_{is} > \infty} \frac{\partial u_i}{\partial y^1_{is}} > 0 \ \forall i,s; \lim_{x^2_{irs} \to \infty} \frac{\partial u_1}{\partial x^2_{irs}} > 0 \ \forall i,r,s; \lim_{v^2_{irs} \to \infty} \frac{\partial u_i}{\partial v^2_{irs}} > 0 \ \forall i,r,s;$$

$$\lim_{y^2_{irs} \to \infty} \frac{\partial u_i}{\partial y^2_{irs}} > 0 \ \forall i,r,s; \lim_{L^0_i \to 0} \frac{\partial u_i}{\partial L^0_i} < 0 \ \forall i; \lim_{L^1_{is} \to 0} \frac{\partial u_i}{\partial L^1_{is}} < 0 \ \forall i,s;$$

and $\lim_{L^2_{irs} \to 0} \frac{\partial u_i}{\partial L^2_{irs}} < 0 \ \forall i,r,s.$

These assumptions ensure that water, food, and the composite good will always be consumed in positive quantities by all consumers and that all consumers will always supply positive amounts of labor. This allows an ex-ante Pareto Optimal allocation of water, food, and the composite good to be described by maximizing a weighted sum of individual utility functions.

Let $W = \sum_{i=1}^{n} \beta_i \{u_i(L_i^0) + \sum_{s=1}^{S} \alpha_{is} [u_i(x_{is}^1, v_{is}^1, y_{is}^1, L_{is}^1) + \delta_i \sum_{r=1}^{R} \alpha_{ir} u_i (x_{irs}^2, v_{irs}^2, y_{irs}^2, L_{irs}^2)]\}$,

where: β_i = welfare weight assigned to i; and

δ_i = i's individual discount rate.

The welfare problem is to maximize W subject to the following constraints.

$$\sum_{i=1}^{n} y_{is}^1 \leq y(L_{ys}^1) \qquad \forall s$$

$$\sum_{i=1}^{n} y_{irs}^2 \leq y(L_{yrs}^2) \qquad \forall r,s$$

$$\sum_{i=1}^{n} v_{is}^1 \leq v(x_{vs}^1, L_{vs}^1) \qquad \forall s$$

$$\sum_{i=1}^{n} v_{irs}^2 \leq v(x_{vrs}^2, L_{vrs}^2) \qquad \forall r,s$$

$$\sum_{i=1}^{n} x_{is}^1 + x_{vs}^1 \leq x(L_{xs}^1) \qquad \forall s$$

$$\sum_{i=1}^{n} x_{irs}^2 + x_{vrs}^2 \leq x(L_{xrs}^2) \qquad \forall r,s$$

$$x(L_{xs}^1) \leq z_s^1 - X_s^1 \qquad \forall s$$

$$x(L_{xrs}^2) \leq X_s^1(1-\theta) + z_r^2 \qquad \forall r,s$$

$$x(L_{xs}^1) \leq \hat{x}(L_{\hat{x}}^0) \qquad \forall s$$

$$x(L_{xrs}^2) \leq \hat{x}(L_{\hat{x}s}^1) \qquad \forall r,s$$

$$X_s^1 \leq F(L_f) \qquad \forall s$$

$$L_f + L_{\hat{x}}^0 \leq \sum_{i=1}^{n} L_i^0$$

$$L_{vs}^1 + L_{xs}^1 + L_{ys}^1 + L_{\hat{x}s}^1 \leq \sum_{i=1}^{n} L_{is}^1 \qquad \forall s$$

$$L_{vrs}^2 + L_{xrs}^2 + L_{yrs}^2 \leq \sum_{i=1}^{n} L_{irs}^2 \qquad \forall r,s$$

Solving the welfare problem outlined above allows the following propositions to be stated:

Proposition 1. An optimal allocation of water equates the expected marginal social value of water used for food production to the expected marginal social value of water used for direct consumption.

Proposition 2. For every combination of future states and future periods, the discounted probability-weighted marginal rate of substitution between water and other goods is equal across consumers.

Proposition 3. For states of the world such that capacity or available inventory is a binding constraint on water deliveries, an ex-ante optimal choice of delivery or storage capacity or an ex-ante inventory policy equates the discounted expected marginal rate of substitution between water and other goods for each consumer and the discounted expected ratio of marginal resource values.

II. Competitive Market Allocations

Turning now to market allocations, a set of competitive markets in water, food, and y might be organized in one of two different ways; either consumers own delivery and storage capacity and store water or firms do. Let us assume for simplicity that there is only one firm producing each good and that each firm behaves as if there is a competitive market (i.e. takes prices as given). The firm delivering water builds both delivery and storage capacity, but when consumers own rights the firm sells all its capacity back to consumers as soon as it is built. When consumers own rights, consumer i can store water equal to his storage capacity shares.

Consumers get all the profits of the firms on a profit sharing basis. When firms own rights, consumers still get all the profits, but capacity and inventory decisions are made by firm managers who may have different subjective probability distributions, discount rates, and attitudes towards risk than consumers have. We can now describe the competitive market by the following set of maximization problems.

If consumers own capacity and inventory, consumer i's decision problem is:

$$\text{Maximize } u_i(L_i^0) + \sum_{s=1}^{S}\alpha_{is}[u_{is}^1, v_{is}^1, y_{is}^1, L_{is}^1) + \delta_i \sum_{r=1}^{R} \alpha_{ir}$$

$$u_i(x_{irs}^2, v_{irs}^2, y_{irs}^2, L_{irs}^2)]$$

Subject to:

$$p_{\hat{x}}^0 \hat{x}^0 + p_f^0 F_i^0 + \sum_{s=1}^{S} q_s^0 [r_{is}^0 - \bar{r}_{is}] \leq w^0 L_i^0 + \theta_{ix} \Pi_x^0$$

$$p_{xs}^1 x_{is}^1 + p_{ys}^1 y_{is}^1 + p_{vs}^1 v_{is}^1 + p_{\hat{x}s}^1 [\hat{x}_{is}^1 - \hat{x}_i^0] + p_{fs}^1 [F_{is}^1 - F_i^0] +$$

$$\sum_{r=1}^{R} q_{rs}^1 [r_{ir}^0 - r_{irs}^1] \leq w_s^1 L_{is}^1 + c_{\hat{x}s}^1 x_{is}^1 + c_{qs}^1 [r_{iss}^1 - X_{is}^1] + \theta_{ix} \Pi_{xs}^1 +$$

$$\theta_{iy} \Pi_{ys}^1 + \theta_{iv} \Pi_{vs}^1 \qquad \forall s$$

$$p_{xrs}^2 x_{irs}^2 + p_{yrs}^2 y_{irs}^2 + p_{vrs}^2 v_{irs}^2 \leq w_{rs}^2 L_{irs}^2 + p_{\hat{x}rs}^2 \hat{x}_{is}^1 + q_{rs}^2 \{X_{is}^1 [1-\theta] + r_{irs}^1\} +$$

$$\theta_{ix} \Pi_{xrs}^2 + \theta_{iy} \Pi_{yrs}^2 + \theta_{iv} \Pi_{vrs}^2 \qquad \forall r, s$$

$$X_{is}^1 \leq F_{is}^1 \qquad \forall s, \text{ where}$$

$p_{\hat{x}}^0, p_{\hat{x}s}^1, p_{\hat{x}rs}^2$ = selling price for a unit of delivery capacity in period 0, period 1, state s, and period 2, state r, given s in period 1;

$\hat{x}_i^0, \hat{x}_{is}^1$ = delivery capacity units purchased by i in period 0 and in period 1, state s;

X_{is}^1 = water stored by i in period 1, state s;

p^0_f, p^1_{fs} = price of a unit of storage capacity in period 0 and period 1, state s;

F^0_i, F^1_{is} = storage capacity units purchased by i in period 0 and period 1, state s;

$q^0_r, q^1_{rs}, q^2_{rs}$ = selling price for a state r delivery right in period 0, period 1, state s and period 2, given s in period 1;

$\bar{r}_{ir}$, r^0_{ir}, r^1_{irs} = i's initial endowment of state r delivery rights and purchases of state r delivery rights in period 0 and period 1, state s;

w^0, w^1_s, w^2_{rs} = wage rate in period 0, period 1, state s, and period 2, state r, given s;

$\theta_{ix}, \theta_{iy}, \theta_{iv}$ = i's share of x,y, and v firm profits;

Π^0_x, Π^1_{xs}, Π^2_{xrs} = firm x profits;

Π^0_y, Π^1_{ys}, Π^2_{yrs} = firm y profits;

Π^0_v, Π^1_{vs}, Π^2_{vrs} = firm v profits; and

$c^1_{\hat{x}s}$, c^1_{qs} = rental rates for delivery capacity and state s delivery rights in period 1, state s.

Firm x's decision problem is:

$$\text{Max } p^0_{\hat{x}}\,\hat{x}(L^0_{\hat{x}}) + p^0_f\,F(L_f) - w^0(L^0_{\hat{x}} + L_f) + \sum_{s=1}^{S}\alpha_{xs}\{\phi_x[p^1_{xs}\,x(L^1_{xs}) + p^1_{\hat{x}s}\,\hat{x}\,(L^1_{\hat{x}s}) - w_0[L^1_{xs} + L^1_{\hat{x}s}] - c^1_{qs}r^1_s - c^1_{\hat{x}s}\hat{x}^1_s] + \delta_i\sum_{r=1}^{R}\alpha_{xr}\,\phi_x\,[p^2_{xrs}x(L^2_{xrs}) - w^2_{rs}L^2_{xrs} - q^2_{rs}\,r^2_{rs} - p^2_{\hat{x}rs}\hat{x}^2_{rs}]\}$$

$$\text{s.t. } x(L^1_{xs}) \leq r^1_s \qquad \forall s$$

$$x(L^2_{xrs}) \leq r^2_{rs} \qquad \forall r,\ s$$

$$x(L^1_{xs}) \leq \hat{x}^1_s$$

$$x(L^2_{xrs}) \leq \hat{x}^2_{rs} \qquad \forall r, s.$$

Firm y's decision problem is:

$$\max \sum_{s=1}^{S} \alpha_{ys} \{\phi_y(p^1_{ys} y(L^1_{ys}) - w^1_s L^1_{ys}) + \delta_y \sum_{r=1}^{R} \alpha_{yr}\phi_y(p^2_{yrs} y(L^2_{yrs}) - w^2_{rs} L^2_{yrs})\}.$$

Firm v's decision problem is:

$$\max \sum_{s=1}^{S} \alpha_{vs}\{\phi_v(p^1_{vs}\ v(L^1_{vs},\ x^1_{vs}) - w^1_s L^1_{vs} - p^1_{xs} x^1_{vs}) + \delta_v \sum_{r=1}^{R} \alpha_{vr}\phi_v(p^2_{vrs} v(L^2_{vrs},$$

$$x^2_{vrs}) - w^2_{rs} L^2_{vrs} - p^2_{xrs} x^2_{vrs})\},$$ where

α_{xs}, α_{ys}, α_{vs} = firm x, y, and v's subjective probability of being in s;

ϕ_x, ϕ_y, ϕ_v = firm x, y, and v's differentiable utility function over profits;

$\delta_x, \delta_y, \delta_v$ = firm x, y, and v's discount rate;

r^1_s, r^1_{rs} = water delivery rights rented by firm x in period 1 state s and purchased by firm x in period 2, state r, given s; and

$\hat{x}^1_s$, $\hat{x}^2_{rs}$ = delivery capacity rights rented by firm x in period 1, state s and purchased by firm x in period 2, state r, given s.

If, on the other hand, firm x owns capacity and storage rights, consumer and firm x maximization problems will be different. In this case, consumer i's problem becomes:

max E U_i as above

$$\text{s.t. } p^1_{xs} x^1_{is} + p^1_{ys} y^1_{is} + p^1_{vs} v^1_{is} \leq w^0 L^0_i + w^1_s L^1_{is} + \theta_{ix}\Pi^1_{xs} + \theta_{iy}\Pi^1_{ys} + \theta_{iv}\Pi^1_{vs}\ \forall s$$

$$p^2_{xrs} x^2_{irs} + p^2_{yrs} y^2_{irs} + p^2_{vrs} v^2_{irs} \leq w^2_{rs} L^2_{irs} + \theta_{ix}\Pi^2_{xrs} + \theta_{iy}\Pi^2_{yrs} + \theta_{iv}\Pi^2_{vrs}\quad \forall r,\ s$$

Firm x's problem becomes:

$$\max - w_0(I^0_{\hat{x}} + L_f) + \sum_{s=1}^{S} \alpha_{xs} \{\phi_x(p^1_{xs}x(L^1_{xs}) - w^1_s [L^1_{xs} + L^1_{\hat{x}s}]) +$$

$$\delta_x \sum_{r=1}^{R} \alpha_{xr}\phi_x(p^2_{xrs}x(L^2_{xrs}) - w^2_{rs}L^2_{xrs})\}$$

$$\text{S.t.} \quad x(L^1_{xs}) \leq z^1_s - X^1_s \qquad \forall s$$

$$x(L^2_{xrs}) \leq X^1_s(1-\theta) + z^2_r \qquad \forall r,s$$

$$x(L^1_{xs}) \leq \hat{x}(L^0_{\hat{x}}) \qquad \forall s$$

$$x(L^2_{xrs}) \leq \hat{x}(L^1_{\hat{x}s}) \qquad \forall r,s$$

$$X^1_s \leq F(L^0_f) \qquad \forall s$$

An analysis of the first order conditions derived from the competitive problems outlined above allows the following additional propositions to be stated.

Proposition 4: When consumers hold title to delivery and storage capacity and stored water, a necessary and sufficient condition for an ex-ante optimal choice of delivery and storage capacity and stored water and ex-ante optimal anticipated allocation of water and other goods is for the ratios of ex-ante marginal utilities of income across all states of the world and all time periods to be equal across consumers.

Proposition 5: The following conditions are sufficient to ensure an ex-ante optimal allocation of water, delivery and storage capacity and stored water when firms hold title to delivery and storage capacity and stored water and sell output in an ex-post competitive spot market:

1. necessary condition in Proposition 4;
2. consumer ex-ante marginal utilities of income

   are discounted constant functions of their subjective probability distributions;

3. consumers and firms owning capacity and inventories have identical discount rates and subjective probability distributions; and

4. firms owning capacity and inventories maximize expected profits.

III. Competitive Markets with Appropriative Water Rights

For this model assume that all the sufficient conditions for proposition 5 hold and that there are m firms delivering water. To describe a market in appropriative water rights with saleable percentage shares of rights which are contingent on an observation

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of the random river flow, we number states of the world in order of increasing river flows and segment the probability distribution into m + 1 blocks, each of the first m corresponding to a firm's original appropriative right and the m + 1 $\underline{st}$ describing the unappropriated portion of the probability distribution. Assuming expected profit maximization and a known discount rate and probability distribution, firm x_j's problem becomes:

$$\text{Max} - w^0[L^0_{j\hat{x}} + L_{jf}] - \sum_{k=1}^{m} \sum_{r=r_{k-1}}^{r_k} [\beta^0_{jkr} - \bar{\beta}_{jkr}] R_k q^0_{kr} + \sum_{\ell=1}^{m+1} \sum_{s=s_{\ell-1}}^{s_\ell} \alpha_s$$

$$\{p^1_{xs}\, x(L^1_{jxs}) - w^1_s\, [L^1_{jxs} + L^1_{j\hat{x}s}] - \sum_{k=1}^{m} \sum_{r=r_{k-1}}^{r_k} [\beta^1_{jkrs} - \beta^0_{jkr}]$$

$$R_k q^1_{krs} + \delta \sum_{q=1}^{m+1} \sum_{r=r_{q-1}}^{r_q} \alpha_r\, [p^2_{xrs}\, x(L^2_{jxrs}) - w^2_{rs}\, L^2_{jxrs} - \sum_{k=1}^{q \neq m+1}$$

$$[\beta^2_{jkrs} - \beta^1_{jkrs}]\, R_k\, q^2_{krs}]\}$$

$$\text{s.t. } x(L^1_{jxs_\ell}) \leq \sum_{k=1}^{\ell} \sum_{r=r_{k-1}}^{r_k} \beta^1_{jkrs} R_k - X^1_{js} \quad \forall s$$

$$x(L^2_{jxr_q s}) \leq X^1_{js}(1-\theta) + \sum_{k=1}^{q} \beta^2_{jkrs} R_k \quad \forall r,s$$

$$x(L^1_{jxs}) \leq \hat{x}\, (L^0_{j\hat{x}}) \quad \forall s$$

$$x(L^2_{jxrs}) \leq \hat{x}\, (L^1_{j\hat{x}s}) \quad \forall r,s$$

$$X^1 \leq F(L\ ) \quad \forall s$$

The consumer, y firm, and v firm problems are unchanged.

An analysis of the first order conditions from these problems allows the following proposition to be stated:

Proposition 6: If all the sufficient conditions outlined in proposition 5 are satisfied, then a competitive market operating under the legal system of appropriative water rights is ex-ante Pareto optimal, given the existence of a competitive market in percentage shares of appropriative water rights which are purchased contingent on the observation of the river flow.

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Footnotes

[1] The information on the Colorado Compact and the river flow is from Hundley (1975).

[2] Boulder Dam became Hoover Dam by an act of Congress in 1947.

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