DISCUSSION PAPER NO. 380

Search, Layoffs and Labor Market Equilibrium

by

Ken Burdett, University of Wisconsin *

and

Dale T. Mortensen, Northwestern University *

revised

September 1979

Many have recognized that the search theoretic explanation of unemployment has little to say about a separation initiated by an employer "without prejudice" to the worker involved, as a layoff is defined. The recent development of the so-called implicit contract theory of layoff behavior is at least in part a response to this inadequacy. [1] Since a necessary condition for layoffs in the contract framework is imperfect interfirm mobility of workers of the kind emphasized by search models, the two approaches are complementary and closely related explanation of unemployment. In spite of the relationship, no successful attempt to combine the two in a single model exists in the literature. [2]

The purpose of this paper is twofold: First, we extend the simple job search model to include the case in which job prospects are characterized by layoff risk as well as the wage. Second, we create a synthesis of the search and contract approaches by using the former to model the supply side and the latter to model the demand side of a labor market. The result is a simple and consistent theory of labor market equilibrium under conditions of imperfect worker information and uncertain derived demand for labor. The theory explains both search and layoff unemployment as market equilibrium phenomena.

Recent empirical research, inspired at least in part by contract theory, has emphasized the distinction between permanent and temporary layoffs. [3] In both cases separations are employer initiated but in the latter the worker has some expectation of being recalled in the future.

Although those temporarily laid off are a small fraction of total unemployment on average, the evidence suggests that they may comprise most of a typical increase in measured unemployment.(See Lilien [1977] and Medoff [1979].) Furthermore, many who are laid off are subsequently rehired suggesting that few search for an alternative employment prospect. The latter property is a characteristic of the equilibrium established in this paper.

The first two sections are devoted to a characterization of an individual worker's optimal sequential search strategy when each job in the set available is characterized by a wage offer and a probability of employment in future periods. [4/] The case of permanent layoff risk is treated in section 1 and the case of temporary layoff risk is treated in section 2. It is of some interest that in both cases a critical wage exists such that any job offering to pay that much or more is acceptable no matter what the probability of layoff in the future might be. Because the worker is indifferent between employment in such a job and unemployed search, the possibility of being laid off in the future is of no consequence in a stationary environment.

That a marginally acceptable job is defined only by its wage offer suggests that workers don't require a "compensating differential" for layoff risk. In section 2, the temporary layoff case, this paradox is resolved by recognizing that a worker will prefer the job with the lower layoff frequency of two that pay the same wage when employed, given that both are strictly acceptable, and by realizing that the same worker has an incentive to seek such a job even after locating an acceptable one. Indeed, given a reasonable specification of search costs, an acceptable job is any that pays a wage at least equal to the opportunity cost of time spent working. The same worker continues to

search while attached to an acceptable job unless the average of the
wage paid when employed and the value of leisure obtained when not, weighted
respectively by the probability of being employed and laid off, is as large
as the analogously defined imputed average income associated with any
alternative available in the market. Hence, an employer must pay a positive
differential for layoff risk to permanently retain a worker's attachment.

Given that the workers have the same value of time and are perfect
substitutes in production, the expected imputed income given attachment,
as defined above, must be equalized across firms if each acts to retain the
members of its labor force. Furthermore, if all other firms retain, the
remaining will as well, given an allocation of the total number
of workers available among firms that is optimal from each firm's perspective
when all retain. The first observation is used in section 3 to derive the
wage and layoff probability offered by an individual firm that wishes to
retain. [5] The second implies that a strategy of retention by all is an
equilibrium in a non-cooperative game of competition for workers.[6] In
section 4, we show that a unique market clearing equilibrium of this kind
exists. Finally, the equilibrium common value of the expected imputed
income obtained by being a member of the labor force of some firm determines
the allocation of all participants between the number unattached and searching
and the number attached per period and the division of the latter between
the average number employed and laid off.

We begin the analysis with a parable all too familiar to many young
economists.

1. Job Search and Permanent Layoffs

Consider the problems faced by Dougal, a bright young economist. Nearing the completion of his graduate studies, Dougal waits for some of the better universities to make job offers. Since he is a well respected graduate student, many universities want his services. Indeed, it is known that he will receive one offer per week until an acceptable one is found. When a university makes him an offer, the chairperson tells Dougal the annual salary he will receive and the probability of renewal at the end of the contract period. The chairperson goes on to explain that the university will only fail to renew his contract if student enrollment, which is unpredictable, falls. Dougal is assured that these variations are such that the probability of a contract renewal is the same at the end of each future contract period. Finally, in an attempt to pressure him, Dougal is asked to give his answer before he receives the next offer. It will come as no surprise that different universities offer different salary/contract renewal probabilities.

While a graduate student, Dougal spent some time studying the market for academic economists and learned the joint distribution of salaries and contract renewal probabilities for someone of his ability. He confidently predicts that (a) this joint distribution will remain the same in all future periods, and (b) each offer can be regarded as a random draw from this distribution. Possessing little sense of history and caring nought for prestige or location, Dougal wants to discover the search strategy that maximized his expected discounted lifetime income. Dougal's expected lifetime is quite long. In fact, he expects to live forever.

Dougal, having taken a graduate course in labor economics, recognizes

that his problem is similar to that faced by unemployed workers in
sequential job search models. The only difference is that all firms
offer the same contract renewal probability but different wage rates in
the existing job search literature. Dougal realizes that if not renewed
at the end of the contract period, he will be laid off and searching for
another job.

Dougal feels that the results from the job search literature can be
applied to determine his optimal search strategy nevertheless. Having been
trained as a theorist, he of course disdains the effort involved in making a
precise numerical solution. Instead, he conjectures.

Conjecture 1:   The optimal strategy if an offer with layoff (non-renewal)
                probability s is received involves the use of a reservation
                salary $w(s)$. An offer with layoff probability s should be
                accepted if and only if the salary, y, is at least as
                great as $w(s)$.

Conjecture 2:   The reservation salary $w(s)$ is a strictly increasing
                function of s.

Dougal is confident conjecture 1 is true but unsure of the validity of
conjecture 2. Ultimately, the following argument persuades him it is correct.
Suppose two offers are received with the same salary attached. Further, assume
that one of these offers involves a greater permanent layoff probability.
If at least one of these offers is acceptable, then the one with the lower
layoff probability yields a higher expected lifetime income and should be
accepted. In fact it is not too difficult to show that even if the offer with
the lower layoff probability involves a slightly smaller salary it may still
be preferred. Feeling he has solved his problem, Dougal waits for the offers

to roll in.

Dougal's first conjecture is correct. Let $W(y,s)$ denote the expected discounted lifetime income if a job with characteristics (y,s) is accepted for at least one period. The expected payoff when Dougal turns down an offer depends on his future search strategy, i.e., the salary/layoff probability combinations that will be accepted if offered. Let V indicate the expected payoff from the search strategy that yields the greatest expected discounted income. It follows immediately that Dougal should accept an offer with characteristics (y,s) if and only if $W(y,s) \geq V$. Letting r denote the discount rate, we have

$$(1) \qquad W(y,s) = \frac{y}{1+r} + \frac{(1-s)}{(1+r)} \max [V, W(y,s)] + \frac{s}{(1+r)} \lambda(y,s)$$

where $\lambda(y,s)$ denotes Dougal's expected discounted lifetime income if laid off from a job with characteristics (y,s) at the end of the period. The restrictions, (a) being laid off is not a signal of incompetence, (b) layoffs are permanent, and (c) Dougal expects to face the same distribution of job offers in any period he is looking for a job, guarantee

$$(2) \qquad \lambda(y,s) = V \quad \text{for any} \quad (y,s).$$

Defining $w(s)$ by $W(w(s),s) = V$ for any given s, and noting from (1) and (2) that $W(y,s)$ increases with y, whereas V is independent of y, establishes the validity of conjecture 1.

Surprisingly, Dougal's second conjecture is false. Manipulation of (1) and (2) demonstrates this claim:

$$(3) \qquad w(s) = rV \quad \text{for any} \quad s.$$

Hence the optimal search strategy for Dougal is such that the layoff
probability offered can be ignored when deciding to accept or reject an
offer. To understand this result more clearly consider the isopayoff
curves on the set of acceptable offers, i.e., the locus of (y,s) combina-
tions that yield the same expected payoff. The slopes of these curves can
be written as

$$
\frac{dy}{ds}\bigg|_{dW(y,s) \,=\, 0} \;=\;
\begin{cases}
\dfrac{1}{(r+s)}\,\{y - rV\} > 0 & \text{if } y > w(s) \\[2em]
0 & \text{if } (y,s) \text{ is such that } y \le w(s) \ .
\end{cases}
$$

Conjecture 2 is false because a marginally acceptable job has the same
payoff as that associated with continuing to search.

Dougal's problem and its solution can be applied to a wide variety of
job search situations. It approximates the problems faced by many unemployed
workers who participate in a labor market in which firms lay off workers
from time to time. Allowing Dougal to return to previously rejected offers
will not change the basic result. There is, however, one obvious limitation
to the possible application of the analysis.

It has been assumed that the job opportunities open to a laid off
worker are unaffected by the fact of being laid off. Suppose this restriction
is dropped. Instead, assume Dougal only fails to get his contract renewed
if the department feels he is incompetent. In this case failure to get a
contract renewal may serve as an adverse signal to potential future employers.
Using the same notation as before, we have

(4) $\qquad \lambda(y,s) = V'$

where V' denotes the expected return to search after being laid off.
Assume $V' < V$, i.e., assume the return to search declines after being laid
off. Let $w(s)$ be defined by $W(w(s),s) = V$ for any given s. Manipulation
of the modified version of (1) yields

(5) $\qquad w(s) = rV + s[V - V']$

for any given s. Hence the reservation salary $w(s)$ is a strictly increasing
function of s. Equation (5) applies whatever the reason for non-stationarity
in the value of search.

<u>Proposition 1:</u> The reservation salary is independent of s (increasing with s)
if the return to search after being laid off is the same as (less than) the
expected return to search before a job was found.

2. Job Search and Temporary Layoffs

The typical laid off worker is not permanently separated from his or her job. Many layoff spells constitute only temporary separations because employers generally rehire laid off workers before hiring workers with no previous employment experience with the firm and laid off workers often wait to be rehired rather than seek out an alternative employment prospect. Recruiting difficulties are sufficient to rationalize the employers' behavior.

There are two questions to be answered concerning the workers' behavior. First, what is an acceptable job to a worker with no other attachment? Second, for which jobs will the same worker wait for recall when laid off rather than search for an alternative? To answer these questions, we adapt the model of the previous section appropriately.

Layoffs are attributed to temporary reductions in the demand for an employer's product and interfirm immobility in the short run. Time is divided into a sequence of short intervals. At the beginning of each interval, the employer announces the fraction of the firm's labor force to be employed during the interval in the light of realized demand. The particular individuals employed are chosen at random and each earns a wage equal to y. The remaining fraction, equal to s on average, is laid off for the duration of the interval. At the beginning of the next period, all members of the labor force have an equal chance of being employed during that period. With this reinterpretation in mind, a job is characterized by the pair (y,s) as in the previous section.

Search is sequential and costly but a worker whether employed, laid off or otherwise unattached to a job has access to the same search technology.

Each worker contingent on his or her status selects a search strategy from the feasible set given the technology; one that maximizes the present value of the expected future income stream. However, in calculating "income" a value is imputed to the time not spent working when unemployed, which is denoted by x. Again the discount rate is r. A worker is said to have a job attachment of some degree if either employed or laid off. The attachment is complete if the worker does not search for an alternative.

Consider an unattached worker. Let V denote the worker's present value of expected future income given an optimal search strategy, hereafter called the value of search. The value of search generally depends on the set of job prospects available, the worker's information about these prospects, and the technology available to augment this information. As in the previous section, we suppose that the worker knows the distribution of prospective contracts but does not know the identity of the employer offering any specific one. Identity information arrives sequentially in a manner described by a Poisson arrival process with

parameter α. Specifically, the expected number of prospects that arrive in a unit period, α, is chosen by the worker subject to a cost $c(\alpha)$. Hence, α measures the intensity of search and $c(\alpha)$ the investment made per period in search. [7]/

Assumption 1 : $c(\alpha)$ is a monotonic strictly convex function such that $c'(0) = 0$ and $\lim\limits_{\alpha \to \infty} c'(\alpha) = \infty$.

In other words, the cost of search increases at the margin, a little search

is virtually costless but the instantaneous location of a job prospect is impossible. The value of $c(\alpha)$ can be interpreted as the sum of any out of pocket cost incurred plus the value of forgone leisure.

For any short time interval of length h , the probability that the worker will find one job prospect is approximately αh and the likelihood of more than one arrival is insignificantly different from zero (of order h^2). Let $W(y,s)$ represent the discounted expected future income stream were the worker attached to a firm whose jobs are characterized by (y,s). As of the beginning of the interval $[0,h]$, the worker regards any prospect found during the interval as a random draw from the set of possibilities. Hence, the worker's optimal choice of α and the present value of net income associated with unattached search given optimal behavior in the future solve

(6) $V = \max_{\alpha \geq 0} [h(x - c(\alpha)) + \alpha h \, E \max [\tilde{W},V] + (1 - \alpha h)V]/(1 + rh)$

by virtue of Bellman's principle of dynamic optimality, where E denotes the expectation operation taken with respect to the distribution of $\tilde{W} = W(\tilde{y},\tilde{s})$ induced by the function $W(\cdot)$ and the known joint distribution of job characteristics. Of course, equation (6) incorporates the fact that an unattached participant accepts any prospect with a value at least as large as the value of search.

An unattached worker is a participant in the labor force if and only if he or she searches at some positive intensity. Given Assumption 1, the optimal search intensity in the unique solution to

(7) $c'(\alpha^o) = E \max [\tilde{W},V] - V,$

that rate which equates the marginal cost and gain attributable to more intensive search. This fact, $c(0) = 0$ and (6) imply

(8) $x/r \leq V \leq \overline{W}$

for every participant, where $\overline{W}$ is the expected present value of the

"best" job prospect or equivalently is the upper bound on the support of $\tilde{W}$.

Equality is strict if and only if $\alpha^o = 0$. In other words, a worker particpates

$(\alpha^o > 0)$ if and only if the present value of attachment to the best job

prospect exceeds the present value of a life time of leisure.

Now, consider the same worker with the opportunity to form an attachment

with a firm with jobs characterized by (y,s). Suppose that the worker's employment

status is reviewed in the manner described earlier at the beginning of every

interval of length h . Whether employed or not, the worker can choose to look

for an alternative at some chosen intensity α. However, if employed during

the interval $[0,h]$, the worker's income net of search cost will be $[y - c(\alpha)]h$.

It will be $[x - c(\alpha)]$ if laid off. The difference, given α, is simply

the surplus attributable to being employed. At the end of the interval, the

worker can choose to either continue the attachment or to form a new one if

an alternative is located. Hence, if we let $W_2(y,s)$ denote the value of

the attachment to (y,s) as of the beginning of the interval $[0,h]$ when the

worker is employed and $W_1(y,s)$ denote the analogous value when laid off, we

have , again by virtue of Bellman's principle ,

(9) $W_1(y,s) = \max_{\alpha \geq 0} [h(x - c(\alpha)) + \alpha h \mathrm{Emax}[\tilde{W},W(y,s)] + (1 - \alpha h) W(y,s)]/(1 + rh)$

and

(10) $W_2(y,s) = \max_{\alpha \geq 0} [h(y - c(\alpha)) + \alpha h \mathrm{Emax}[\tilde{W},W(y,s)] + (1 - \alpha h)W(y,s)]/(1 + rh).$

Finally, the expected present value of continuing the attachment as of the

end of the interval is simply the average of the two weighted by the

probability of layoff and employment in the next interval; i.e.,

(11) $W(y,s) = sW_1(y,s) + (1-s)W_2(y,s)$.

By inspecting (9), (10), and (11), we see that $W(y,s)$ is a continuous strictly increasing function of y given any $0 \leq s < 1$. Hence, the prospect (y,s) is acceptable when the worker is unattached if and only if $y \geq w_1(s)$ given s where the reservation wage $w_1(s)$ is the unique solution to $W(w_1(s),s) = V$. Because $y = x$ implies $V = W_1(x,s) = W_2(x,s)$ for all s by virtue of (6), (9), (10), and (11), the reservation wage is simply the opportunity cost of time; i.e. $w_1(s) = x$.

Although the conclusion is similar to that obtained in the permanent layoff case in the sense that the reservation wage is independent of the layoff probability, the value of the reservation wage differs to the extent that the imputed value of search per period, rV, exceeds the opportunity cost of time, x . This difference arises because we have assumed that the worker can search while attached whether employed or laid off. Therefore, any job that just compensates for the leisure forgone while employed is equivalent to unattached search. However, a worker "attached" to a firm offering such a job is obviously indifferent to employment now or in the future.

A worker currently attached to the job (y,s) accepts an alternative when located if the latter exceeds $W(y,s)$ in value. Furthermore, an attached worker's optimal search intensity satisfies

$$c'(\alpha^o) = E \max [\tilde{W}, W(y,s)] - W(y,s) .$$

Hence, the worker will eventually find and accept an alternative if

$W(y,s)$ is less than the best prospect available, its expected present
value $\overline{W}$. Equivalently, an employer can permanently retain the attachment
of the worker if and only if $y \geq w_2(s)$ given s where $w_2(s)$ is the unique
solution $W(w_2(s),s) = \overline{W}$.

Proposition 2: The reservation wage, $w_1(s)$, is independent of the layoff
probability but the wage required to retain a worker, $w_2(s)$, is a
strictly increasing function of the layoff probability. Specifically,

$$(12) \qquad (1 - s)w_2(s) + sx = w \equiv r\overline{W} \geq x = w_1(s)$$

where x is the value of the worker's time.

Proof: Since $W(y,s) \geq \overline{W}$ implies $\alpha^o = c(\alpha^o) = 0$, the equations
(9), (10), and (11) at $(y,s) = (w_2(s),s)$ form a linear system with
the following unique solution:

$$(13) \qquad (1 + rh)rW_1(w_2(s),s) \;=\; (rh + s)x + (1 - s)w_2(s)$$

$$(1 + rh)rW_2(w_2(s),s) \;=\; sx + (1 + rh - s)w_2(s)$$

$$rW(w_2(s),s) \;=\; sx + (1 - s)w_2(s) \;.$$

The first equation of (12) is implied by $W(w_2(s),s) = \overline{W}$ and the last equation
of (13), the inequality is implied by (8) and the last equality is implied by
$W(w_1(s),s) = V$ as already demonstrated.

The parameter w is the flow equivalent of the expected present value
of the best alternative prospect. It can be interpreted as the wage paid
in the best job were the layoff probability zero. Hence, if the prospect

(y,s) has an equivalent present value, then the difference

$y - w = s(w - x)/(1 - s)$ is the differential needed to compensate a

worker with value of time x for the positive layoff probability s .

3. Optimal Retention Contracts

Implicit contract theory is based on two postulates. First, inter-firm mobility is imperfect, at least in the short run. Second, the demand for each firm's product is uncertain. Because workers are allocated to firms before the demand state is realized as a consequence of the first assumption, a centralized expost spot auction for workers is not possible. Expost the only alternatives to employment with the firm to which any worker is attached are leisure and search.

The contract approach views the employing firms as competing for workers exante by offering contracts that specify the wage to be paid and the likelihood of employment in each future period of a worker's tenure. The market for contracts is said to be in equilibrium when each firm is able to attract and retain a labor force of the desired size given the contracts offered by all the other firms. This notion of equilibrium, though often stated, is not particularly well defined in the existing literature because the nature of worker immobility is not explicitly modelled. In this section we review the theory of optimal contract choice when worker immobility is attributable to the cost of search as modelled in the previous section. In the concluding section we show that the concept of equilibrium outlined above can be viewed as a steady state of a competitive process for workers in which information about the identities of the firm's offering different contracts is imperfect.

The workers are not only perfect substitutes in production, they also have

the same opportunity cost of time, x . Let i = 1, ..., m denote the set of employing firms. In general the firms produce different products. The output of firm i per period is $f_i(\cdot)$, a function of the number of workers employed per period. The output produced in a period is sold at price p_i . Uncertain demand is modelled by assuming that the vector of future prices is generated by some stochastic process. We assume that the sequence of future price vectors are identically and independently distributed. Specifically, the uncertainty in any future price for each firm's output is described by a c.d.f. $F_i(\cdot)$ defined on the set of possible prices $P_i \subset R_+$ which is known to the firm. Finally, each firm ranks alternatives according to the expected profit criterion.

As both workers and employers are neutral with respect to income risk, the wage paid workers when employed, y_i , can be regarded as independent of the output price realized in any period without loss of generality.[8]/ The contingent fraction of a firm's labor force to be employed during any particular period is determined by a chosen rule $q_i:P_i \rightarrow [0,1]$. Individual members of the labor force to be employed are selected at random. Hence, the exante probability of layoff is

$$(14) \qquad s_i = 1 - E q_i(p) = 1 - \int_{P_i} q_i(p) dF_i(p).[9]/$$

The supply consequences of offering an arbitrary pair (y_i, s_i) are illustrated in Figure 1 . To attract unattached workers, the employer must pay a wage at least as large as $w_1(s) = x$, the common value of

worker time. The typical firm can attract attached workers with a wage/layoff probability combination that lies above $y = w_1(s)$ and below $y = w_2(s)$ if the others do not retain. However, to retain the attachment of its own labor force members, the employer must offer a combination on or above the curve $y = w_2(s)$. In other words, retention requires

(15) $\quad (1 - s_i)y_i + s_i x \geq w$,

where w is the given "full employment" wage equivalent associated with the best alternative available to its workers.

Figure 1: Supply Conditions
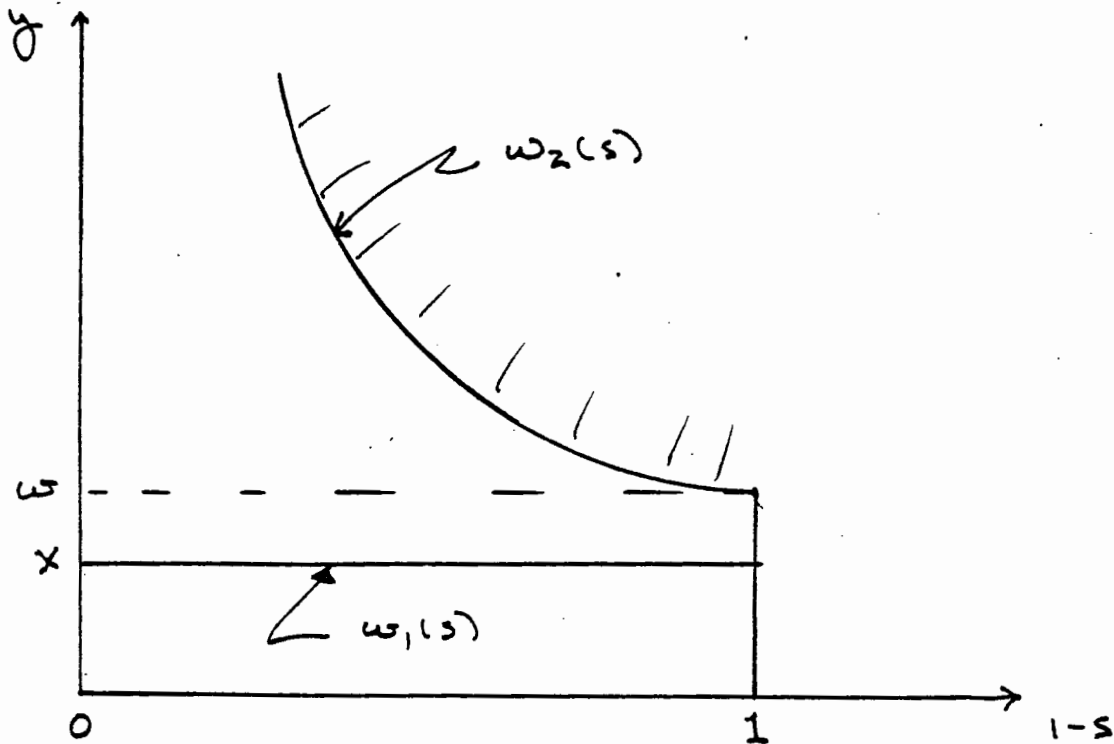
Given that employer i retains, a wage y_i , an average layoff probability s_i , a desired steady state labor force size n_i , and a contingent employment rule $q_i : P_i \to [0,1]$ are chosen to maximize expected profit

$$\pi_i = \int_{P_i} p f_i (q(p)n) dF_i(p) - (1 - s) yn$$

subject to (14) and (15) . Since in a steady state the firm has no incentive to satisfy (15) as an inequality, the problem is equivalent to choosing a number n_i and a function $q_i : P_i \to [0,1]$ to solve

$$(16) \qquad \pi_i(w,x) = \max \int_{P_i} [p f_i (q(p)n) - q(p)xn - (w - x)n] \, dF_i(p)$$

<u>Assumption 2</u>: (a) $f_i(\cdot)$ is a strictly increasing and strictly concave function such that $f_i'(0) = \infty$, and

(b) P_i is bounded away from both 0 and ∞ for all $i = 1, \ldots, m$.

Given the assumptions, a solution to (16), n^o and $q^o : P_i \to [0,1]$, is unique and satisfies the following Kuhn-Tucker conditions:

$$(17.a) \quad \int_{P_i} \{ q^o(p) p f_i'(q^o(p)n^o) - q^o(p)x - (w - x) \} dF_i(p) = 0$$

$$(17.b) \quad p f_i'(q^o(p)n^o) - x - \eta(p) = 0 \qquad \forall \; p \in P_i$$

$$(17.c) \quad \eta(p) [1 - q^o(p)] = 0 \quad \text{and} \quad \eta(p) \geqq 0 \;\; \forall \; p \in P_i \; .$$

The multiplier $\eta(p)$ is that associated with the upper bound on $q(p)$.

By substituting appropriately from (17.b) and (17.c), (17.a) can be rewritten as

$$\int_{P_i} [q^o(p)(x+\eta(p)) - q^o(p)x - (w-x)]dF_i(p) = 0 .$$

or, equivalently,

(18.a) $\int_{P_i} \max [0, pf'_i(n^o) - x]dF_i(p) = w - x .$

Together, conditions (17.b) and (17.c) imply

$$q^o_i(p) = \min[1, d_i(x/p)/n_i]$$

where $d_i(\cdot) \equiv [f'_i(\cdot)]^{-1}$. Hence,

(18.b) $s^o_i = 1 - \int_{P_i} \min [1, d_i(x/p)/n^o_i]dF_i(p)$

and

(18.c) $(1 - s^o_i)y^o_i + s^o_i x = w$

by virtue of (14) and (15) respectively.

Clearly, if there is no uncertainty in the output price, P_i is a singleton, then the firm's labor force is always fully employed. The firm pays a wage equal to the given value of w and demands a labor force of the size that equates the value of marginal product to the full employment required wage w . In the general case, some fraction of the labor force is

laid off when the realized value of marginal product given full employ-
ment $pf_i'(n^o)$ is less than the value of worker time, x . Indeed, in
these demand states, the fraction employed is chosen to equate the expost
value of marginal product of those employed, $pf_i'(q^o(p)n^o)$, to the
value of time, x . The desired labor force size is chosen exante to balance
the expected loss associated with not having a large enough labor force
expost, the left side of (18.a) , to the additional cost of having
another worker in the labor force, $w - x$. This condition is similar
to that found in the literature on the "stock out" model of optimal
inventory holding. Since $w = x$ implies that $\overline{p}f_i'(n^o) = x$, where $\overline{p}$
is the largest possible price, by virtue of (18.a), layoffs will occur
in low price states if the value of time is positive by virtue of (18.b)
and the difference between the "full employment" wage required to retain
and the value of time is not too large.

The conditions of (18) generally define both the desired labor force
size and the optimal average layoff probability as functions of the given
"full employment" wage required.

Proposition 3: If firm i acts to retain the attachment of its workers,
then the number desired $n_i^o(w,x)$ and the optimal layoff probability
$s_i^o(w,x)$ are both continuous functions, decreasing in w and increasing
in x . Furthermore, $n_i^o(x,x) > 0$.

Proof: The first claim is implied by (18.a) and the assumption

that $f_i'(\cdot)$ is continuous and strictly decreasing. Given this result, the second claim is implied by (18.b) . The final claim is implied by $f'(0) = \infty$.

When all employers retain, each locates somewhere along the curve $y = w_2(s)$ in Figure 1 . The combination chosen by a particular firm depends on the peculiarities of its technology and distribution of its output price. Although wage offers generally differ between firms, any differential reflects the compensation required for a difference in layoff probabilities.

4. Labor Market Equilibrium

If every firm acts to retain the attachment of its labor force members, then the labor market can be said to clear at a common contract value when the aggregate desired labor force size is equal to the steady state number of workers that desire attachment. This solution to the problem of coordinating the plans of the many employers and workers in a labor market, which we call a retention equilibrium, is ·valid if, and really only if, retention by all employers is a non-cooperative solution to the game of contract choice. In this section we show that a unique retention equilibrium exists and argue that it is a non-cooperative solution to the game if the following condition holds.

Assumption 3: The conditional probability that an offer received by a searching worker is that of firm i equals n_i/n .

The assumption asserts that information flows about each firm are proportional to the size of the firm's labor force. Although it is admittedly ad hoc, the assumption is consistent with the observation that most workers find out about the employment opportunities at a particular firm from the firm's own labor force members. [10]/

Let $w = r\overline{W}$ denote the average imputed income obtained by a worker attached to a firm offering a contract of the largest value, $\overline{W}$, in the market. As noted above, w is the wage that a firm that never lays off workers must pay to retain their attachment, the "full employment" wage required. If all firms act to retain, then the aggregate number of workers desired by all firms is the positive, continuous and strictly decreasing function of the full employment wage defined by

$$(19.a) \quad n^d(w,x) = \sum_{i=1}^{m} n_i^o(w,x) \ .$$

Given uncertain output prices, some firms lay off workers from time to time. The total number laid off on average when all retain attachment is given by the function

$$(19.b) \quad \ell(w,x) = \sum_{i=1}^{m} s_i^o(w,x) n_i^o(w,x)$$

It too is continuous and decreasing in w from Proposition 3 .

On the supply side, let n_o denote the total number of parti-cipants in the labor market. By assumption, the number that retire per period equals the number of new entrants in each period so that n_o is a constant. The fraction that retire is independent of labor market status and is denoted by δ . In a steady state, the flow of attached workers that retire is just balanced by the flow of unattached workers that find job prospects, given that all employers act to retain the attachment of their respective labor force members. Given Assumption 3, the following steady state condition holds for every firm

$$(20) \quad (n_i/n)\, \alpha^o [n_o - n] = \delta n_i \quad , \quad i = 1,\ldots,m,$$

where α^o is the optimal search intensity common to unattached workers, the rate at which an unattached worker finds job prospects, and n_i/n is the fraction of unattached workers that contact firm i .

The optimal search intensity of an unattached worker, α^o , is the solution to (7) . Hence, we have

<u>Proposition 4</u>:   If every employing firm retains the attachment of its labor force, then the optimal search intensity of an unattached worker $\alpha^o(w,x)$ is a continuous function, increasing in w and decreasing in x, such that $\alpha^o(x,x) = 0$.

Proof. Because the value of every offer is w/r under the hypothesis, (6) can be rewritten as

$$rV = \max_{\alpha \geq 0} \; [x - c(\alpha) + (\alpha/r)(w - rV)] \; .$$

Hence,

$$rc'(\alpha^o) = w - rV \; .$$

A differentiation of the value function yields

$$\frac{\partial rV}{\partial w} = \frac{\alpha^o}{\alpha^o + r} < 1 \quad \text{and} \quad \frac{\partial rV}{\partial x} = \frac{r}{\alpha^o + r} > 0 \; .$$

This fact and $c'(\alpha)$ continuous and strictly increasing implies that α^o is strictly increasing in w, decreasing in x and continuous on $w \geq x$. Finally, since $w = x$ implies $rV = x$, $c'(0) = 0$ implies $\alpha^o(x,x) = 0$.   Q.E.D.

The proposition and and equation (20) imply that the aggregate steady state "supply" of attached workers,

(2.1) $n^s(w,x) = \alpha^o(w,x)n_o/[\delta + \alpha^o(w,x)]$,

is zero at $w = x$, continuous and strictly increasing in w for all $w \geq x$ decreasing in x and everywhere less than n_o. The reader should be well prepared for the following.

<u>Definition</u>: A retention equilibrium is a pair (w^*, n^*) such that

$$n^d(w^*, x) = n^s(w^*, x) = n^* .$$

The following is an immediate implication of Propositions 3 and 4 and the definition.

<u>Proposition 5</u>: A unique retention equilibrium exists.

Figure 2
Retention Equilibrium

As illustrated in Figure 2, the equilibrium "full employment" wage required to retain, w^*, exceeds the common value of time in order to motivate search by new entrants and the equilibrium number of attached workers is less that the total number of participants as a consequence of the time required to find a job by new entrants. In equilibrium, average layoff unemployment is represented by the distance from the origin to ℓ^* along the horizontal axis and search unemployment is represented by the distance from n^* to n_o .

A retention equilibrium is closely related to the notion of a competitive equilibrium under conditions of certain demand and perfect interfirm mobility. As the variances in output prices vanish, $\ell(\cdot)$ tends to zero and $n^d(\cdot)$ limits to the classical market demand for workers. As either search costs vanish or the turnover rate tends to zero, $n^s(\cdot)$ converges to the "reverse L" shaped curve $n = n_o$ for all $w > x$ with a perfectly elastic segment at $w = x$, the supply curve of the standard theory.

This observation suggests that the pursuit of a retention strategy, heretofore assumed, is individually rational in the sense that each firm finds the strategy in its own interest under the expectation that the others will continue to offer the market clearing "full employment" wage w^* in the future. We know that this property obtains in the limiting case of costless search (perfect interfirm mobility). Specifically, given $w_j = w^* \; \forall \; j \neq i$, $w_i > w^*$ implies that firm i would instantaneously attract the entire labor force and $w_i < w^*$ implies that all members of the firm's own labor force would instantaneously vanish. Hence, the profit associated with an offer of w^* and the desired labor force size given that offer is positive and exceeds the maximal profit obtainable given any higher or lower offer.

Given our assumption that a little search is costless $(c'(0) = 0)$, the _eventual_ responses to a differential $w^* - w_i$ are the same. They simply take time to materialize. Formally, Proposition 2 and the steady state condition (20) imply that the flow of workers that seek attachment with firm i is permanently larger (smaller) than the flow of leavers given a positive (negative) differential under the expectation that the other firms continue to offer w^* and that the strategy of firm i has no appreciable effect on the aggregate number of attached workers. In other words, given non-cooperative behavior, each firm perceives that

it faces a long run or steady state supply curve for attached workers that is perfectly elastic at $w_i = w^*$. Hence, a permanent policy of retention given that all other firms offer w^* in the future and that the firm's current labor force size is that desired, given $w_i = w^*$, maximizes the firm's expected profit per period over an infinite future on the class of all stationary strategies.

The equilibrium levels of both search and layoff unemployment are "efficient" given the search technology specified and the assumed stochastic nature of product demand. Within any sufficiently short time interval, expost reallocation of workers designed to take advantage of realized differences in the value of marginal products across firms is technically infeasible. However, total income expost, which properly includes an imputation for the value of time not spent working by laid off workers, is maximized by the optimal layoff rules given the feasible set of alternatives. Given intertemporal independence of product demand, expected exante income net of search costs is also maximized in equilibrium by the equalization of the expected values of offers across firms. Wasteful search by attached workers is eliminated and the information needed by new entrants to seek a job at an efficient rate is provided. Nevertheless, search cost saving innovations and product price variance reducing policies will reduce unemployment and, consequently, increase total income both exante and expost.

Although our theory of equilibrium is not much more complicated than the standard perfectly competitive story, its comparative static impli- cations are much richer. Two examples suffice to make the point. An increase in the cost of search at the margin shifts $n^s(\cdot)$ everywhere to the left in Figure 2 . Since the demand side of the market is not directly

affected, the "full employment" wage is larger and the number of attached workers is smaller in the new equilibrium. Because the optimal layoff probability is decreasing in w , the fraction of those attached who are laid off on average is smaller as a consequence. Alternatively, an increase in the dispersion of output prices can be shown to shift $n^d(\cdot)$ to the right in Figure 2 . As a consequence, the new equilibrium is characterized by a higher "full employment" wage, a larger number of attached workers and, consequently, less search unemployment. These examples also illustrate the nature of the supply and demand interactions that determine the equilibrium levels of both forms of unemployment as well as the wage level.

Furthermore, the model is well suited for the purpose of analyzing the equilibrium effects of unemployment insurance policy. Consider the case of an employment benefit payment of b per period paid only to laid off workers and financed by a tax system that is not experience rated. Since new entrants don't qualify for the benefit, an increase is equivalent to an increase in the opportunity cost of time spent not working by attached workers. $\underline{11/}$ Hence, the equilibrium "full employment" wage required and the equilibrium aggregate number of attached workers associated with each non-negative benefit b satisfy the market clearing condition

$$n^*(b) = n^d(w^*(b), x+b) = n^s(w^*(b), x).$$

Because the number of attached workers desired by every firm increases with the opportunity cost of their workers' time by virtue of Proposition 3, both $n^*(b)$ and $w^*(b)$ increase with b.

The effect of an increase in the benefit on the equilibrium levels of

search and layoff unemployment is more complicated. The optimal layoff
probability for an individual firm $s_i^o(w, x+b)$ increases with the
benefit level, given the wage, by virtue of Proposition 3. [12/] However,
because Proposition 3 also implies that the layoff probability decreases
with the "full employment" wage required, this effect is only part of the
story. An increase in the benefit induces a negative feed back on the
layoff probability through its effect on the equilibrium "full employment"
wage. Indeed, the equations of (18) imply that the offsetting feed back
effect can dominate at the level of an individual firm if the marginal
product curve is sufficiently elastic. [13/] However, to obtain a decrease
in the aggregate fraction of the equilibrium number of attached workers
laid off, the aggregate steady state number of workers available $n^s(w,x)$
must be relatively elastic. Nevertheless, an increase in the benefit
unambiguously decreases the level of search unemployment in equilibrium
because the induced increase in the wage stimulates more intensive search
by new entrants. [14/]

Summary

We argue that the search theoretic and implicit contract approaches to labor market analysis can be fruitfully combined. The synthesis developed in this paper yields an equilibrium theory of both search and temporary layoff unemployment, of the wage level, and of the relationship between wage rates and layoff rates across firms. The environment considered is characterized by imperfect interfirm mobility in the short run and by uncertain derived demand for labor by firms in the future.

Given short run immobility, every participant is either attached to a firm at a moment in time or is an unemployed worker seeking an attachment. Each firm is viewed as offering a contract to its attached labor force that determines the wage to be paid if employed and the probability of being employed in the future. Employed workers are assumed to work a fixed number of hours per period but otherwise the production technology satisfies standard conditions. Lack of interfirm mobility is attributed to incomplete worker information about the identities of the firms offering alternative contracts and the cost of acquiring such information. The search technology assumed allows for a search intensity that can be varied at a strictly increasing marginal cost but is independent of a worker's labor market status. The principal results of the paper follow. First, in the case of workers who are perfect substitutes in production and have identical preferences for contracts, an employer can retain the attachment of its labor force members

if and only if its contract has a value to the worker which is equivalent
to that of the best offer in the market. Second, if all employers
choose contracts that retain, a unique market clearing contract value
exists. By market clearing, we mean that the aggregate steady state number
of attached workers available given optimal search is equal to the total
number desired by all the employers given the optimal contract choice of each.
Third, given the market clearing contract value, every firm retains if the
others do. Finally, the equilibrium has the following properties. The search
unemployed are new entrants in the process of finding an attachment, layoff
unemployment occurs at each firm to the extent needed to equate the realized
value of marginal product to the value of time spent not working, and
any difference between the wage rates offered by any two firms is the
compensation for the difference in their layoff rates required to keep
the values of the two offers equal.

Although the model is no more special than almost any other in the
related literature, a number of restrictions must be relaxed before
the theory can lay claim to generality in any important sense. This
observation suggests several topics for future research.

The assumption that there is no autocorrelation in the demand for
labor can be given up with ease in principle. However, the basic simplicity
of the results, particularly the diagrammatic representation of equilibrium
given in Figure 2, must be foregone. If demand states across firms (sectors
or industries) are not perfectly correlated, interfirm movement of workers

occurs in equilibrium. Even if perfectly correlated, movements in and out
of the labor force occur. The complexity of the generalized theory is sug-
gested by the work of Lucas and Prescott [1974].

A more important restriction from the point of view of pure theory
is the assumption that workers are identical both in production
and with regard to preferences. Initial efforts by the authors suggest that
the basic results can be preserved if differences in the workers'
values of time can be distinguished. A recent paper by Loewenstein
[1979] suggests similar results in the case of productivity differences
that are observable. However, in practices both kinds of differences are im-
perfectly observed at best. In this case the analysis is complicated by
selection and sorting problems of the kind considered by Grossman [1978].

Finally, in this paper the process of matching unattached workers
is coordinated in such a way that in equilibrium no searching worker is
denied attachment for lack of a vacancy and no firm has to wait to replace workers
that leave by assumption. Hence, there are no vacancies and search unemploy-
ment is voluntary. The hand that directs such a marvel of coordination is
still most invisible.

FOOTNOTES

[1] The standard references include Azariadis [1975], Baily [1974], and Gordon [1974].

[2] Baily [1977] takes a step which suggested the direction pursued in this paper.

[3] The literature includes Feldstein [1975], Lilien [1977], and Medoff [1976].

[4] The model used is an adaptation of that developed by the authors in Mortensen [1978] and Burdett [1978].

[5] The model of optimal contract formation used is virtually identical to that developed by Feldstein [1976] and Baily [1977] except that the length of the work week is fixed in our formulation.

[6] This idea is often expressed but is never fully exploited in the literature on labor contracts.

[7] This way of modelling "search intensity" seems to have originated with Mortensen [1976] and Wilde [1977]. Alternative approaches are reviewed in Lippman and McCall [1976].

[8] Recent contributions by Feldstein [1976] and Baily [1977] clearly demonstrate that aversion to risk in earnings is not necessary for a theory of layoffs. Akerlof and Miyazaki [1978], Mortensen [1978], and others show that risk aversion is not sufficient either.

[9] If the process generating prices is autocorrelated, then the expectation must be conditional on information about past price realizations. The theory is more complicated in this case because intertemporal interfirm movements of workers will occur in "equilibrium" as in Lucas and Prescott [1974].

[10] See Rees and Schultz [1970].

[11] The same result holds in Feldstein [1975] and Baily [1976].

[12] This is precisely the effect derived by Feldstein [1975] and by Baily [1976].

[13] In Feldstein's [1975] analysis, this effect can't dominate because an increase in the benefit increases each firm's fixed cost per attached worker given a non-experience rated unemployment insurance tax and because firms instantaneously enter and leave to maintain profits at zero. In our analysis, the number of firms is fixed and all earn positive profits at least when $b = 0$. Hence, for small values of b none leave. Which specification is the appropriate one would seem to be an empirical question.

[14] For a more extensive analysis of this issue, although in a partial equilibrium context, see Mortensen [1977]. Note that the new entrant is the only unemployed type of participant that searches in equilibrium in our model. There is evidence that the duration of search by permanently separated workers is adversely affected by larger benefits (see Katz [1977]) These don't exist here, but would if derived demand were autocorrelated.

REFERENCES

Akerlof, G.A., and Miyazaki, H. "The Implicit Contract Thoery of
   Unemployment Meets the Wage Bill Argument." Memorandum No. 217,
   Center for Research in Economic Growth, Stanford University, 1978.

Azariadis, C. "Implicit Contracts and Underemployment Equilibria."
   Journal of Political Economy 83 (December 1975) : 1183-1202.

Baily, M.N. "On the Theory of Layoffs and Unemployment." Econometrica
   45 (July 1977): 1043-1063.

Baily, M.N. "Wages and Unemployment Under Uncertain Demand." Review
   of Economic Studies 41 (January 1974): 37-50.

Barro, R.J. "Long Term Contracting, Sticky Prices, and Monetary Policy."
   Journal of Monetary Economics 3(July 1977): 305-316.

Burdett, K. "Employee Search and Quits." The American Economic Review 68
   (March 1978): 212-220.

Feldstein, M. "Temporary Layoffs in the Theory of Unemployment." Journal
   of Political Economy 84 (October 1976): 937-957.

Feldstein, M. "The Importance of Temporary Layoffs: An Empirical Analysis."
   Brookings Papers of Economic Activity 3 (1975): 725-745

Gordon, D.F. "A Neo-Classical Theory of Keynsian Unemployment." Economic
   Inquiry 12 (December 1974): 431-459.

Grossman, H. "Risk Shifting, Layoffs, and Seniority." Journal of Monetary
   Economics 4 (November 1978): 661-686.

Katz, A.(ed.) "The Economics of Unemployment Insurance, A Symposium."

    Industrial and Labor Relations Review 30 (July 1977)

Lilien, D. "The Cyclical Pattern of Temporary Layoffs in U.S. Manufacturing."

    UCSD Working Paper No. 77-24.

Lippman, S.A., and McCall, J.J. "The Economics of Job Search: A

    Survey." Economic Inquiry 14 (June 1976): 155-189.

Loewenstein, M."Worker Heterogeneity, Hours Restrictions, and Temporary

    Layoffs." Discussion Paper No. 378, Center for Mathematical Studies,

    Northwestern University, April 1979.

Lucas, R.E., and Prescott, E.C. "Equilibrium Search and Unemployment."

    Journal of Economic Theory (February 1974): 188-209.

Medoff, J.L. "Layoffs and Alternatives Under Trade Unions in U.S.

    Manufacturing." American Economic Review 69 (June 1979): 380-395.

Mortensen, D.T. "Specific Capital and Labor Turnover." The Bell Journal

    9 (Autumn 1978): 572-586.

Mortensen, D.T. "On the Theory of Layoffs." Discussion Paper No. 322,

    Center for Mathematical Studies in Economics and Management Science,

    Northwestern University, March 1978.

Mortensen, D.T. "Unemployment Insurance and Job Search Decisions."

    Industrial and Labor Relations Review 30 (July 1977): 505-517.

Rees, A. and Shultz, G.P. Workers and Wages in an Urban Labor Market,

    Chicago: The University of Chicago Press, 1970.

Wilde, L.L. "Labor Market Equilibrium Under Non-Sequential Search."

    Journal of Economic Theory 16 (December 1977): 373-393.