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Do Bequests Offset Social Security?

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ABSTRACT

In a recent paper Barro (1974) argues that the existence of a bequest motive vitiates the effect of intergenerational transfer policies. The utility function he uses has as an argument the utility of the household's direct heirs. It is shown that Barro's conclusion may not hold if utility depends directly on the size of the bequest. A model is presented in which a stronger bequest motive implies a larger long-run impact on consumption and on the capital-labor ratio.

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I. Introduction

In a recent paper Barro (1974) argues that the existence of a bequest motive, with bequests operative, vitiates the effect of intergenerational transfer policies. Briefly, he claims that government-initiated transfer policies will be exactly offset by changes in bequests, so that neither the consumption of any household nor the path of capital accumulation is affected.

This result depends crucially on the utility function Barro uses, which has as its arguments the household's own lifetime consumption and the lifetime utility of its direct heirs. Barro's conclusion may not hold if utility depends directly on the size of the bequest. On the contrary, in the example below a stronger bequest motive implies a larger long-run impact on consumption and on capital accumulation.

II. A Model of Social Security

Using a continuous-time model with overlapping generations, balanced budget transfer policies formulated in terms of flows are considered. For simplicity it is assumed that labor is inelastically supplied and that factor prices are unaffected. The latter assumption holds for a "small open economy," with the rest of the world following a balanced growth path, or for a closed economy with the production function

$$F(K,L,t) = rK + e^{gt}wL \ , \tag{1}$$

where g is the (constant) rate of Hicks-neutral technological progress. In either case changes in aggregate consumption and capital accumulation affect neither the rate of return on capital, r, nor the wage rate per unit of effective labor, w.

The population, which grows at the constant rate n, consists of a continuum of households. Each lives for one unit of time and is characterized by its time of formation, τ. The utility function of each household over its lifetime consumption stream $\{C(a)\}$ and its bequest B is:

$$U(\{C(a)\},B) = \int_0^1 \rho(a)C(a)^\alpha da + \beta B^\alpha , \tag{2}$$

$$\alpha \cdot \rho(a) \geq 0, \ 0 \leq a \leq 1; \ \alpha \cdot \beta \geq 0, \qquad \alpha < 1.$$

This specification postulates unit elasticities of consumption and bequests, but allows an age-dependent consumption profile. Under this assumption the optimal consumption stream and bequest of a household with net worth W at its time of formation are given by:

$$C(a) = c(a)W , \quad 0 \leq a \leq 1; \tag{3a}$$

$$B = bW,$$

with

$$\int_0^1 e^{-ra}c(a)da + e^{-r}b = 1. \tag{3b}$$

Attention will be restricted to policies for which per capita taxes and subsidies are age-dependent and grow at the rate g, and for which the government's budget is balanced at each instant. Any such policy can be written as $\{z(a), 0 \leq a \leq 1\}$ satisfying:

$$\int_0^1 e^{-(n+g)a} z(a)da = 0. \tag{4}$$

The tax paid $(z < 0)$ or transfer received $(z > 0)$ by household τ at age a is $e^{g\tau} z(a)$. Let $Z(\tau)$ denote the discounted value of the transfer policy to household τ, evaluated when the household is formed.

$$Z(\tau) \equiv \begin{cases} e^{g\tau} \int_{-\tau}^{1} e^{-ra} z(a)da , & -1 \leq \tau \leq 0 \\ e^{g\tau} Z(0) , & 0 < \tau . \end{cases} \tag{5}$$

A transfer policy's direct impact on each household's net worth is given by $Z(\tau)$. In addition, the policy has indirect consequences if there are bequests.

If a transfer program is initiated unexpectedly at time $t = 0$, a household in cohort $-1 \leq \tau < 0$ cannot change its consumption over the age interval $0 \leq a < -\tau$. Therefore, its MPC over the age interval $-\tau \leq a \leq 1$ and its MBP are larger than in (3) by the factor:

$$R(\tau) \equiv \left[\int_{-\tau}^{1} e^{-ra} c(a)da + e^{-r} b \right]^{-1}, \quad -1 < \tau < 0. \tag{6}$$

Using this expression, the direct and indirect impact of the policy on net worth per unit of effective labor in generation τ is given by:

$$\Delta V(\tau) \equiv \begin{cases} e^{-g\tau} Z(\tau) & , \quad -1 < \tau \leq 0 \\ Z(0) + \hat{b} R(\tau-1) \Delta V(\tau-1) & , \quad 0 < \tau \leq 1 \\ Z(0) + \hat{b} \Delta V(\tau-1) & , \quad 1 < \tau \end{cases} \tag{7}$$

where

$$\hat{b} \equiv e^{-(n+g)} b$$

An unfunded social security program is a policy that transfers income from workers to retirees. Formally, it is a tax-transfer schedule $\{z(a)\}$ satisfying (4), and for which there is an age of retirement a_R such that:

$$z(a) \leq 0 \;, \qquad 0 \leq a \leq a_R$$

$$z(a) \geq 0 \;, \qquad a_R < a \leq 1 \,.$$

No transfers are received before retirement and no taxes are paid after retirement. Assume that $\{z(a)\}$ is not identically zero.

If $n + g \geq r$, it is possible to construct transfer policies that raise the net worth of every household. In order to rule out these "free lunch" policies, assume that $r > n + g$.

Under this assumption, an obvious but important steady-state property of unfunded social security programs is that for some $t_0 \in (-1,0)$:

$$\begin{aligned} Z(\tau) &\geq 0 , \quad -1 < \tau \leq t_0 , \\ Z(\tau) &\leq 0 , \quad t_0 \leq \tau . \end{aligned} \tag{8}$$

Cohorts $-1 < \tau < 1 - a_R$ pay no taxes, so that their net benefit is non-negative. On the other hand, it is known from investment theory that a benefit stream that incurs costs early and yields profits late, with only one change in sign in the benefit stream, is monotonically decreasing in the discount rate. Hence if $r > g + n$, using (4):

$$Z(\tau) < e^{g\tau} \int_0^1 e^{-(g+n)a} z(a)da = 0 , \qquad 0 \leq \tau .$$

Since $Z(\tau)$ is continuous and $z(a)$ changes sign only once, (8) follows.

The effect of any intergenerational transfer program on capital accumulation can be found as follows. Let $\{W(t,\tau), \ \tau \leq t \leq \tau+1\}$ denote the asset holdings of generation τ over its lifetime. Then the change in the capital stock at date t is:

$$\Delta K(t) = \int_{t-1}^{t} \Delta W(t,\tau)d\tau , \qquad 0 \leq t, \tag{9}$$

where $\Delta W(t,\tau)$ is the change in asset holdings induced by the policy. This change consists of three parts: the change in the inheritance received, the value of transfers received, and the change in consumption expenditures.

$$\Delta W(t,\tau)\, e^{-(n+g)\tau} = \begin{cases} \int_{\tau}^{t} e^{r(t-s)}\{z(s-\tau) - c(s-\tau)R(\tau)\Delta V(\tau)\}\, ds, \\ \qquad\qquad 0 \leq t < \tau + 1, \quad -1 < \tau \leq 0; \\ e^{r(t-\tau)}\hat{b}R(\tau-1)V(\tau-1) + \int_{\tau}^{t} e^{r(t-s)}\{z(s-\tau) - c(s-\tau)\Delta V(\tau)\}\, ds, \\ \qquad\qquad \tau \leq t \leq \tau + 1, \quad 0 < \tau \leq 1; \\ e^{r(t-\tau)}\hat{b}V(\tau-1) + \int_{\tau}^{t} e^{r(t-s)}\{z(s-\tau) - c(s-\tau)\Delta V(\tau)\}\, ds \\ \qquad\qquad \tau \leq t \leq \tau + 1, \quad 1 < \tau \quad . \end{cases} \tag{10}$$

Next, note that:

$$\int_{t-1}^{t} e^{(n+g)\tau}\, e^{r(t-\tau)}\, \hat{b}V(\tau-1)d\tau = \hat{b}e^{(n+g)t} \int_{0}^{1} e^{ms}\, \Delta V(t-s-1)ds, \tag{11}$$

and that:

$$\begin{aligned} &\int_{t-1}^{t} e^{(n+g)\tau} \int_{\tau}^{t} e^{r(t-s)}\{z(s-\tau) - c(s-\tau)\Delta V(\tau)\}ds\, d\tau \\ &\quad = e^{rt} \int_{t-1}^{t} e^{(n+g)\tau} \int_{0}^{t-\tau} e^{-r(\tau+a)}\{z(a) - c(a)\Delta V(\tau)\}da\, d\tau \\ &\quad = e^{(n+g)t} \int_{0}^{1} e^{ms} \int_{0}^{s} e^{-ra}\{ z(a) - c(a)\Delta V(t-s)\}da\, ds, \end{aligned} \tag{12}$$

where

$$m \equiv r - n - g .$$

First consider the effect of an unfunded social security program when there are no bequests ($\beta=0$). Since there are no indirect effects, the change in each household's net worth is $Z(\tau)$. Using (8), net worth increases for households $\tau < t_0$ and decreases for households $t_0 < \tau$. In the short run aggregate consumption increases and capital accumulation declines. However, by date t_0+1 consumption declines for all living households.

The change in the path of capital accumulation can also be found. If $\beta=0$ then $b = \hat{b} = 0$, and from (7), $\{\Delta V(\tau) = Z(0),\ 0 \leq \tau\}$. Substituting (10) and (12) into (9) and using (3b) and (5):

$$\begin{aligned} \Delta K(t)\cdot e^{-(n+g)t} &= \int_0^1 e^{-ra}\{z(a) - c(a)Z(0)\}\int_a^1 e^{ms}\, ds\, da \\ &= \frac{1}{m}\int_0^1 \{e^m e^{-ra} - e^{-(n+g)a}\}\ \{z(a) - c(a)Z(0)\}\ da \qquad (13) \\ &= \frac{1}{m} Z(0)\int_0^1 e^{-(n+g)a}\ c(a)\ da \quad , \qquad 1 \leq t\ . \end{aligned}$$

By date t=1 the capital-labor ratio has reached its new steady-state value. For unfunded social security programs $Z(0)$ is negative and the capital-labor ratio declines.

Consider now the case where bequests are positive. First, inspecting (7) and (8), note that $V(\tau)$ displays damped oscillations. Consequently, looking across generations, consumption per unit of effective labor also displays damped oscillations. The consumption of generations $-1 < \tau < t_0$ is increased; the consumptions of generations $(\tau + i)$ for $-1 < \tau < t_0$, $i = 1,\ldots,I$ may also be increased due to the "echo" effect of bequests.

However, in the long run echos are swamped by the direct effect of the transfers. A necessary and sufficient condition for convergence to a steady state in the absence of transfers is that $\hat{b} < 1$. Under this restriction, the difference equations in (7) are stable, so that the decrement in net worth per labor unit asymptotically approaches its equilibrium level, $Z(0)/(1 - \hat{b})$. Hence, in the long run consumption by every living generation falls.

Next, consider the long-run effect on the capital stock. Define,

$$\Delta V^{\infty} \equiv \lim_{\tau \to \infty} \Delta V(\tau) = Z(0)/(1 - \hat{b}).$$

Using (9)-(12),

$$\lim_{\tau \to \infty} \Delta K(t) \cdot e^{-(n+g)t}$$

$$= \hat{b} \;\; \Delta V^{\infty} \int_0^1 e^{ms}\, ds + \int_0^1 e^{-ra} \left(z(a) - c(a) \;\; \Delta V^{\infty}\right) \int_a^1 e^{ms}\, ds\, da$$

$$= \frac{1}{m} \left\{\hat{b}\, \Delta V^{\infty} \cdot (e^{m}-1) + \int_0^1 (e^{m}e^{-ra} - e^{-(n+g)a}) \left(z(a) - c(a) \;\; \Delta V^{\infty}\right) da\right\}$$

$$= \frac{1}{m} \; \frac{Z(0)}{1 - \hat{b}} \int_0^1 e^{-(n+g)a} \; c(a) \; da. \tag{14}$$

The change in the capital-labor ratio asymptotically approaches the value in (14). For unfunded social security programs $Z(0) < 0$ and the capital-labor ratio declines.

To examine the effect of the bequest motive, it is useful to consider the normalized marginal propensity to consume. Define

$$\hat{c}(a) \equiv \frac{c(a)}{\int_0^1 e^{-rs}\, c(s)\, ds} = \frac{c(a)}{1 - e^{-r}b} \;\; , \quad 0 \leq a \leq 1 \; . \tag{15}$$

An increase in β with $\{p(a)\}$ unchanged increases b, but does not affect $\{\hat{c}(a)\}$.

Substituting (15) into (14),

$$\lim_{t\to\infty} \Delta K(t)\, e^{-(n+g)t} = \frac{1}{m} Z(0) \frac{1 - e^{-r_b}}{1-e^{-(n+g)_b}} \int_0^1 e^{-(n+g)a}\, \hat{c}(a)\, da. \tag{16}$$

Since $r > n + g$, the factor $(1 - e^{-r_b})/(1 - e^{-(n+g)_b})$ is increasing in b. It is clear from (16) that the no-bequest case ($b = 0$) minimizes the long-run effect on cohort net worths and on the capital-labor ratio. As the bequest motive becomes stronger the long-run impact of the policy increases, although the speed of adjustment declines. This conclusion, while surprising at first, is illuminated by the following example.

Suppose that there is neither population growth nor productivity increase. Then in the steady state established after the adoption of a social security program, each cohort's inheritance exactly equals its bequest. Hence the change in lifetime, discounted consumption per cohort is exactly equal to the net tax paid, Z(0). The stronger the bequest motive (the smaller the propensity to consume), the larger is the change in net worth required to elicit the required change in consumption. (Note that otherwise-identical economies with different bequest motives will start from different stationary states. After the adoption of an unfunded social security program, the final capital-labor ratio--although it has fallen farther--is still larger in the economy where the bequest motive is stronger.)

III. Conclusion

The model above illustrates that Barro's conclusion is very sensitive to the assumptions he makes. The example above is also restrictive, but not pathological. Although the model becomes analytically intractable, the qualitative results can be expected to carry over for at least some ranges of parameters when more general utility functions and production function are considered. Therefore, sensible policy recommendations can be formulated only on the basis of empirical evidence about household behavior, the sensitivity of factor returns to input ratios, etc.

Finally, note that models like the one above can also be used to examine the effects of changes on the wage structure. For example, increasing the wage rates of older workers at the expense of younger ones has the same effects as an unfunded social security program.

References

Barro, Robert J., "Are Government Bonds Net Wealth?" _Journal of Political Economy_, Vol. 82, No. 6 (Nov./Dec. 1974), 1095-1117.