Discussion Paper No. 370

A CRITIQUE OF TIEBOUT'S THEORY OF LOCAL PUBLIC EXPENDITURES

by

Truman Bewley

Department of Economics
Northwestern University
Evanston, Illinois 60201

Revised April, 1980

1) Introduction

The goal of this paper is to point out that Tiebout's notion of equilibrium with local governments does not have the nice properties of general competitive equilibrium, except under very restrictive assumptions. Tiebout [39] suggested that there are competitive forces which tend to make local governments allocate resources in a Pareto optimal fashion. Consumers choose to live in those towns with the mix of taxes and public goods they prefer. Local governments choose this mix so as to attract inhabitants. This idea may seem intriguing, for it suggests that the invisible hand solves an important part of Samuelson's perplexing public goods problem [32]. Tiebout, in fact, makes an argument which is nearly rigorous. I give a rigorous version of his argument at the end of the paper. However in this rigorous version, so many restrictive assumptions are made that public goods become essentially private. In the body of the paper, I give a series of examples with which I try to convince the reader that one is forced to adopt Tiebout's restrictive assumptions. The idea is that if one changes any of his assumptions, then either equilibria may not exist or may not be Pareto optimal.

My examples are presented in the context of a general class of Tiebout models. I consider several subclasses, one of which is the special case considered by Tiebout. In each of the subclasses except that considered by Tiebout, I give a counterexample either to the existence of equilibrium or to its Pareto optimality. The subclasses are so chosen that the difficulties they reveal would be shared by any reasonable Tiebout model which differed from his special case. I believe that my examples controvert Tiebout's suggestion ([39] , last paragraph) that his theory compares favorably with competitive equilibrium theory.

Most of the examples in this paper have already appeared in the literature. I cite related work as I go along. What is new here is that I assemble the examples in a unified argument.

I wish to point out that no effort is made to make examples realistic. The examples are made as simple as possible so as to reveal logical difficulties. Tiebout's ideas may well apply in realistic special cases. If no more is true, then Tiebout's ideas lead not to a general theory but to a possibility which requires empirical verification.

2. Tiebout Models

I now describe informally the class of models which I use to formulate Tiebout's idea. Such models I call Tiebout models. They are all Arrow-Debreu general equilibrium models which in addition have distinct regions of habitation. Each region has a government which provides local public goods and collects local taxes to pay for them. There is perfect consumer mobility between regions. Consumers are fully informed about prices, taxes and public goods in each region and choose to live in that region where they can enjoy the highest level of utility. They do not believe that prices, taxes or the provision of public goods are influenced by their own choices.

There are discontinuities associated with migration, so that equilibria may not exist in Tiebout models with finitely many consumers. This technical problem may be overcome by assuming that consumers form a continuum. For this reason, whenever I give a counterexample to the existence of equilibrium, I assume that consumers do form a continuum.

I assume that governments cannot discriminate among consumers by name or according to taste when levying taxes. If governments could do so, they could charge Lindahl prices and the local public good problem would largely reduce to the question of how governments could determine these prices. (See Samuelson [32] and Foley [16]). I do allow governments to know consumer initial endowments and to base taxes on them. I do so in order to avoid any dead weight losses resulting from taxes. I wish to separate problems associated with tax distortion from those of local public goods.

Many of my counterexamples could be eliminated by assuming that all consumers have distinct initial endowments. One could argue that with probability one endowments would be distinct. However, if no two consumers had identical endowments, then governments could again collect Lindahl prices. Since I give governments the power to distinguish endowments, I feel free to specify any arrangement of endowments when I construct examples. If I assumed that endowments were random, then I would feel free to restrict governments' power of taxation. Otherwise, there would be no local public goods problem.

Taxes on initial endowments should be distinguished from income taxes or taxes on land. Income taxes may be avoided by not selling. Land taxes may be avoided by selling land. A tax on initial endowments cannot be avoided.

In order to give Tiebout's ideas the best chance possible, I assume that there are no spill-over effects between regions. The inhabitants of one region are affected by what happens in other regions only through prices and migration. Spill-over effects between regions have been discussed, for instance, in Pauly [28] and Williams [42].

The relation between the cost of public goods and the size of a region's population has a profound effect on the outcome of a Tiebout model. I consider two extreme cases, that in which the cost is independent of population and that in which it is proportional to population. The first case I call the case of pure public goods. The second I call the pure public service case. Occasionally, I discuss a third case often appearing in the literature. In this case, the *per capita* cost of public goods is a U-shaped function of population.

The role of regions in the production process is very important. I again consider two extreme cases. In one case, all production takes place inside regions and there is no trade between them. I call such regions autarkic. The other case is that of free trade. In this case, production is completely independent of the regional distribution of population.

I make a number of simplifying assumptions which deprive regions of almost all their natural characteristics as geographical areas. I assume that regions do not directly affect utility or production and that there are no commodities associated with a particular region. For instance, there is no land. Initial endowments are attached to their possessors and move with them from region to region. Initial endowments must be thought of as various forms of labor. Eliminating these assumptions would not help Tiebout's theory. It is much easier to construct counterexamples when regions are dissimilar and have their own resources. (See example I.5, in Appendix I.) If there are indivisibilities associated with land, especially serious problems arise. These problems occur when only one factory or consumer can occupy one location. Koopmans and Beckman [22] showed that in this case it may not be possible to characterize a Pareto optimum as a competitive equilibrium. Starrett [37] proved that an equilibrium may not exist.

I now give informal definitions of some basic concepts I use in discussing Tiebout equilibrium. An allocation in a Tiebout model specifies the following items: the consumption bundle of each consumer, the input-output vector of each firm, the bundle of public goods provided by each regional government, and the region of residence of each consumer. An allocation is feasible if the goods absorbed by consumers and governments may be produced or supplied directly from the consumers' initial endowments.

Pareto optimality is defined in the usual way. A feasible allocation is Pareto optimal if there exists no other feasible allocation which makes every consumer at least as well off and some better off.

I now specify the choices available to the actors in a Tiebout model. Firms choose their input-output vectors. Consumers choose their consumption bundles and regions of residence. Each government chooses the bundle of public goods or services that it provides. It also chooses a tax system. A tax system is a function from the commodity space to the real numbers. It specifies each inhabitant's tax payment as a function of his initial endowment. There is also an implicit assumption that governments can contact residents of other towns in order to convince them to move. This issue is discussed in section 4.

A Tiebout equilibrium consists of a feasible allocation, a price for each commodity, and a tax system for each region. These must satisfy the following conditions.

2.1) Each consumer's consumption bundle satisfies his budget constraint and there is no other bundle in his budget set which he prefers.

2.2) Each consumer inhabits the region he prefers. When a consumer compares regions, he assumes that tax systems, public expenditures and prices would not change if he moved.

2.3) Each firm maximizes profit.

2.4) Each regional government spends as much money on public goods as it receives in taxes.

2.5) Each government's expenditures, tax system and choice of inhabitants are consistent with whatever its objective may be .

The last condition requires that the motivation of the government be specified. A number of distinct motivations are conceivable. I divide these into two classes, the motivations of democratic governments and of entrepreneurial governments. Democratic governments seek to maximize the welfare of their own citizens. Entrepreneurial governments have objectives which are independent of the welfare of their citizens. Entrepreneurial governments may try to repel some inhabitants or attract new ones. The various forms of government are discussed in the next two sections.

In reading the examples that follow, it may help to keep in mind the tables given below. The tables represent the various cases I consider. The number in each box is the number of the example which eliminates that case. The box marked "Tiebout" corresponds to the case considered by Tiebout. There are other examples in the following sections and in Appendix I, which deal with issues of secondary importance. The most important of these is example 8.2, which shows that in Tiebout's case, one must have at least as many regions as types of consumers. Another important example is (6.1). This example eliminates one possible motivation one could give to regional governments in the case marked "Tiebout" in the diagram below.

Democratic governments

	Pure public good	Pure public service
Autarky	3.1	3.2
Free trade	3.1	3.2

Entrepreneurial governments

	Pure public good	Pure public service
Autarky	5.1	7.1
Free trade	5.1	Tiebout

3. Democratic Governments

In this section, I show that Tiebout equilibria may not be Pareto optimal if local governments are democratic in that they try to promote the welfare of their own citizens. I can be vague about how such governments should act, for in any examples it is obvious what the governments should do. I give two examples. The first deals with the case of pure public goods, and the second deals with the case of pure public services. Both examples are such that it makes no difference whether regions are autarkic or there is free trade.

3.1) Example (Pure Public Goods) There are two identical consumers and two regions. One type of public good may be provided. There is one private good, called labor or leisure. Each consumer is endowed with one unit of labor. The utility of a consumer if he inhabits either region is $u(\ell,g) = g$, where ℓ is the time he spends on leisure and g is the quantity of public good provided in the region he inhabits. Production possibilities are expressed by the equations, $g_j = L_j$, $j = 1, 2$, where g_j is the quantity of public good provided in region j and L_j is the quantity of labor devoted to the production of the public goods for region j.

The following situation describes an equilibrium. The price of labor and the price of the public good are both 1. The tax system in each region assigns a tax of 1 to any initial endowment, so that all consumer income is taxed away. One consumer lives in each region, each consumer sells all his labor to producers of public goods, and one unit of public good is provided in each region. This situation clearly satisfies conditions 2.1 - 2.4 for a Tiebout equilibrium. Also, each government is doing the best it can for its own citizen, so that 2.5 is also satisfied. Yet both consumers would be better off if they lived together in one region and if two units of public good were provided in that region and none in the other.

The problem brought out by this example has been much discussed in the literature. The problem is that consumers do not take into account the economies of scale which result when they move into a region. This problem has been discussed in the context of regions in which land is a factor or production. In this case, immigration simultaneously reduces the per capita cost of the public good and the per capita return from working the land. It may be advantageous for one region to subsidize another in order to inhibit emigration. This problem has been discussed by Buchanan and Wagner [9], Buchanan and Goetz [8], Flatters, Henderson and Mieszkowski [15], Stiglitz [38] and Starrett [36], pp. 20-21.

The problem brought out by example (3.1) could also arise if the per capita cost of public goods were less than proportional to regional population.

The problem in the next example is that consumers are mismatched and cannot sort themselves out by migration alone.

3.2) Example (Pure Public Service)

There are two identical regions, four consumers and four types of public service. Labor or leisure is the only private good and each consumer is endowed with one unit of labor. Regions are labeled by 1 and 2. Consumers and public services are labeled by A, B, C, and D. Production relations are expressed by the equations

$$n_j(g_{jA} + g_{jB} + g_{jC} + g_{jD}) = 2L_j,$$

for $j = 1,2$, where n_j is the number of consumers in region j, L_j is the quantity of labor used to produce public services for region j, and g_{jk} is the quantity of public service k provided in region j, for $j = 1,2$, and $k = A, B, C, D$.

The utility of consumer A when he inhabits region j is

$$u_A(\ell, g_{jA}, g_{jB}, g_{jC}, g_{jD}) = 2g_{jA} + g_{jB},$$

for $j = 1,2$, where ℓ is the quantity of leisure he consumes. Similarly, the utility of consumer B when he inhabits region j is

$$u_B(\ell, g_{jA}, g_{jB}, g_{jC}, g_{jD}) = g_{jA} + 2g_{jB}.$$

The utility of consumer C is

$$u_C(\ell, g_{jA}, g_{jB}, g_{jC}, g_{jD}) = 2g_{jC} + g_{jD}.$$

That of consumer D is

$$u_D(\ell, g_{jA}, g_{jB}, g_{jC}, g_{jD}) = g_{jC} + 2g_{jD}.$$

Thus, consumers A and B are natural partners, as are consumers C and D.

The following situation describes a Tiebout equilibrium in the above example. Consumers A and C live in region 1 and consumers B and D live in region 2. $(g_{1A}, g_{1B}, g_{1C}, g_{1D}) = (1,0,1,0)$ and $(g_{2A}, g_{2B}, g_{2C}, g_{2D}) = (0,1,0,1)$. The price of labor and the price of each type of public service are 1 in both regions. The tax system in each region assigns a tax of one to any endowment, so that all consumer income is taxed away. This is certainly an equilibrium if the regions are democratic. Consumers assume that if they move, the allocation of public services will remain unchanged. Hence, each consumer believes that moving will reduce his utility from 2 to 1.

However, the above situation is not Pareto optimal. Suppose that consumers B and C switched regions, so that A and B were together in region 1 and C and D lived in region 2. Suppose also that $(g_{1A}, g_{1B}, g_{1C}, g_{1D}) = (1,1,0,0)$ and that $(g_{2A}, g_{2B}, g_{2C}, g_{2D}) = (0,0,1,1)$. Then, each consumer would receive three units of utility, which is more than in the previous equilibrium.

This example is reminiscent of Samuelson's marriage example ([33], appendix), though he made a quite different point.

Brueckner [6] shows that Tiebout equilibria may not be Pareto optimal if local communities are governed by majority rule. In his examples, regions are autarkic and public goods are neither pure public goods nor pure public services. Roughly speaking, the _per capita_ cost of public goods is U-shaped as a function of population.

The problems appearing in examples (3.1) and (3.2) would not arise if governments initiated changes in the supply of public goods in anticipation of the migration that would ensure. Such governments are discussed in the next section.

4. Entrepreneurial Governments

It appears that Tiebout's theory requires that governments initiate changes in policy which break up economic states which are not Pareto optimal. Such governments must be given a reasonable goal.

Tiebout [39] was not very helpful on the subject of local government motivation. He assumed that regions competed for inhabitants in order to reach an optimal size. The optimal size is determined by U-shaped cost functions for public goods. If regions already had the optimal number of inhabitants, then their governments would have no reason to change policy. Hence, one can easily construct an example for this case of an equilibrium which is not Pareto optimal. It is sufficient to imitate example (3.2). In order to overcome this problem, one could assume that empty regions were always available and eager to have inhabitants. But this approach seems too far-fetched.

One can overcome this problem by assuming that regional governments want to maximize the population of the region they control. This is one assumption I experiment with. One can motivate it by assuming that each government is controlled by landlords, who direct it to maximize the value of their land. Property value maximization has appeared in the literature, as in Pauly [29] and Sonstelie and Portney [34].

The idea of landlord control is not without problems. For instance, one is obliged to assume that all landlords in a region have identical interests. Also, one must assume that landlords do not benefit directly from public goods. All these objections go away if one simply assumes that local governments want to maximize population.

Another way to give life to local governments is to assume that they maximize profits. Their revenues come from taxes and their expenses are the cost of public goods. This is certainly a bizarre assumption. It seems most appropriate for a model of feudal society in which governments are owned by individuals. However, this idea is consistent with Tiebout's attempt to create a theory of local public goods analogous to general equilibrium theory. Profit maximizing local governments have already appeared in the literature, as in Margolis ([23], p. 549).

Still another assumption would be that policy changes are made by discontented consumers from various regions who move into a region and take control of its government. This idea leads to the same definition of equilibrium as does profit maximization, as will become clear.

I ignore several difficulties in the interpretation of policy changes. For instance, it may be necessary to exclude certain consumers from a region in order to make a new policy worthwhile. Such exclusion is in conflict with the basic assumption that consumers are free to move where they like. One could avoid this difficulty by assuming that there are always empty regions in reserve, in which to organize new governments. However, this seems silly.

Other problems relate to the fact that governments must calculate the effect of policy changes. Part of this ambiguity is removed by assuming that each government believes that no other government will change policy when it changes its own. But nevertheless, in order that a policy change succeed, it might be necessary for consumers to immigrate from all over the economy. How would the regional government know that these consumers existed? Also, in the autarkic case, changes in policy lead to changes in prices in the region. If competitive equilibrium in the region is not unique, how could the regional government predict what the new prices would be? This problem is illustrated by example (I.2) in Appendix I.

I ignore all these difficulties in order to give the Tiebout model the best chance possible.

It must be assumed that profit maximizing local governments are perfect competitors. Otherwise, monopoly power on the part of local governments could lead to equilibria which would not be Pareto optimal. Example (I.1) in Appendix I illustrates this point. The obvious way to formulate the condition that regions be perfect competitors is to require that they make zero profit. This is condition (2.4). Since governments make no profits, it is not necessary to specify the distribution of profits to consumers.

I label as <u>entrepreneurial</u> those governments which can initiate policy changes. A change of policy accompanied by a migration of consumers is called a change of <u>regime</u>.

I now give informal definitions of equilibrium when governments are entrepreneurial. The definitions depend on whether regions are autarkic or trade freely and on whether governments maximize profit or population. This makes four cases in all. In each case, a Tiebout equilibrium is defined as in section 2. However, I must now describe condition 2.5 more precisely. This condition simply asserts that governments act in their own interest. The precise forms of this condition are related to the notion of blocking in the theory of the core. Roughly speaking, condition 2.5 is satisfied if no regime can block.

Before dealing with condition 2.5, I need some terminology. An <u>allocation for an autarkic region</u> specifies the following: the inhabitants of the region, the consumption bundle of each inhabitant, the input-output vector of each firm in the region, and the bundle of public goods provided by the regional government. The allocation is <u>feasible</u> if the goods absorbed by the consumers and government of the region may be provided by the firms or inhabitants of the region using the inhabitants' initial endowments.

A <u>regime for an autarkic region</u> consists of a feasible allocation for the region, a price for each commodity traded in the region, and a regional tax system. These must satisfy the analogues of conditions 2.1, 2.3 and 2.4 which apply to a single region. Notice that a regime for an autarkic region includes prices. I include prices because it seems natural to suppose that the government of an autarkic region would realize that changes in policy would affect prices.

If there were free trade, then governments should not believe that they could affect prices. For in this case, governments would buy goods on economy-wide markets, and migration would not affect production. So, assume that there is free trade and that prices are given and fixed. An allocation for a region specifies the inhabitants of the region as well as the consumption bundle of each inhabitant and the bundle of public goods provided by the regional government. A <u>regime</u> for a region consists of an allocation for the region and a regional tax system. These must satisfy 2.1, 2.3 and 2.4. Notice that the allocation does not specify an input-output vector and need not be feasible.

I now turn to the notion of blocking. I say that a regime for a given region blocks an allocation for the whole economy if (1) the regime assigns at least one consumer to the region and (2) every consumer in the region is better off than he was under the original allocation. I say that a regime for a given region strictly blocks an allocation for the whole economy if (1) the regime blocks the allocation and (2) the population of the region under the regime is at least as great as it was under the original allocation. These definitions apply whether regimes are autarkic or trade freely.

I now make condition 2.5 more precise. If governments are profit maximizing then the condition becomes the following.

4.1) (Profit maximizing governments) The equilibrium allocation cannot be blocked by any regional regime.

It is easy to see why this condition must apply if governments maximize profits. Recall that in equilibrium, governments make zero profit. This is condition 2.4 and reflects the idea that governments are perfect competitors. If condition 4.1 were not satisfied, then the government of a blocking regime could make a profit by establishing the new regime and then raising taxes slightly. The new inhabitants of the region would not object to the tax increase, for they would be better off than before.

I now turn to the case of population maximizing governments. In this case, condition 2.5 becomes the following.

4.2) (Population maximizing governments) The equilibrium allocation cannot be strictly blocked by any regional regime.

The motivation for this definition should be clear.

Observe that an equilibrium with profit maximizing governments is also an equilibrium when governments maximize population.

5. Pure Public Goods

In this section, I show that in the case of pure public goods, a Tiebout equilibrium may not be Pareto optimal, even when local governments are entrepreneurial. The counterexamples are such that if regions were autarkic, relative prices would be the same in all regions. Hence, the examples apply to the case of free trade as well as to that of local production. I consider only the case of equilibrium with profit-maximizing governments, since such equilibria are also equilibria when governments maximize population.

5.1) Example

There are two regions and two consumers, one private good, called labor or leisure, and one public good. Each consumer has an initial endowment of one unit of labor. Production possibilities are expressed by the equation $g = L$, where g is the quantity of public good produced and L is the quantity of labor used. The utility function of consumer 1 is $u_1(\ell,g) = g$. Here, ℓ is his consumption of leisure and g is the provision of public good in the region where he lives. The utility function of consumer 2 is $u_2(\ell,g) = 3\ell + g$.

The following situation is an equilibrium in this example. Consumer i lives in region i, for $i = 1,2$. In region 1, consumer 1 devotes all his labor to the production of the public good and so consumes no leisure and one unit of public good. Consumer 2 devotes no labor to the production of the public good and consumes one unit of leisure and none of the public good. In each region, the price of each good is 1. The tax system in region 1 requires that all consumers pay a tax of 1. The tax system in region 2 requires that no consumer pay any tax. Observe that the utility level of consumer 1 is 1 and that of consumer 2 is 3.

This situation is clearly a Tiebout equilibrium. Each consumer is doing the best he can alone. Suppose that a new regional regime were to be established involving both consumers. They would have to pay the same taxes, since their initial endowments would be the same. Let the tax be τ units of labor, where $0 \leq \tau \leq 1$. Then, 2τ units of public good would be provided. The utility of consumer 1 would be 2τ, so that he would prefer to live alone if τ were less than 1/2. The utility of consumer 2 would be $3(1-\tau) + 2\tau = 3 - \tau$. Hence, consumer 2 would prefer to live alone if τ exceeded zero. Hence, the consumers can be in equilibrium only if they live separately.

Now suppose that the consumers lived together and that consumer 1 paid all his income in taxes and consumer 2 paid none. Then, one unit of public good would be produced, consumer 1 would consume no leisure and consumer 2 would consume one unit of leisure. The utility of consumer 1 would be 1, just as in equilibrium, but that of consumer 2 would be 4, which is one unit more than in equilibrium. Hence, the equilibrium is not Pareto optimal.

The problem in the above example is that it is impossible to tax the consumers at different rates. Of course, consumer 1 would want to pay taxes in this example. But if there were one million consumers of each type living in one region and if a tax were imposed only on those consumers of type 1, then each of them would have an incentive to pretend that he was of type 2.

One could get around the difficulty in the example by imposing a tax on the sale of labor, since consumer 2 does not sell his labor in the Pareto optimum, and so could escape this tax. However, it is easy to modify the example so as to block this way out. One simply splits the private good into two. The initial endowments and utility functions of the modified example are given in the following table.

$$
\begin{array}{ll}
\omega_1 = (1,0) & u_1(x_1,x_2,g) = g \\
\omega_2 = (0,1) & u_2(x_1,x_2,g) = g \\
\omega_3 = (1,0) & u_3(x_1,x_2,g) = 3x_2 + g \\
\omega_4 = (0,1) & u_4(x_1,x_2,g) = 3x_1 + g
\end{array}
$$

Here, ω_1 is the initial endowment vector of consumer i, and u_i is his utility function, for all i. x_k is the quantity of private good k consumed, and g is the quantity of public good provided. Production is expressed by the equation $g = \min(y_1,y_2)$ where y_k is the input of private good k. There are two regions.

The difficulty appearing in example 5.1 appears in this example as well, and all consumers must trade in order to enjoy positive utility. Hence, a tax on sales would not discriminate against those who prefer the public good.

It should be observed that the difficulty occurring in example (5.1) could arise even if the cost of the public good were less than proportionate to the population served. One may see that this is so simply by making slight modifications in the example.

Johnson ([21], pp. 49-51), gives an example of a pure public goods Tiebout economy, which has an inefficient equilibrium. There are a number of differences between his example and mine. He uses a version of Lindahl equilibrium in his definition of equilibrium and he allows trade between regions.

Stiglitz ([38], pp. 294-5) proved that an equilibrium may not exist in a model with pure public goods, autarkic regions, and entrepreneurial governments. I have not used his example, for he associates a factor of production, namely land, with regions. The gist of his example is contained in example I.3 in the appendix. I do not know whether equilibrium generally exists in the cases dealt with in this section.

Guesnerie and Oddou [18] analyze in detail a model with pure public goods, autarkic regions and entrepreneurial governments. They use a game theoretic approach.

6. Public Service and Population Maximizing Governments

The example of this section demonstrates that in the pure public service case, Tiebout equilibria with population maximizing regions may not be Pareto optimal. The example is so arranged that it applies to both the case of free trade and autarky.

6.1) Example

There are two regions, four consumers, one public service and one private good, called labor or leisure. Each consumer is endowed with one unit of labor. The production function of the public service is $n_j g_j = L_j$, where g_j is the level of public service in region j, n_j is the number of people there, and L_j is the quantity of labor devoted to the production of the public service for region j. The utility function of consumer 1 is $u_1(\ell, g) = g$, where ℓ is the quantity of leisure consumed and g is the level of public service in his region. The utility function of the other consumers is $u_i(\ell, g) = \ell$, $i = 2,3,4$.

The following situation describes an equilibrium with population maximizing regions. Consumers 1 and 2 live in region 1 and consumers 3 and 4 live in region 2. In both regions, there are no taxes and no public service. This arrangement clearly satisfies conditions (2.1)-(2.4) and (4.2). However, it is not Pareto optimal. If consumer 2 were to move to region 2, he would not lose. But then, consumer 1 could produce one unit of public service and gain one unit of utility.

7) Public Service and Autarkic Governments

The next example shows that in the case of pure public services, a Tiebout equilibrium may not exist if regions are autarkic and local governments are entrepreneurial. I consider only the case of population maximizing governments, since an equilibrium with profit maximizing governments is also one when governments maximize population.

7.1) Example

There are two types of consumers, two regions, two private goods and one public service. A consumer of type i is endowed with one unit of private good i, for $i = 1,2$. The utility function of any consumer is $u(x_1,x_2,g) = g$, where x_k is the quantity of private good k consumed and g is the quantity of public good provided in the region where he lives.

Consumers form a continuum, and two thirds of the consumers are of type 1. One third are of type 2.

Production possibilities in one region are determined by the equation $rg = \sqrt{y_1 y_2}$, where r is the proportion of consumers living in the region and g is the quantity of public good provided. y_k is the input of private good k, for $k = 1,2$.

I now show that the above example has no equilibrium when governments maximize population. Such an equilibrium must satisfy the analogues of conditions (2.1) - (2.4) and (4.2) which apply to an economy with a continuum of consumers.

Suppose the example had an equilibrium. Let r_{ij} be the proportion of consumers of type i in region j and let g_j be the level of public service in region j.

If each region contains consumers of only one type, then none of the public service can be produced in either region and the utility level of every consumer is zero. But then, all consumers could form a new regime in one region and enjoy positive utility. This would contradict (4.2).

Hence, at least one region, say region 1, contains consumers of both types. Consumers derive no utility from consuming private goods. Hence, by condition (4.2) all wealth in both regions is taxed away and used to produce the public good. Hence, $g_1 = \sqrt{r_{11}r_{21}}\ (r_{11}+r_{21})^{-1}$. Also, if some consumers live in region 2, then $g_2 = \sqrt{r_{12}r_{22}}\ (r_{12} + r_{22})^{-1}$. Since consumers in region j enjoy utility level g_j and since consumers are free to migrate, it follows that either $g_1 = g_2$, or no consumers live in region 2. This implies that either $r_{11}r_{21}^{-1} = r_{12}r_{22}^{-1} = (r_{11} + r_{12})(r_{21} + r_{22})^{-1} = 2$ or $r_{11} = 2/3$, $r_{21} = 1/3$ and $r_{12} = r_{22} = 0$. In either case, the level of utility enjoyed by all consumers is $\sqrt{2}/3$. One region, say region 2, contains no more than half of all consumers. A new regime may be established in region 2 with all the consumers of type 2 and half of those of type 1. The level of public service would be $\sqrt{(1/3)(1/3)}\ /\ (2/3) = 1/2 > \sqrt{2}/3$. Region 2 would have 2/3 of all consumers, which would be more than before. This contradicts condition (4.2). Hence, there exists no equilibrium.

It might be thought that the problems encountered in the above example arose only because local governments knew that their policies affected prices. However, even if governments ignored their influence on prices, equilibria still might not exist. Example I.5 in the appendix demonstrates this point. One should realize that if governments do not take price changes into account, then they may undertake changes of regime which are impossible. For this reason, it does not seem reasonable to assume that governments of autarkic regions believe that prices are fixed.

An example very similar to (7.1) was discussed by Berglas [3]. However, he did not point out that it was a counterexample to the existence of equilibrium. Rather, he used the example to argue that Tiebout's analysis can be extended so as to include heterogenous communities. Tiebout's analysis leads to the conclusion that consumers will sort themselves out into homogeneous communities, each containing one type of consumer. One of the points I wish to make is precisely that one cannot extend Tiebout's analysis to include heterogeneous communities.

Ellickson [13] produced an example of a Tiebout model with autarkic regions and entrepreneurial governments and with no equilibrium. There are a number of differences between his example and example (7.1) above. He has three consumers, and here consumers form a continuum. His costs of production are such that the public good is neither a pure public good nor a pure public service. He allows utility to be transferable and I do not.

Tiebout models with local production and entrepreneurial local governments have often appeared in the literature. I have already mentioned Berglas [3], Ellickson [13] and Stiglitz [38]. Additional works include Ellickson [14], McGuire [24], Pauly [26],[27], Rothenberg [30] and Sorenson, Tschirhart and Whinston [35].

8) Free Trade

In this section, I endeavor to convince the reader that in the free trade case one is almost obliged to confine attention to a particularly simple model in which regional governments provide pure public services and there are at least as many regions as types of consumers.

I first defend the assumption that public goods are pure public services. The case of pure public goods was taken up in section 5. A common assumption in the literature is that the *per capita* cost of public services is a U-shaped function of the number served, an idea inherited from Tiebout. I now argue that this assumption leads to difficulties which may be overcome only by making very strong hypotheses. The following example illustrates the difficulties which arise.

8.1) Example

There are two regions, three consumers, one public good and one private good, called labor or leisure. Each consumer is endowed with one unit of labor and his utility function is $u(\ell,g) = g$. Here, ℓ is the quantity of leisure consumed and g is the quantity of public good offered in his region. Let $c(n)$ be defined by $c(2) = -1$ and $c(n) = 0$, otherwise. The production constraint is $(2n_1 + c(n_1))g_1 + (2n_2 + c(n_2))g_2 = L$, where, for $j = 1,2$, n_j is the number of consumers living in region j, g_j is the level of public service there, and L is the quantity of labor used.

Observe that the labor cost of public goods for region j is

$(2n_j + c(n_j))g_j$. If each region spends all of its n_j units of labor on the public good, then $n_j = (2n_j + c(n_j))g_j$, for $j = 1,2$.

This example has no equilibrium, even when governments maximize population. Suppose that two consumers lived in region 1 and one in region 2. Then, $g_1 = 2/3$ and $g_2 = 1/2$, so that the consumer in region 2 would enjoy less utility and would migrate to region 1. On the other hand, if all three consumers lived together in region j, then $g_j = 1/2$, so that any two consumers would have an incentive to form a separate regional regime.

Clearly, a similar example could be made with any number of consumers.

U-shaped cost functions for public goods have been assumed by Tiebout [39], Buchanan [7], Jerome Rothenberg [30], McGuire [24], Sonstelie and Portney [34], Berglas [3], Sorenson, Tschirhart, and Whinston [35], and Wooders [44]. Pauly [27] pointed out the difficulty illustrated by example (8.1).

The only way to overcome the problem brought out by example (8.1) is to assume that there are so many consumers that after giving each region its optimal number of people, there would be relatively few people left over. In fact, as will become clear in the next section, there would have to be many consumers of each type and many regions for each type of consumer. These assumptions are much stronger than the corresponding assumption I make in the next section, which is simply that there are at least as many regions as types of consumers. I can make this weaker assumption because I assume that the public good is a pure public service. It therefore seems more reasonable to replace Tiebout's assumption of U-shaped costs by the assumption that public services are pure. In fact, empirical evidence seems to indicate that the *per capita* cost of public services is not U-shaped at all, but is indeed flat. (See Borcherding and Deacon [5] and Bergstrom and Goodman [4].)

The next example shows that if public services are pure, then there may exist no equilibrium if there are fewer regions then types of consumers.

8.2) Example

There are two regions and three types of consumers. The consumers form a continuum and one-third of the consumers are of each type. There is one public service and one private good, called labor or leisure. Each consumer is endowed with one unit of labor. The production function of the public good is $r_1g_1 + r_2g_2 = L$, where g_j is the level of public service in region j, r_j is the proportion of the population living in region j, and L is the quantity of labor devoted to production. The utility function of consumers of type i is $u_i(\ell,g) = \ell^{3-i}g^{i-1}$, for $i = 1,2,3$, where ℓ is the quantity of leisure consumed and g is the level of public service provided.

This economy has no equilibrium if governments maximize profits. Since all consumers have the same initial endowment, all those in one region must pay the same taxes. If all three types lived together and none of the public good were provided, then those of types 2 and 3 would prefer to live by themselves. If some of the public good were provided, then those of type 1 would prefer to live by themselves. A similar story applies of consumers of type 1 live with consumers of either type 2 or 3. If consumers of types 2 and 3 lived together without anyone of type 1, then those of type 2 would prefer to live by themselves if g exceeded 1/2, and those of type 3 would prefer to live alone if g were less than 1. Hence, they could not live together and be in equilibrium. But it is impossible for all three types to live separately, since there are only two regions. Hence, there is no equilibrium.

Clearly, an example similar to the above can be made with any number of types of consumers, provided that the number of types exceeds the number of regions.

The point made by the above example has been made in a somewhat different context by Goldstein and Pauly [17],pp. 83-88 and by Pauly [29], pp. 240-1.

Westhoff [40] studies a Tiebout model with a continuum of consumers and finitely many regions. His definition of equilibrium is quite different from my own in that he requires that regional government policy be determined by the votes of residents. He avoids social choice ambiguities by assuming that there is only one public good and one private good, so that the choice is between more or less public good. A variant of the single-peakedness assumption guarantees the existence of equilibrium. He gives a counterexample to the existence of equilibrium if this assumption is not satisfied.

Thus far, I have assumed that consumers' utility functions do not depend on the region inhabited. I now point out a good reason for making this assumption. If the utility functions of consumers did depend on the region inhabited, then there might be no equilibrium even if there were more regions than types of consumers. In order to see that this is so, modify example (8.2) by introducing two new regions. Let consumers' utility functions be as in (8.2) if they inhabit regions 1 or 2. If a consumer inhabits regions 3 or 4, let his utility function be $u_i(x,g) - 2$, for all i, where u_i is as in example (8.2). Then, no one would want to inhabit regions 3 or 4, so that there still exists no equilibrium. It seems that if utility functions did depend on the region of habitation, then one would have to make very elaborate assumptions indeed in order to guarantee the existence of equilibrium.

One might think that the division of consumers into types need be made only according to preferences for public goods. But it is easy to see by means of examples that the division must be according to initial endowments and preferences over private goods as well.

If it is assumed that the cost of public goods depends on the number of people served, it is also natural to assume that it depends on the type of people served. For instance, education costs more per family in communities of young families than in communities of old people. However, in cases like this, one meets the sort of problems pointed out by Rothschild and Stiglitz [31] in the context of adverse selection in insurance markets. The only way to overcome these problems is to allow direct control of the nature of the inhabitants living in a community, which would be contrary to the spirit of the Tiebout model.

The following example illustrates the problem

8.3) Example

There are two regions, one public good and one private good, called labor or leisure. There are two types of consumers. Consumers form a continuum, and one half the consumers are of each type. Each consumer has an initial endowment of one unit of labor. The production function of the public good is $(r_{11} + 2r_{21})g_1 + (r_{12} + 2r_{22})g_2 = L$, where, for all i and j, r_{ij} is the proportion of consumers who are of type i and inhabit region j, and g_j is the level of public service in region j. L is the quantity of labor used. The utility function of any consumer of either type is $u(\ell,g) = g$, where ℓ is the quantity of leisure consumer and g is the level of public service.

This example has no equilibrium, even if governments maximize population. Clearly, all labor is devoted to the production of public goods. By the free migration condition (2.2), all consumers of the same type enjoy the same utility, no matter where they live. Hence, in each region half the consumers are of each type, so that each consumer enjoys 2/3 units of utility. One of the regions has at most half the consumers. If this region took all the consumers of type 1 and one half of those of type 2, it would have 3/4 of all consumers, and each of them would enjoy 3/4 units of utility, contradicting condition (4.2). Hence, there exists no equilibrium.

If in this example entrepreneurial regions were allowed to vary their offer of public goods, but not to exclude people from a region, then the situation would be the same as in Rothschild and Stiglitz [31].

In order to obtain a counterexample to the existence of equilibrium in this case, let the utility function of consumers of type 1 be $u_1(\ell,g) = \ell g$ and let that of consumers of type 2 be $u_2(\ell,g) = \ell g^2$ Let the proportion α of consumers be of type 1, where $0 < \alpha < 1$. Now the situation is formally the same as in the Rothschild-Stiglitz paper. A fully rigorous analysis of the model is contained in Charles Wilson [43].

Goldstein and Pauly have pointed out that the Rothschild-Stiglitz problem arises in the local public goods context. (See Goldstein and Pauly [17], pp. 78 and 79 and Pauly [29], p. 238.) Rothschild and Stiglitz also mention that their ideas could find application in the theory of local goods [31], p. 648.. Johnson [21], pp. 67-68, gives an example somewhat related to the Rothschild-Stiglitz problem. In Johnson's model, the poor chase the rich from region to region. The cost of production of public goods is U-shaped, and his definition of equilibrium involves the concept of Lindahl equilibrium.

9) Tiebout's Theory

I hope I have convinced the reader that there is only one situation in which Tiebout's theory really makes sense. That is in the free trade, pure public service case, and when there are at least as many regions as types of consumers. In this section, I give Tiebout's theory rigorous form in this special case.

I assume that governments maximize profits. Recall that example (6.1) shows that if governments maximize population instead of profits, then Tiebout equilibria may not be Pareto optimal, even if there is free trade and governments offer pure public services. Example 6.1 also satisfies a key assumption of this section, which is that there are as many regions as types of consumers.

Prices vary over $\Delta^{L+N-1} = \{p \in R_+^{L+N} \mid \sum_k p_k = 1\}$. Vectors in Δ^{L+N-1} are denoted by (p,q), where the components of p refer to private goods and those of q to public goods.

A tax system for region j is a function $\tau_j = R_+^L \to [0,\infty)$. The budget set of consumer of type T if he lives in region J is $\beta_t(p,\tau_j) = \{x \in R_+^L \mid p \cdot x \leq p \cdot \omega_t - \tau_j(\omega_t)\}$, where $p \in R_+^L$ is the vector of private good prices.

I assume that there are L private goods, N types of public service and J regions, where L, N and J are positive integers. Also, there are T types of consumers, where T is a positive integer. $u_t(x,y)$ denotes the utility function of a consumer of type t, where x is his bundle of private goods and g is the bundle of public services provided in the region he inhabits. ω_t denotes the initial endowment vector of a consumer of type t. I assume that consumers are endowed only with private goods.

R^n denotes n-dimensional Euclidean space and $R^n_+ = \{x \in R^n \mid x_k \geqq 0,$ for all $k\}$. Thus, bundles of private goods belong to R^L_+ and bundles of public services belong to R^N_+.

For simplicity, I assume that production exhibits constant returns to scale. I let production possibilities be described by a single social production possibility set, Y. Since there are no profits in equilibrium, it is not necessary to specify the distribution of profits.

There are I_t consumers of type t, for $t = 1,\ldots,T$. Consumers are indexed by (t,i), where t indicates the type and i distinguishes consumers of the same type. $\mathcal{J} = \{(t,i) : i=1,\ldots,I_t,\ t = 1,\ldots,T\}$ denotes the set of all consumers.

An <u>allocation</u> for the economy is of the form $(r, \vec{x}, \vec{g}, y)$. $r : \mathcal{J} \to \{1,\ldots,J\}$ is a function which assigns consumers to their regions of residence. $\vec{x} = (x_{(t,i)})_{(t,i) \in \mathcal{J}}$ is the vector which describes the allocation of private goods, where $x_{(t,i)} \in R^L_+$, for all (t,i). $\vec{g} = (g_1,\ldots,g_J)$ describes the allocation of public services. $g_j \in R^N_+$ is the allocation to region j. $y \in Y$ is the social production vector.

Given a residence function, $r: \mathcal{J} \to \{1,\ldots,J\}$, n^r_j denotes the number of consumers assigned to region j by r. That is,

$n^r_j = |\{(t,i) : r(t,i) = j\}|$, where $|\cdot|$ denotes cardinality.

An allocation (r, x, g, y) is <u>feasible</u> if

$$\left(\sum_{(t,i) \in \mathcal{J}} (x_{(t,i)} - \omega_t),\ \sum_{j=1}^{J} n^r_j\, g_j\right) = y.$$

A Tiebout equilibrium consists of an allocation $(r, \vec{x}, \vec{g}, y)$, a price system $(p,q) \in \Delta^{L+N-1}$ and tax systems $(\tau_1,\ldots,\tau_J)$. These must satisfy the following conditions.

9.1) $(r, \vec{x}, \vec{g}, y)$ is feasible.

9.2) (Profit maximization) $(p,q)\cdot y = \max\{(p,q)\cdot z \mid z \in Y\}$

9.3) (Utility maximization and free mobility)

For all $(t,i) \in \mathcal{I}$, $u_t(x_{(t,i)}, g_{r(t,i)})$

$$= \max \{u_t(x,g_j) \mid x \in \beta_t(p,\tau_j),\ j=1,\ldots,J\}$$

and $x_{(t,i)} \in \beta_t(p,\tau_{r(t,i)})$, for all $(t,i) \in \mathcal{I}$.

9.4) (Perfectly competitive governments)

$$n_j^r\, q\cdot g_j = \sum_{(t,i)\,:\,r(t,i)=j} \tau_j(\omega_t).$$

9.5) (Government profit maximization)

The allocation $(r, \vec{x}, \vec{g}, y)$ cannot be blocked by an regional regime.

Blocking was defined informally in section 4. I now define it more precisely. An allocation for a region is of the form $(A, \vec{x}, g)$, where A is a non-empty subset of $\mathcal{I}$, $\vec{x} = (x_{(t,i)})_{(t,i) \in A}$ is a consumption allocation for the consumers in A, and $g \in R_+^N$ is the bundle of public services provided by the regional government. Let a price system $(p,q) \in \Delta^{L+N-1}$ be given and fixed. A regime for a region consists of an allocation for the region, $(A, \vec{x}, g)$, and a tax system $\tau : R_+^L \to [0,\infty)$. These must satisfy the following two conditions.

$$u_t(x_{(t,i)},g) = \max\{u_t(x,g) \mid x \in \beta_t(p,\tau)\} \text{ and } x_{(t,i)} \in \beta_t\ (p,\tau)\},$$

$$\text{for all } (t,i) \in A.$$

$$|A|\ q\cdot g = \sum_{(t,i)\in A} \tau(\omega_t),$$

where A denotes the cardinality of A.

Let an allocation $(r, \vec{x}, \vec{g}, y)$ for the economy be given. A regime $(A, \underline{x}, \underline{g})$ for some region blocks $(r, \vec{x}, \vec{g}, y)$ if

$$u_t(\underline{x}_{(t,i)}, \underline{g}) > u_t(x_{(t,i)}, g_{r(t,i)}), \quad \text{for all } (t,i) \in A$$

I make the following assumptions.

9.6) $T \leqq J$.

That is, there are at least as many regions as types of consumers.

9.7) $u_t : R_+^{L+N} \to R$ is continuous, strictly quasi-concave and strictly monotone, for all t.

$u : R_+^n \to R$ is said to be strictly quasi-convex if $u(x) \geqq u(y)$ implies

$u(\alpha x + (1-\alpha)y) > u(y)$, for all α such that $0 < \alpha < 1$.

u is strictly monotone if $x \geqq 0$ and $x \neq 0$ imply that $u(x+y) > u(y)$, for all y.

9.8) For all t, $\omega_t \in R^L_+$ and $\omega_t \neq 0$.

9.9) $Y \subset R^{L+N}$ and Y is a closed convex cone with apex zero.

9.10) $Y \cap R^{L+N}_+ = \{0\}$.

9.11) There exists $\bar{y} \in Y$ such that every component of $\bar{y} + (\sum_{i=1}^{I} \omega_i, 0)$ is positive.

I now prove the following theorems.

9.12) Theorem (Existence) If assumptions (9.6) - (9.11) are satisfied, then there exists a Tiebout equilibrium satisfying conditions(9.1 - 9.5).

9.13) Theorem (Pareto optimality) If assumptions (9.6) - (9.11) are satisfied, then every Tiebout equilibrium allocation is Pareto optimal.

Proof of theorem 9.12

I prove this theorem by defining a corresponding general equilibrium economy with no regions or public services. I apply a standard existence theorem to show that the new economy has a competitive equilibrium. I then show that a Tiebout equilibrium corresponds to this competitive equilibrium. The standard equilibrium existence theorem I apply is stated in Appendix II.

The new economy is defined as follows. The set of consumers is $\mathcal{I}$. I treat all commodities as private goods. The utility function of consumer $(t,i) \in \mathcal{I}$ is $u_t\colon R_+^{L+N} \to R$. His initial endowment is $(\omega_{(t,i)}, 0) \in R_+^{L+N}$. The production set is $Y \subset R^{L+N}$. Assumptions (9.6) - (9.11) imply that the assumptions of the theorem in Appendix II apply. Hence by this theorem, the new economy has a competitive equilibrium. Let $(p,q) \in \Delta^{L+N-1}$ be the price system of this equilibrium and let $(((x_{(t,i)}, g_{(t,i)}))_{(t,i) \in \mathcal{I}}, y)$ be the allocation, where $(x_{(t,i)}, g_{(t,i)}) \in R_+^{L+N}$, for all $(t,i) \in \mathcal{I}$, and where $y \in Y$. These satisfy the following conditions:

9.14) $$\sum_{(t,i)\in\mathcal{I}} (x_{(t,i)} - \omega_t, g_{t,i}) = y,$$

9.15) $$(p,q) \cdot y = \max \{(p,q) \cdot z \mid z \in Y\}, \text{ and}$$

9.16) for all (t,i), $(x_{(t,i)}, g_{(t,i)})$ solves the problem

$$\max \{u_t(x,g) \mid (x,g) \in R_+^{L+N} \text{ and } p \cdot x + q \cdot g \leqq p \cdot \omega_t\} .$$

I now define the corresponding Tiebout equilibrium. First of all, observe that $(x_{(t,i)}, g_{(t,i)})$ does not depend on i. This fact follows from (9.16), since u_t is strictly quasi-concave. Hence, I may write $(x_{(t,i)}, g_{(t,i)}) = (x_t, g_t)$, for all (t,i). Let $\vec{x} = (x_t)_{(t,i)\in\mathcal{J}}$.

Because there are at least as many regions as types (assumption 9.6), I may place all the consumers of type t in region $j = t$, for $t = 1,\ldots,T$. That is, I define r by $r(t,i) = t$, for all (t,i).

For $j \leq T$, I define τ_j by $\tau_j(\omega) = q \cdot g_j$, for all $\omega \in R_+^L$. If $j > T$, I let $\tau_j(\omega) = 0$, for all ω. Finally, let $\vec{g} = (g_1,\ldots,g_T,0,\ldots,0)$. That is, region j provides public service g_j, for $j = 1,\ldots,T$. For $j > T$, region j provides no public services.

I claim that $(r,\vec{x},\vec{g},y),(p,q)$ and $(\tau_1,\ldots,\tau_J)$ form a Tiebout equilibrium. It should be clear that conditions (9.14) - (9.16) imply that conditions (9.1) - (9.3) are satisfied. Condition (9.4) follows from the definition of the τ_j. Condition 9.5 may be proved by the argument which proves that the allocation of a competitive equilibrium lies in the cone. (See Debreu and Scarf [12].)

Q.E.D.

Proof of Theorem 9.12

By using the condition implied by government profit maximization, (9.5), and by using the strict quasi-concavity of utility functions, it is easy to show that each consumer receives the same allocation of private and public goods that he would if he were alone in a region. Hence, as in the previous proof, the equilibrium is exactly the same as it would be if all public goods were private. Thus, the standard theorem proves that the equilibrium is Pareto optimal. (See Debreu [10], p. 94.)

Q.E.D.

Remark: If one drops the entrepreneurial government condition (9.5), then a variation of example (3.2) shows that equilibria need not be Pareto optimal. Let there be four types of consumers and let them be as in example (3.2). Let there be two consumers of each type and suppose that there are four regions. In each of two regions, place a consumer of type A together with one of type C, as in example (3.2). In each of the other two regions, place a consumer of type B together with a consumer of type D. This arrangement satisfies (9.1) - (9.4) and yet is not Pareto optimal.

When one thinks of a public service, one thinks of a commodity which must be produced in the region where it is provided. It is possible to prove the previous theorems for this case, provided that private goods remain perfectly transportable between regions. The general conclusion remains the same. Consumers segregate themselves into homogeneous communities.

In order to formalize this version of the model, it is necessary to assign a production vector to each region. That is, an allocation is of the form $(r,\vec{x},\vec{g},y)$, where $r,\vec{x}$ and $\vec{g}$ are as before and $\vec{y} = (y_1,\ldots,y_j)$, with $y_j \in Y$, for all j. I write vectors

$y \in Y$ as $y = (y^1, y^2)$, where $y^1 \in R^L$ and $y^2 \in R^N$. The components of y^1 refer to private goods and those of y^2 to public services. The allocation $(r, \vec{x}, \vec{g}, \vec{y})$ is <u>feasible</u> if

$$\sum_{t=1}^{T} \sum_{i=1}^{I_t} (x_{(t,i)} - \omega_t) = \sum_{j=1}^{J} y_j^1 \text{ , and}$$

$n_j^r g_j = y_j^2$ for all j. This definition of feasibility expresses the notion that private goods are freely transportable and public services are immobile.

An equilibrium now consists of an allocation $(r, \vec{x}, \vec{g}, \vec{y})$, a price vector $p \in \Delta^{L-1}$, price vectors $q_j \in R_+^N$, for $j = 1, \ldots, J$, and tax systems $\tau_1, \ldots, \tau_J$. These must satisfy the appropriate analogues of conditions (9.1) - (9.4). They must also satisfy an analogue of condition 9.5 which reflects the assumption that governments bélieve that prices of private goods are constant, though the cost of public services may be affected by a change of regime. The cost of public services depends on the fixed prices of private inputs and on the quantitites of public services purchased by the regional government.

If assumptions (9.6) - (9.11) are satisfied, then theorems (9.12) and (9.13) remain true. The proofs in this case are much like those that have already been given. One defines a private economy with no public goods such that equilibria for this economy correspond to equilibria for the Tiebout economy with no two consumers of different types in the same region. In the private economy, there are $L + NT$ distinct private goods. Goods $k = 1, \ldots, L$ correspond to the private goods of the Tiebout economy. Goods $k = L + (t-1)N+1,\ldots,L+tN$ correspond to the public services consumed by consumters of type t. The consumption set of consumers of type t is $X_t = \{x \in R_+^{L+NT} \mid x_k > 0$ only if $k = 1,\ldots,L$ or $k = L + (t-1)N+1,\ldots,L+N\}$. There are T production sets. The t^{th} production set for $t = 1,\ldots,T$, is

$$Y_t = \{y \in R^{L+NT} \mid (y_1,\ldots,y_k,y_{L+(t-1)N+1},\ldots,y_{L+t_N}) \in Y$$
$$\text{and } y_k \neq 0 \text{ only if } k = 1,\ldots,L \text{ or } k = L+(t-1)N+1,\ldots,L+tN\}.$$

The private economy is described by the following data:

$$\{(X_t,u_{(t,i)},\omega_{(t,i)}),Y_t: i = 1,\ldots,I_t;\ t = 1,\ldots,T\}.$$

I do not give further details.

Those papers in the literature which prove the efficiency of Tiebout equilibria deal essentially with the model of theorems (9.12) and (9.13). These include Tiebout himself [39], Buchanan [7], Jerome Rothenberg [30], Barr [2], McGuire [24], and Sonstelie and Portney [34]. The essential difference between their work and that above is that they assume that the cost of providing a certain level of public service is U-shaped as a function of regional population. This introduces a complication in that optimal community size and the choice of public service level are interdependent. The conclusion is the same as in theorems (9.12) and (9.13). Regional populations would and should be homogeneous. Rothenberg discusses the tension between the need for homogeneous community populations and the economies of scale enjoyed by larger communities.

Wooders [44] discusses a model which is similar to the versions of theorems (9.12) and (9.13), discussed above, with local production of public services. She, however, does not assume that public goods are public services. Instead, she allows costs to vary with population size in an arbitrary way. For this reason, in order to prove that equilibrium exists she makes a very special assumption that total population can be exactly divided up into communities of optimal size.

Papers of Hamilton [19], [20] and Wheaton [41] deal with a model which is essentially that of Theorems (9.12) and (9.13). They discuss the inefficiencies which arise if local governments raise revenue using distortionary taxes. Hamilton considers proportional property taxes on houses. Wheaton considers proportional income taxes and Lindahl prices. Hamilton suggests that the distortion he discusses could be overcome by zoning. Wheaton asserts that the presence of distortions refutes the "Tiebout hypothesis." This is unfair. Tax distortions are a separate issue.

Negishi [25] discusses Pareto optimal equilibria in a model which he asserts is an expression of Tiebout's ideas. But in fact his model is not of a Tiebout economy at all. In his model, each consumer's consumption set is simply the non-negative cone of a Euclidean space. The components of vectors in this space correspond to land and public goods in each region as well as to private goods. Consumption sets are therefore convex. But in a Tiebout model, consumption sets are essentially non-convex. In fact, they are not even topologically connected. When a consumer chooses his residence, he chooses from a discrete set. In a Tiebout model, the region of residence is implicitly, if not explicitly, an argument in every consumer's utility function.

10) Conclusion

In the models of the previous section, regions play little more than a formal role. No resources are associated with them, they play no role in the production of private goods, and utility functions do not depend on location. I explained in section 8 why I could not allow utility functions to depend on location. Resources cannot very well be associated with regions unless regions play a role in the production of private goods. As soon as regions play a role in such production, one meets the problems explained in section 7. These are extreme examples in that regions are autarkic. But a special case of a model with productive regions is one with autarkic regions. Also, it should be clear that the examples illustrate problems which would arise if regions played some role in the production of private goods, even if inter-regional trade were possible.

A key assumption in the previous section is that the cost of public goods is proportional to the number served. Example (5.1) shows what can go wrong if costs are less than proportional to population size. Example (8.3) shows that costs cannot depend on the type of consumer served.

One thus seems to be more or less obliged to use the models of section 9. Tiebout seems to have had a model like these in mind when he wrote the more precise parts of his paper. Perhaps the reader finds this model satisfactory. I find a model with homogeneous communities and profit maximizing governments startling and strikingly in conflict with my everyday experience. Tiebout seems to imply that this narrow model is only meant to illustrate an idea with wider application. It seems that no wider model exists.

Tiebout's narrow model is essentially a general equilibrium model. General equilibrium theory was designed so as to solve a very specific allocation problem using a specific mechanism. Tiebout considers a different allocation problem and suggests a different mechanism. But when one considers his idea critically, one finds that in order to solve his problem in the way he suggests, one is obliged to strip the problem of all its distinguishing characteristics and to reduce it to the problem already solved in general equilibrium theory.

APPENDIX I

Here, I collect some examples which seem informative yet do not fit well into the body of the paper. The first shows that local government monopoly power could lead to inefficient equilibria.

I.1) Example

The economy has one region and one consumer, one public good and one private good, called labor or leisure. The consumer owns his regional government and instructs it to maximize profits as measured in labor. The consumer is endowed with one unit of labor and the production function is $g = L$, where g is the quantity of public good and L is the quantity of labor devoted to production. The utility function of a consumer is $u(\ell,g) = \ell + 2g$, where ℓ is the quantity of leisure consumed and g is the quantity of public good provided.

The following situation describes a monopolistic equilibrium in the above example. None of the public good is produced. All of the consumer's income is taxed away. One unit of leisure is paid to the consumer as profits. This arrangement is clearly profit maximizing, but is not Pareto optimal. The consumer would gain one unit of utility if he used all his labor to produce the public good.

The next example illustrates a difficulty in the concept of Tiebout equilibrium for economies with autarkic regions. The problem has to do with condition (4.1). Regional economies might have several competitive equilibria. In this case, a local government would not necessarily know whether it would be worth while to change policy unless it knew which equilibrium would prevail.

I.2) Example

There are two regions, two consumers, three private goods and two pure public services. Consumer i is endowed with one unit of good i and one unit of good 3, for $i = 1,2$. The production function in either region is $n(g_1 + g_2) = y_3$, where g_k is the quantity of public service k provided in the region, n is the size of the region's population, and y_3 is the quantity of good 3 used in production. Regional economies are autarkic. The utility function of consumer i is $u_i(x_1,x_2,x_3,g_1,g_2) = v_i(x_1,x_2) + g_i$, for $i = 1,2$, where x_k is the quantity of private good k consumed and g_k is the quantity of public service k provided.

Consider the private goods economy with private goods 1 and 2. That is, consumer i has utility function v_i and is endowed with one unit of good i, for $i = 1,2$. I assume that this has two equilibria, A and B. Let (x^c_{i1},x^c_{i2}) be the allocation to consumer i in equilibrium c, where $i = 1,2$ and $c = A,B$. I assume that

I.3) $$v_1(x^A_{11},x^A_{12}) - v_1(x^B_{11},x^B_{12}) > 1$$

and that

I.4) $$v_2(x^B_{21},x^B_{22}) - v_2(x^A_{21},x^A_{22}) < 1.$$

Clearly, v_1 and v_2 may be chosen so as to satisfy these conditions.

Suppose that both consumers lives in region 1, that the economy is at equilibrium B and that all of good 3 is taxed away in order to provide 1 unit of public service 1 to each consumer and none of public service 2. By (I.3) and (I.4), both consumers would be better off if they moved to equilibrium A and if private good 3 were used to produce 1 unit of public service 2 instead of one unit of public service 1. But how could this change be arranged in a competitive economy? (Imagine that there are one million consumers of each type.)

The next example concerns the assumption that entrepreneurial governments of autarkic regions take into account the impact of policy changes on prices. It is hard to imagine how governments could not be aware of this impact, but it is important to try to see what matters when one goes from free trade to the autarkic case. Is it the presence of localized production or is it the fact that governments no longer believe that prices are fixed? The following example shows that equilibria still may not exist in the autarkic case, even if governments do not take into account price changes. This is so even if governments act so as to maximize the population of their regions.

I.5) Example

There are three regions, two types of consumers, two private goods and two pure public services. Consumers form a continuum, and half are of each type. Each consumer of type i is endowed with one unit of good i. Regions are autarkic and production is governed by the equation $r(g_1 + g_2) = 2\sqrt{y_1 y_2}$, where r is the proportion of the population in the region, g_k is the level of public service k provided there, and

y_k is the quantity of private good k devoted to production. The utility function of a consumer of type i is $u_i(x_1,x_2,g_1,g_2) = g_i$, where x_k is the quantity of private good k consumed and g_k is the level of public service k provided.

Suppose that local governments maximize population and that they believe that when they make policy changes prices will remain constant. I now show that in this case the economy of the above example has no equilibrium. Suppose that there were an equilibrium. By the free migration condition, all consumers of the same type must enjoy the same utility level, no matter where they live. Now suppose that in some region there were more consumers of one type than another. Then, by attracting additional consumers of the type in a minority, the government could raise the utility level of each consumer in its region and also increase the population of the region. The government would see that this was so, even if it believed that prices would remain fixed. Thus the economy would not be in equilibriuum. I conclude that in each region half the population is of each type. It follows that everywhere the price of each private good is, say, equal to one and the price of each public good equals one. Since consumers everywhere receive the same utility, the levels of both public services are the same everywhere. The level of one of these services, say the first, must be less than one. Also, one of the regions must have no more than one third of the population. It seems fair to assume that if there were no people in this region, then its government would believe that relative prices in its region were as elsewhere. If this government believes that all prices would remain constant, then it would believe that it could invite all the consumers of type 1 to its region and offer to provide one unit of public service 1 and none of public service 2. This would make consumers of type 1 better off than before. (Such a new regime would be impossible, of course.) Thus, the government would believe it could increase its share of the population from not more than one third to one-half. Hence, the economy is not in equilibrium. This contradiction shows that no equilibrium exists.

The following example seems to contain the gist of a counterexample given by Stiglitz [38], pp. 294-5. It is a counterexample to the existence of Tiebout equilibrium. In the example, it is implicitly assumed that each region has a fixed amount of land available for production. The land is owned by the regional government. The presence of land makes it possible to assume that production exhibits diminishing returns to scale. Diminishing returns are crucial in the example. The example is of a Tiebout economy with autarkic regions, a pure public good and a continuum of consumers. By an equilibrium I mean an allocation, prices and taxes which satisfy conditions (2.1) - (2.4) and (4.1). Governments maximize profits.

I.6) Example

There are two autarkic regions, one type of consumer, two private goods and one pure public good. Consumers form a continuum. The private goods may be thought of as labor and food. Each consumer has an initial endowment of one unit of labor. His utility function is $u(\ell,x,g) = x + g$, where ℓ is is leisure consumed, x is the food consumed, and g is the quantity of public good provided in the region inhabited. Food and the public good are produced jointly in either region according to the production function

$g = x = L$, where g and x are the quantities of public good and food produced, respectively, and L is the proportion of all labor available which is used in production in the given region.

Observe that if r_i is the proportion of the population living in region i, for $i = 1,2$, and if each consumer in region i contributes his full one unit of labor, then r_i units of food and public goods are produced. Per capita consumption of food in region i is r_i

I now show that no equilibrium exists. Suppose that there were an equilibrium. I claim that the proportion of the population living in region 1, r_1, is either 1/4 or 3/4. Clearly, it may be assumed that all labor in region 1 is devoted to production, so that the utility of each consumer is $x + g = r_1^{-3/7} + r_1^{4/7}$. This quantity is maximized when and only when $r_1 = 3/4$. Hence, if $r_1 > 3/4$, it pays to exclude some of the inhabitants of region 1 from the region. Similarly, if $r_1 < 1/4$, then $r_2 > 3/4$ and the same argument applies to region 2. If $1/4 < r_1 < 3/4$, then $r_2 \neq 3/4$ and it would pay to form a new region in region 1 with 3/4 of the population there, some being drawn from each region. Hence, $r_1 = 1/4$ or $3/4$.

If $r_1 = 1/4$, then the consumers of region 1 enjoy less utility than those of region 2. If $r_1 = 3/4$, then those of region 2 enjoy less utility. But if consumers are perfectly mobile, they must all be enjoying the same utility. This contradiction proves that no equilibrium exists.

Appendix II

In this appendix I state the equilibrium existence theorem which I use in section 9.

Consider an economy with I consumers, n commodities and one firm. There are no regions or public commodities. Let the utility function of the i^{th} consumer be u_i and let his initial endowment be ω_i, for $i = 1,\ldots,I$. Y denotes the production possibility set of the unique firm. Assume that

II.1) $u_i : R^n_+ \to R$ is continuous, strictly quasi-concave and strictly monotone, for all i;

II.2) $\omega_i \in R^n_+$ and $\omega_i \neq 0$, for all i;

II.3) $Y \subset R^n$ is a closed convex cone with apex zero;

II.4) $Y \cap R^n_+ = \{0\}$; and

II.5) there exists $\bar{y} \in Y$ such that every component of $\bar{y} + \sum_{i=1}^{I} \omega_i$ is positive.

An allocation for the economy consists of a vector $(x_1,\ldots,x_I;y)$, where $x_i \in R^n_+$, for all i, and $y \in Y$. It is feasible if

$$\sum_{i=1}^{I}(x_i - \omega_i) = y.$$

An <u>equilibrium</u> consists of $(x_1,\ldots,x_I;y)$ and $p \in \Delta^{n-1}$, where

II.6) $(x_1,\ldots,x_I;y)$ is a feasible allocation;

II.7) $p \cdot y = \max \{p \cdot z \mid z \in Y\}$, and

II.8) for all i, $p \cdot x_i = p \cdot \omega_i$ and $u_i(x_i) \geqq u_i(x)$, for all $x \in R^n_+$ such that $p \cdot x \leqq p \cdot \omega_i$.

<u>Theorem</u> If the economy satisfies assumptions (II.1) - (II.5), then there exists an equilibrium.

The assumptions of this theorem are almost those of the existence theorem on p. 83 of Debreu [10]. In order to apply Debreu's theorem, it would be necessary to assume that every component of ω_i was positive, for all i. One can easily prove the theorem directly by applying a theorem of Debreu [11], p. 259, in order to prove that the economy has a quasi-equilibrium. It is then easy to prove that the quasi-equilibrium is an equilibrium. One can also simply check that the economy satisfies all the assumptions of the existence theorem on p. 119 of Arrow and Hahn [1].

Acknowledgement

I am very grateful to Gerald Goldstein, John Ledyard and Mark Pauly for guidance to the literature on local public goods. I am also grateful to the Center for Mathematical Studies in Economics and Management Science at Northwestern University for providing a very congenial research environment.

REFERENCES

[1] Arrow, Kenneth and Frank Hahn, General Competitive Analysis (Holden-Day, Inc. San Francisco: 1971).

[2] Barr, James, "Tiebout Models of Community Structure," Regional Science Association Papers, 30 (1973), 113-139.

[3] Berglas, Eitan, "Distribution of Tastes and Skills and the Provision of Local Public Goods," Journal of Public Economics, 6 (1976), 409-23.

[4] Bergstrom, Theodore and Robert Goodman, "Private Demands for Public Goods," American Economic Review, 63 (June 1973), 280-296.

[5] Borcherding, Thomas, and Robert Deacon, "The Demand for the Services of Non-Federal Governments," American Economic Review, 62 (1972), 891-901.

[6] Brueckner, Jan, "Equilibrium in a System of Communities with Local Public Goods," Economic Letters, 2 (1979), 387-393.

[7] Buchanan, James, "An Economic Theory of Clubs," Economica, 32 (1965), 1-14.

[8] Buchanan, James and Charles Goetz, "Efficiency Limits of Fiscal Mobility: An Assessment of the Tiebout Model," Journal of Public Economics, 1 (1972), 25-43.

[9] Buchanan, James and Richard Wagner, "An Efficiency Basis for Federal Fiscal Equalization," in The Analysis of Public Output, Julius Margolis, ed. (National Bureau of Economic Research, New York: 1970).

[10] Debreu, Gerard, Theory of Value (Wiley and Sons, New York: 1959).

[11] Debreu, Gerard, "New Concepts and Techniques for Equilibrium Analysis," International Economic Review, 3 (Sept. 1962), 257-273.

[12] Debreu, Gerard and Herbert Scarf, "A Limit Theorem on the Core of an Economy," International Economic Review, 4 (1963), 235-246.

[13] Ellickson, Bryan, "A Generalization of the Pure Theory of Public Goods," American Economic Review, 63 (June 1973), 417-432.

[14] Elickson, Bryan, "The Politics and Economics of Decentralization," Journal of Urban Economics, 4 (1977), 135-149.

[15] Flatters, Frank and Vernon Henderson and Peter Mieszkowski, "Public Goods, Efficiency, and Regional Fiscal Equalization," Journal of Public Economics, 3 (1974), 99-112.

[16] Foley, Duncan, "Lindahl's Solution and the Core of an Economy with Public Goods," Econometrica, 38 (1970), 66-72.

[17] Goldstein, Gerald and Mark Pauly, "Group Health Insurance as a Local Public Good," in The Role of Health Insurance in the Health Services Sector, Richard Rosett, ed., (National Bureau of Economic Research, New York: 1976).

[18] Guesnerie, Roger and Claude Oddou, "Second Best Taxation as a Game," Discussion Paper No. 7919, CEPREMAP, 142 Rue du Chevaleret, 75013 Paris, France.

[19] Hamilton, Bruce, "Zoning and Property Taxation in a System of Local Governments," Urban Studies, 12 (1975), 205-211.

[20] Hamilton, Bruce, "Capitalization of Intrajurisdictional Differences in Local Tax Prices," American Economic Review, 66 (1976), 743-753.

[21] Johnson, Michael, "A Pure Theory of Local Public Goods," (Ph.D. Dissertation: Northwestern University, 1977).

[22] Koopmans, Tjalling and Martin Beckman, "Assignment Problems and the Location of Economic Activities," Econometrica 25, (Jan. 1957), 53-76.

[23] Margolis, Julius, "The Demand for Urban Services," Issues in Urban Economics, Harvey Perloff and Lowdon Wings, Jr., eds., (Johns Hopkins Press, Baltimore, Maryland: 1968).

[24] McGuire, Martin, "Group Segregation and Optimal Jurisdictions," Journal of Political Economy, 82 (1974), 112-132.

[25] Negishi, Takoshi, "Public Expenditure Determined by Voting with One's Feet and Fiscal Profitability," Swedish Journal of Economics, 74 (1972), 452-458.

[26] Pauly, Mark, "Clubs, Commonality, and the Core: An Integration of Game Theory and the Theory of Public Goods," Economica, 34 (1967), 314-324.

[27] Pauly, Mark, "Cores and Clubs," Public Choice, 9 (1970), 53-65.

[28] Pauly, Mark, "Optimality, 'Public' Goods, and Local Governments: A General Theoretical Analysis," Journal of Political Economy, 78 (1970), 572-585.

[29] Pauly, Mark, "A Model of Local Government Expenditure and Tax Capitalization," Journal of Public Economics, 6 (1976), 231-242.

[30] Rothenberg, Jerome, "Local Decentralization and the Theory of Optimal Government," The Analysis of Public Output, Julius Margolis, ed., (National Bureau of Economic Research: 1970).

[31] Rothschild, Michael, and Joseph Stiglitz, "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," Quarterly Journal of Economics, 40 (1976), 629-649.

[32] Samuelson, Paul, "The Pure Theory of Public Expenditures," Review of Economics and Statistics, 36 (1954), 387-9.

[33] Samuelson, Paul, "Aspects of Public Expenditure Theories," Review of Economics and Statistics, 40 (1958), 332-338.

[34] Sonstelie, Jon, and Paul Portney, "Profit Maximizing Communities and the Theory of Local Public Expenditures," Journal of Urban Economics, 5 (1978), 263-277.

[35] Sorenson, John and John Tschirhart and Andrew Whinston, "Private Clubs and the Core," Journal of Public Economics, 10 (1978), 77-95.

[36] Starrett, David, "Welfare Measurement for Local Public Finances," IMSS Technical Report No. 228 (Dec. 1976), Stanford University.

[37] Starrett, David, "Market Allocations of Location Choice in a Model with Free Mobility," Journal of Economic Theory, 17 (1978), 21-37.

[38] Stiglitz, Joe, "The Theory of Local Public Goods," in The Economics of Public Services, Martin Feldstein and Robert Inman, eds., (Macmillan, London: 1977).

[39] Tiebout, Charles, "A Pure Theory of Local Expenditures," Journal of Political Economy, 64 (1956), 416-424.

[40] Westhoff, Frank, "Existence of Equilibria in Economies with a Local Public Good," Journal of Economic Theory, 14 (1977), 84-112.

[41] Wheaton, William, "Consumer Mobility and Community Tax Bases," Journal of Public Economics, 4 (1975), 377-384.

[42] Williams, Alan," The Optimal Provision of Public Goods in a System of Local Governments," Journal of Political Economy, 74 (1966), 18-33.

[43] Wilson, Charles, "Model of Insurance Markets with Incomplete Information," Journal of Economic Theory, 16 (1977), 167-207.

[44] Wooders, Myrna, "Equilibria, the Core, and Jurisdiction Structures in Economies with a Local Public Good," Journal of Economic Theory, 18 (1978), 328-348.