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Information, Incentives, and Growth Under Uncertainty

by

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ABSTRACT

Uncertainty in the aggregate production function implies uncertainty in the returns to at least one factor, and, consequently, variability in the personal distribution of income. A model with overlapping generations is used to examine tax policies that simultaneously control the distribution of consumption among age cohorts and the path of capital accumulation while maintaining a decentralized economy--i.e., maintaining private ownership of all capital. Situations of both perfect and imperfect information about the random variable affecting production are considered. It is shown that any consumption-investment plan that does not discriminate among members of an age cohort can be controlled, and that no information is needed about individuals' subjective probability distributions over the possible states of the world.

1. Introduction

Uncertainty in the aggregate production function implies uncertainty in the returns to at least one factor and, via the national income identity, in aggregate income. Unless ownership of each factor is equally distributed, variability in the personal distribution of income and in the distribution of consumption follow as well. This paper considers tax policies that enable individuals to pool these risks.

When the production function is stochastic, the forces determining the division of income between wages and rents determine, concomitantly, the burden of risk on labor and on owners of capital. Since uncertainty about the rate of return affects the incentives to invest, in a decentralized economy a new channel of influence from income distribution to capital accumulation appears.

The effects on investor behavior of uncertainty in the returns to various capital goods are familiar. To the extent that risks on different assets are independent or offsetting, portfolio diversification offers "insurance" to investors. As Diamond [1967] has shown, under certain conditions the allocation of risk among investors achieved by a stock market is Pareto-efficient. Under these conditions there are no further welfare gains available from risk-pooling among market participants. However, in the presence of real uncertainty in the aggregate return to capital there is a residual risk that must be borne by all investors as a group. Similarly, uncertainty in the wage rate falls inevitably on workers.

However, uncertainties in the returns to capital and labor may offset each other, at least to some extent. Furthermore, looking over a longer time horizon the groups of workers and investors are constantly changing. This opens up the possibility for risk-pooling among workers and/or investors who are active in different markets--that is, for intergenerational risk-pooling.

The portfolio selection problem will be ignored here, and a model with overlapping generations will be used to examine tax policies that simultaneously control the distribution of consumption among age cohorts and the path of capital accumulation, while maintaining a decentralized economy--i.e., maintaining private ownership of all capital. Only balanced-budget tax policies are considered, so that there is no role for government debt. Real capital, which has an uncertain rate of return, is the only asset available to savers. Since age cohorts overlap, changing the distribution of consumption among generations is possible; and since both wages and returns to capital are uncertain, such changes are desirable in the sense of expected utility. Government intervention is called for because private market institutions cannot allocate sequential risks efficiently. As in other instances where a "contract" among age cohorts is required (see, for example Samuelson [1958]), only through government institutions can the desired result be realized, since only in that way can participation by unborn generations be guaranteed.

The problem of implementing consumption-investment plans under uncertainty is related to two strands in the theoretical literature. The first is the work on growth under uncertainty (Phelps [1961], Mirrlees [1971], Brock and Mirman [1972], Mirman [1973], and Mirman and Zilcha [1975]). Here interest has centered on deriving conditions for convergence of the economy to a (stochastic) steady state, and conditions for the existence, uniqueness and stability of optimal paths of capital accumulation. The second is the literature on policies for implementing desired paths of consumption and capital accumulation in a decentralized economy when there is no uncertainty (Arrow and Kurz [1970], Samuelson [1975], Chamley [1977], and others). The objective here has been to construct incentive schemes that induce the desired type of behavior on the individual level.

Weaving together these two strands introduces a new difficulty: in a world of uncertainty an individual's plans for the future depend on his information about the uncertain events. Under any fixed incentive structure, including the absence of all government intervention, individuals who attach different subjective probabilities to various outcomes will, in general, choose different courses of action. Consequently, even if tastes and objective circumstances are constant across the population, if beliefs vary it may be difficult or impossible to realize a desired consumption path through a decentralized mechanism. This problem is similar to the one that arises if individuals differ in tastes or in any other unobservable characteristic. Any such difference causes a fixed incentive structure to impinge differentially on individuals' actions. This is in contrast to the situation where individuals vary in a characteristic that is observable, for example in age or health. In the latter cases group-specific policies can be used to deal with the incentive problem.

When information varies among individuals, incentive structures that are group-specific in a certain sense can be achieved by setting state-dependent values for policy variables. This property can be exploited to advantage for the problem at hand.

Of primary concern here will be the existence of government policies that implement particular plans, the extent of public agreement about policies' objective consequences, and the extent of public agreement about policies' welfare implications. The latter issues are important if policies cannot be imposed by "the government," but rather must be agreed upon by the public or its representatives. It will be assumed throughout that all individuals in all age cohorts have identical tastes and endowments, so that differences in information about the random variable affecting production are the source of all differences in behavior and in social welfare calculations.

Before proceeding, some preliminary terminology must be introduced. The term "instrument" will be used here, in the usual way, to refer to a decision variable in the hands of the government. For a fixed set of instruments another definition follows.

<u>Definition 1</u>: A <u>policy</u> is a set of values for the instruments. These values may depend on the values of state variables and on the realization of the random variable.

A plan, defined precisely below, is a set of functions describing consumption and investment. Using a model in which the production function is stochastic, this paper looks at the problem of constructing one or more policies that implement a given plan. This is done first for situations of complete information about the stochastic process, and then for situations of incomplete information. ("Complete information" will mean that the probabilities governing the stochastic process are known; "incomplete information" will mean that the probabilities are unknown, so that subjective probabilities must be used in all calculations.) In the latter case information may vary among individuals. The conclusions are as follows.

a) Any feasible plan that allocates identical consumption bundles to all members of an age cohort can be implemented under any information structure, without knowing either the objective probabilities governing the random variable or the information structure in the population. Therefore, if individuals with different information (i.e., with different subjective probabilities) can agree on such a plan, they can agree on a policy to implement it. The instruments used are a consumption tax, a tax on interest income, and age-dependent lump-sum subsidies. Furthermore, plans that allocate identical consumption bundles to all members of an age cohort are the <u>only</u> ones that can be implemented, using this set of instruments, without knowing the information structure.

b) If the random variable affecting production is i.i.d. or if it follows a Markov process, plans that are optimal under the utilitarian social welfare function--if any exist--are among the plans considered here.

c) Under the utilitarian criterion, for any given investment plan, the optimal allocation of the corresponding levels of aggregate consumption requires that in each period the marginal utilities of all individuals be equal. Since this requirement is independent of beliefs about the random variable, given any investment plan there can be no disagreement about the optimal allocation of aggregate consumption.

d) Plans that equate marginal utilities in each period are implemented by policies in which the tax on interest income completely offsets fluctuations in the return to capital, i.e., completely stabilizes the rate of interest net of taxes at the social rate of discount. Hence, if the utilitarian criterion is accepted, the optimal interest tax policy is determined.

e) Even if aggregate consumption is always allocated optimally, comparisons among different investment plans involve the distribution of the random variable. Therefore individuals who have different information will disagree about a policy's welfare implications, even if they agree about its (contingent) objective consequences.

In Section 2 the model is presented and the problem of implementation is examined under the assumption that the random variable is i.i.d.; the extension to Markov processes and other types of random variables is discussed in Section 3; welfare considerations are examined in Section 4; and in Section 5 the conclusions are discussed.

2. Implementation

Each individual lives for two periods. He works, consumes and saves when young, and consumes his assets, including the accrued returns, when old. For simplicity it is assumed that labor is inelastically supplied, and that each individual supplies one unit of labor when young. Utility then depends only on the consumption bundle (c^y, c^o). All individuals in all age cohorts have the same strictly concave, additively separable utility function:

$$U(c^y, c^o) = U^y(c^y) + U^o(c^o)$$

$$U_c^y > 0, \quad U_c^o > 0, \quad U_{cc}^y < 0, \quad U_{cc}^o < 0 \tag{1}$$

$$\lim_{c \to 0} U_c^y(c) = \lim_{c \to 0} U_c^o(c) = \infty \quad .$$

The population grows at the constant rate n, and all variables are measured on a per capita basis.

Homogeneous output is used for both consumption and investment. Uncertainty enters through the production function:

$$y(t) = f[k(t); r(t)] \quad , \qquad t = 0, 1, \ldots,$$

where

$y(t)$ = output (net of depreciation) per retiree in period t;

$k(t)$ = capital per retiree in period t;

$\tilde{r}$ is an i.i.d. random variable that takes on the values $(r_1, r_2, \ldots, r_I)$ with probabilities $(p_1, p_2, \ldots, p_I)$ respectively, with $p \in S_+^I \equiv \{\underline{s} > 0 | \sum_i s_i = 1\}$; and

$r(t)$ = the realization of the random variable in period t.

Thus it is as if there are I different production functions, and in each period one of them is chosen, at random, to describe production. Note that the functions need not be ordered:

$$f(k;r_i) > f(k;r_j) \not\Longrightarrow f(k';r_i) > f(k';r_j) \quad , \quad \text{for } k \neq k',$$

nor are they required to have any other particular properties. In fact, they need not be concave or differentiable, or even continuous or monotonic in k.

The division of income between labor and capital is described by the wage and rental functions, $w(k;\tilde{r})$ and $R(k;\tilde{r})$. Since k is the ratio of capital to retirees, factor payments must satisfy:

$$(1+n)\cdot w(k;r_i) + k\cdot R(k;r_i) = f(k;r_i) \quad , \quad k \geq 0;\ i = 1,\ldots,I. \qquad (2)$$

Factors may be paid their marginal products, but this is not required.

In any period the effects of past decisions are reflected only in the current capital stock. Therefore, attention will be limited to autonomous plans for consumption and investment.

<u>Definition 2</u>: A <u>feasible plan</u> is a set of functions $[\gamma^y(k;r_i)$, $\gamma^o(k;r_i)$, $\xi(k;r_i)$, $k \geq 0;\ i = 1,\ldots,I]$ that specify the consumption of each young individual , the consumption of each old individual, and net investment per young individual, contingent on the existing capital stock and on the current realization of the random variable. These functions must satisfy:

$$\begin{aligned}&\gamma^o(k;r_i) + (1+n)[\gamma^y(k;r_i) + \xi(k;r_i)] + nk = f(k;r_i),\\&\gamma^y(k;r_i) \geq 0,\ \ \gamma^o(k;r_i) \geq 0,\ \ k + \xi(k;r_i) \geq 0,\\&\qquad\qquad\qquad\qquad k \geq 0;\ i = 1,\ldots,I.\end{aligned} \tag{3}$$

Note that under this definition the consumption levels of both young and old depend only on the current realization or $\tilde{r}$. Notice, too, that this definition of a feasible plan includes as a subset plans in which consumption and investment depend only on the total quantity of goods available (capital plus output), and not on the capital stock and the realization of the random variable separately.

An individual's expectations about the capital stock, as well as his beliefs about the random variable, affect his decisions. Although many models of expectation formation could be incorporated, attention here will be confined to the case where expectations are "rational," i.e., where individuals correctly foresee the size of the capital stock one period ahead. Since the capital stock in period t is completely determined by decisions made in period t-1, it can be assumed that the government announces its plan and that all individuals believe with certainty that it will be carried out.

<u>Complete Information</u>

First consider the situation where all individuals have complete information about $\tilde{r}$. In general plans can be implemented only through policies that use state-contingent values of the instruments. Proposition 1 suggests one sufficient set of instruments.

Proposition 1: If all individuals have complete information about the random variable $\tilde{r}$ and have rational expectations about the capital stock, then any feasible plan can be implemented using the following set of instruments:

a) a consumption tax, $[t^c(k;r_i),\ k \geq 0;\ i = 1,\ldots,I]$;

b) a tax on interest income, $[t^r(k;r_i),\ k \geq 0;\ i = 1,\ldots,I]$; and

c) age-dependent lump-sum subsidies, $[s^y(k;r_i),\ s^o(k;r_i),\ k \geq 0;\ i = 1,\ldots,I]$.

(Both "taxes" and both "subsidies" may be either positive or negative.)

Proof: Let $[\gamma^y(k;r_i),\ \gamma^o(k;r_i),\ \xi(k;r_i),\ k \geq 0;\ i = 1,\ldots,I]$ be any feasible plan, and define the marginal utilities:

$$\begin{aligned} u(k;r_i) &\equiv U^y_c[\gamma^y(k;r_i)]\ , \\ v(k;r_i) &\equiv U^o_c[\gamma^o(k;r_i)]\ , \quad k \geq 0;\ i = 1,\ldots,I; \end{aligned} \tag{4}$$

and the tax factors:

$$\begin{aligned} T^c(k;r_i) &\equiv [1 + t^c(k;r_i)]^{-1} \\ T^r(k;r_i) &\equiv 1 + [1 - t^r(k;r_i)]\cdot R(k;r_i)\quad ,\quad k \geq 0;\ i = 1,\ldots,I. \end{aligned} \tag{5}$$

Since there is a one-to-one correspondence between the schedules $[t^c(k;r_i)]$ and $[T^c(k;r_i)]$, and between the schedules $[t^r(k;r_i)]$ and $[T^r(k;r_i)]$, from this point on T^c and T^r will be referred to as tax rates.[1]

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[1] If the interest rate is zero in any state of the world ($R(k;r_i) = 0$ for some k,i), a tax on interest is obviously ineffective. In that state, however, a tax can be levied on wealth, t^w, with:

$$t^w(k;r_i) = 1 - T^r(k;r_i).$$

A policy that implements the desired plan can be constructed by noting that it must satisfy three requirements. First, since each retiree consumes all of his assets, the lump-sum subsidy to the old must be related to the consumption and interest taxes by:

$$s^o(k;r_i) = \gamma^o(k;r_i)/T^c(k;r_i) - k\cdot T^r(k;r_i) \ , \ k \geq 0; \ i = 1,\dots,I. \quad (6)$$

Second, since the assets accumulated by each worker must be equal to the current per capita stock of capital plus net investment per capita, the lump-sum subsidy to the young must be related to the consumption tax by:

$$s^y(k;r_i) = k + \xi(k;r_i) - w(k;r_i) + \gamma^y(k;r_i)/T^c(k;r_i) \ , \quad k \geq 0; \ i = 1,\dots,I. \quad (7)$$

Finally, each worker must choose his assigned level of consumption voluntarily, i.e., $\gamma^y(k;r_i)$ must be the solution of:

$$\begin{aligned} \underset{c}{\text{Max}}\{U^y(c) + \underset{j}{E}U^o(T^c(k + \xi(k;r_i);r_j)[T^r(k + \xi(k;r_i);r_j) \\ \cdot \ [w(k;r_i) + s^y(k;r_i) - c/T^c(k;r_i)] \\ + s^o(k + \xi(k;r_i);r_j)])\} \quad , \quad k \geq 0; \ i = 1,\dots,I. \end{aligned} \quad (8)$$

Substituting from (6) and (7), this requires:

$$u(k;r_i)\ T^c(k;r_i) = \sum_j p_j T^r(k + \xi;r_j)\ T^c(k + \xi;r_j)\ v(k + \xi;r_j) \quad (9)$$
$$k \geq 0; \ i = 1,\dots,I.$$

A policy that implements the given plan is defined by any pair of consumption and interest tax schedules satisfying (9), together with lump-sum subsidy schedules given by (6) and (7).

Only strictly positive values for T^c and non-negative values for T^r are economically meaningful. A zero value for T^c corresponds to an infinitely high tax on consumption, and a negative value to a subsidy of more than 100%; a negative value for T^r corresponds to an interest tax that takes more than 100% of interest plus principle. A zero value for T^r corresponds to a tax that confiscates all interest and principle, and will be included as a possibility.

Policies that implement the given plan can be constructed in the following way. Consider the set of consumption tax schedules $\mathcal{T}^c$ defined by:

$$\mathcal{T}^c([u(k;r_i),\ k \geq 0;\ i=1,\ldots,I]) \equiv \{[T^c(k;r_i),\ k \geq 0;\ i=1,\ldots,I] \mid$$
$$u(k;r_i)T^c(k;r_i) = u(k';r_j)T^c(k';r_j)\ ,$$
$$k,k' \geq 0;\ i,j = 1,\ldots,I\}.$$

Substituting any of the schedules in the set $\mathcal{T}^c$ into (9), the problem of constructing a policy is reduced to finding an interest tax schedule that satisfies:

$$1 = \sum_j p_j T^r(k;r_j)v(k;r_j)/u(k;r_j)\quad,\quad k \geq 0\ .$$

Let $\mathcal{T}^r$ be defined as this set of interest tax schedules.

$$\mathcal{T}^r([u(k;r_i),v(k;r_i),\ k \geq 0;\ i=1,\ldots,I];\ \underline{p})$$
$$\equiv \{[T^r(k;r_i),\ k \geq 0;\ i=1,\ldots,I] \mid \sum_j p_j T^r(k;r_j)v(k;r_j)/u(k;r_j) = 1,$$
$$k \geq 0;\ i = 1,\ldots,I\}.$$

Any interest tax schedule in $\mathcal{T}^r([u,v],\underline{p})$ together with any consumption tax schedule in $\mathcal{T}^c([u])$ satisfy (9). A policy implementing the given plan is completed by adding schedules for the lump-sum subsidies that are defined by (6) and (7). Q.E.D.

Using the set of instruments specified in Proposition 1, the government can implement any feasible plan through a wide variety of policies. Risk can be pooled among age cohorts in any way that society wishes. Both the level of investment and the distribution of consumption can be completely divorced--except for the adding up constraint--from the wage and rent functions, i.e., from the pre-tax distribution of income. Since all individuals in all generations have complete information about the probabilities governing the random variable, all agree, ex ante, about both the objective consequences and the welfare implications of any policy.

Incomplete Information

Thus far it has been assumed that all individuals have complete information about the random variable. If, instead, individuals are assumed to have identical but incomplete information, the analysis above needs to be modified only slightly. Let $\underline{p}^h \in S_+^I$ denote individuals' (common) subjective probability distribution for $\tilde{r}$. Using the same set of instruments, clearly the government can still implement any feasible plan.

Corollary 1: If all individuals have identical beliefs about $\tilde{r}$, described by the probabilities $\underline{p}^h$, and have rational expectations about the capital stock, any feasible plan can be implemented using a consumption tax, a tax on interest income, and age-specific lump-sum subsidies.

Proof: Given any feasible plan, policies that implement it can be constructed exactly as in the proof of Proposition 1, substituting $\underline{p}^h$ for $\underline{p}$. Q.E.D.

In general the policies in Proposition 1 or Corollary 1 can be constructed only if the government knows individuals' probability distribution over the random variable. However, some of those policies do not depend on individuals' information.

Proposition 2: If all individuals have identical beliefs about $\tilde{r}$, described by the probabilities $\underline{p}^h$, and have rational expectations about the capital stock, any feasible plan can be implemented, using the set of instruments in Proposition 1, without knowing $\underline{p}^h$.

Proof: Let $[\gamma^y(k;\tilde{r}),\ \gamma^o(k;\tilde{r}),\ \xi(k;\tilde{r})]$ be any feasible plan, and let $[u(k;\tilde{r})]$ and $[v(k;\tilde{r})]$ be defined as in (4). A policy implements the given plan for all subjective probability distributions if and only if it is in the joint intersection of all the sets of policies that implement it for particular values of $\underline{p}^h$.

Since the set of consumption tax schedules $\mathcal{T}^c([u])$ is independent of $\underline{p}^h$, this joint intersection consists of the policies:

$$[T^r(k;r_i),\ k \geq 0,\ i = 1,\ldots,I] \in \bigcap_{\underline{p}^h \in S_+^I} \mathcal{T}^r([u,v];\underline{p}^h)$$

$$= [T^r(k;r_i) = u(k;r_i)/v(k;r_i),\ k \geq 0,\ i = 1,\ldots,I], \qquad (10)$$

$$[T^c(k;r_i),\ k \geq 0,\ i = 1,\ldots,I] \in \mathcal{T}^c([u])\ ,$$

and schedules for the lump-sum subsidies that are defined by (6) and (7). Q.E.D.

Notice that the interest tax schedule in (10) is determined by the allocation of consumption; the consumption tax schedule is determined, up to the choice of a scalar, by the consumption of the young; and the schedules of lump-sum taxes depend on this scalar.

Proposition 2 implies that the informational requirements for implementing a plan are minimal; only information about the utility function is needed, none about the probabilities. This leads in turn to the conclusion that any plan that can be implemented under Proposition 2 can also be implemented when individuals attach different subjective probabilities to the random variable. However, when individuals' beliefs about $\tilde{r}$ differ, a broader definition of a feasible plan is needed.

Assume now that there are H types of individuals, indexed by $h = 1,\ldots,H$, and characterized by their differing information about $\tilde{r}$. Assume, further, that the information structure is stationary over time (among cohorts), and define:

$\underline{p}^h \in S_+^I$, $h = 1,\ldots,H$ — the subjective probabilities of a type h individuals over $\tilde{r}$, with $\underline{p}^h \neq \underline{p}^{h'}$ for $h \neq h'$, and

$\underline{\alpha} \in S_+^H$ — a vector describing the proportions of different types of individuals in the population.

Using this notation, without loss of generality, it can be assumed that H and $[\underline{p}^h, h = 1,\ldots,H]$ are fixed. The information structure is then completely described by $\underline{\alpha}$.

It will be assumed throughout that the government cannot distinguish among different types of individuals, i.e., that it cannot levy type-specific taxes. However, different types of individuals within each age cohort will, in general, choose different actions when they are young. Therefore a plan can specify levels of consumption for the young that are contingent on type as well as the capital stock and the realization of $\tilde{r}$. Furthermore, because of its effect on his saving, the consumption of an old individual can now depend on the realization of $\tilde{r}$ when he was young, as well as its realization in the current period, the capital stock, and his type. However, if net investment depends only on the current--not the past--realization or $\tilde{r}$, then the total consumption of an old generation can depend only on the current value of $\tilde{r}$. Only the extent and direction of the variations in consumption among the old depend on the value of $\tilde{r}$ when they were young. These considerations lead to the following definition of a feasible plan when there are different types of individuals.

Definition 2': Given $\underline{\alpha}$, a feasible plan is a set of functions $[(\gamma^{hy}(k;r_i), [\gamma^{ho}(k;r_i,r_j), j = 1,\ldots,I], h = 1,\ldots,H), \xi(k;r_i), k \geq 0;$ $i = 1,\ldots,I]$ that specify values for the consumption of a young individual, contingent on his type, the capital stock, and the current realization of $\tilde{r}$; for the consumption of an old individual, contingent on his type, the capital stock, the current realization of $\tilde{r}$, and the realization of $\tilde{r}$ in the previous period; and for net investment per young individual, contingent on the capital stock and the current realization of $\tilde{r}$. These functions must satisfy:

$$\sum_h \alpha^h \left[(1+n)\cdot\gamma^{hy}(k;r_i) + \gamma^{ho}(k;r_i,r_j)\right] + (1+n)\cdot\xi(k;r_i) + nk = f(k;r_i), \quad (11)$$

$$\gamma^{hy} \geq 0, \quad \gamma^{ho} \geq 0, \quad h = 1,\ldots,H; \; k + \xi(k;r_i) \geq 0,$$

$$k \geq 0; \; i,j = 1,\ldots,I.$$

Using the notation in Definition 2', let the average consumption of the young and old be denoted by:

$$\bar{\gamma}^y(k;r_i) \equiv \sum_h \alpha^h \gamma^{hy}(k;r_i) \quad ,$$

$$\bar{\gamma}^o(k;r_i) \equiv \sum_h \alpha^h \gamma^{ho}(k;r_i,r_j) \; , \; j = 1,\ldots,I \qquad (12)$$

$$k \geq 0; \; i = 1,\ldots,I.$$

While in general constructing a feasible plan requires that the information structure, $\underline{\alpha}$, be known, some plans are feasible for any value of $\underline{\alpha}$.

Proposition 3: A plan $[\gamma^{hy},\gamma^{ho},\xi]$ is feasible for any information structure if and only if it satisfies:

$$(1+n)\cdot\gamma^{hy}(k;r_i) + \gamma^{ho}(k;r_i,r_j) + (1+n)\cdot\xi(k;r_i) + nk = f(k;r_i) \; , \qquad (13)$$

$$k \geq 0; \; h = 1,\ldots,H; \; i,j = 1,\ldots,I \; .$$

Proof: A plan is feasible for any information structure if and only if it satisfies (11) for all $\underline{\alpha} \in S^H_+$. Obviously any plan satisfying (13) satisfies (11) for all $\underline{\alpha}$; and if a plan satisfies (11) for all $\underline{\alpha}$, letting $\underline{\alpha} = e^h \equiv (0,\ldots,0,1,0,\ldots,0)$, h = 1,...,H, gives (13). Q.E.D.

When individuals have different information, not all feasible plans can be implemented using the set of instruments in Proposition 1. Therefore it is interesting to begin by examining the set of plans that can be implemented.

Proposition 1': Given $\underline{\alpha}$, and assuming that all individuals have rational expectations about the capital stock, a feasible plan $[\gamma^{hy}, \gamma^{ho}, \xi]$ can be implemented using a consumption tax, a tax on interest income, and age-specific lump-sum subsidies, if and only if there is a set of values $[(\Delta\gamma^h(k;r_i),\ h = 1,\ldots,H),\ T(k;r_i),\ S(k;r_i),\ k \geq 0;\ i = 1,\ldots,I]$ satisfying:

$$\gamma^{hy}(k;r_i) = \bar{\gamma}^y(k;r_i) + \Delta\gamma^h(k;r_i) \quad , \quad h = 1,\ldots,H; \tag{14a}$$

$$\begin{aligned}\gamma^{ho}[k + \xi(k;r_j);r_i,r_j] = {} & \bar{\gamma}^o[k + \xi(k;r_j);r_i] \\ & - \Delta\gamma^h(k;r_j)S[k + \xi(k;r_j);r_i]/T(k;r_j) \ , \\ & j = 1,\ldots,I;\ h = 1,\ldots,H;\end{aligned} \tag{14b}$$

$$\begin{aligned}T(k;r_i)u(k;r_i) = \sum_j p_j^h\, v[k + \xi(k;r_i);r_j,r_i]S[k + \xi(k;r_i);r_j] \ , \\ h = 1,\ldots,H;\end{aligned} \tag{14c}$$

$$\sum_h \alpha^h \Delta\gamma^h(k;r_i) = 0 \tag{14d}$$

$$T(k;r_i) > 0, \quad S(k;r_i) \geq 0, \quad k \geq 0;\ i = 0,\ldots,I, \tag{14e}$$

where $[\bar{\gamma}^y]$ and $[\bar{\gamma}^o]$ are as defined in (12), and $[u]$ and $[v]$ as in (4).

Proof: Suppose that a feasible plan $[\gamma^{hy},\gamma^{ho},\xi]$ and a set of values $[\Delta\gamma^{h}, T, S]$ satisfying (14) are given. Then the policy

$$T^{c}(k;r_i) = T(k;r_i) ,$$

$$T^{r}(k;r_i) = S(k;r_i)/T(k;r_i) ,$$

$$s^{y}(k;r_i) = k + \xi(k;r_i) - w(k;r_i) + \bar{\gamma}^{y}(k;r_i)/T(k;r_i) ,$$

$$s^{o}(k;r_i) = \bar{\gamma}^{o}(k;r_i)/T(k;r_i) - k\cdot S(k;r_i)/T(k;r_i) ,$$

$$k \geq 0;\ i = 1,\ldots,I,$$

satisfies both (9) and the budget constraint for every consumer type. Hence it implements the plan.

Conversely, suppose that a feasible plan and a policy implementing it, $[T^{c},T^{r},s^{y},s^{o}]$, are given. The required set of values contains:

$$\Delta\gamma^{h}(k;r_i) = \gamma^{hy}(k;r_i) - \bar{\gamma}(k;r_i),\quad k \geq 0;\ h = 1,\ldots,H;\ i = 1,\ldots,I.$$

To complete the list note that the only difference between the consumption levels of two retirees of different types, in the same cohort, is due to differences in their savings when young, net of taxes.

$$\gamma^{ho}[k + \xi(k;r_j);r_i,r_j] = \gamma^{h'o}[k + \xi(k;r_j);r_i,r_j]$$
$$- [\Delta\gamma^{h}(k;r_j) - \Delta\gamma^{h'}(k;r_j)]T^{r}[k + \xi(k;r_j);r_i]T^{c}[k + \xi(k;r_j);r_i]/T^{c}(k;r_j)$$
$$k \geq 0;\ h,h' = 1,\ldots,H;\ i,j = 1,\ldots,I.$$

Multiplying each side by $\alpha^{h'}$ and summing over h':

$$\gamma^{ho}(k+\xi;r_j,r_i) = \bar{\gamma}^{o}(k+\xi;r_j) - \Delta\gamma^{h}(k;r_i)T^{r}(k+\xi;r_j)T^{c}(k+\xi;r_j)/T^{c}(k;r_i),$$

$$k \geq 0;\ h = 1,\ldots,H;\ i,j = 1,\ldots,I.$$

Therefore,

$$T(k;r_i) = T^{c}(k;r_i)\ ,$$

$$S(k;r_i) = T^{r}(k;r_i)T^{c}(k;r_i)\ ,\ k \geq 0;\ i = 1,\ldots,I,$$

complete the required set of values. Q.E.D.

Plans in which all individuals within a cohort receive the same contingent consumption levels are especially interesting. These plans will be called equable.

Definition 3: A plan is equable if

$$\gamma^{hy}(k;r_i) = \bar{\gamma}^{y}(k;r_i)\ ,$$

$$\gamma^{ho}(k;r_i,r_j) = \bar{\gamma}^{o}(k;r_i)\ ,\ j = 1,\ldots,I;\ k \geq 0;\ i = 1,\ldots,I.$$

It is clear from Proposition 2 that any feasible, equable plan can be implemented without knowing $\underline{\alpha}$. For the set of instruments given there, the converse is also true.

Corollary 2: If all individuals have rational expectations about the capital stock, any feasible, equable plan can be implemented, without knowing $\underline{\alpha}$, using a consumption tax, a tax on interest income, and age-specific lump-sum subsidies. Conversely, among the plans that are feasible for any value of $\underline{\alpha}$, only the equable plans can be implemented using that set of instruments.

Proof: Given any feasible, equable plan, the policies described in the proof of Proposition 2 implement it, and the arguments there apply here as well.

Consider now any plan that is feasible for all values of $\underline{\alpha}$, i.e., satisfying (13). By Proposition 1', it is possible to implement the plan only if it satisfies (14) for some set of values $[\Delta\gamma^h,T,S]$. From (13), feasibility for all $\underline{\alpha}$ implies:

$$\gamma^{ho}(k;r_i,r_j) = \gamma^{ho}(k;r_i,r_{j'}) \quad , \quad j,j' = 1,\ldots,I;$$

$$k \geq 0;\ h = 1,\ldots,H;\ i = 1,\ldots,I.$$

Substituting this expression into (14b), implementation requires:

$$\Delta\gamma^h(k;r_j)\cdot S[k + \xi(k;r_j);r_i] = 0 \quad , \quad k \geq 0;\ h = 1,\ldots,H;\ i,j = 1,\ldots,I.$$

Therefore, if

$$\Delta\gamma^h(k;r_j) \neq 0 \quad \text{for some } h,k,j \ ,$$

then

$$S[k + \xi(k;r_j);r_i] = 0 \quad , \quad i = 1,\ldots,I, \text{ for that } k,j.$$

However, (14c) could not then be satisfied. Hence the plan can be implemented only if it is equable. Q.E.D.

Policies implementing equable plans can be formulated without any information about the probabilities governing the stochastic process affecting production, or about the distribution of information in the population. Furthermore, even if his own information about $\tilde{r}$ is incomplete and even if he knows nothing about the beliefs of others, any individual will agree that any equable plan can be implemented through the policies in Proposition 2. In short, there can be no disagreement about the (contingent) objective consequences of the policies in Proposition 2. These are the only policies with that property; hence only equable plans can be the subject of such agreement.

3. Other Stochastic Processes

The probabilities governing the stochastic process enter into neither the description of a feasible plan (Definition 2') nor the description of the policies in (10). Hence, given any feasible, equable plan, the policies given by (10) implement that plan under any information structure and regardless of the properties of the stochastic process. For example, $\tilde{r}(t)$ may be serially correlated or non-autonomous, or the information structure may vary among cohorts. The policies in (10) are, in this sense, dominant.

Still, depending on the nature of the random variable, the form of the social welfare function, and the planning horizon, in general there are welfare gains from considering more complicated plans. For example, if $\tilde{r}$ is non-autonomous, non-autonomous plans should be considered. These and other welfare considerations will be examined next.

4. Welfare

It will be assumed throughout this section that all individuals agree that society should use an infinite planning horizon and that social welfare should be evaluated using the utilitarian criterion. Hence they all agree that the feasible plan should be chosen that maximizes:

$$\underset{\tilde{r}(0),\tilde{r}(1),\ldots}{E} \{ \sum_{t=-1}^{\infty} (\frac{1+n}{1+\rho})^t \sum_h \alpha^h [U^y[\tilde{c}^{hy}(t)] + U^o[\tilde{c}^{ho}(t)]]\}$$

$$= (\frac{1+n}{1+\rho})^{-1} \sum_h \alpha^h U^y[c^{hy}(-1)] + \sum_{t=0}^{\infty} (\frac{1+n}{1+\rho})^t \underset{\tilde{r}(0),\ldots,\tilde{r}(t)}{E} \{ \sum_h \alpha^h [U^y[\tilde{c}^{hy}(t)]$$

$$+ (\frac{1+\rho}{1+n}) U^o[\tilde{c}^{ho}(t-1)]]\}$$

where ρ is the social rate of discount, and $\tilde{c}^{hy}(t)$ and $\tilde{c}^{ho}(t)$ are the levels of consumption of a type h member of the cohort born in period t, when he is young and old respectively. Obviously $[\tilde{c}^{hy}(t), \tilde{c}^{ho}(t), h = 1,\ldots,H;\ t = -1,0,1,\ldots]$ are random variables whose distribution depends on the distribution of $\tilde{r}$ and on the choice of a plan.

First note that under the utilitarian welfare function, if $\tilde{r}(t)$ is i.i.d. or follows a Markov process, then:

1) if a unique optimal plan exists it is among those covered by Definition 2' (i.e., it is autonomous, and levels of consumption and investment depend only on the current values of k and $\tilde{r}$); and

2) if multiple optimal plans exist at least some of them are among those covered by Definition 2'.

For if $\tilde{r}(t)$ is i.i.d. or Markov, the existing capital stock and the current realization of $\tilde{r}$ summarize all information about the economy. The preceding claims can then be verified by assuming the opposite and deriving a contradiction.

It will be assumed in the rest of this section that $\tilde{r}$ is i.i.d. or Markov, and the discussion will be confined to autonomous plans.

Next note that using the utilitarian welfare function draws attention to equable plans, since under it all optimal plans are equable. In fact, a decomposition argument can be used to show that individuals will agree, regardless of their information about $\tilde{r}$, on the socially optimal allocation--between young and old and among different types--of any given level of aggregate consumption. Unanimity about consumption allocations occurs even though differences in information will, in general, lead to differences in opinion about the optimal investment path.

This can be seen by considering any exogenously given investment plan $[\xi(k;r_i),\ k \geq 0;\ i = 1,\ldots,I]$ satisfying:

$$k + \xi(k;r_i) \geq 0 \quad ,$$

$$f(k;r_i) - nk - \xi(k;r_i) \geq 0 \quad , \quad k \geq 0;\ i = 1,\ldots,I.$$

If investment follows the specified path, an individual whose information about $\tilde{r}$ is represented by $\underline{p}^h$ believes that social welfare is maximized if, for each value of $(k;r_i)$, consumption is allocated by choosing $[\gamma^{hy}(k;r_i),\ \gamma^{ho}(k;r_i),\ h = 1,\ldots,H]$ according to:

$$\max_{[c^{hy}, c^{ho}, h=1,\ldots,H]} \sum_h \alpha^h [U^y(c^{hy}) + (\tfrac{1+\rho}{1+n}) U^o(c^{ho})]$$

$$\text{s.t.} \qquad \sum_h \alpha^h [(1+n)c^{hy} + c^{ho}] = f(k;r_i) - nk - \xi(k;r_i).$$

The solution, which satisfies:

$$U_c^y[\gamma^{hy}(k;r_i)] = (1+\rho)U_c^o[\gamma^{ho}(k;r_i)] \ , \ h = 1,\ldots,H; \ k \geq 0; \quad i = 1,\ldots,I; \tag{15}$$

is an equable plan that is independent of $\underline{p}^h$.

However, even if aggregate consumption is allocated according to (15), individuals who have different beliefs about $\tilde{r}$ will disagree about the welfare implications of different plans. This can be seen by letting $V(C)$ denote the level of social welfare, undiscounted and on a per capita basis, that is achieved by the allocation in (15) when per capita consumption, averaged over all types and cohorts, is C.

If (15) is used to allocate consumption, an individual with information reflected in the probabilities $\underline{p}^h$ believes that the investment plan should be chosen by:

$$\max_{[\xi(k;r_i), k\geq 0; i=1,\ldots,I]} \sum_{t=0}^{\infty} (\tfrac{1+n}{1+\rho})^t \underset{\tilde{r}(0),\ldots,\tilde{r}(t)}{E^h} V[\tilde{C}(t)]$$

where

$$\tilde{k}(t) = \tilde{k}(t-1) + \xi[\tilde{k}(t-1);\tilde{r}(t-1)] \quad , \quad t = 1,\ldots$$

$$\tilde{C}(t) = f[\tilde{k}(t);\tilde{r}(t)] - n\tilde{k}(t) - \xi[\tilde{k}(t);\tilde{r}(t)], \ t = 0,1,\ldots$$

$$k(0) = k_0 \ ,$$

and the expectation is relative to the probabilities $\underline{p}^h$. Clearly the solution depends on $\underline{p}^h$.

One further area of agreement deserves mention. As was pointed out above, the tax on interest income in the "no information" policies depends only on the allocation of consumption. To be precise, it is equal to the ratio of the marginal utilities of the young and the old. This ratio of marginal utilities is equal to $(1+\rho)$ in any plan that maximizes social welfare and also in any plan that is believed (perhaps mistakenly) to maximize social welfare. Moreover, this ratio is equal to $(1+\rho)$, and all individuals will agree that it is $(1+\rho)$, in any second-best solution where the path of investment is given exogenously and the allocation of aggregate consumption is given by (15). Therefore, in any first-best or second-best policy $[T^r(k;r) \equiv 1+\rho]$. That is, the optimal allocation of aggregate consumption requires a tax on interest income that stabilizes the after-tax rate of return on capital at ρ, the social rate of discount. This condition is independent of the probabilities governing $\tilde{r}$.

The "risk-pooling" aspect of these policies is illustrated in the following example. Assume that I = 2, and that the two production functions cross, as shown in Figure 1, when the capital/labor ratio is $\hat{k}$. Assume further that each factor is paid its marginal product. Now consider the situation when $k(t) = \hat{k}$. Since $k(t)$ is determined by decisions made through period t-1, its value is known at the end of period t-1. However, at that time the realization of $\tilde{r}$ for period t is unknown. Hence, at the end of period t-1 total output for period t is known with certainty, but wages and the return to capital are uncertain. Thus investment and

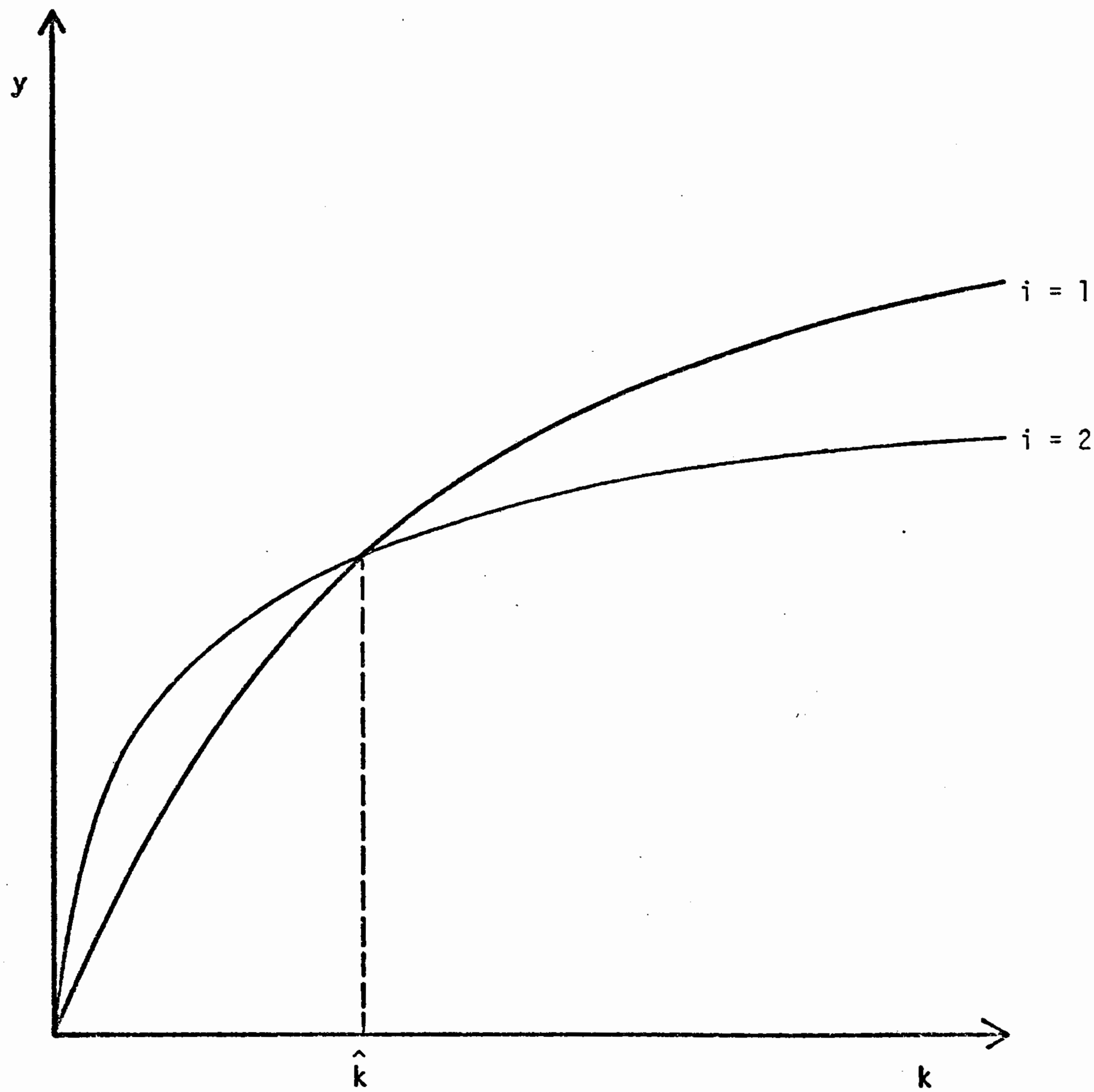

Figure 1

the levels of consumption of both young and old are uncertain. The policies considered here enable society to insure completely against this type of risk.

The welfare gains are clear. In fact, as has been shown (see Brock and Mirman [1972] and Mirman and Zilcha [1975]), if the random variable is i.i.d. all optimal plans have the property that consumption and investment depend only on the quantity of goods available (capital plus output), and not on the capital stock and the value of the random variable separately. Hence in this example the optimal plan provides complete insurance, and allocates the same quantities to investment and to consumption by each age cohort, for either realization of $\tilde{r}$ in period t.

5. Conclusion

The results above suggest that neither uncertainty in the production process nor diversity of individual information about that uncertainty is an insuperable barrier to decentralized policies for growth. Any feasible, equable plan can be implemented using a consumption tax, a tax on interest income, and age-dependent lump-sum subsidies. Regardless of the information structure, policies exist that implement the plan and require no information about the random variable or about individuals' (subjective) beliefs. All individuals, regardless of their information about the stochastic process (and regardless of their information about the beliefs of others) will agree about the objective consequences of these policies.

Furthermore, given any path for investment, all will agree on the optimal (utilitarian) allocation of aggregate consumption in each period. Any policy which is unanimously agreed to provide an optimal allocation of consumption contains a tax on interest income that stabilizes the after-tax rate of return on capital at $(1+\rho)$, the social rate of discount. Hence all individuals will agree on the optimal interest tax policy.

Diversity of information gives rise, however, to disagreement about the welfare implications of different investment plans. This, in turn, leads individuals with different information to disagree about the optimal schedules for the consumption tax and lump-sum subsidies. For example, Mirman and Zilcha [1975] have shown that under certain assumptions on the production function there is a unique optimal growth path (with aggregate consumption allocated by (15)). Thus, if information is incomplete and varies among individuals, those individuals will believe different paths to be (uniquely) optimal.

The set of instruments used above deserves some attention. As usual other instruments are equivalent to some of those employed here. For example, a tax on wealth could be used instead of the interest tax, and a tax on saving could be used instead of the consumption tax. A point that must be stressed, however, is that age-specific lump-sum subsidies are always needed. The tax on the young can, in this model, be replaced with a wage tax, since only the young have wage income and labor supply is completely inelastic. A wage tax could not be used if the model were extended to include many-period lifetimes or elastic labor supply.

The time structure of the model must also be mentioned. Changes in the wage rate and in the interest rate, as well as individual decisions about consumption, in fact occur at frequent intervals within each person's lifetime. Consider first changes in the interest rate. If it is admitted that the rate of return on capital fluctuates over short intervals, the gains from using tax policies to pool risk among age cohorts might seem to disappear. The average rate of return experienced by each individual over his lifetime would then be a function of many random variables, and hence subject to the law of large numbers. Although some residual gains from risk-pooling would remain, they could be expected to be small. There are two flaws in this reasoning. First, looking at a fairly short period, like a year, the rate of return on capital is not independently distributed over time; rather it displays a high degree of serial correlation. Thus a typical individual is subject to a large number of highly correlated, rather than uncorrelated risks. Furthermore, the "average" rate of return calculated over T periods is $\prod_{t=1}^{T} [1 + \tilde{r}(t)]$; this is different from receiving the average rate of return for each of the T periods, $[1 + \frac{1}{T} \sum_{t=1}^{T} \tilde{r}(t)]^T$. The former expression has a smaller expected value and a larger variance than the latter. Thus, admitting that the rate of return fluctuates frequently does not eliminate the role for risk-pooling.

Allowing more frequent consumption decisions involves more serious problems. Chamley [1977] has shown that even when there is no uncertainty, with many-period lifetimes allocations away from the steady state can be implemented only through age-specific lump-sum taxes. A second-best solution with only two tax schedules, applying to the working and to the retired populations, might then be worth exploring.

Still, it should be emphasized that the possibility of frequently revising consumption decisions does not eliminate the rationale for risk-pooling. First, although information about past and current wage rates and rates of return on capital can be used to revise consumption plans, uncertainty about future rates will not be eliminated entirely. Second, since individuals save more heavily during the period of time immediately preceding retirement (after raising a family, etc.), fluctuations in the interest rate that occur at the beginning of the working lifetime are of secondary importance. The rates of return occurring just before retirement will receive most of the weight in an individual's calculations. Third, and most important, to the extent that different age cohorts are subject to different real risks, what is required is a transfer of purchasing power between generations. Information can help an individual to plan, but does not relieve him of all the consequences of future events. A program of intergenerational transfers, which allows risks to be shared, can be beneficial even when those risks are accurately anticipated.

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