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Worker Heterogeneity, Hours Restrictions,
and Temporary Layoffs

by

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1. Introduction

Over the years, economists have devoted considerable effort to the problem of providing a convincing explanation of layoff unemployment. The results have not been very successful for the most part. However, a recently developed approach, the so-called "implicit-contract theory," seems promising.¹ Implicit-contract models are based on the realistic assumption that there are long-term attachments between employers and employees. Furthermore, some of these models seem to suggest that layoffs and sticky wages may arise as part of an optimal strategy on the part of a firm and its workers to cope with fluctuations in output demand. In much of the implicit-contract literature, there is confusion over the types of assumptions that are required to generate layoffs. Fortunately, this state of affairs is remedied in a recent paper by Mortensen.² Mortensen shows that, in the absence of outside payments to unemployed workers, layoffs are never optimal if workers dislike risk in consumption and leisure and if output is solely a function of man-hours worked (and, hence, does not depend negatively on the number of workers employed.) In addition, Mortensen provides examples which show that layoffs may be optimal if either one of these assumptions is violated.

Practically all the implicit-contract models appearing in the literature make the assumption that all workers are exactly alike. This is a strange assumption in view of the fact that the implicit-contract

approach makes sense only if there are long-term attachments between firms and workers. These attachments presumably exist as a result of costs to workers of finding alternative employment and costs to firms of replacing workers who quit -- costs which would be minimal in a world of homogeneous workers. In this paper, I introduce worker heterogeneity into the implicit contract model, using Mortensen's model as the basic frame of reference.

After introducing worker heterogeneity into the model, I ask the question: Under what conditions will the existence of a heterogeneous labor force make layoffs an efficient strategy response to a randomly fluctuating output demand? The answer turns out to be quite simple. If workers dislike risk in consumption and leisure, in the sense that they have utility functions which are concave functions of consumption and leisure, and if a firm is able to costlessly vary hours across its workers, then an efficient contract between the firm and its workers will be characterized by full employment for all workers. If, however, the firm is limited in its ability to vary hours across workers, then layoffs may become an optimal response to low output demand. The layoffs generated by the model are similar in nature to the layoffs explained by the Mortensen model -- in both cases layoffs "reflect a rational response to fluctuations in the value of labor productivity given tastes and technology."

The rest of this paper is organized as follows. Section 2 provides some motivation for the assumption that firms cannot costlessly vary

hours across workers. Section 3 presents the basic model of the paper. Worker heterogeneity is incorporated into the model in a simple and yet intuitive manner. Two polar cases are contrasted in this section. In the first case, hours can be costlessly varied across workers. In the second, all workers are constrained to work the same number of hours. It will be shown that layoffs are never optimal in the first case, but they may be in the second. Section 4 then presents a very simple example in which layoffs are an optimal response to low output demand. Finally, section 5 shows how the model of section 3 can be generalized.

2. Motivation For an Hours Restriction Assumption

In the introduction, I indicated that the effect of worker heterogeneity on a firm's optimal layoff policy will depend crucially on whether or not the firm can costlessly vary hours across its workers. Before turning to a formal model, it seems worthwhile to briefly discuss whether it is reasonable to believe that some firms may be limited in their ability to vary hours across workers.

What kinds of conditions are likely to make it difficult and costly for a firm to vary hours across its workers? Or, conversely, under what circumstances will a firm find it easy to vary hours across its employees? A very simple case, in which hours can be costlessly varied across employees, is that in which every worker produces the final product by himself, and total output is simply equal to the sum of the amounts produced by the individual workers. A firm with this type of production technology obviously has no difficulty in varying hours across its workers. The essential reason for this is the fact that there is no interaction among the workers. However, it is precisely this lack of worker interaction which makes the example unrealistic. Certainly, most real-world production processes require a considerable amount of worker interaction (with the prime examples perhaps being those processes which involve the use of an assembly line). In fact, one of the functions performed by the firms in an economy is simply the coordination of the activities of workers.

So it seems that if a firm's production process is such that there are substantial costs in varying hours across workers, then that production process must require some type of worker interaction. Is the converse true? Is the existence of worker interaction sufficient to produce real costs in varying hours? Probably not. To point out the additional conditions required to produce such costs, let us consider a simple example. Suppose a particular firm's production process is such that the production of a unit of final output requires the cooperation of two types of workers, say, of type 1 and type 2, each performing a separate task. What methods can be used to vary hours across these two worker types? One possibility is to store unfinished output, produced by one of the workers, for the later use of the other worker. A second method is to always have a type 1 and a type 2 employee working together, even though these workers do not work the same number of hours. This can be done by resorting to job shift changes across one of the worker classes. If there are costs involved in holding inventory and in changing shifts, then the firm in this example will find it costly to vary hours across worker types.

In conclusion, it seems reasonable to believe that there are many firms in our economy which are limited in their ability to (costlessly) vary hours across their work force. Such limitations can be generated by weak assumptions: first, a production technology which requires worker interaction, and, second, the assumption that a firm finds it costly, or perhaps impossible, to store unfinished output and to change job shifts.

3. The Basic Model

The model to be presented in this section is similar to the other models in the implicit-contract literature, and is especially close to Mortensen's model. The new feature being added is the introduction of worker heterogeneity.

Consider a labor market with a given set of employing firms and a given set of workers. Workers may differ according to both their productive capacities and their tastes, although in this section we will emphasize variations in productive capabilities. At the beginning of the period each worker joins a firm, and is attached to the firm the entire period. Workers select among firms according to the contracts offered by the different firms.

A firm offers a contract whose provisions differ across worker types. The contract specifies the wages the firm will pay, the hours employed workers will work, and the proportion of workers that will be employed in each of the contingencies that may arise during the period. These contingencies are assumed to be completely characterized by the price the firm receives for its final product. Furthermore, both a firm and its workers know the probability distribution on the set of possible prices, although neither knows with certainty what the realized price will be.

Under the same terminology as appears in the Mortensen paper, we will say that a contract is efficient for a firm with a given labor force

if there is no other contract preferred by both the firm and its workers. Clearly, it is always in the interest of both the workers and the employer to agree to an efficient contract. The major questions of concern to us are: Will efficient contracts provide for layoffs? Under what assumptions can layoffs be ruled out? What assumptions are sufficient to generate layoffs?

Consider a given firm with a fixed labor force. Let $p \in P$ denote the realized price of the firm's output, and let $F(p)$ be the price probability distribution defined over the price space P . Assume that the firm's production function takes the simple form:

$$(1) \quad X = f(L^S), \quad f' > 0, \quad f'' < 0,$$

where L^S denotes standardized, or quality-adjusted, labor services.

Workers are allowed to differ in their productive ability. Let $a \in A$ be a worker ability index, and let $G(a)$ denote the ability distribution of the firm's workers over the ability space A .³ Also, let $h(a)$ represent the fraction of the period worked by an employed worker of ability a . It can then be assumed that an employed type a worker's contribution standardized labor services L^S is simply $ah(a)$. If $q(a)$ denotes the proportion of type a workers employed during the period, then the total value of standardized labor services supplied by the firm's labor force is

$$(2) \quad L^S = \int_A ah(a)q(a)dG .$$

Combining (1) and (2) , the total output produced during the period will be:

$$(3) \quad X = f\left(\int_A ah(a)q(a)dG\right) .$$

Several comments concerning equation (3) are in order. First of all, notice that worker heterogeneity has been introduced in a very special way. The production function in (3) assumes constant marginal rates of technical substitution between different types of workers. This particular specification has been chosen because it is easy to work with and yields very intuitive results. Fortunately, as we will see in section 5 , the production function can be generalized without substantially changing the nature of our results.

Also notice the very simple way in which worker productivity variations are captured -- through variations in a . There are two possible explanations for these worker productivity differences. First, workers may be different before they come to the firm. They may have different natural abilities and may have invested in different amounts of human capital. In this case, at the beginning of the period, the firm can choose the distribution of workers that it desires. Presumably, this distribution will be chosen so as to maximize the firm's expected profit. In this context, it is natural to speak of a labor market

equilibrium. We can say that the labor market is in equilibrium when all workers have joined firms and all firms are content with their worker distribution.

The second explanation of the worker productivity differences is the well-known specific human capital story. According to this story, workers are equally productive when they initially come to the firm. Over time, their productivity increases as they acquire specific human capital. At any point in time, productivity will differ among workers according to their job tenure. Thus, the distribution of workers over ability is identical to their distribution according to tenure.

To complete the specification of the model, it is necessary to introduce worker preferences. We will assume that workers' preferences can be represented by utility functions defined over the set of consumption-leisure combinations. These utility functions may differ across worker classes, although it is not necessary that they do so. Allowing variations in utility functions is simply a means of introducing another kind of worker heterogeneity into the model. However, this will have no effect on the results we obtain. A type a worker has a utility function of the form:

$$(4) \quad U^a = U^a(c, \ell) ,$$

where c is the worker's real consumption and ℓ is his leisure during the period (or the fraction of the period he is not working). We will

make the standard assumption that U^a is a concave function of c and l .

At the beginning of the period, workers do not know for certain what utility level they will realize since workers cannot be certain as to the compensation they will receive and the number of hours they will work. These will depend on the price the firm realizes for its output, a price which is not known with certainty at the beginning of the period. We will assume that workers desire to maximize their expected utility, and therefore rank contracts according to the value of expected utility they offer.

We are at last ready to analyze the characteristics of efficient contracts. As was mentioned in the introduction, these characteristics will depend crucially on a firm's ability to vary hours across its workers. Section 2 suggested that it is not unreasonable to believe that real-world firms may be limited in their ability, or may at least find it costly, to vary hours across their labor force. In this section, we will consider two polar cases. In the first, it will be assumed that the firm can costlessly vary hours across its workers. Because of the connotations of the term, we will say that in this case hours are a "private good." In the second case, all employed workers must work the same number of hours. This case may be described as one in which hours are a "public good." Section 5 will analyze the intermediate case, in which hours can be varied, but at some real cost.

Case 1: Hours are a Private Good

As I have already mentioned, a contract specifies for all possible prices the compensation, which will be denoted by w , paid to employed workers of each type, the number of hours employed workers of each type will work, and the proportion of workers of each class that will be employed. Formally, a contract is a function defined on $P \times A$:⁴

$$(5) \quad (w, h, q) : P \times A \rightarrow \mathbb{R}_+^1 \times [0, 1] \times [0, 1] .$$

The expected profit yielded by such a contract is:

$$(6) \quad E \Pi = \int_P \left\{ p f \left(\int_A a h(p, a) q(p, a) dG \right) - \int_A w(p, a) q(p, a) dG \right\} dF .$$

If workers cannot self-insure their income, then the value of their consumption is always equal to the compensation they receive.⁵

The expected utility yielded by the contract to a type a worker will then be :

$$(7) \quad EU^a = \int_P \left\{ q(p, a) U^a(w(p, a), 1-h(p, a)) + (1-q(p, a)) U^a(0, 1) \right\} dF .$$

An efficient contract maximizes the joint welfare of a firm and its workers. Mathematically, it solves the problem:

(8) Max $E \Pi$

subject to $EU^a \geq \bar{U}(a)$, where $\bar{U}(a)$ is a given function of a .

A solution to this problem satisfies the following necessary conditions.

There exists a function $\lambda(a)$ such that:

(9.a) $\int_P \{q(p,a)U^a(w(p,a),1-h(p,a)) + (1-q(p,a))U^a(0,1)\} dF = \bar{U}(a) \quad \forall a,$

and a.e. on $P \times A$,

(9.b) $q[\lambda U_1^a(w,1-h) - 1] \leq 0$, with equality if $w > 0$,

(9.c) $q[pf'(\cdot)a - \lambda U_2^a(w,1-h)] \leq 0$, with equality if $h > 0$,

(9.d) $\eta = pf'(\cdot)ah - w + \lambda(U^a(w,1-h) - U^a(0,1)) \begin{cases} \leq 0 \\ > 0 \end{cases}$ as $\begin{cases} q = 0 \\ 0 < q < 1 \\ q = 1 \end{cases}$ 7/

The proof that no efficient contract provides for layoffs when hours are a "private good" will be a direct consequence of the first-order conditions (9.b), (9.c), and (9.d). Before proceeding with the proof, it is necessary to state formally what we mean by layoffs in terms of the model.

Definition A contract is said to provide for layoffs if $\{(p,a): 0 < q(p,a) < 1\}$ has positive measure.

This definition is analogous to the one used by Mortensen. Notice that a no-layoff contract does not necessarily guarantee that all employees will work a positive number of hours. In particular, it does not rule out the possibility that there are possible output prices at which all workers of a given type work zero hours. This case, however, only occurs when these workers' value of marginal product falls below their marginal rate of substitution of consumption for leisure at full-leisure, and it may not be reasonable to call such workers unemployed. A no-layoff contract does, however, rule out the possibility that there are output prices at which some workers of a given class will be employed and others of the same class will be unemployed. Furthermore, a no-layoff contract requires that output demand fluctuations be met solely by variations in hours worked; variable- q policies are ruled out.

We are now ready to prove that layoffs are never optimal if a firm's workers are risk averse and if the firm is able to (costlessly) vary hours across its workers.

Proposition 1. If hours are a private good and if $U^a(c, \ell)$ is strictly concave for all a , then no efficient contract provides for layoffs.

Proof.⁸ Define $C^a(w, h, x, y)$ in the following way:

$$(10) \quad U^a(x,y) = U^a(w,1-h) + (x-w)U_1^a(w,1-h) + (y-(1-h))U_2^a(w,1-h) - C^a(w,h,x,y) .$$

The first three terms on the right-hand side of (10) are simply the plane tangent to the utility function U^a at the point $(w,1-h)$. Since a tangent plane lies above a strictly concave function, $C^a(w,h,x,y) > 0$ if $(w,1-h) \neq (x,y)$. In particular, let

$$(11) \quad C^a(w,h) = C^a(w,h,0,1) .$$

Then $C^a(w,h) > 0$, provided that $(w,h) \neq (0,0)$.⁹

Now a.e. where $q \neq 0$:

$$(12.a) \quad w = w\lambda U_1^a(w,1-h) \text{ from (9.b)}$$

and

$$(12.b) \quad pf'(\cdot)ah = h\lambda U_2^a(w,1-h) \text{ from (9.c) .}$$

Substituting (11.a) and (11.b) into (9.d), we get:

$$(12.c) \quad \eta = \lambda [-wU_1^a(w,1-h) + hU_2^a(w,1-h) + U^a(w,1-h) - U^a(0,1)] \\ = \lambda C^a(w,h)$$

> 0 , which implies $q = 1$.

Q.E.D.

The explanation for this result is straightforward. Layoffs are not efficient in this model for essentially the same reason that they are not efficient in Mortensen's model. The assumption that workers have concave utility functions implies that "no worker prefers one consumption-leisure combination during the fraction of his work life that he spends employed and another combination during the remaining fraction spent unemployed to the average of the two combinations all the time." And the assumption that a firm can costlessly vary hours across its work force implies that "there is no real cost in catering to this preference." Shortly we shall see that layoffs are not generally inefficient when the second assumption is relaxed. First, however, it is of some interest to examine some comparative-static implications of the model we are presently working with. We will briefly digress from the main line of argument and attempt to determine how hours and wage compensation vary across prices and across ability types. For simplicity, it will be assumed that all workers have the same utility function. Also, since layoffs are never optimal, we can disregard equation (9.c) and assume that $q = 1$ a.e.¹⁰

From equations (9.b) and (9.c) we see that an efficient contract requires that each worker's value of marginal product always equal the rate at which he is willing to substitute consumption for leisure. The same condition appears in the traditional formulation of labor market equilibrium: the demand price for labor, the value of the marginal product, must equal the supply price, workers' marginal rate of substitution between consumption and leisure. In a contract model, however, a change

in the demand price produces no income effect because an employer insures the income of his workers (that is, the marginal utility of consumption is equalized across all possible prices, as is evidenced from (9.b)). As a result, hours worked are always an increasing function of price.

Proposition 2 . Under the assumptions of this section, an efficient contract has the following properties:

- i) $\frac{\partial h}{\partial p} > 0$ a.e.
- ii) $\frac{\partial w}{\partial p} \begin{matrix} \geq \\ < \end{matrix} 0$ as $U_{12} \begin{matrix} \leq \\ > \end{matrix} 0$.

Proof. Differentiating (9.b) and (9.c) with respect to p , we have:

$$(13.a) \quad \lambda U_{11} \frac{\partial w}{\partial p} - \lambda U_{12} \frac{\partial h}{\partial p} = 0$$

$$(13.b) \quad -\lambda U_{12} \frac{\partial w}{\partial p} + \lambda U_{12} \frac{\partial h}{\partial p} = -f'(L^S) a + pf''(L^S) \frac{\partial L^S}{\partial p} .$$

Solving (13.a) and (13.b) for $\frac{\partial w}{\partial p}$ and $\frac{\partial h}{\partial p}$, we find:

$$(14.a) \quad \frac{\partial w}{\partial p} = [-U_{11} a(f'(L^S) + pf''(L^S) \frac{\partial L^S}{\partial p})] / \lambda D$$

$$(14.b) \quad \frac{\partial w}{\partial p} = \frac{U_{12}}{U_{11}} \frac{\partial h}{\partial p} ,$$

where $D = \begin{vmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{vmatrix} > 0$ since U is concave.

Since $q = 1$ a.e., $L^S = \int_A ahdG$. Differentiating L^S with respect to p and using (14.a), we find:

$$\frac{\partial L^S}{\partial p} = \int_A a \frac{\partial h}{\partial p} dG$$

$$(15) \quad \frac{\partial L^S}{\partial p} = \int_A \{ [-a^2 U_{11} \{ f'(L^S) + pf''(L^S) \} \frac{\partial L^S}{\partial p}] / \lambda D \} dG,$$

from which we can solve for $\frac{\partial L^S}{\partial p}$:

$$(16) \quad \frac{\partial L^S}{\partial p} = \frac{-f'(L^S) \int_A \frac{a^2 U_{11}}{\lambda D} dG}{1 + pf''(L^S) \int_A \frac{a^2 U_{11}}{\lambda D} dG} .$$

(16) can now be substituted into (14.a):

$$\frac{\partial h}{\partial p} = \frac{-U_{11} a}{\lambda D} (f'(L^S) + pf''(L^S) [\frac{-f'(L^S) \int_A \frac{a^2 U_{11}}{\lambda D} dG}{1 + pf''(L^S) \int_A \frac{a^2 U_{11}}{\lambda D} dG}])$$

$$(17) \quad \frac{\partial h}{\partial p} = \frac{-U_{11} af'(L^S)}{\lambda D (1 + pf''(L^S) \int_A \frac{a^2 U_{11}}{\lambda D} dG)} > 0 .$$

ii) now follows from (14.b), (17), and the fact that $U_{11} < 0$.

Q.E.D.

A second question of interest is: How do wages and hours worked vary across worker types? As we shall see, there does not seem to be a clear-cut answer to this question. First of all, notice that at present we do not have sufficient information to even begin to analyze the problem. The optimal wage-employment strategy will clearly depend on the expected utility that must be offered to the different worker types -- something which has received relatively little attention in our analysis up to this point. Let us take the following approach.

Recall that one possible explanation for productivity variations among a firm's workers is simply that the workers are different before they come to the firm because they have different natural abilities and have acquired different amounts of human capital. In this case, the firm is able to choose the number of workers of each type that it desires, subject to the constraint that it offers its workers the same expected utility as other firms. It may be assumed that the firm will choose its worker distribution so as to maximize its expected profit. Furthermore, we may say that labor market equilibrium occurs when all workers are attached to firms and all firms are content with their work force.

Now consider a particular firm with worker distribution $G(a)$. Let $g(a) = G'(a)$. Market equilibrium requires that g be chosen so as to maximize the firm's expected profit Π , where

$$(18) \quad \Pi = \underset{w,h}{\text{Max}} \int_p \left\{ pf \left(\int_A ahgda \right) - \int_A wgda \right\} dF$$

$$\text{s.t.} \quad EU^a \geq \bar{U}(a) \quad \forall a .$$

Maximization of Π with respect to $g(a)$ requires that $\frac{\partial \Pi}{\partial g} = 0$ a.e. on A .

Furthermore, by the envelope theorem,

$$(19) \quad \frac{\partial \Pi}{\partial g(a)} = \int_p pf'(\cdot) ah dF - \int_p w dF .$$

Thus, market equilibrium requires that

$$(20) \quad \int_p pf'(\cdot) ah dF = \int_p w dF .$$

Using (20) together with the first-order conditions characterizing efficient contracts, we can determine how hours and wages vary across the firm's work force. Differentiating (9.b) and (9.c) with respect to a , and then solving for $\frac{\partial w}{\partial a}$ and $\frac{\partial h}{\partial a}$, we find

$$(21.a) \quad \frac{\partial w}{\partial a} = [-\lambda U_{12} pf'(L^S) + \frac{\partial \lambda}{\partial a} (\lambda U_{212} - U_{22})] / \lambda^2 D ,$$

$$(21.b) \quad \frac{\partial h}{\partial a} = [-\lambda U_{11} pf'(L^S) + \frac{\partial \lambda}{\partial a} (\lambda U_{211} - U_{12})] / \lambda^2 D .$$

Next, let us differentiate (20) with respect to a to get

$$(22) \int_p \{pf'(L^S)h + pf'(L^S)a \frac{\partial h}{\partial a}\} dF = \int_p \frac{\partial w}{\partial a} dF .$$

We can now substitute (21.a) and (21.b) into (22) and rearrange terms to arrive at:

$$(23) \frac{\partial \lambda}{\partial a} [\int_p \{[pf'(\cdot)a(\lambda U_{211} - U_{12}) + U_{22} - \lambda U_{212}] / \lambda^2 D\} dF] \\ = \int_p \{-pf'(\cdot)h + [p^2 f'(\cdot)^2 a \lambda U_{11} - \lambda U_{12} pf'(\cdot)] \lambda^2 D\} dF .$$

Although the expressions for $\frac{\partial w}{\partial a}$, $\frac{\partial h}{\partial a}$, and $\frac{\partial \lambda}{\partial a}$ cannot be given unambiguous signs, it is nevertheless possible to interpret them. In the standard model of an individual's labor supply decision, leisure is a normal good if and only if $U_{12} - \lambda U_{211} \geq 0$, and consumption is normal if $\lambda U_{212} - U_{22} > 0$. So, from (23), we see that $\lambda(a)$, the real cost to the firm of increasing a type a worker's expected utility, is an increasing function of a if consumption and leisure are both normal. The expressions for $\frac{\partial w}{\partial a}$ and $\frac{\partial h}{\partial a}$ each contain two terms. The second term may be called an "income effect." If consumption and leisure are normal, then this effect is positive in (21.a) and negative in (21.b). The first term in (21.b) may be called the "substitution effect." The positive substitution effect in (21.b) reflects the fact that the real value of work time varies positively with a . Whether $\frac{\partial h}{\partial a}$ is positive or

negative depends on whether the substitution or the income effect dominates. Similarly, the first term in (20.a) may be called a "cross-substitution effect." Its sign is negative (positive) if consumption and leisure are complements (substitutes) in the utility function. The sign of $\frac{\partial w}{\partial a}$ depends on which effect, the income effect or the cross-substitution effect, is dominant.

Our inability to sign $\frac{\partial w}{\partial a}$ and especially $\frac{\partial h}{\partial a}$ should not be terribly surprising. On the one hand, the fact that workers indexed by high a 's are more productive suggests that it may be desirable for them to work longer hours than those workers indexed by lower a 's. On the other hand, the market equilibrium condition requires that higher a workers be rewarded for their higher productivity -- in the form of higher wage compensation, or more leisure, or both. The net effect of these considerations will depend on the particular form of the workers' utility function.

This marks the end of our digression, and we now return to the major consideration of this paper -- the determination of the optimality of layoffs in the presence of worker heterogeneity. We have shown that no efficient contract provides for layoffs when hours are a "private good." We now consider the other polar case -- the case in which hours are a "public good."

Case 2: Hours are a Public Good

In this case, the firm is unable to vary hours across its work force. Since all employed workers must work the same number of hours, a contract offered by the firm now has the form:

$$(24) \quad \begin{aligned} (w, q) &: P \times A \rightarrow \mathbb{R}_+^1 \times [0, 1] \\ h &: P \rightarrow [0, 1] \end{aligned}$$

As before, an efficient contract maximizes the joint welfare of the firm and its workers, and therefore solves (8) .

A solution to (8) now satisfies the following necessary conditions.¹¹

There exists a function $\lambda(a)$ such that:

$$(25.a) \quad \int_P \{qU^a(w, 1-h) + (1-q)U^a(0, 1)\} dF = \bar{U}(a) \quad \forall a \in A ,$$

$$(25.b) \quad q(\lambda U_1^a(w, 1-h) - 1) = 0 \quad \text{a.e. on } P \times A ,$$

$$(25.c) \quad \int_A pf'(\cdot) a q dG - \int_A \lambda U_2^a(w, 1-h) q dG = 0 \quad \text{a.e. on } P ,$$

$$(25.d) \quad \eta = pf'(\cdot) a h - w + \lambda(U(w, 1-h) - U(0, 1)) \begin{matrix} \leq \\ \geq \end{matrix} 0 \text{ as } \begin{matrix} q = 0 \\ 0 < q < 1 \\ q = 1 \end{matrix} \text{ a.e. on } P \times A .$$

Note that (25.a), (25.b), and (25.d) are identical to (9.a), (9.b), and (9.d), the corresponding first-order conditions arising in the case

where hours are a "private good." The hours condition, (25.c), however, differs from its case 1 counterpart, (9.c) . The explanation for this is simple. When the firm is able to vary hours across its workers, optimality requires that each worker's value of marginal product equal his marginal rate of substitution of consumption for leisure. However, when hours cannot be varied among workers, the best that can be done is to equate the average value of the marginal product to the average marginal rate of substitution of consumption for leisure. This result should not be surprising since it is the standard optimality condition in the case of a public good.

It is important to emphasize that when hours are a "public good," an efficient contract is not generally characterized by equality between each worker's value of marginal product and marginal rate of substitution of consumption for leisure (although such equality must hold on the average over all employed workers.) This point can be made clearer by rewriting (25.c) in the form:

$$(26) \quad \int_A (pf'(\cdot)a - \frac{U_2}{U_1}) qdG = 0 \quad \text{a.e. on } P .$$

From (26) , we see that for some worker types, the value of the marginal product may exceed the marginal rate of substitution of consumption for leisure, and for others, the value of the marginal product may fall short of the marginal rate of substitution of consumption for leisure. However,

when added together over the entire employed work force, differences between the value of the marginal product and the rate of substitution of consumption for leisure must sum to zero.

The fact that the firm is unable to equate the value of the marginal product to the marginal rate of substitution between consumption and leisure for all workers simultaneously has important consequences for the optimality of layoffs.

Proposition 3. If hours are a public good, then an efficient contract may provide for layoffs.

Proof. Strictly speaking, the proof of proposition 3 requires the construction of an example in which an efficient contract provides for layoffs. Such an example will be presented in the next section. Here we will simply show that layoffs cannot obviously be ruled out of efficient contracts as they could in case 1, where hours could be varied among workers.

Layoffs are obviously suboptimal if a.e. on $P \times A$, $q \neq 0$ implies $\eta > 0$.

When $q > 0$, $w = w \lambda U_1(w, 1-h)$ from (24.b). If we substitute $w \lambda U_1$ for w in (24.d), add and subtract $h \lambda U_2$, and then rearrange terms, we get:

$$\eta = (pf'(\cdot)a - \lambda U_2^a(w, 1-h))h + \lambda(U^a(w, 1-h) - wU_1^a(w, 1-h) + hU_2^a(w, 1-h) - U^a(0, 1))$$

$$(27) \quad \eta = (pf'(\cdot)a - U_2^a / U_1^a)h + \lambda C^a(w, h) .$$

Although $C^a(w, h) \geq 0 \quad \forall q$, the first term in (27) is negative for some a and p . It is not true that η is necessarily strictly positive (unlike case 1).

Q.E.D.

Equation (27), the layoff condition for type a workers when the output price is p , has the following interpretation. The term λC can be interpreted as a measure of workers' risk aversion, and it represents the real cost of laying off an additional worker. $\lambda(a)C^a$ essentially measures the degree of concavity of the type a workers' utility function. If workers are risk-neutral, in the sense that they have a linear utility function, then λC is zero (the tangent surface is identical to the function $U^a(c, \ell)$.) If U^a is strictly concave, then $\lambda(a)C^a$ is positive.¹²

The first term in (27) is simply the product of the number of hours worked by employed workers and the difference between a type a worker's value of marginal product of work time and rate of substitution of consumption for leisure. If the firm were able to vary hours among its workers, then this term would always vanish.¹³ However, when hours are a "public good," this term is not generally equal to zero, and will, in fact, be negative for some worker types. It then

represents the cost of keeping such workers employed. Optimality requires that the two effects balance each other out (unless $q(p,a)$ is at a corner solution with $q(p,a) = 0$ or $q(p,a) = 1$).

In summary, layoffs may be an efficient strategy if the firm is unable to vary hours across its labor force. When layoffs are optimal, it is because they are a means by which the firm can adjust the labor supply of certain subgroups of workers. More precisely, layoffs will be used to reduce the labor input of those workers whose value of the marginal product falls below the marginal rate of substitution of consumption for leisure.

4. An Example

In this section, I will present a simple example in which an efficient contract provides for layoffs. Although certain restrictive assumptions will be utilized, it will be possible to generate some useful results -- results which will have interesting implications for more general cases.

Consider a firm with a production function of the form:

$$(28) \quad X = f(L^S), \text{ with } f' > 0, f'' < 0, f'(0) = \infty, \text{ and } f'(\infty) = 0.$$

Let us suppose that the firm has two types of workers with abilities a_1 and a_2 , respectively, and with $a_1 > a_2$. Let m_1 be the number of type 1 workers, m_2 be the number of type 2 workers, q_1 be the proportion of type 1 workers who are employed, q_2 be the proportion of type 2 workers employed, and let h be the fraction of the period worked by all employed workers. Then total standardized labor services L^S are given by:

$$(29) \quad L^S = \sum_{i=1}^2 h a_i q_i m_i .$$

Also, let us assume that all workers have the same utility function $U(c, l)$ and that $U(c, l)$ is a linear function of c and l . The assumption that U is linear is subject to criticism on the grounds that it implies that workers do not dislike risk in consumption and leisure. ^{14/} Thus, when we assume that U is linear, we are, in effect, assuming that workers no longer have a basic dislike for employment arrangements which provide for layoffs. On the other hand, the assumption that U is linear does permit a great simplification of the analysis. The fact that we will

obtain an interesting result which has implications for more general cases is sufficient justification for its use.

Finally, let w_i denote the wage compensation paid to workers of type i .

An efficient contract solves:

$$(30) \quad \text{Max}_{h(p), w_i(p), q_i(p)} \int_P \left\{ \text{pf} \left(\sum_{i=1}^2 h_i q_i m_i \right) - \sum_{i=1}^2 w_i q_i m_i \right\} dF$$

subject to $\int_P \{q_i U(w_i, 1 - h_i) + (1 - q_i) U(0, 1)\} dF = \bar{U}_i, \quad i = 1, 2$

$0 \leq h \leq 1$

$0 \leq q_i \leq 1, \quad i = 1, 2 .$

Let λ_i be the shadow price corresponding to the expected utility constraint for workers of type $i, i = 1, 2$. Then a solution to (30) satisfies the following first order conditions: 15/

$$(31.a) \quad \int_P \{q_i U(w_i, 1 - h) + (1 - q_i) U(0, 1)\} dF = \bar{U}_i, \quad i = 1, 2.$$

$$(31.b) \quad q_i (\lambda_i U_1 - 1) = 0, \quad i = 1, 2.$$

$$(31.c) \quad m_1 q_1 (\text{pf}'(L^S) a_1 - \lambda_1 U_2) + m_2 q_2 (\text{pf}'(L^S) a_2 - \lambda_2 U_2) \geq 0,$$

with equality if $h < 1$.

$$(31.c) \quad m_i (\text{pf}'(L^S) a_i - w_i + \lambda_i (U(w_i, 1 - h) - U(0, 1))) \begin{cases} \leq 0 & \text{as } q_i = 0 \\ > 0 & \text{as } 0 < q_i < 1 \\ & q_i = 1 \end{cases}, \quad i = 1, 2$$

As was done in the preceding section, (31.d) may be rewritten as:

$$(31.d) \quad m_i (\text{pf}'(L^S) a_i - \lambda_i U_2) h \begin{cases} \leq 0 & \text{as } q_i = 0 \\ > 0 & \text{as } 0 < q_i < 1 \\ & q_i = 1 \end{cases}, \quad i = 1, 2.$$

It is easy to verify that the optimal employment strategy may be characterized in the following way:

$$\text{Let } p_1 = \frac{U_2/\dot{U}_1}{f'(a_1 m_1) a_1}, \quad p_2 = \frac{U_2/U_1}{f'(a_1 m_1) a_2}, \quad p_3 = \frac{U_2/U_1}{f'(a_1 m_1 + a_2 m_2) a_2}$$

Then

- (a) For $0 < p < p_1$, $q_2 = 0$, $0 < h q_1 < 1$, and $\frac{d(h q_1)}{dp} > 0$.
- (b) For $p_1 \leq p \leq p_2$, $q_2 = 0$ and $1 = h = q_1$.
- (c) For $p_2 < p < p_3$, $0 < q_2 < 1$, $1 = h = q_1$ and $\frac{dq_2}{dp} > 0$.
- (d) For $p_3 \leq p$, $1 = h = q_1 = q_2$.

So we see that an efficient contract must provide for layoffs in our present example (Optimality requires that type 2 workers be laid off at low output prices.). In addition to this basic result, there are other characteristics of the efficient employment strategy, as described by (a) - (d), which deserve mention.

First of all, we may observe that all type 1 workers are employed full-time, with no leisure, before any type 2 workers are ever employed. This unrealistic result is due to our strong and unreasonable assumption that the workers' marginal rate of substitution of consumption for leisure is fixed (that is, independent of c and l).

A second characteristic of the efficient employment strategy is that all adjustments in the labor input of type 2 workers to changes in the output price come through changes in q_2 , the proportion of type 2 workers employed. This result is most interesting and occurs for a simple reason: Our

assumption that all employed workers must work the same number of hours, and the fact that the type 1 workers are employed full-time whenever type 2 workers work, together imply that the fraction of the period that type 2 workers must work is fixed at 1. Note, however, that what is really important here is not that the fraction of the period that type 2 workers can work is set at 1, but that the number of hours these employees can work is effectively bounded away from zero. In this respect, our example is similar to the examples of layoffs that appear in Mortensen's paper. Furthermore, this consideration suggests that the assumption of a constant marginal rate of substitution of consumption for leisure can be modified somewhat without affecting the basic result that adjustments in the labor input of type 2 workers in response to output price changes are accomplished mainly through changes in the number of type 2 workers who are employed. In particular, if the marginal rate of substitution of consumption for leisure were relatively constant for relatively high values of ℓ , and then rapidly became large as ℓ approached zero, a similar result should obtain.

5. A Generalization of the Basic Model

In the introduction to this paper, I suggested that, given worker heterogeneity, efficient contracts may provide for layoffs if firms find it costly to vary hours across their workers. And in section 2, I argued that real-world firms are, in fact, likely to incur real costs in varying hours across their work force. In the formal analysis of sections 3 and 4, however, only two extreme cases were considered. In the first, the firm could costlessly vary hours across employed workers. In the second, the firm was completely unable to vary hours among its workers. In this section, I show how the basic model can be generalized to allow for the intermediate case in which hours can be varied, but at some real cost.

Consider a firm with n types of workers. Let $U^k(c, \ell)$ denote the utility function of type k workers. Also, let m_k denote the number of workers of type k and let q_k represent the proportion of workers of type k who are employed. Finally, let h_k represent the fraction of the period worked by employed workers of type k and define h by:

$$(32) \quad h = \min (h_1, h_2, \dots, h_n).$$

Now in section 2 it was suggested that if a firm's production technology requires interaction among workers, and if it is costly for the firm to store unfinished output and to change job shifts, then it is likely that hours can be varied across workers only at some real cost. Furthermore, this cost should vary positively with the extent to which hours are varied across the firm's labor force. The following production

function has precisely this property:

$$(33) \quad X = f(hq_1m_1, hq_2m_2, \dots, hq_n m_n, (h_1 - h)q_1m_1, (h_2 - h)q_2m_2, (h_n - h)q_n m_n)$$

$$\text{where } f_i \geq f_{n+i} \geq 0 \text{ for } i = 1, 2, \dots, n.$$

Unlike the production function we have been using up till now, (33) does not assume constant marginal rates of technical substitution between the different worker types. Furthermore, note that the two extreme possibilities considered in section 3 arise as special cases of (33). If $f_i \equiv f_{n+i}$ for all i , then varying hours across workers imposes no real cost. On the other hand, if $f_{n+i} \equiv 0$ for all i , then it is "infinitely" costly to vary hours across workers, in the sense that increasing the hours worked by some workers beyond those worked by others yields absolutely no addition to output.

Let us now characterize the efficient contract when the firm has a production function of the form specified in (33). As we shall see, the basic results are virtually identical to those obtained with the simpler model of section 3.

As before, an efficient contract maximizes the joint welfare of the employer and the employees. The problem that must be solved may be specified as:

$$(34) \quad \begin{array}{l} \text{Max} \\ h_k(p), w_k(p) \\ q_k(p), h \end{array} \int_P \{ pf(hq_1m_1, \dots, hq_n m_n, (h_1 - h)q_1m_1, \dots, (h_n - h)q_n m_n) - \sum_{k=1}^n w_k q_k m_k \} dF$$

subject to $\int_P \{ q_k U^k(w_k, 1 - h_k) + (1 - q_k) U^k(0, 1) \} dF = \bar{U}_k,$

$k = 1, 2, \dots, n,$

$h_k \geq h, \quad k = 1, 2, \dots, n,$

provided that in the solution $h_k = h$ for some k . ^{16/}

Let λ_k be the shadow price corresponding to the expected utility constraint for workers of type k and let μ_k be the non-negative shadow price corresponding to the constraint $h_k \geq h$. A solution to (34) must then satisfy:

$$(35.a) \quad \int_P \{q_k U^k(w_k, 1 - h_k) + (1 - q_k) U^k(0, 1)\} dF = \bar{U}_k, \quad k = 1, 2, \dots, n,$$

and a.e. on P ,

$$(35.b) \quad q_k m_k (\lambda_k U_1^k(w_k, 1 - h_k) - 1) = 0, \quad k = 1, 2, \dots, n,$$

$$(35.c) \quad \sum_{k=1}^n q_k m_k p (f_k - f_{n+k}) - \mu_k = 0,$$

$$(35.d) \quad q_k m_k (p f_{n+k} - \lambda_k U_2^k(w_k, 1 - h_k)) + \mu_k = 0, \quad k = 1, 2, \dots, n,$$

$$(35.e) \quad \mu_k (h_k - h) = 0, \quad k = 1, 2, \dots, n,$$

$$(35.f) \quad \eta_k = p h f_k + p (h_k - h) f_{n+k} - w_k + \lambda_k (U^k(w_k, 1 - h_k) - U^k(0, 1)) \begin{matrix} \leq \\ > \end{matrix} 0$$

$$\begin{matrix} q_k = 0 \\ \text{as } 0 < q_k < 1 \\ q_k = 1 \end{matrix} \quad , \quad k = 1, 2, \dots, n.$$

In analyzing the first-order conditions, it is useful to distinguish the more general case in which $f_k \neq f_{n+k}$ for at least one k from the special case in which $f_k = f_{n+k}$ for all k .

Case 1 $f_k \neq f_{n+k}$ for some k , $1 \leq k \leq n$.

The first thing to note is that $h_j = h$ for some j , $1 \leq j \leq n$, as desired.

Proposition 4. If $f_k \neq f_{n+k}$ for some k , $1 \leq k \leq n$, then $h_j = h$ for some j , $1 \leq j \leq n$.

Proof. The proof is trivial. If $h_j > h$ for all j , $1 \leq j \leq n$, then $\mu_j = 0$ for all j , $1 \leq j \leq n$, from (35.e). Therefore, from (35.c),

$$\sum_{k=1}^n q_k m_k p(f_k - f_{n+k}) = 0.$$
 But this is impossible since $f_k \geq f_{n+k}$ for all k and $f_k > f_{n+k}$ for at least one k .

Q.E.D.

Recall from section 5 that when all employed workers had to work the same number of hours, optimality required that these common hours be set so as to equate the average value of the marginal product to the average marginal rate of substitution of consumption for leisure. Interestingly, the same result holds in our present model. To see this, note from (35.d) that:

$$(36) \quad -\mu_k = q_k m_k (p f_{n+k} - U_2^k(w_k, 1 - h_k)), \quad k = 1, 2, \dots, n.$$

If we now substitute (36) into (35.c), we obtain the desired result:

$$(37) \quad \sum_{k=1}^n p f_k q_k m_k = \sum_{k=1}^n \lambda_k U_2^k q_k m_k.$$

Now let us consider the question of the optimality of layoffs. In sections 3 and 4, we saw that when all workers must work the same hours, efficient contracts may provide for layoffs. The same thing is true in the model of this section, in which hours can be varied across workers, but at the real cost of some output loss.

Proposition 5. Assume $U^k(c,1)$ is strictly concave. Then the following is true of an efficient contract.

If $h_k > h$, then workers of type k are not laid off.

However, if $h_k = h$, then it is possible that workers of type k are laid off.

Proof. The proof is similar to the proofs of propositions 1 and 3 in section 3. Recall that in section 3, we saw that the basic layoff condition required that two factors -- a risk factor and a factor equal to the difference between a worker's value of marginal product and rate of substitution of consumption for leisure -- be balanced against each other. The same result obtains in our present model:

Lemma. If $q_k > 0$, then

$$(38) \quad \eta_k = h(pf_k - \lambda_k U_2^k) + \lambda_k C^k(w_k, h_k),$$

$$\text{where } C^k(w_k, h_k) = U^k(w_k, 1 - h_k) - w_k U_1^k(w_k, 1 - h_k) + h_k U_2^k(w_k, 1 - h_k) - U^k(0, 1). \quad \underline{17/}$$

Proof. First suppose $h_k > h$. (35.f) can be rewritten as:

$$(39) \quad \eta_k = ph(f_k - f_{n+k}) + ph_k f_{n+k} - w_k + \lambda_k (U^k(w_k, 1 - h_k) - U^k(0, 1)).$$

$h_k > h$ implies that $\mu_k = 0$. $\mu_k = 0$ and (35.d) imply:

$$(40) \quad pf_{n+k} = \lambda_k U_2^k(w_k, 1 - h_k).$$

Also, (35.d) implies:

$$(41) \quad w_k = w_k \lambda_k U_1^k(w_k, 1 - h_k).$$

Substituting (40) and (41) into (39) and rearranging terms gives us (38).

Now suppose $h_k = h$. Then (35.f) becomes:

$$(42) \quad \eta_k = h p f_k - w_k + \lambda_k (U^k(w_k) - U^k(0,1)).$$

Note that (41) still holds. Substituting (41) into (42) and adding and subtracting $h \lambda_k U_2^k$ then gives us (38).

This proves the lemma.

Using (38) it is easy to complete the proof of proposition 5.

Suppose $h_k > h$. Then, as we saw in the proof of the lemma, $p f_{n+k} = \lambda_k U_2^k(w_k, 1 - h_k)$. Since $f_k \geq f_{n+k}$, it follows that $p f_k \geq \lambda_k U_2^k(w_k, 1 - h_k)$. Therefore, the first term in (38) is non-negative. The term $\lambda_k C^k$ is strictly positive since U^k is strictly concave. Hence, η_k is strictly positive, and workers of type k are not laid off.

Now suppose $h_k = h$. Then it is no longer generally true that $p f_k \geq \lambda_k U_2^k(w_k, 1 - h_k)$. (In fact, if $p f_j > \lambda_j U_2^j(w_j, 1 - h_j)$ for some j , then (37) tells us that there is an i such that $h_i = h$ and $p f_i < \lambda U_2^i(w_i, 1 - h_i)$).^{18/} So it is no longer generally true that $\eta_k > 0$ and the possibility that some type k workers are laid off cannot be ruled out.

Q.E.D.

Case 2 $f_k = f_{n+k}$ for $k = 1, 2, \dots, n$.

This is the special case in which hours can be varied across workers with no loss in productive efficiency. It is easy to verify that the basic results of case 1, section 3, all hold in the present case. In particular, layoffs are never optimal.

Summary

This paper has developed a simple implicit-contract model which takes worker heterogeneity into account. This worker heterogeneity might arise for a number of reasons. For example, if it takes time for workers to acquire firm-specific skills, productivity will vary among a firm's workers in accordance with their tenure.

An important result of this paper is that layoffs are never optimal if a firm is able to vary hours among its workers without any loss in productive efficiency. The story changes, however, when such hours variations can be achieved only at some real cost. It is then no longer efficient to equate each worker's value of marginal product with his marginal rate of substitution of consumption for leisure. Instead, such a relationship will only hold on average. Furthermore, layoffs may now be an efficient means of reducing the labor input of those workers whose value of the marginal product falls below the rate at which they are willing to substitute consumption for leisure.

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FOOTNOTES

- 1/ The earliest implicit-contract papers -- Azariadis, Baily (1974), and Gordon -- emphasize worker risk aversion and the fact that contracts permit the transfer of risk from workers to employers. Later papers by Baily -- Baily (1976) and Baily (1977) -- downplay the importance of worker risk aversion. Feldstein has used implicit-contract models to analyze the effect of unemployment insurance on layoffs -- Feldstein (1976) and Feldstein (1977).
- 2/ Mortensen (1978)
- 3/ Notice that $\int_A dG = N$, where N is the total number of workers attached to the firm.
- 4/ Notice that we are imposing a particular constraint on allowable contracts, since we are ruling out compensation to unemployed workers. This is done primarily for simplicity, and as will be obvious from the proofs, will have no effect on our results concerning the optimality of layoffs. The assumption that the firm cannot compensate unemployed workers might perhaps be justified by the fact that such arrangements are seldom observed in the real-world (perhaps because of enforceability difficulties), although it certainly would be more in the spirit of the basic contract idea to allow compensation to unemployed workers. The interested reader may refer to Mortensen's paper, in which such compensation is allowed.
- 5/ The assumption that workers cannot self-insure their income is made for convenience, and does not affect the basic results we will obtain. Allowing workers to costlessly self-insure their income does not affect

the efficient employment strategy; it simply means that efficient wage rules are determined only up to expected wage payments. A lengthier discussion of this point can be found in Mortensen's paper.

6/ It is being assumed that all workers of a given type are equally likely to be laid off. In interpreting (7), note that $q(p,a)$ is the probability that a type a worker is employed given that the output price is p , and $1 - q(p,a)$ is the probability that he is laid off.

7/ For simplicity, it is being assumed that $h < 1$. $h = 1$ can be ruled out by appropriate assumptions on the utility function (for example, $h = 1$ would be ruled out if $\lim_{l \rightarrow 0} U_2^a(c,l) = \infty \forall a,c$).

8/ This proof is essentially the same as the one appearing in Mortensen's paper.

9/ The case in which $(w,h) = (0,0)$ is uninteresting. If $(w(a),h(a)) = (0,0)$, then employed type a workers work zero hours and receive no compensation, and, hence, the distinction between employed and unemployed workers is not meaningful.

10/ It has not been shown that $q = 0$ is never optimal. However, for every efficient contract in which $q = 0$ on some subset B of $P \times A$, there is an equivalent contract in which $q = 1$ a.e. and $h = w = 0$ on B .

11/ For convenience, we are assuming interior solutions for w and h .

12/ It is interesting to note the similarity between the measure of risk aversion used here, λC , and the Pratt-Arrow measure, which

is defined for a utility function $V(x)$ with a single argument.

By Taylor's theorem,

$$\begin{aligned} 2\lambda C &= - (w^2 U_{11}(\hat{w}, 1-\hat{h}) - 2wh U_{12}(\hat{w}, 1-\hat{h}) + h^2 U_{22}(\hat{w}, 1-\hat{h})) \\ &= - (w^2 U_{11}(\hat{w}, 1-\hat{h}) - 2wh U_{12}(\hat{w}, 1-\hat{h}) + h^2 U_{22}(\hat{w}, 1-\hat{h})) / U_1 , \end{aligned}$$

where $0 < \hat{w} \leq w$ and $0 < \hat{h} < h$.

The Pratt-Arrow measure, on the other hand, is $-V''(x)/V'(x)$.

Our measure can be thought of as a generalization of the Pratt-Arrow measure to a utility function with two arguments.

13/ And then we would have: $\eta = \lambda C > 0$, which implies layoffs are never optimal.

14/ In terms of the discussion of the preceding section, $\lambda_i C_i$ vanishes for $i=1$ and $i=2$.

15/ Note that U_1 and U_2 are independent of w and h since U is linear.

16/ It is intuitively obvious that this condition will be satisfied since $f_k \geq f_{n+k}$ for all k . If $h_k > h$ for all h , then output can be costlessly increased simply by increasing h .

17/ Note that (38) suggests layoffs are most likely for low productivity,

high time preference, low risk averse workers.

18/ We know that $h_i = h$ because $pf_1 \geq \lambda_1 U_2^1$ for all i such that $h_i > h$.