Discussion Paper No. 384

"The Matching Process as a
Non-Cooperative/Bargaining Game"

by

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May, 1979

This paper is to be presented at the Universities-National Bureau Conference on the Economics of Information and Uncertainty, Boston, Mass., June 14-16. Financial support from the National Science Foundation (Grant SOC-77-08896) is acknowledged. This research was initiated while the author was visiting as a Fellow of the Institute for Advanced Studies, Hebrew University, Jerusalem, Israel.

1. Introduction

The term matching refers to any process by which persons and/or objects are combined to form distinguishable entities with some common purpose that none can accomplish alone. The allocation of apartments to tenants, the assignment of jobs to workers or factories to sites, the pairing of men and women in marriage and the formation of collections of agents known as firms are all examples. Problems of interest are those in which matchings take place voluntarily, substitution possibilities exist in the sense that no individual agent is an essential member of any coalition, and the "value" of the joint activity engaged in by a coalition can be divided among its members in many ways. There are two questions of interest. For a given environment described by the set of agents, the "value" of each possible coalition and the technology by which coalitions can form, what is the "equilibrium" coalition structure? Second, is an equilibrium coalition structure "efficient" in any meaningful sense?

At this level of generality, there is a small but diverse literature. The topics include location problems, the theory of coalition production economies, labor managed firms, marriage and divorce, and the theory of local public goods. That the value of a coalition's activities depends on the identities of its members and that the willingness of the members of a coalition to participate depends on the division of that value are essential ingredients. A further complication arises when the identities and/or locations of potential members are not known with certainty ex ante. In this case the existence of recruiting and search costs create quasi-rents. How these are divided affects the incentives that individual agents have to allocate resources to the process of coalition formation. The focus of the paper is on this aspect of the problem.

The problem of coalition formation under conditions of imperfect and costly information is most closely related to the search theoretic approach to market analysis. There are two recent papers on the topic, one by the author [1978] and another by Diamond and Maskin [1979]. Both papers are attempts to extend existing search theory in ways that allow equilibrium analysis. The relatively simple problem of bilateral matching, pairing, is treated. The divisions of the surplus attributable to the existence of a match is by nature a bilateral bargaining problem. A particular solution to this problem determines the value of the match to each member of a pair. If values associated with the potential pairings are not identical, then an individual agent neither holds out for the best possible match nor sticks with an existing one if a better opportunity presents itself. In the absence of a requirement to compensate each other in the event of a separation, separations occur too frequently. In a partial equilibrium context, I show that any matched pair maximizes their joint wealth, however they choose to divide it, if each is required to compensate the other for the lost share of the surplus in the event of a separation initiated by the former.

Diamond and Maskin, using the descriptive language of contract law, call an agreement concerning the division of the value of a match a "contract", a separation initiated by one of the two parties a "breach of contract", and required compensation equal to lost rent "compensatory damages". Compensation for breach voluntarily written into a contract is called "liquidated damages". By taking into account interactions that I ignore, they show that liquidated damages are sometimes greater than compensatory damages. They also study the issue of the efficiency of the matching process under both damage regimes when the surplus attributable to any match is divided equally between the members of the pair.

The focus of the paper is on the relationship between the bargaining outcome expected by the as yet unmatched pairs and the incentive of each unmatched agent to invest in the process of forming matches. This focus is resolved by using a model based on two distinguishing assumptions. First, no search by matched agents is allowed. Second, the aggregate rate at which matches form is endogenously determined by the search intensities chosen by individual unmatched agents. The breach of contract issue is ignored given the first assumption, but the divisions of the value of a match that agents expect to be written into contracts is crucial as a consequence of the second. Finally, following Diamond and Maskin, both "linear" and "quadratic" matching technologies are considered.

The method of analysis follows. Given a particular individually rational solution to the bargaining problem that any two agents of opposite type face when they meet, the problem of determining the search intensity choices is formulated as a many person repeated game. The game is played by all the unmatched agents of the two types. A constant steady state fraction of matched agents of each type exists given a bargaining outcome, any solution to the game and a specification of the technology. Each agent's payoff function is the discounted flow of expected future net benefits and benefits are transferable across agents. The non-cooperative Nash solution to the game of search intensity choice is imposed.

Not surprisingly the joint Nash search intensity choices and hence the matching process that is induced by it are generally inefficient in the sense that another possibility exists which would make all players better off. If the probability that a match will form in a short time interval is independent of the number of unmatched agents, the "linear" technology case, no unmatched agent searches intensively enough given any fixed division of the value of a match. The externality involved can be described as follows. If an unmatched agent searches more intensely, he and <u>some agent of the opposite type</u> will form a match more quickly on average. However, in contemplating his search intensity choice, the agent only takes account of his own expected benefit which is proportional to his share of the surplus obtained in the future match. The share to be obtained by his future partner is ignored. An alternative contract exists that will solve this incentive problem. Specifically, when the agent responsible for the formation of a particular match is allocated all the surplus attributable to it, then the joint wealth of all players is maximized by the Nash solution to the game of search intensity choice.

Given a "quadratic" technology, the probability that a match will form in a short time interval is proportional to the number of unmatched pairs. The contingent contract just described does not yield an efficient matching outcome in this case. Although the externality discussed still exists, more intensive search by all other agents reduces the number of agents of the opposite type that each individual can expect to find in the future. As a consequence of this second externality alone, unmatched agents search too intensively. Interestingly, the effects of the two externalities in combination cancel, given an appropriate bargaining outcome. In one limiting case of the model, the Nash solution to the game of search intensity choice maximizes the total wealth of all the searching agents if every partnership divides the surplus equally. More generally, the agent responsible for the formation of each match must be allocated a share of its surplus that lies between one-half and unity.

In sum, matching outcomes depend on the bargains that agents not yet matched expect to negotiate. Although there is no reason to believe that one individually rational outcome will occur rather than another, the incentives induced by virtually all motivate inefficient search. However, a particular bargaining outcome does exist that yields an efficient matching process in each example considered in the paper. The imposition of this contract can be viewed as an assignment of property rights that would induce a cooperative solution to the game of matching.

2. Matching Technologies

In this section we sketch an aggregate matching model, formally a stochastic process of the "birth/death" type. Following Diamond and Maskin, the problem is one of forming pairs composed of agents of two different types for the special case in which the numbers of agents of both types are equal. Let m denote the common number of agents of the two types or, equivalently, the number of possible pairs. Let n denote the number of unmatched pairs. The state space for the matching process is the set of all possible values that n can take on, the set $\{0,1,\ldots,m\}$.

Let a(n) denote the average instantaneous rate at which new matches form and b(n) denote the average instantaneous rate at which new unmatched pairs enter the process given that there are n unmatched pairs at the moment. (Both of these functions are specified in detail later.) Hence, the probability that exactly one new match will form in a short time interval of length Δt is approximately $a(n)\Delta t$ and the probability that one new unmatched pair will enter the system is $b(n)\Delta t$. Since either or neither of these two possibilities will occur during the interval with virtual certainty for sufficiently small values of Δt, we have

$$P_{t+\Delta t}(0) = \Delta t a(1) P_t(1) + (1-\Delta t b(0)) P_t(0) + 0(\Delta t)!$$

$$P_{t+\Delta t}(n) = \Delta t a(n+1) P_t(n+1) + \Delta t b(n-1) P_t(n-1)$$

$$+ [1-\Delta t a(n) - \Delta t b(n)] P_t(n) + 0(\Delta t)!,$$

$$n = 1, 2, \ldots, m$$

$$\sum_{n=0}^{m} P_t(n) = 1,$$

where $P_t(n)$ is the probability that there will be n unmatched pairs at time t and $0(\Delta t)/\Delta t \to 0$ as $\Delta t \to 0$. The first equation reflects the fact that there can be no unmatched pairs at the end of the interval $(t,t+\Delta t)$ only if either there were one at the beginning and a match formed during the interval or there were none at the beginning and none entered during the interval. The second equation reflects the fact that either a "birth" or a "death" can occur or neither does when $n > 0$. The last requirement reflects the fact that $[P_t(0), P_t(1),\ldots,P_t(m)]$ is the probability distribution over possible states at time t.

Divide both sides of the first two equations by Δt, rearrange terms appropriately and take the limits as $\Delta t \to 0$. The result is the system of differential equations

$$\dot{P}(0) = a(1)P(1) - b(0)P(0)$$

$$\dot{P}(n) = a(n+1)P(n+1) + b(n-1)P(n-1) - [a(n)+b(n)]P(n),$$

$$n = 1,2,\ldots,m$$

$$\sum_{n=0}^{m} P(n) = 1.$$

It is well known that the solution to this system converges to a unique steady state as $t \to \infty$ if $(a(n),b(n)) \geq 0$. (See Feller 1968, pp. 454-458 .) The limiting distribution is the particular solution to the difference equation

(1) $$a(n+1)P(n+1) + b(n-1)P(n-1) = [a(n)+b(n)]P(n)$$

associated with the boundary conditions

(2a) $$a(1)P(1) = b(0)P(0)$$

and

(2b) $$\sum_{n=0}^{m} P(n) = 1$$

For each n, the limiting probability is the relative frequency with which the process is in state n along any sample path of infinite length.

I consider two alternative specifications of the matching rate $a(n)$, linear and quadratic. In the linear case, the probability that some one of the n unmatched pairs meet in a short time interval is independent of the number of unmatched pairs. Hence, the average instantaneous rate at which pairs form is proportional to n; i.e.,

(3) $$a(n) = \alpha n$$

where $\alpha\Delta t$ is the probability that a particular pair of the n possibilities form a match. In the quadratic case, the probability that a particular unmatched pair will form a match during a short time interval is proportional to the fraction of agents of either type that are not matched. Hence,

(3') $$a(n) = \alpha(n/m)n = \alpha n^2/m.$$

These alternative specifications can be interpreted as follows. Let α_1 denote the frequency with which each unmatched agent of type 1 meets agents of type 2 and let α_2 denote the frequency with which each unmatched agent of type 2 meets agents of type 1. The contact frequency per unmatched pair is the sum

(4) $$\alpha = \alpha_1 + \alpha_2.$$

If matched agents of the opposite type are never met (3) obtains. However, if all matched and unmatched agents of the opposite type are contacted with equal probability, then (3') obtains because n/m is the probability that a contact made will be unmatched. Hence, in the quadratic case matched and unmatched agents can't be distinguished ex ante.

For the specification of b(n), the rate at which new unmatched pairs enter the system, we suppose that existing matches dissolve at an exogenous average rate β. Hence

(5) $$b(n) = \beta(m-n)$$

where $1/\beta$ is the expected duration of a match. In other words, β is the "turnover" rate.

In principle one can solve (1) for the explicit functional form of the limiting probability distribution over the states of the process for each specification of a(n) and b(n). For our purpose, it is enough to derive an expression for the expected fraction of unmatched pairs. Since we are primarily interested in the large numbers of agents case, this task is facilitated by an appeal to the law of large numbers.

Given (2a), an inductive argument applied to (1) yields

(6) $$P(n) = [b(n-1)/a(n)]P(n-1), \quad n = 1,\ldots,m,$$

in general. Of course,

$$E\,n \equiv \sum_{n=0}^{m} n\,P(n) = \sum_{n=1}^{m} n\,P(n) = \sum_{n=1}^{m+1} n\,P(n)$$

$$= \sum_{n=0}^{m} (n+1)P(n+1)$$

by virtue of the fact that (2b) implies $P(m+1) = 0$. Hence, in the linear case (3), (5) and (6) imply

$$E\,n = \sum_{n=0}^{m} (n+1)[\beta(m-n)/\alpha(n+1)]P(n)$$

$$= (\beta/\alpha) \sum_{n=0}^{m} (m-n)P(n) = (\beta/\alpha)[m-E\,n]$$

or, equivalently,

(7) $$E\,n/m = \beta/(\alpha+\beta).$$

Indeed, experts will recognize that $P(n)$ is the binomial distribution with "probability of success" $\beta/(\alpha+\beta)$ and "sample" size m. Hence, the variance of n/m,

$$\frac{m}{m^2}[\beta/(\alpha+\beta)][1-\beta/(\alpha+\beta)],$$

vanishes as $m \to \infty$.

The explicit form of the distribution function is not so transparent in the quadratic case but the law of large numbers still applies. The latter fact allows us to derive the limiting value of $E\,(n/m)$ using the following argument. First, note that

$$E\,n^2 \equiv \sum_{n=0}^{m} n^2 P(n) = \sum_{n=1}^{m} n^2 P(n) = \sum_{n=1}^{m+1} n^2 P(n)$$

$$= \sum_{n=0}^{m} (n+1)^2 P(n+1)$$

by virtue of (2b). Consequently, (3'), (5) and (6) imply

$$E\ n^2 = \sum_{n=0}^{m} (n+1)^2[m\beta(m-n)/\alpha(n+1)^2]P(n)$$

$$= (m\beta/\alpha) \sum_{n=0}^{m} (m-n)P(n) = (m\beta/\alpha)(m-E\ n)$$

or equivalently

$$E\ (n/m)^2 = (\beta/\alpha)[1-E\ \ n/m].$$

As the variance of n/m, $E\ (n/m)^2 - [E\ n/m]^2$, vanishes as $m \to \infty$, the mean is approximately equal to the positive root of the quadratic

$$[E\ n/m]^2 + (\beta/\alpha)\ E\ n/m - (\beta/\alpha) = 0$$

when m is large. In other words,

(7') $$E\ (n/m) = \frac{1}{2}\left[(\beta/\alpha)^2 + 4(\beta/\alpha)\right]^{1/2} - \frac{1}{2}(\beta/\alpha)$$

Equations (7) and (7') imply that

(8) $$E\ (n/m) = f(\beta/\alpha)$$

in both cases where f(x) is a strictly increasing concave function such that f(0) = 0 and f(∞) = 1. Furthermore, the elasticity $\eta(x) = xf'(x)/f(x)$ is decreasing and tends to zero as $x \to \infty$ in both cases, but

$$\eta(0) = \begin{cases} 1 & \text{if linear,} \\ \frac{1}{2} & \text{if quadratic.} \end{cases}$$

In other words, the expected fraction of unmatched agents is approximately β/α in the linear case and $(\beta/\alpha)^{1/2}$ in the quadratic when the turnover rate, β, is small relative to the contact rate, α. The specification assumed by Diamond and Maskin [1979] is equivalent to this approximation. As the observed fraction of unmatched agents is small in many contexts, the unemployment rates in labor markets and the vacancy rates in the markets for apartments are examples, its consideration is not without interest.

3. Matching Equilibria

An equilibrium theory of search intensity choice by unmatched agents is developed in this section. Since these choices determine the stochastic rate at which matches form, specifically the parameter α in the previous section, the theory provides a behavioral foundation for studying bilateral matching processes. The model is special in the sense that only unmatched agents are permitted to search. This restriction is imposed to permit a clearer view of issues relating to efficiency of matching processes.

An agent's search intensity is defined as the expected frequency with which agents of the opposite type are contacted. The cost of search per unit time period, $c_i(s)$, i = 1 and 2, is an increasing strictly convex function defined on the positive real line with the property that $c_i(0) = 0$. The argument s_i is the expected number of contacts made by the agent per unit time period. Hence, $s_i\Delta t$ is (approximately) equal to the probability that agent i will initiate a contact with an agent of type $j \neq i$ in a short time interval of length Δt. Hence, $\Delta t(s_1+s_2)$ is the probability that a particular unmatched pair will meet during the interval in the linear matching technology case. In the quadratic case, $\Delta t(s_1+s_2)n/m$ is the same probability.

Ex ante all unmatched pairs are identical in the sense that the expected total value of any match is the same for all possible pairings. Prior to a face to face meeting no one has information on which to base an inference concerning how the value of a particular match will differ from that of any other. However, ex post a statistic $x \in [0,1]$, which we interpret as the "quality" or "fit" of the match, is observed. It determines the value of the match $w(x)$. In other words, at the actual meeting of the two agents the "goodness of fit" is determined. This process of "getting to know one another" is viewed as a random draw from a distribution characterized by the c.d.f. $F(x)$. This formalization of ex post heterogeneity is due to Wilson [1979].

Consistent with the interpretation of x as an indicator of quality, $w(x)$ is a positive increasing continuous function on $[0,1]$. The distribution function $F(x)$ is also assumed to be continuous.

A division of the value of a match between the members of a partnership contingent on the fit realized is a vector function $(w_1(x), w_2(x))$ where $w_i(x)$, $i = 1$ and 2, is the allocation obtained by the agent of type i. Ultimately, the division is determined as an outcome of the bargaining that takes place between the members of actual pairs after they meet. For now, the division and the c.d.f. $F(x)$ are regarded as given, the same for all potential pairs, and known to all unmatched agents.

Let $v_i(t)$, $i = 1$ and 2, denote the expected present value of an agent's future net stream of benefits given that he pursues an optimal search strategy. The agent's choice problem is one of dynamic programming and $v_i(t)$ is the value of the agent's optimal program at time t. We wish to apply Bellman's principle of dynamic optimality. To do so, we must specify the outcomes of all events that can occur during a small future time interval of length Δt.

I start with the case of a linear matching technology. The probability that a particular agent of type i will meet some unmatched agent of type j is $\Delta t(s+s_j(t))$, $j \neq i$, where s is the search intensity to be chosen and $s_j(t)$ is the search intensity common to all agents of the opposite type. Suppose that the latter is known to our agent and is regarded as given. If the agent doesn't meet another of opposite type during the interval, then he continues to search which has expected value $v_i(t+\Delta t)$ by definition. If a prospective partner is met during the interval, then a fit $x \in [0,1]$ is realized and the pair considers the split $(w_1(x),w_2(x))$. An individually rational match is consummated if and only if

$$(w_1(x),w_2(x)) \geq (v_1(t+\Delta t),v_2(t+\Delta t)). \tag{9}$$

Call $A(t+\Delta t) \subset [0,1]$, the subset of qualities defined by these inequalities, the set of acceptable fits. Bellman's principle then requires that our agent's optimal strategy and its value, given the same for agents of the opposite type, satisfy

$$\begin{aligned} v_i(t) = \max_{s \geq 0} \{&-\Delta t c_i(s) + \frac{\Delta t}{1+r\Delta t}(s+s_j)[\Pr\{x \in A(t+\Delta t)\} \\ &\cdot E\{w_i(x) | x \in A(t+\Delta t)\} + \Pr\{x \notin A(t+\Delta t)\}v_i(t+\Delta t)] \\ &+ \frac{1}{1+r\Delta t}[1-\Delta t(s+s_j)]v_i(t+\Delta t)\}, \qquad j \neq i,\ i = 1 \text{ and } 2, \end{aligned} \tag{10}$$

where $\Delta t\, c_i(s)$ is the cost of search incurred by the agent during the interval and r is the discount rate common to all agents.

A joint search strategy $(s_1^o(t), s_2^o(t))$ that solves (10) for both i = 1 and 2 is a candidate for a Nash solution to the game of search strategy choice played by unmatched agents. Because the supergame is a sequence of the same instantaneous game continuously repeated, the solution is stationary. By requiring $v(t) = v(t+\Delta t)$ for all $(t,\Delta t)$ and by making the obvious limiting argument, (10) can be made to yield the following necessary and sufficient conditions for a non-cooperative stationary Nash search intensity pair denoted as (s_1^o, s_2^o). Letting (v_1^o, v_2^o) denote the associated payoffs obtained,

$$\text{(10.a)}\quad rv_1^o = \max_{s_1 \geq 0} [(s_1+s_2^o)\Pr\{x \in A^o\}[E\{w_1(x)|x \in A^o\}-v_1^o] - c_1(s_1)]$$

and

$$\text{(10.b)}\quad rv_2^o = \max_{s_2 \geq 0} [(s_1^o+s_2)\Pr\{x \in A^o\}[E\{w_2(x)|x \in A^o\}-v_2^o] - c_2(s_2)]$$

where A^o is the set of acceptable fits defined by (9) when $v_i(t+\Delta t) = v_i^o$, i = 1 and 2. In a Nash equilibrium every unmatched agent selects his own search intensity to maximize the expected net benefit flow attributable to his own search given the optimal choices made by all other unmatched agents.

Now consider the bilateral bargaining problem that two agents of opposite type face when they meet. Because the division $(w_1(x), w_2(x))$ is arbitrary at this point, it can happen that the realized fit x is not in the acceptable set A^o even though w(x), the total value of a match, exceeds the sum of both agents' values of continued search, $v_1^o + v_2^o$. However, in this situation an alternative division of the value of the match exists which would make both agents better off by inducing a consummation of the match even if both expect the division $(w_1(x),w_2(x))$ to obtain for any alternative matching opportunity. In other words, only divisions that are feasible and both individually and jointly rational; i.e.,

(11.a) $w(x) = w_1(x) + w_2(x)$

and

(11.b) $w(x) \geq v_1^o + v_2^o \Rightarrow (w_1(x),w_2(x)) \geq (v_1^o,v_2^o)$, $\forall x \in [0,1]$,

can be equilibrium outcomes of the bilateral bargaining problem that unmatched agents face when they meet.

The existing theory of symmetric bilateral bargaining does not provide any generally accepted restrictions on outcomes beyond those given in (11). Hence we must be content with the following definition of equilibrium.

Definition 1. An allocation of the value of every possible match $(w_1^o,w_2^o) : [0,1] \to \mathbb{R}_+^2$ and a search strategy pair $(s_1^o,s_2^o) \in \mathbb{R}_+^2$ is an equilibrium solution to the combined non-cooperative/bargaining game of matching if they satisfy

(12.a) $rv_1^o = \max_{s_1 \geq 0} [(s_1+s_2^o)E \max[w_1^o(x)-v_1^o,0] - c_1(s_1)]$

(12.b) $rv_2^o = \max_{s_2 \geq 0} [(s_1^o+s_2)E \max[w_2^o(x)-v_2^o,0] - c_2(s_2)]$

and

(13.a) $w_1^o(x) + w_2^o(x) = w(x), \quad \forall x \in [0,1]$

(13.b) $w(x) \geq v_1^o + v_2^o \Rightarrow (w_1^o(x),w_2^o(x)) \geq (v_1^o,v_2^o)$

given a linear matching technology.

The equations of (12) are implied by (10) and (11) and the equations of (13) are a restatement of feasibility and individual rationality respectively. In sum, an equilibrium search intensity pair is a Nash strategy relative to a bargaining outcome and the bargaining outcome is feasible and individually rational given the non-cooperative Nash payoffs induced by it.

Because (13.a) implies that the converse of (13.b) is true, the set of equilibrium acceptable fits is

$$A^o = \{x \in [0,1] \mid w(x) \geq v_1^o + v_2^o\}.$$

Because $w(x)$ is non-decreasing in x, a critical reservation fit $x^o \leq 1$ exists such that all fits $x \geq x^o$ are acceptable. The minimally acceptable fit is the smallest solution to

$$(14) \qquad w(x^o) = v_1^o + v_2^o.$$

As a consequence of the well known indeterminacy of the bilateral bargaining problem, many equilibria exist in general. To illustrate this point, consider the following family of divisions of the value of every possible match as candidates for equilibrium bargaining outcomes:

$$(15.a) \qquad w_1^o(x) = v_1^o + \theta[w(x) - v_1^o - v_2^o] \qquad \forall\ x \in [0,1],$$

$$(15.b) \qquad w_2^o(x) = v_2^o + (1-\theta)[w(x) - v_1^o - v_2^o] \qquad \forall\ x \in [0,1].$$

This rule satisfies both conditions of (13) for every choice of $\theta \in [0,1]$. Obviously the family is the class of rules--divide the surplus of the match between the two types of agents according to the shares θ and $(1-\theta)$. The special case of $\theta = 1/2$ is Nash's [1950] solution to symmetric bilateral bargaining problems.

<u>Proposition 1</u>: Given a linear matching technology, a unique non-trivial matching equilibrium exists for every $\theta \in [0,1]$ if either (i) $c_1'(0) < \theta E\, w(x)$ or (ii) $c_2'(0) < (1-\theta)E\, w(x)$.

Proof. Combine (12) and (15) to obtain

$$\text{(16.a)} \quad rv_1^o = \max_{s_1 \geq 0} \left[(s_1+s_2^o)\theta E \max[w(x)-v^o,0] - c_1(s_1) \right]$$

$$\text{(16.b)} \quad rv_2^o = \max_{s_2 \geq 0} \left[(s_1^o+s_2)(1-\theta)E \max[w(x)-v^o,0] - c_2(s_2) \right]$$

where $v^o \equiv v_1^o + v_2^o$. Since every element of the class of rules defined by (15) satisfies (14), we need only show that unique strategy/payoff pairs exist that solve (16) for every $\theta \in [0,1]$. As the cost functions are strictly convex, the solutions to the two optimization problems implicit in (16) are unique for an arbitrary value of v^o, call it v. Let $(s_1,s_2) = (\sigma_1(v),\sigma_2(v))$ denote the functions implicitly defined by the following sufficient conditions for optimality:

$$\text{(17.a)} \quad c_1'(s_1) \geq \theta\, E \max[w(x)-v,0], \qquad \text{equality if } s_1 > 0,$$

$$\text{(17.b)} \quad c_2'(s_2) \geq (1-\theta)\, E \max[w(x)-v,0], \qquad \text{equality if } s_2 > 0.$$

Since $c_1'(s_1)$ and $c_2'(s_2)$ are both continuous and increasing, the implicit functions defined by (17), $\sigma_1(v)$ and $\sigma_2(v)$, are both continuous and non-increasing. Furthermore, $c_1'(s_1) \geq 0$, $c_2'(s_2) \geq 0$ and $w(1) \geq w(x)$ $\forall\, x \in [0,1]$ together with (17) imply $\sigma_1(w(1)) = \sigma_2(w(1)) = 0$. Finally, the hypothesis implies either $\sigma_1(0) > 0$, $\sigma_2(0) > 0$ or both.

An inspection of (16) and (17) reveals that $v^o = v_1^o + v_2^o$ is a fixed point of the continuous function $\phi(v)$ defined by

$$(18)\qquad r\phi(v) = \max_{s_1 \geq 0} [(s_1+\sigma_2(v))\theta\ E[w(x)-v,0] - c_1(s_1)]$$

$$+ \max_{s_2 \geq 0} [(\sigma_1(v)+s_2)(1-\theta)\ E[w(x)-v,0] - c_2(s_2)].$$

Since $(s_1^o,s_2^o) = (\sigma_1(v^o),\sigma_2(v^o))$, it suffices to establish that $\phi(v)$ has a unique fixed point. Because $E\,w(x) > 0$ and $c_1(0) = c_2(0) = 0$, the fact that either $\sigma_1(0) > 0$, $\sigma_2(0) > 0$ or both implies $\phi(0) > 0$. Furthermore, $\phi(w(1)) = 0$ because $\sigma_1(w(1)) = \sigma_2(w(1)) = 0$. Hence, the continuity of $\phi(v)$ is sufficient to guarantee a $v^o = \phi(v^o)$ $\varepsilon\ (0,w(1))$. Finally, the fixed point is unique because (18) and $\sigma_i(v)$, $i = 1$ and 2, non-increasing imply that $\phi(v)$ is decreasing.

The hypothesis is necessary as well as sufficient for a non-trivial equilibrium. If both (i) and (ii) fail, then the equilibrium is $(s_1^o,s_2^o) = 0$. No unmatched agent searches because the marginal cost is too high relative to the expected benefit of trying to find a match.

In equilibrium, the matching rate is

$$(19)\qquad \alpha^o = (s_1^o+s_2^o)\Pr\{x\ \varepsilon\ A^o\} = (s_1^o+s_2^o)[1 - F(x^o)]$$

where x^o is the marginally acceptable fit as defined by (14). In other words, the equilibrium matching rate is equal to the product of the equilibrium meeting rate and the equilibrium probability that a random meeting of an unmatched pair will yield an acceptable match. Both of these and, hence, the equilibrium steady state fraction of unmatched agents $E\,(n/m) = \beta/(\alpha+\beta)$ vary with θ, the shares of the surplus obtained by the two agent types.

Given an appropriate modification of the equations of (12), the existence of a matching equilibrium can also be established for the quadratic matching technology. During a short interval of length Δt, the probability that an individual agent of type i will either find or be found by some unmatched agent of the other type, $j \neq i$, is

$$\Delta t[s+s_j](n/m)$$

Here s is the agent's own search intensity, s_j is the common intensity at which agents of the other type search, and n/m, the fraction of unmatched agents of each type, is both the probability that an agent found by our individual is not matched and the probability that some one of the n unmatched agents of the other type will find our individual. With large numbers of agents, n/m is (almost) nonstochastic and equal to $f(\beta/\alpha)$ in a steady state, where $f(\cdot)$ is the function defined by (7').

By virtue of Bellman's principle, a particular agent of type i selects an intensity that solves

$$v_i = \max_{s \geq 0} \{-\Delta t\ c_i(s) + \frac{\Delta t}{1+r\Delta t}(s+s_j)f(\beta/\alpha)\ E \max[w_i(x), v_i^o] + \frac{1}{1+r\Delta t}[1 - \Delta t(s+s_j)f(\beta/\alpha)]v_i\},$$

providing that bargaining outcomes are individually rational. If the search intensities chosen by all other agents are known and regarded as given, then the joint solution for all agents is the non-cooperative Nash search intensity pair (s_1^o, s_2^o) with associated payoff that satisfy

$$(20.a)\quad rv_1^o = \max_{s_1 \geq 0} [(s_1+s_2^o)f(\beta/\alpha^o)\ E\ \max[w_1(x)-v_1^o,0] - c_1(s_1)]$$

$$(20.b)\quad rv_2^o = \max_{s_2 \geq 0} [(s_1^o+s_2)f(\beta/\alpha^o)\ E\ \max[w_2(x)-v_2^o,0] - c_2(s_2)]$$

where

$$(21.a)\qquad \alpha^o = (s_1^o+s_2^o)[1 - F(x^o)]$$

and

$$(21.b)\qquad x^o = w^{-1}(v^o).$$

Replacing the conditions of (12) with (20) we obtain sufficient conditions for a matching equilibrium in the quadratic case.

Consider again the family of feasible and individually rational bargaining outcomes that divide the surplus of every match according to fixed shares, those defined by (15) for all values of $\theta \in [0,1]$.

Proposition 2: Given a quadratic matching technology, a non-trivial equilibrium exists for every $\theta \in [0,1]$ if either (i) $c_1'(0) < \theta\ E\ w(x)$ or (ii) $c_2'(0) < (1-\theta)\ E\ w(x)$.

Proof. Given (15), the equations of (20) can be rewritten as

$$(22.a)\qquad rv_1^o = \max_{s_1 \geq 0} [(s_1+s_2^o)f(\beta/\alpha^o)\theta\ E\ \max\ [w(x)-v^o,0]-c_1(s_1)]$$

$$(22.b)\qquad rv_2^o = \max_{s_2 \geq 0} [(s_1^o+s_2)f(\beta/\alpha^o)(1-\theta)\ E\ \max\ [w(x)-v^o,0]-c_2(s_2)]$$

where $v^o = v_1^o+v_2^o$. Again consider the necessary and sufficient conditions for an optional pair (s_1,s_2) given an arbitrary v^o, denoted as v. These are

(23.a) $c_1'(s_1) \geqq f(\beta/\alpha)\,\theta\, E \max[w(x)-v,0]$, equality if $s_1 > 0$,

(23.b) $c_2'(s_2) \geqq f(\beta/\alpha)(1-\theta) E \max[w(x)-v,0]$, equality if $s_2 > 0$.

Because $f(\beta/\alpha)$ is increasing and $\alpha = (s_1 + s_2)[1-F(w^{-1}(v))]$,

the equations define continuous functions $(\sigma_1(v), \sigma_2(v))$ such that $(\sigma_1(0), \sigma_2(0)) > 0$ and $\sigma_1(w(1)) = \sigma_2(w(1)) = 0$.

Hence, $v(w(1)) = 0$ and $v(0) > 0$ so that a fixed point $v^o = \phi(v^o)$

$\epsilon\ (0,w(1))$ exists where $\phi(v)$ is the function defined by

$$r\phi(v) = \max_{s_1 \geqq 0} [\ (s_1 + \sigma_2(v) f(\beta/\alpha(v))\ \theta\ E \max\ [w(x) - v,0] - c_1(s_1)\ ]$$

$$+ \max_{s_2 \geqq 0} [\ (\sigma_1(v) + s_2) f(\beta/\alpha(v))\ (1-\theta)\ E \max\ [w(x) - v,0] - c_2(s_2)\ ]$$

and

$$\alpha(v) = [\sigma_1(v) + \sigma_2(v)]\ [1-F(w^{-1}(v))\ ]\ .$$

Because the functions $(\sigma_1(v), \sigma_2(v))$ need not be non-increasing, the argument used to establish uniqueness in Proposition 1 does not go through. Nevertheless, for every fixed point $v^o, (s_1^o, s_2^o) = (\sigma_1(v^o), \sigma_2(v^o))$ is an equilibrium search intensity pair.

4. Matching Efficiency

In the linear matching technology case, unmatched agents do not search intensively enough in any of the equilibria identified in the previous section. Specifically, an intensity pair $(s_1, s_2) > (s_1^o, s_2^o)$ exists that is strictly preferred by every unmatched couple. Because the matching frequency is determined by the search intensities of all agents, an increase in that of one type augments the value of search to every member of the other type. However, no individual agent takes account of this external economy. In this section, I show that this externality is internalized by the bargaining outcome that allocates all the surplus attributable to every match to the agent responsible for making the match.

Although this same externality is present given a quadratic technology, there is another with a countervailing effect. It arises because the expected meeting rate is proportional to the fraction of unmatched pairs which is itself endogenously determined as a decreasing function of the sum of the intensities with which the two agent types search. In the absence of the first externality, more intensive search by all other agents reduces the return to search for each individual by reducing the probability that an agent met will be unmatched. Interestingly, if the surplus attributable to every match is shared equally, then the effect of the second externality just cancels that of the first in the limit as the fraction of unmatched agents tends to zero. In the general case, joint wealth maximization requires that the matchmaker receive the larger share of the surplus attributable to each match.

The principal purpose of this section, then, is to show that most equilibria are inefficient but that joint wealth maximizing equilibria exist if a more general class of feasible and individually rational bargaining outcomes is allowed. The class includes those that make the division of the surplus attributable to every match between the partners contingent on the identity of the agent responsible for making the match.

To formally establish that every equilibrium identified in the previous section is inefficient given a linear technology, we use the fact that the conditions of (16) implicitly define two functions $v_1(s_2)$ and $v_2(s_1)$ such that $(v_1^o, v_2^o) = (v_1(s_2^o), v_2(s_1^o))$. Both are clearly continuous and strictly increasing due to the external economy already discussed. If $(s_1^o, s_2^o) > 0$, then these functions and the first order conditions for a Nash strategy choice by members of each agent type implicitly define the equilibrium intensity pair (s_1^o, s_2^o) as the intersection of the two reaction curves. Formally, (16) and (17) imply

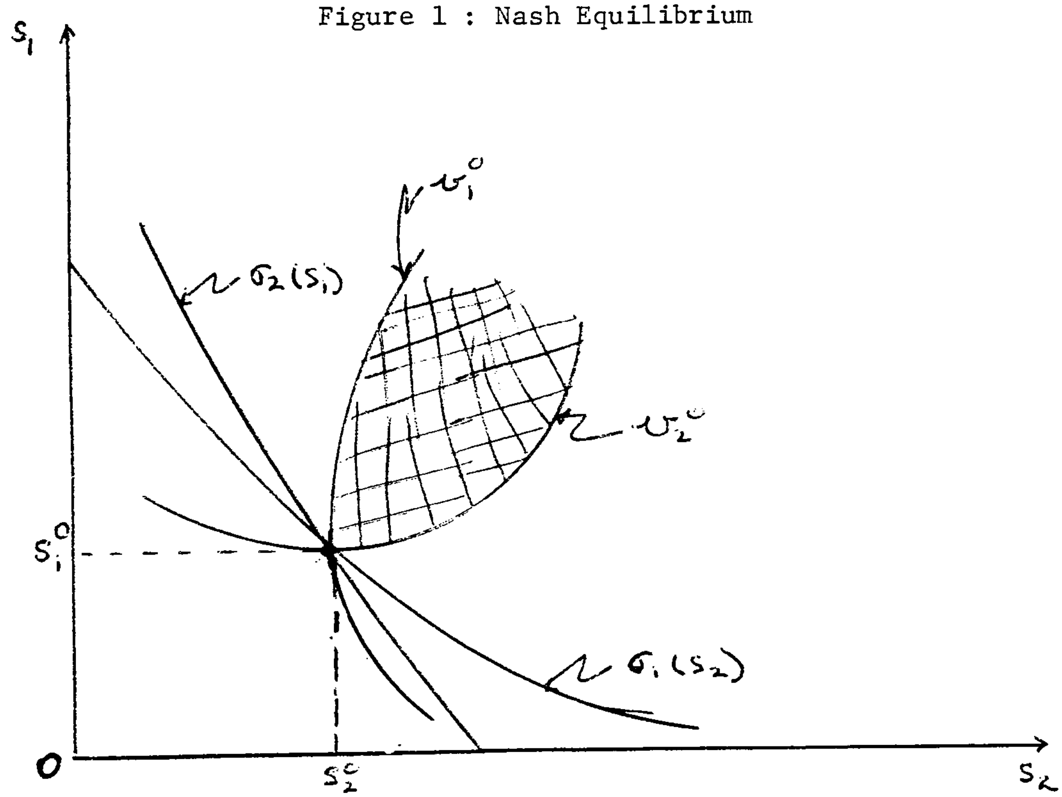
$$c_1'(s_1^o) = E \max [w_1^o(x) - v_1(s_2^o), 0]$$

and

$$c_2'(s_2^o) = E \max [w_2^o(x) - v_2(s_1^o), 0].$$

Let $g_i(s_j)$, $j \neq i$, denote the two reaction curves implicitly defined by these two equations. As $c_i'(s_i)$ and $v_i(s_j)$ are all strictly increasing, the optimal choice by one type given the other's intensity $g_i(s_j)$ is continuous and decreasing as illustrated in Figure 1. The curves labeled v_1^o and v_2^o in the Figure represent the intensity pairs that yield the same value of search to agents of type i = 1 and 2 as that obtained at (s_1^o, s_2^o). Since the payoff realized by each type increases with the other's search intensity, all intensity pairs in the shaded region in Figure 1 are strictly preferred by agents of both types to the equilibrium (s_1^o, s_2^o).

Figure 1 : Nash Equilibrium



The average quality of the matches that form in equilibrium is also too low. Since $v = v_1 + v_2 > v^o = v_1^o + v_2^o$ for any intensity pair in the preferred region, the minimally acceptable fit $w^{-1}(v)$ is larger than in equilibrium. As a consequence, the matching rate

$\alpha = (s_1+s_2)[1 - F(w^{-1}(v))]$ can be too small even though $(s_1+s_2) > (s_1^o+s_2^o)$. In other words, the existence of the externality does not unambiguously imply that the equilibrium fraction of unmatched agents $\beta/(\alpha^o+\beta)$ is too large except in the special case in which all matches have identical values ex post $(w(x) = w \ \forall \ x \in [0,1])$.

No unmatched agent searches intensively enough because none expects to receive the net social benefit attributable to the formation of a match, $w(x)-v_1^o-v_2^o$, in the future in return for the marginal investment required to seek out some agent of the opposite type. Viewed from this perspective, it would appear that the externality could be internalized by allocating the entire net benefit, the surplus attributable to such a match, to the agent who succeeded in making the contact responsible for the formation of the match. This particular allocation rule is a special case of the class of bargaining outcomes that are contingent on this random event.

Let $w_{ij}(x)$ denote the value of a match with fit $x \in [0,1]$ to the agent of type i given that the pair met as a consequence of a contact made by the agent of type j. The argument provided in the previous section justifies the following generalization of the equilibrium concept.

Definition 2. An allocation rule $(w_{1j}^o, w_{2j}^o) : [0,1] \to \mathbb{R}_+^2$, j = 1 and 2, and a search intensity pair $(s_1^o, s_2^o) \in \mathbb{R}_+^2$ is an equilibrium solution to the non-cooperative/bargaining game of matching given a linear technology if

$$\text{(24.a)} \qquad rv_1^o = \max_{s_1 \geq 0} \{s_1 \, E \max [w_{11}^o(x)-v_1^o,0] + s_2^o \, E \max [w_{12}^o(x)-v_1^o,0] - c_1(s_1)\}$$

$$(24.b) \qquad rv_2^o = \max_{s_2 \geq 0} \{s_1^o \text{ E max } [w_{21}^o(x)-v_2^o,0] + s_2 \text{ E max}[w_{22}^o(x)-v_2^o,0] - c_2(s_2)\}$$

and for j = 1 and 2,

$$(25.a) \qquad w(x) = w_{1j}^o(x) + w_{2j}^o(x) \qquad \forall\, x \in [0,1]$$

$$(25.b) \quad w(x) \geq v_1^o+v_2^o \Rightarrow (w_{1j}^o(x),w_{2j}^o(x)) \geq (v_1^o,v_2^o) \qquad \forall\, x \in [0,1].$$

The conditions of (24) define a non-cooperative Nash search intensity pair and reflect the fact that the surplus obtained by each party to a match is contingent on who made the contact. The conditions of (25) require that the contingent allocation of the value is feasible and individually rational. One can easily establish existence in the sense of Proposition 1 for every rule that divides the surplus attributable to every match according to shares contingent on the name of the agent making the match.

An inspection of (24) reveals that the externality is still present except in the special case

$$(26) \qquad w_{ij}^o(x) = \begin{cases} v_i^o + w(x) - v_1^o - v_2^o & \text{if } j = i, \\ v_i^o & \text{if } j \neq i. \end{cases}$$

This rule obviously allocates all the surplus of every match to the agent responsible for the contact that led to its formation. Given (24) and (26), we have

$$r\,v^o = \max_{(s_1,s_2) \geq 0} [(s_1+s_2) \text{ E max } [w(x)-v^o,0] - c_1(s_1) - c_2(s_2)]$$

where $v^o = v_1^o+v_2^o$. Hence,

<u>Proposition 3</u>. Given a linear technology, the joint wealth of every unmatched couple is maximum in equilibrium if and only if all

the surplus associated with every match is allocated to the agent responsible for its formation.

Given a quadratic matching technology, the analogous definition of an equilibrium is obtained by replacing the conditions of (24) by

$$\text{(27.a)} \qquad rv_1^o = \max_{s_1 \geq 0} \{s_1 f(\beta/\alpha^o)\ E\ \max[w_{11}^o(x)-v_1^o,0] - c_1(s_1)$$

$$+ s_2^o f(\beta/\alpha^o)\ E\ \max\ [w_{12}^o(x)-v_1^o,0]\}$$

$$\text{(27.b)} \qquad rv_2^o = \max_{s_2 \geq 0} \{s_2 f(\beta/\alpha^o)\ E\ \max[w_{22}^o(x)-v_2^o,0] - c_2(s_2)$$

$$+ s_1^o f(\beta/\alpha^o)\ E\ \max[w_{21}^o(x)-v_2^o,0]\}$$

where $f(\beta/\alpha)$ is the increasing function defined by (7') and

$$\text{(28)} \qquad \alpha^o = (s_1^o+s_2^o)[1 - F(w^{-1}(v^o))]$$

is the equilibrium rate at which acceptable matches form. Again equilibrium can be established for any rule that allocates the surplus according to shares contingent on the name of the agent responsible for the contact using the argument of Proposition 2.

An inspection of (27) reveals the following fact. Were the efficient allocation rule for the linear case ($w_{ij}^o = v_i^o$, $j \neq i$) adopted, then every agent searches too intensely. The reduction of the probability that an agent contacted in the future will be unmatched attributable to more intensive search by all ($f'(\cdot) > 0$) is not taken into account by any individual. This observation suggests that some rule that allocates less than the entire surplus to the agent responsible for making a particular match might have the desired incentive properties.

The joint wealth maximizing problem is

$$(29.a) \qquad rv^* = \max_{(s_1,s_2) \geq 0} \{(s_1+s_2)f(\beta/\alpha)\ E\max\ [w(x)-v^*,0] - c_1(s_1) - c_2(s_2)\}$$

$$= (s_1^*+s_2^*)f(\beta/\alpha^*)\ E\max\ [w(x)-v^*,0]-c_1(s_1^*)-c_2(s_2^*)$$

where

$$(29.b) \qquad \alpha = (s_1+s_2)[1 - F(w^{-1}(v^*))].$$

As $f(\cdot)$ is an increasing concave function such that $f(0) = 0$ by virtue of (7'), the right hand side of (29.a) is strictly concave in (s_1,s_2). Hence, the following first order conditions are sufficient to determine the search strategy pair (s_1^*,s_2^*) that maximize the sum of the values of search v_1+v_2.

$$(30) \qquad c_i'(s_i^*) \geq \left[f(\beta/\alpha^*) + (s_1^*+s_2^*)f'(\beta/\alpha^*)\frac{\partial(\beta/\alpha)}{\partial(s_1+s_2)}\right] \cdot \frac{\partial(s_1+s_2)}{\partial s_i}\ E\max\ [w(x)-v^*,0]$$

$$= [1 - \eta(\beta/\alpha^*)]f(\beta/\alpha^*)\ E\max\ [w(x)-v^*,0],$$

with strict equality holding if $s_i^* > 0$, $i = 1$ and 2, where $\eta(x) = xf'(x)/f(x)$ is the elasticity of $f(\cdot)$. As $\eta(0) = 1/2$ and $f(0) = 0$ while $\eta(\infty) = 0$ and $f(\infty) = 1$ by virtue of (7'), one can establish that v^* exists by applying the now familiar fixed point argument.

Equations (29) and (30) imply that the joint wealth maximizing intensity pair is a Nash solution given the following feasible and individually rational contingent bargaining outcome:

$$(31)\qquad w_{ij}(x) = \begin{cases} v_i^o + [1 - \eta(\beta/\alpha^*)][w(x)-v_1^o-v_2^o] & \text{if } j = i, \\ v_i^o + \eta(\beta/\alpha^*)[w(x)-v_1^o-v_2^o] & \text{if } j \neq i, \end{cases}$$

$i = 1$ and 2. Given this rule, every Nash solution (s_1^o, s_2^o) satisfies

$$(32)\qquad rv^o = \max_{(s_1,s_2) \geq 0} \{(s_1+s_2)[1 - \eta(\beta/\alpha^*)]f(\beta/\alpha^o) \cdot E \max [w(x)-v^o,0] - c_1(s_1) - c_2(s_2)\} + (s_1^o+s_2^o)\eta(\beta/\alpha^*)f(\beta/\alpha^o)\; E \max [w(x)-v^o,0]$$

where $v^o = v_1^o+v_2^o$ by virtue of (27). Consequently,

$$(33)\qquad c_i'(s_i^o) \geq [1 - \eta(\beta/\alpha^*)]f(\beta/\alpha^o)\; E \max [w(x)-v^o,0]$$

with strict equality holding it $s_i^o > 0$, $i = 1$ and 2.

Clearly, every solution to (29) and (30) satisfies (32) and (33). Hence,

<u>Proposition 4</u>. Given a quadratic technology and a contingent bargaining outcome that allocates to the agent responsible for making every match the share $1-\eta(\beta/\alpha^*)$ of its surplus, a search intensity pair that maximizes the joint wealth of every unmatched couple is a Nash solution to the game of search intensity choice.

Because of the possibility of multiple equilibria (see Proposition 2), the converse isn't guaranteed. However, if there is an inefficient equilibrium, neither agent type searches intensively enough.

<u>Proposition 5</u>. Given the hypothesis of Proposition 4, the joint wealth maximizing search intensity pair (s_1^*, s_2^*) is unique and at least as large as (s_1^o, s_2^o), any Nash solution associated with the allocation rule (31).

Proof. Because $1/2 \geq \eta(\beta/\alpha) \geq 0$ and $c_1'(s_1)$ and $c_2'(s_2)$ are continuous and strictly increasing, the functions $v(s_1,s_2)$ defined by

$$rv(s_1,s_2) = [s_1c_1'(s_1) + s_2c_2'(s_2)]/[1 - \eta(\beta/\alpha^*)] - c_1(s_1) - c_2(s_2)$$

is continuous and strictly increasing.

$$v^* = v(s_1^*,s_2^*)$$

and

$$c_1'(s_1^*) = c_2'(s_2^*) \qquad \text{if } (s_1^*,s_2^*) > 0$$

by virtue of (29) and (30), while

$$v^o = v(s_1^o,s_2^o)$$

and

$$c_1'(s_1^o) = c_2'(s_2^o) \qquad \text{if } (s_1^o,s_2^o) > 0$$

by virtue of (32) and (33). Hence, the fact that v^* is unique and such that $v^* \geq v^o$ by definition implies (s_1^*,s_2^*) unique and $(s_1^*,s_2^*) \geq (s_1^o,s_2^o)$. Q.E.D.

Furthermore, Proposition 5 provides the means needed to establish the following converse of Proposition 4.

<u>Proposition 6</u>. Given the hypothesis to Proposition 4, a Nash solution to the game of search intensity choice maximizes the joint wealth of every unmatched couple if all matches are identical ex post ($w(x) = w \ \forall \ x \in [0,1]$).

Proof. Because all matches are acceptable ($w \geq v^o$) in equilibrium $\alpha^o = \left(s_1^o+s_2^o\right)[1 - F(w^{-1}(v^o))] = s_1^o+s_2^o$. Hence, under the hypothesis, (33) can be rewritten as

$$(34.a) \qquad c_1'(s_1^o) \geq [1 - \eta(\beta/\alpha^*)]f(\beta/(s_1^o+s_2^o))[w-v^o],$$

equality if $s_1^o > 0$,

and

$$(34.b) \qquad c_2'(s_2^o) \geq [1 - \eta(\beta/\alpha^*)]f(\beta/(s_1^o+s_2^o))[w-v^o],$$

equality if $s_2^o > 0$.

Because $f(\beta/\alpha)$ is strictly increasing and continuous and $c_1'(s_1)$ and $c_2'(s_2)$ are both strictly increasing and continuous, the solution to (34) is unique for every choice of v^o and decreases as v^o increases. As (s_1^*,s_2^*) solves (34) when $v^o = v^*$, $v^* > v^o$ implies $(s_1^*,s_2^*) < (s_1^o,s_2^o)$, which contradicts Proposition 5. Q.E.D.

One way to interpret these results follows. When the agent who makes each match receives the entire surplus, a joint wealth maximizing equilibrium is possible if the agent's share $[w(x)-v^o]$ is taxed at the proportional rate $\eta(\beta/\alpha^*)$ and if the proceeds of the tax are redistributed to the other agent. The optimal tax rate depends on the joint wealth maximizing meeting rate $\alpha^* = (s_1^*+s_2^*)[1 - F(w^{-1}(v^*))]$. To calculate it, one would have to solve explicitly the joint wealth maximizing problem. However, because $E(n/m) = f(\beta/\alpha)$, $f(0) = 0$ and $\eta(0) = 1/2$ and $f(\infty) = 1$ and $\eta(\infty) = 0$, the optimal tax rate is approximately 1/2 (the surplus is shared equally) if the equilibrium fraction of unmatched agents is near zero and unique and is zero (the agent who makes a match gets all the surplus) if the equilibrium fraction of unmatched agents is near one and unique. Finally, uniqueness of equilibrium is guaranteed if matches are not too heterogeneous ex post.

5. A Summary and a Reinterpretation

A unique feasible and individually rational division of the surplus attributable to every match that motivates all unmatched agents to search efficiently exists given either technology. The allocation has the property that a larger share is received by the agent responsible for making the match. The sum of the ex ante present values of the future net incomes accruing to the members of the typical unmatched pair is maximum when they expect this allocation rule to obtain. However, no individual once contacted by another has an incentive to agree to that division ex post. Furthermore, the agent who made the contact has no special bargaining position as a consequence once the meeting takes place. Hence, there is no reason to believe that ex post bilateral bargaining will yield the efficient agreement.

Agents who are as yet unmatched might precommit. Each may well be willing to agree ex ante to assign the unknown agent who will make the match the appropriate share of the surplus. However, there exists no means by which the typical unmatched pair can meet ex ante for this purpose. Once the pair meets, the two no longer have the incentives required to obtain the agreement that might have motivated their meeting. The fact of having met only presents them with the bilateral bargaining problem as we formulated it in the text.

This paradox might be resolved by introducing a class of third parties, brokers or middlemen, who supply matching services and by so doing have a continuing interest in the bargaining outcomes. Of course, brokers exist in many market contexts in which matching is important. Labor markets, markets for housing and at various times and places the "marriage" market all serve as examples. The presumed ability of specialists to provide matching services of better quality and at a lower cost is the usual explanation given for the existence of such middlemen. Although these advantages may be necessary to explain the existence of brokers, another possible role is suggested by the following reinterpretation of the model.

Suppose that there are two types of principals that can be matched as pairs for some purpose. However, assume that the cost of self search by each principal is prohibitive relative to the expected benefit attributable to a future match. A principal can hire a broker to search in his stead at a reasonable price because the latter can search more economically. Given that none of the principals search for themselves, w(x) is the difference between the total value of a match with fit x and the sum of the opportunity cost that the two would incur were they matched. Given this interpretation any match with fit x such that $w(x) - p(x) \geq 0$ is acceptable to the pair where p(x) is the sum of the contingent commissions that the two principals pay to their brokers. If the sum of the opportunity costs of being matched is the same for every unmatched pair, then competition among the many unmatched principals for the scarce matching services supplied by brokers would bid the sum of the commission up to w(x). Given this price structure, the agents in our model can be interpreted as the brokers who represent the 2n unmatched principals.

Because all the matches are equivalent from each principal's perspective and each is indifferent to the length of time required to obtain a match, the search intensities and the criterion for an acceptable match are discretionary decisions taken by the brokers. Hence,

s_i is the intensity of search chosen by a broker who represents a principal of type i and v_i is the present value of the profit that the broker can expect in return for his effort to locate a match for that principal. An allocation of w(x) between the two agents who meet to form a match is now a division of the commission, that both principals are willing to pay, between their respective brokers.

The one difference is that the brokers have a continuing interest in the market for matching services that principals searching for themselves would not have. Having formed one match, they look forward to the prospect of doing the same for other principals in the future. They not only have an incentive to precommit themselves to the efficient allocation rule; as third parties they also have the means to do so. The fact that in some market contexts the broker responsible for creating a match receives the entire finder's fee while in others commissions are split between the principals' brokers in a prescribed manner is suggestive in the light of our results concerning the dependence of the efficient allocation rule on the form of the matching technology.

This reinterpretation of the model is obviously a very special case once brokers are introduced. The opportunity costs of being matched isn't the same for all principals of the same type. This kind of heterogeneity will create inframarginal rents for some and hence an interest in the intensity with which the broker searches. A general model must also allow for search by the principals as well as the brokers. These complications may yield quite different results. Nevertheless, the reinterpretation suggests a fruitful path for further research.

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