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INVESTMENT POLICY, OPTIMALITY, AND THE MEAN-VARIANCE MODEL

by

D. P. Baron

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ABSTRACT

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This paper examines the source of the Pareto inefficiency of the value-maximizing investment allocation in a mean-variance model with homogeneous expectations. With price-taking behavior by investors perceived-value maximization is shown to be Pareto optimal and unanimously supported by shareholders. The actual value- maximizing allocation is however Pareto inefficient because shareholders are assumed to behave strategically in anticipating how their implicit prices will be affected by a change in the investment allocation. The principal assumptions used to eliminate this inefficiency are examined and a perspective on the search for a general theory of investment for shareholder-owned firms is provided.
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David P. Baron*

The mean-variance approach to the analysis of securities market performance has enabled researchers to summarize the performance of securities in terms of easily interpretable parameters and to develop and test hypotheses concerning the relative values of securities. While most of these studies treat the investment decisions of firms as exogenous, the model may also be used to characterize the optimal investment decisions of firms. The central methodological step in the determination of the optimal investment policy, given mean-variance preferences and homogeneous expectations, is the maximization of the market value of the firm as expressed by the difference between the expected return and a risk premium that is a function of the covariance between the firm's return and a market portfolio. Stiglitz (1972) and Jensen and Long (1972) have demonstrated that the investment allocation determined by maximizing this market value is not a constrained Pareto optimum, and Leland (1974) and Ekern and Wilson (1974) have shown that the value-maximizing allocation is not in the best interests of final shareholders.

The difference between the investment allocation resulting from value maximization and the allocation that is in the best interests of shareholders continues to be a source of discussion. For example, in a model of an international firm operating in two segmented national markets, Adler (1974) used the value maximization approach and concluded that market segmentation required "compensation" to achieve optimality. In a recent comment on that article

Goldberg and Lee (1977) employ the "reaction principle" of Fama and Laffer (1972) and Fama (1972) in each national market of Adler's model and conclude that optimality is directly obtained without need for compensation. Both of these conclusions are correct based on the assumptions from which they are derived, as Adler (1977) observes, but the difference between the value-maximizing and the unanimously-supported, Pareto optimal allocations in a mean-variance model remains an issue.

Recently, a number of authors, including Brenner and Subrahmanyam (1977), Grossman and Stiglitz (1977), Krouse (1978), Le Roy (1976), Mayshar (1978), Mossin (1977), Nielsen (1976), Rubinstein (1978), Svensson (1977), and Yawitz (1977), have dealt with a variety of aspects of this difference. The objective of this paper is to assess and analyze this difference by 1) characterizing in a self-contained manner both the investment allocation that is in the best interests of shareholders and the allocation resulting from value maximization in a mean-variance model, 2) examining the cause of the Pareto inefficiency of the value-maximizing investment allocation, 3) illustrating the difference between the objectives of perceived and actual value maximization, 4) considering the conditions proposed in the literature to eliminate the Pareto inefficiency of the value-maximizing allocation, and 5) providing a brief perspective on the problems that remain in developing a general theory of investment for shareholder-owned firms.

In the next section a one-period model is introduced that is used to characterize the unanimously-preferred investment allocation. When a spanning condition is satisfied and individuals behave in accord with a competitivity assumption, the investment allocation that is unanimously supported by all initial shareholders is shown to be consistent with the objective of perceived value maximization. When the competitivity assumption is not made, unanimity among final shareholders results when an equilibrium is evaluated at a "steady-state," but perceived value maximization does not obtain because no assumption is made regarding how the market value of a firm responds to changes in the investment allocation.

The value-maximizing allocation in a mean-variance model with homogeneous expectations is characterized in Section II and the objective of value-maximization is shown to be opposed by all shareholders. Furthermore, this allocation is Pareto inefficient because shareholders do not behave as price-takers but instead behave strategically by taking into account the effect of changes in the investment allocation on their implicit valuations of returns. This strategic behavior may be interpreted as a moral hazard problem, as the result of monopoly power, or as an "inappropriate" consideration of the consumption effects associated with the investment allocation. An example presented in Section III indicates however that an initial shareholder can be better off with the value-maximizing investment allocation than with the unanimously-supported allocation resulting from competitive price-taking behavior even though the former allocation is Pareto inefficient and the latter is Pareto efficient.

One reaction to the Pareto inefficiency resulting from value maximization in a mean-variance model is to attempt to restore efficiency by adding assumptions so that strategic behavior is not possible. Two classes of such assumptions are considered in Section IV. The first class achieves efficiency by making firms negligible in size relative to the aggregate economy, so that a change in the investment allocation to a firm will have a negligible effect on an individual's implicit valuations. A second approach taken by Fama and Laffer is to consider an economy with fixed aggregate supplies in each state of nature so that implicit valuations are fixed and then to employ a reaction principle that applies to large well as to small firms. The objective of this approach is to demonstrate a result analogous to the Modigliani-Miller (1958) theorem that all individuals are indifferent to the allocation among firms of a fixed supply of investment. In addition to not yielding a theory of investment allocation, the reaction principle cannot provide a characterization of the allocation of endowments between consumption and investment that determines the aggregate supply of investment to firms.

The analysis to be presented in the first four sections indicates that a general theory of investment policy is not yet available. The final section is concerned with some as yet unresolved aspects of the search for a general theory of investment.

I. The Securities Market and Optimality

A. The Model

The model to be utilized is a simple one involving a one-commodity economy in which individuals allocate their budgets at time zero between consumption, investment in firms, and a portfolio of securities. There are J firms, $j=1,\ldots,J$, in the economy, each characterized by a production function f_{js} that gives the output $x_{js} = f_{js}(x_{j0})$ at time 1 in state of nature s, $s=1,\ldots,S$, when an investment x_{j0} is made at time 0. The investment x_{j0} is provided by the initial shareholders of the firm, and the output is distributed to the final shareholders at time 1. The vector $x_j = (x_{j1},\ldots,x_{jS})$ will be referred to as the return vector of firm j, and to simplify the analysis, the return vectors x_j, $j=1,\ldots,J$, are assumed to be linearly independent. The production functions $f_{js}(x_{j0})$ are assumed to incorporate nondecreasing returns for small x_{j0} and eventually nonincreasing returns. In addition, $f_{js}(0) = 0$ for all s and j. A special case that will be considered is $x_{js} = \gamma_{js}f_j(x_{j0})$, where γ_{js} can be interpreted as the per-unit yield resulting from the firm's planned output $f_j(x_{j0})$.[1] Firm J will be assumed to be riskless so that $f_{Js}(x_{J0}) = f_J(x_{J0})$ for all s.

At time 0 an individual i, $i=1,\ldots,I$, is endowed with an amount y_{i0} of the commodity and ownership shares $\overline{\alpha}_{ij}$, $j=1,\ldots,J$, of the firms. Investment in a firm is provided by its initial shareholders, so the value of the individual's endowments is $(y_{i0} + \sum_j \overline{\alpha}_{ij}(V_j - x_{j0}))$, where V_j is the market value of the time 1 return x_j, and hence $V_j - x_{j0}$ is the net value of the firm. An individual may purchase new ownership shares α_{ij} in a securities market, and the return vector R_i on his portfolio is then $R_{is} = \sum_j \alpha_{ij} x_{js}$, s=1,...,S. At time 1 the individual consumes his return, $c_{is} = R_{is}$, s=1,...,S, while time 0 consumption c_{i0} is constrained by the budget condition

$$c_{i0} + \sum_j \alpha_{ij} V_j \leq y_{i0} + \sum_j \bar{\alpha}_{ij}(V_j - x_{j0}).$$

The individual is assumed to have state-dependent preferences represented by continuously differentiable, strictly increasing, strictly concave, utility functions $U_{is}(c_{i0}, c_{is})$, and expectations represented by probabilities $g_{is} > 0$, s=1,...,S. The individual thus chooses a vector $z_i = (c_{i0}, \alpha_{i1}, \ldots, \alpha_{iJ})$ to maximize expected utility $E_i U_i = \sum_s g_{is} U_{is}(c_{i0}, c_{is})$. The securities market is assumed to be "perfectly competitive" in the sense that there are no transactions costs or taxes, short sales are permitted, and individual trades do not affect security prices. The market however is not "perfect" in the sense that the total amount of investment available to firms or the total supply of the commodity in each state is fixed, since individuals can allocate their endowments between consumption and investment and thus alter the total investment and the total supply of the commodity at time 1.

The optimal z_i for individual i corresponding to a proposed investment allocation $(x_{10}, \ldots, x_{J0})$ satisfies

$$\sum_s g_{is} \frac{\partial U_{is}}{\partial c_{i0}} - \lambda_i = 0 \tag{1}$$

$$\sum_s p_{is} f_{js}(x_{j0}) = V_j, \; j=1,\ldots,J, \tag{2}$$

where λ_i is a multiplier and p_{is} is the implicit state-contingent claim price.[2] This implicit price is the marginal rate of substibution of a unit of the commodity to be received at time 1 if and only if state s occurs for a unit of the commodity at time 0. The implicit prices are defined as

$$p_{is} = g_{is} \frac{\partial U_{is}}{\partial c_{is}} / \lambda_i, \ s=1,\ldots,S, \tag{3}$$

and depend on the aggregate investment and its allocation among the firms. For the riskless firm J, $\sum_s p_{is} = V_J / f_J(x_{J0})$ for $x_{J0} > 0$, so the interest rate r can be defined by $\sum_s p_{is} \equiv 1/(1+r)$. Consequently, $(1+r)p_{is}$ may be interpreted as consumer i's "probability" associated with state s. Multiplying by the initial share endowment $\bar{\alpha}_{ij}$ in firm j and summing over all consumers yields $\sum_s ((1+r)\sum_i \bar{\alpha}_{ij} p_{is}) = 1$, which represents the sum of the "market probabilities" $p_{ms} \equiv (1+r)\sum_i \bar{\alpha}_{ij} p_{is}$. The value V_j of the firm may thus be interpreted as the discounted market expected value of the firm.[3]

To determine the "preferred" investment in firm j, let $E_i U_i^o$ denote individual i's expected utility at his optimal portfolio $\hat{z}_i$ satisfying (1) and (2). His preferences regarding revisions in the investment allocation to firm j are determined by differentiating $E_i U_i^o$ with respect to x_{j0} to obtain[4]

$$\frac{\partial E_i U_i^o}{\partial x_{j0}} = \lambda_i [\hat{\alpha}_{ij} \sum_s p_{is} f'_{js}(x_{j0}) + \sum_k (\bar{\alpha}_{ik} - \hat{\alpha}_{ik}) \frac{\partial V_k}{\partial x_{j0}} - \bar{\alpha}_{ij}], \tag{4}$$

where the usual Nash assumption is made that the individual behaves as if a change in the investment in firm j will not affect the investment in firm k so $\partial x_{k0}/\partial x_{j0} = 0$ for all $k \neq j$. The preferences of the individual reflect both his role as a consumer and as an investor. The first term $(\hat{\alpha}_{ij} \sum_s p_{is} f'_{js}(x_{j0}))$ depends on final shareholdings $\hat{\alpha}_{ij}$ and thus represents a consumption effect reflecting the impact of the investment on time-one consumption. The last two terms represent the effect of the revision in investment on the capital gains or losses on shareholdings as realized at time zero. Those terms are composed of the change $(\sum_k \overline{\alpha}_{ik} \frac{\partial V_k}{\partial x_{j0}} - \overline{\alpha}_{ij})$ in the net value of his endowments and the change $\sum_k \hat{\alpha}_{ik} \frac{\partial V_k}{\partial x_{j0}}$ in the cost of purchasing the optimal portfolio.

To evaluate the individual's preferences as indicated in (4), it is necessary for the individual to forecast the changes in the values of the firms. An individual knows that for any investment allocation a securities market equilibrium will be established and hence that the value of the firm will satisfy (2). Consequently, the perceived change $\partial V_k^i/\partial x_{j0}$ in the value of a firm can be determined by differentiating (2) to obtain

$$\sum_s \frac{\partial p_{is}}{\partial x_{j0}} f_{ks}(x_{k0}) + \sum_s p_{is} \frac{\partial f_{ks}(x_{k0})}{\partial x_{j0}} = \frac{\partial V_k^i}{\partial x_{j0}}, \quad k=1,\ldots,J. \tag{5}$$

The second term gives the valuation of the change in time 1 output using the implicit prices p_{is}, while the first term reflects the change in value due to the change in implicit prices. The evaluation of the expression in (5) and its implications for (4) are considered in the next two subsections.

B. Competitivity and Unanimity Among Initial Shareholders

In the complete market models of Arrow (1964) and Debreu (1959) and in the incomplete market models of Diamond (1967), Leland (1973), and Radner (1974), the evaluation of alternative investment allocations is based on a competitivity assumption under which consumers are assumed to act as price takers with respect to their implicit prices. More formally, the competitivity assumption means that consumers evaluate a proposed revision in an investment plan from the currently proposed x_{j0} to $x_{j0} + dx_{j0}$ using the implicit prices corresponding to the investment allocation $(x_{10},\ldots,x_{J0})$ at which the securities market equilibrium conditions in (1) and (2) are evaluated.[5] Those implicit prices will be referred to as "current" implicit prices, since they correspond to the "currently" proposed investment allocation. Using current implicit prices to evaluate proposed revisions in investment plans is equivalent to the condition that there are no gains to arbitrage from scale change in risks. That is, a proportionate increase δ in the "composite good" $f_{js}(x_{j0})$ is assumed to result in a proportionate increase in the value of that output or[6]

$$\sum_s p_{is}(1+\delta)f_{js}(x_{j0}) = (1+\delta)V_j.$$

Consideration of the strict correctness of the competitivity assumption will be reserved for Section IV, and at this point only its implications will be investigated.

An individual behaving as a price-taker with respect to his implicit prices (determined at a securities market equilibrium corresponding to an allocation $(x_{10},\ldots,x_{j0})$) will evaluate (5) as

$$\sum_s p_{is} f'_{js}(x_{j0}) = \frac{\partial V^i_j}{\partial x_{j0}} \tag{6}$$

$$\frac{\partial V^i_k}{\partial x_{j0}} = 0, \quad k \neq j. \tag{7}$$

Substituting these forecasts into (4) yields

$$\frac{\partial E_i U_i^o}{\partial x_{j0}} = \lambda_i \bar{\alpha}_{ij} \left(\frac{\partial V_j^i}{\partial x_{j0}} - 1\right) = \lambda_i \bar{\alpha}_{ij} \left(\sum_s p_{is} f'_{js}(x_{j0}) - 1\right), \tag{8}$$

so every initial or ex ante shareholder ($\bar{\alpha}_{ij} > 0$) prefers a change in x_{j0} that is perceived to increase the net market value of the firm.[7]

The process by which an investment plan is selected may be thought of as a voting process in which each initial shareholder votes his shares in favor of or against the proposed revision in the investment plan depending on the sign of the right-side of (8). Any proposed revision will be unanimously approved or rejected if all initial shareholders evaluate $(\partial V_j^i / \partial x_{j0})$ identically. This will be the case if the state distribution of returns, the set of return vectors $(R_{i1}, \ldots, R_{iS})$ that can be generated by the purchases of securities, spans the marginal return vector $\Delta x_j \equiv (f'_{j1}(x_{j0}), \ldots, f'_{jS}(x_{j0}))$. That is, the return vectors $(x_1, \ldots, x_J)$ span a subspace T of dimension J, and when the marginal return vector lies in this subspace, unanimity can be demonstrated. Spanning implies that there exist β_{jk} such that[8]

$$f'_{js}(x_{j0}) = \sum_{k=1}^{J} \beta_{jk} x_{ks}, \quad s=1,\ldots,S,$$

and using this expression to evaluate the forecast in (6) yields

$$\begin{aligned} \frac{\partial V_j^i}{\partial x_{j0}} = \sum_s p_{is} f'_{js}(x_{j0}) &= \sum_s p_{is} \sum_k \beta_{jk} x_{ks} \\ &= \sum_k \beta_{jk} \sum_s p_{is} x_{ks} \\ &= \sum_k \beta_{jk} V_k. \quad \text{(from (2))} \end{aligned} \tag{9}$$

Since the V_k are market observable prices and the β_{jk} are determined from the technologies of firms, all individuals have the same forecast of the change in the value of the firm. Substituting (9) into (8) then yields

$$\frac{\partial E_i U_i^o}{\partial x_{j0}} = \lambda_i \bar{\alpha}_{ij} (\sum_k \beta_{jk} V_k - 1), \tag{10}$$

indicating that all initial shareholders unanimously prefer the investment plan such that the term in parentheses in (10) is zero.[9,10] Initial or <u>ex ante</u> shareholder welfare is thus consistent with <u>perceived</u> (net) value maximization.[11]

One case in which the spanning property is satisfied is that of a complete market ($J = S$), since then the entire space is spanned. In this case the J equations in (2) can be solved to demonstrate that all individuals have identical implicit prices, $p_{is} = p_s$ for all i and s, so the left side of (6) is independent of i implying that $\partial V_j^i / \partial x_{j0}$ is independent of i.[12]

When defined in sufficient detail to capture all of the production and consumption effects, the number of states of nature is certainly far greater than the number of securities available for trade.[13,14] When the market is incomplete ($J < S$), individuals do not have sufficient instruments in which to trade to cause the implicit prices to be the same for all individuals. Thus, the left-side of (6) may not be evaluated identically by all individuals. The spanning condition however is sufficient to establish unanimity.[15]

As an example, if $f_{js}(x_{j0}) = \gamma_{js} f_j(x_{j0})$, the spanning relationship is $\beta_{jk} = 0$, $k \neq j$, and $\beta_{jj} = f_j'(x_{j0})/f_j(x_{j0})$. Then,

$$\frac{\partial V_j^i}{\partial x_{j0}} = \sum_s p_{is} \gamma_{js} f_j'(x_{j0}) = (f_j'(x_{j0})/f_j(x_{j0})) V_j,$$

which implies that

$$\sum_s p_{is} \gamma_{js} = V_j / f_j(x_{j0}).$$

The right side may be interpreted as the value of a unit of the composite commodity γ_{js} or as the market certainty equivalent of the yield γ_{js}, s=1,...,S. Evaluating (10) indicates that the optimal $\hat{x}_{j0}$ satisfies

$$f_j'(\hat{x}_{j0}) = f_j(\hat{x}_{j0})/V_j. \tag{11}$$

If the firm can generate no rents for its owners (i.e. $V_j = \hat{x}_{j0}$ in equilibrium), then (11) indicates that the average return will equal the marginal return. This will be the case with free entry and perfectly mobile resources, and in this case all firms would have the same average and marginal returns in equilibrium.

C. Equilibrium and the Competitivity Assumption

The equilibrium in an incomplete market with spanning and competitive behavior may be thought of as a rational expectations equilibrium in which individuals form correct expectations regarding the equilibrium market values of firms and consequently prefer to vote for the investment allocation that in fact fulfills those expectations.[16] The process by which such an equilibrium is attained can be thought of as involving a sequence of investment proposals by firms and votes by shareholders. Suppose that firms initially propose investment plans. Shareholders then evaluate those plans using their implicit prices determined, with the aid of an auctioneer, at a securities market equilibrium corresponding to those plans. With spanning and the competitivity assumption all initial shareholders unanimously vote for either a decrease or increase in the investment level for each firm. The firm then revises its plan accordingly, announces the revision, and the process continues until all firms propose investment levels such that no initial shareholder votes for a revision.[17]

Central to this process is the assumption that individuals evaluate proposed investment plans using their corresponding implicit prices and specifically do not engage in strategic behavior by attempting to anticipate how a proposed revision in the investment allocation will affect their prices. This behavior may be warranted in a complete market or in an incomplete market with spanning, since the opportunity set of an individual is invariant to the investment decisions of firms. The individual however may recognize that his budget set could be affected by the decisions of firms and hence that his implicit prices can change in response to changes in market values. Such an individual will generally not know what the equilibrium market values or implicit prices will be, and for lack of better information he may use "current" marginal rates of substitution or implicit prices to forecast market value changes. If all individuals behave in this way, unanimity among initial shareholders and a competitive Pareto optimal allocation will result. In Section II however the mean-variance model will be shown to possess a special property that enables investors to determine how their implicit prices, and those of all other individuals, will change, and value maximization in this model is shown to assume that individuals take this into account in evaluating investment proposals.

D. Unanimity in the Absence of the Competitivity Assumption

Ekern and Wilson (1974), Leland (1974), Ekern (1975)(1976), and Nielsen (1976) have analyzed an incomplete market model without making the competitivity assumption. Instead, they evaluate an individual's preferences at a securities market equilibrium in which the initial endowment of shares ($\bar{\alpha}_{ij}$) are optimal given the currently proposed investment allocation. That is, when given the opportunity to make trades, all consumers choose shareholdings such that $\hat{\alpha}_{ij} = \bar{\alpha}_{ij}$, for all j. In this case the individual's preferences for a change in the investment as given by the expression in (4) are evaluated as

$$\frac{\partial E_i U_i^o}{\partial x_{j0}} = \lambda_i \hat{\alpha}_{ij} \left(\sum_s p_{is} f'_{js}(x_{j0}) - 1\right), \tag{12}$$

which is identical to (8) except that $\hat{\alpha}_{ij}$ replaces $\bar{\alpha}_{ij}$. This expression results because $\hat{\alpha}_{ij} = \bar{\alpha}_{ij}$ implies that the change in the value of the endowments of shares equals the change in the cost of purchasing the optimal portfolio so initial wealth effects are cancelled. Consequently, no assumption regarding forecasts of changes in market values is required, and unanimity among final or <u>ex post</u> shareholders is implied by the spanning condition in (9).[18] The optimal investment allocation thus satisfies the same condition as with the competitivity assumption.

The process corresponding to this <u>ex post</u> case has been described as follows by Drèze (1974). Consider a sequence of hypothetical "days" where each morning individuals with the aid of an auctioneer solve their consumption-portfolio problems given a proposed investment allocation, and each afternoon firms evaluate their investment using the market values established in the morning. If $(\sum_k \beta_{jk} V_k - 1)$ is positive (negative), firm j proposes a small increase (decrease) in investment x_{j0}. The next morning individuals determine their optimal consumption-portfolio decisions given the revised investment plan, and the process continues. The process terminates when the investment plan proposed the previous afternoon causes no trades to be made the next morning. This process is clearly myopic, since each morning investors make no attempt to predict how investment plans will be modified in the afternoon or how market values and implicit prices will be affected. This *ex post* case is thus perhaps best thought of as a characterization of a steady-state in which individuals hold their preferred portfolios and firms continually employ the unanimously-preferred level of investment.

E. Pareto Optimality

An investment allocation must satisfy the minimal condition of constrained Pareto optimality for it to be judged efficient,[19] since if a reallocation through the available market instruments could make at least one individual better off without making any individual worse off, that reallocation would be socially desirable. Two forms of constrained Pareto optimality can be defined corresponding to whether or not the competitivity assumption is made. First, a "competitive" or *ex ante* Pareto optimal allocation is defined as one corresponding to the case in which market value forecasts are made in accord with the competitivity assumption. Diamond demonstrated that the allocation unanimously supported by initial shareholders is a competitive Pareto optimum, and Leland (1973) and Forsythe (1975) have shown that the competitivity assumption and spanning are necessary conditions for a competitive Pareto optimum given general preferences and expectations. Second, an "*ex post*" Pareto optimal allocation is defined as one corresponding to the case in which the initial endowments of shares are optimal, i.e., $\hat{\alpha}_{ij}=\bar{\alpha}_{ij}$ for all i and j,given the equilibrium investment allocation. The allocation unanimously preferred by final shareholders constitutes an *ex post* Pareto optimum as indicated by Leland (1974). Pareto optimality as considered by Diamond, Radner, and Leland (1973) is "competitive," while that considered by Ekern and Wilson, Leland (1974), Ekern, and Nielsen is "*ex post*."

Both the competitive and the myopic processes lead to an equilibrium that is in the best interests of shareholders, but it is important to distinguish between initial (or *ex ante*) and final (or *ex post*) shareholders. The competitive process acts in the best interests of initial shareholders, since no trades occur until the equilibrium production plans are announced, and if the myopic process involves only hypothetical trades, the best interests of initial shareholders are also served. If real trades in securities are made at each step in the myopic process, however, the best interests of final shareholders are served.[20]

II. Value Maximization in a Mean-Variance Model

Stiglitz, Jensen and Long, and Long (1972) have shown that the value-maximizing allocations in a mean-variance model with homogeneous expectations is not a competitive Pareto optimum, and Leland and Ekern and Wilson show that when there is spanning all *ex post* shareholders prefer that the firm not maximize its value. This section analyzes this value-maximizing investment allocation and compares it with the unanimously-supported allocation characterized in the previous section.[21] In order to make this comparison, the equilibrium conditions for the capital asset pricing model are derived as a special case of the model in Section I.

The mean-variance model developed by Sharpe (1964), Lintner (1965a)(1965b), and Mossin (1966)(1969) expresses the expected utility of an individual in terms of the mean μ_i and the variance σ_i^2 of the portfolio return R_i as

$$W_i(c_{i0},\mu_i,\sigma_i^2) \equiv \sum_s g_{is}U_{is}(c_{i0},c_{is}),$$

where W_i is a mean-variance utility function. The case to be considered is that in which $x_{js} = \gamma_{js} f_j(x_{j0})$, $s = 1,\ldots,S$, so [22]

$$\mu_i = \sum_j \alpha_{ij}\bar{\gamma}_j f_j(x_{j0})$$

$$\sigma_i^2 = \sum_j \sum_k \sigma_{jk} f_j(x_{j0}) f_k(x_{k0}) \alpha_{ij}\alpha_{ik},$$

where with homogeneous expectations $\bar{\gamma}_j$ is the expected yield for firm j and σ_{jk} is the covariance of the yields for firms j and k.

The market valuation conditions corresponding to (2) may be written letting $x_s = \sum_k x_{ks}$, $s=1,\ldots,S$, and $\text{cov}(\gamma_{js},x_s) = \sum_k \sigma_{jk} f_k(x_{k0})$, as[23]

$$[\bar{\gamma}_j - \eta \,\text{cov}(\gamma_{js},x_s)]\, f_j(x_{j0}) = (1+r)V_j,\ j=1,\ldots,J-1, \tag{13}$$

$$f_J(x_{J0})/V_J = \lambda_i/\frac{\partial W_i}{\partial \mu_i} \equiv 1+r, \tag{14}$$

where $\eta = 1/\sum_i (1/2\eta_i)$ is the harmonic mean of the individuals' marginal rates of substitution $\eta_i = -(\partial W_i/\partial\sigma_i^2)/(\partial W_i/\partial\mu_i)$ of the variance for the mean of the return. Comparison of (13) with the portfolio optimality condition for individual i indicates that $\hat{\alpha}_{ij} = \eta/2\eta_i$ for all $j \leq J-1$, so the individual holds the same percentage of all risky securities, which is the familiar strong separation property of a mean-variance model with homogeneous expectations.

The expression in (13) can be rewritten by dividing by V_j to obtain the more familiar form

$$E(\tilde{R}_j) - (1+r) = \eta^* \text{cov}(\tilde{R}_j,\tilde{R}_M), \tag{15}$$

where $E(\tilde{R}_j) = \bar{\gamma}_j f_j(x_{j0})/V_j$, $\eta^* = \eta \sum_k V_k$, and $\text{cov}(\tilde{R}_j,\tilde{R}_M)$ is the covariance between the return on the j^{th} firm and the return $R_{ms} = \sum_k \gamma_{ks} f_k(x_{k0})/\sum_k V_k$

on the market portfolio. The form in (13) is more useful however for evaluating the investment decision and will be employed in investigating the efficiency of the value-maximizing investment allocation.

Value maximization as employed by the above-mentioned authors involves differentiating (13), with η and r held constant, to obtain

$$(1+r)\,\frac{\partial V_j}{\partial x_{j0}} = [\bar{\gamma}_j - \eta\,\mathrm{cov}(\gamma_{js},x_s) - \eta\,\mathrm{cov}(\gamma_{js},x_{js})]f_j'(x_{j0}), \quad (16)$$

which may be interpreted as the mean-variance market forecast of the change in the value of the firm. Dividing (13) by $f_j(x_{j0})$, substituting into (16), and equating $\partial V_j/\partial x_{j0}$ to one, yields

$$[V_j^o/f_j(x_{j0}^o) - \eta^o\mathrm{cov}(\gamma_{js},x_{js}^o)/(1+r^o)]f_j'(x_{j0}^o) = 1, \quad (17)$$

where x_{j0}^o is the value-maximizing investment level and V_j^o, η^o, and r^o are the corresponding market parameters.

This expression differs from the condition in (11) characterizing the unanimously- preferred investment level by half the marginal variance $\eta^o\mathrm{cov}(\gamma_{js},x_{js}^o)f_j'(x_{j0}^o)/(1+r^o)$ of the firm's output. To investigate whether the value-maximizing allocation characterized by (17) is Pareto optimal, substitute (17) into the condition equivalent to (4) to obtain[24]

$$\frac{\partial W_i}{\partial x_{j0}} = \frac{\partial E_i U_i^o}{\partial x_{j0}} = \lambda_i[\hat{\alpha}_{ij}\eta^o\mathrm{cov}(\gamma_{js},x_{js}^o)f_j(x_{j0}^o)/(1+r^o) + \sum_{k\neq j}(\alpha_{ik}-\hat{\alpha}_{ik})\frac{\partial V_k^o}{\partial x_{j0}}], \quad (18)$$

where from (13) with r^o held constant

$$(1+r^o)\,\frac{\partial V_k^o}{\partial x_{j0}} = -\,\eta^o\,\mathrm{cov}(\gamma_{js},x_{ks}^o)f_j'(x_{j0}^o),\ k=1,\ldots,J,\ k\neq j.$$

This latter condition indicates that the investment in the j^{th} firm affects the value of the k^{th} firm through the covariance between their returns. Even if the condition in (18) is evaluated _ex post_ ($\bar{\alpha}_{ik} = \hat{\alpha}_{ik}$) or if $\sigma_{jk} = 0$ for $k \neq j$, this expression is not zero, and hence, the value-maximizing allocation is neither a competitive nor an _ex post_ Pareto optimum. In either of these cases

$$\frac{\partial E_i U_i^o}{\partial x_{j0}} = \lambda_i \hat{\alpha}_{ij} \eta^o \operatorname{cov}(\gamma_{js}, x_{js}^o) f_j(x_{j0}^o)/(1 + r^o). \tag{19}$$

Consequently, if the firm proposed the investment level x_{j0}^o, all initial shareholders, who will also be final shareholders (since $\hat{\alpha}_{ij} = \eta^o/(2\eta_i^o) > 0$), would vote in favor of an increase in investment for the firm. The value maximization criterion thus would be unanimously rejected by the final shareholders. The analysis of Stiglitz, Jensen and Long, Ekern and Wilson, and Leland thus should not be interpreted as demonstrating the inefficiency of securities markets but instead as indicating that the value maximization objective is not appropriate in an _ex post_ analysis, as first observed by Wilson (1972).

The Pareto inefficiency of the value-maximizing allocation results because the mean-variance model has the special property that a consumer can determine exactly how his implicit prices, and those of all other individuals, will change as the investment in a firm is changed. Comparing (13) with (2) for the case of homogeneous expectations and $x_{j0} = \gamma_{js} f_j(x_{j0})$ indicates that

$$p_{is} = g_s[1 - \eta \sum_k (\gamma_{ks} - \bar{\gamma}_k) \; f_k(x_{k0})]/(1 + r), \tag{20}$$

so all individuals have the same implicit prices as in a complete market. Differentiation of the implicit price p_{is}, again under the assumption that η and r are held constant, yields [25]

$$\frac{\partial p_{is}}{\partial x_{j0}} = - \eta g_s(\gamma_{js} - \overline{\gamma}_j) f_j'(x_{j0})/(1+r). \tag{21}$$

Multiplying by $\gamma_{js} f_j(x_{js})$ and summing over s yields

$$\sum_s \frac{\partial p_{is}}{\partial x_{j0}} \gamma_{js} f_j(x_{j0}) = - \eta \ \mathrm{cov}(\gamma_{js}, x_{js}) f_j'(x_{j0})/(1 + r), \tag{22}$$

which is the first term in the forecast condition in (5) and is the negative of the term representing the Pareto inefficiency in (19).[26]

As opposed to the situation in Sections I-B, the individual in a mean-variance model with homogeneous expectations can explicitly determine how his implicit prices, and thus those of all other individuals, will be affected by a change in investment. Value maximization thus assumes that individuals behave strategically by taking into account this effect in making the value-maximization forecast in (16).[27,28] This is possible because of the special structure of the mean-variance model that results both from the homogeneous expectations assumption and from the ability to represent by a single market observable parameter η the market aggregate of consumers' marginal rates of substitution of the variance of return for the mean return.

The divergence between the value-maximizing investment level and the investment that is in the best interests of shareholders can be illustrated by considering the "market line" relationship in (15). The competitivity assumption implies that if the scale of the firm is increased by a factor $(1+\delta)$ then (15) is

$$E(\tilde{R}_j)(1+\delta) - (1+r)(1+\delta) = \eta^*(1+\delta)\mathrm{cov}(\tilde{R}_j,\tilde{R}_M),$$

so the expected return and the risk as measured by the covariance increase proportionately yielding a linear market line. The value maximization forecast in (16) implies a relationship given by

$$E(\tilde{R}_j)(1+\delta) - (1+r)(1+\delta) = \eta^*(1+\delta)\mathrm{cov}(\tilde{R}_j,\tilde{R}_M) + \eta^*\delta(1+\delta)\mathrm{var}(\tilde{R}_j),$$

which is not linear because of the anticipation of the changes in implicit prices. Linearity requires that consumers behave as if their implicit prices are unaffected by changes in investments in firms.[29]

Value maximization as in (17) does not result in a Pareto optimal allocation because, even though the value of the firm is maximized, that maximization leads to a less desirable pattern of time-one consumption than does the investment level $\hat{x}_{j0}$ resulting from the competitive process. The term in (19) represents a consumption effect associated with the individual's final shareholdings (since $\eta^o = \hat{\alpha}_{ij} 2\eta_i^o$), and indicates that the investment is less than the preferred level which reduces the amount of the commodity available for consumption. Mossin (1977) ends his book with a quotation from Marshall (1920,p.486) that indicates an analogous consideration

> "...a railway company...finds its own interests so closely connected with those of the purchasers of its services, that it gains by making some temporary sacrifice of net revenue with the purpose of increasing consumers' surplus."

Another interpretation of the Pareto inefficiency can be given by recalling that the implicit prices, when multiplied by $(1+r)$, can be interpreted as probabilities. The divergence between a Pareto efficient allocation and the actual value maximization allocation is thus seen to be due to "moral hazard" or an externality associated with the ability of a firm to affect the market probabilities.[30]

In general, moral hazard prevents a Pareto optimal resource allocation (see Helpman and Laffont (1975)).

An alternative interpretation of the absence of Pareto efficiency with the value maximization objective in a mean-variance model with homogeneous expectations is to attribute it to the presence of monopoly power on the part of the firm. This interpretation focuses on the ability of a firm to affect (implicit) prices through the level of investment it receives from its shareholders.[31] As will be indicated in Section IV, if the number of firms offering the same yield γ_{js} is increased, the monopoly power is eliminated and the value-maximizing allocation approaches the competitive allocation.

When the individual predicts the change in the implicit prices as in (21), the individual's preferences regarding the level of investment in the firm is

$$\frac{\partial E_i U_i^o}{\partial x_{j0}} = \lambda_i [\bar{\alpha}_{ij} \{ [\bar{\gamma}_j - \eta \operatorname{cov}(\gamma_{js}, x_s)] f_j'(x_{j0}) - 1\} - \sum_k (\bar{\alpha}_{ik} - \hat{\alpha}_{ik}) \eta \operatorname{cov}(\gamma_{js}, x_{ks}) f_j'(x_{j0})] / (1 + r).$$

While this expression is zero at the competitive allocation based on the forecasts in (6) and (7) or evaluated at an <u>ex post</u> equilibrium, it is otherwise nonzero. Consequently, initial as well as the final shareholders will not be unanimous regarding the investment level unless the competitive forecasts in (6) and (7) are used or the <u>ex post</u> evaluation is made. As indicated in (19) value maximization is not preferred by the initial or final shareholders, and in general, there is no unanimously-supported objective for the firm unless all individuals have identical share endowments ($\hat{\alpha}_{ij} = \hat{\alpha}_{kj}$ for all i and k and for all j).

III. Actual Versus Perceived Value Maximization - An Example

While the value-maximizing allocation in a mean-variance model with homogeneous expectations is not Pareto optimal, it does not follow that shareholders would be better off if they behaved competitively as price takers instead of behaving strategically. The welfare comparison of two equilibria is difficult at best, so examples will be used to illustrate the possible outcomes. The examples indicate that there can be gains to strategic, value-maximizing behavior, but those gains can be eliminated if all individuals behaved strategically. It is also show that strategic behavior can be a dominant strategy for the shareholders of a firm.

As an example consider the case analyzed by Ekern and Wilson and Mayshar in which all risky firms have production functions characterized by constant returns to scale, $f_j(x_{j0}) = x_{j0}$, $j=1,\ldots,J-1$. The utility functions of individuals are assumed to be of the form

$$W_i(c_{i0},\mu_i,\sigma_i^2) = H_i(2c_{i0}^{\frac{1}{2}} + \beta_i(\mu_i - \tfrac{1}{2}\beta_i\sigma_i^2)),$$

where H_i is a strictly increasing function, and β_i is a risk aversion parameter.[32] For the case in which there are only two individuals, time-zero consumption satisfies

$$c_{10} = c_{20}(\beta_2/\beta_1)^2 \ .$$

Adding the budget constraints of the two individuals yields

$$\hat{c}_{10} + \hat{c}_{20} = y_{10} + y_{20} - \sum_j x_{j0},$$

so

$$c_{10} = \beta_2^2 A$$

$$c_{20} = \beta_1^2 A$$

where

$$A \equiv (y_{10} + y_{20} - \sum_j x_{j0})/(\beta_1^2 + \beta_2^2).$$

The value V_J of the riskless firm is then

$$V_J = \beta_1\beta_2 f_J(x_{J0})A^{\frac{1}{2}} ,$$

and the interest rate satisfies

$$1 + r = (\beta_1\beta_2)^{-1}A^{-\frac{1}{2}} .$$

Also, $\eta = (\beta_1\beta_2)/(\beta_1 + \beta_2)$, $\hat{\alpha}_{1j} = \beta_2/(\beta_1 + \beta_2)$, and $\hat{\alpha}_{2j} = \beta_1/(\beta_1 + \beta_2)$.

The following table summarizes the value maximization and the competitive allocations for the risky firms.

	Actual Value Maximization	Competitive Perceived Value Maximization
Investment	$x_{j0}^o = \dfrac{\overline{\gamma}_j - (1 + r^o)}{2\eta\sigma_{jj}}$	$\hat{x}_{j0} = \dfrac{\overline{\gamma}_j - (1 + \hat{r})}{\eta\sigma_{jj}}$
Market value	$V_j^o = \dfrac{\gamma_j^2 - (1 + r^o)^2}{4\eta\sigma_{jj}}$	$\hat{V}_j = \hat{x}_{j0}$
Net market value	$V_j^o - x_{j0}^o = \dfrac{(\gamma_j - (1 + r^o)^2}{4\eta\sigma_{jj}}$	$\hat{V} - \hat{x}_{j0} = 0$

For the same interest rate the competitive investment is twice as great as the value-maximizing investment reflecting the anticipated increase in the interest rate r that results from increased investment. The equilibrium interest rate is determined by substituting these values into the expression for $1 + r$ and solving. The remaining endogenous variables are $\hat{\alpha}_{1J}$ and $\hat{\alpha}_{2J}$ which can be determined from the budget constraints. The first example summarized in Table 1 involves two individuals and three firms and pertains to the symmetric case in which each individual initially owns half of each firm

($\overline{\alpha}_{i1} = \overline{\alpha}_{i2} = \overline{\alpha}_{i3} = .5$, $i=1,2$). The equilibrium is characterized for four cases: 1) both firms make the competitive investment, 2) both firms make the value-maximizing investment, 3) firm one makes the competitive and firm 2 the value-maximizing investment, and 4) the converse of 3). The expected utility is greatest with the competitive allocation, least with the value-maximizing allocation, and is in between these two levels for the other two cases. This example indicates that competitive behavior can be strictly Pareto superior to value maximizing behavior and suggests that the greater the number of firms that value maximize the lower is expected utility.

[INSERT TABLE 1]

The example in Table 1 however does not indicate if the shareholders of one firm could gain if they made the value-maximizing investment in their firm while other firms made the competitive investment. To investigate this issue, the case in which the first individual owns the first firm and the second individual owns the other two firms will be considered. The interest rates, investments, market values,and time 0 consumption are the same as in Table 1, and only the other endogenous variables are summarized in Table 2. The example indicates that competitive behavior is not Pareto superior to value-maximizing behavior and that if one individual operates his risky firm as a value maximizer while the other operates competitively, the former gains while the latter loses. When they both value maximize, the expected utility of each individual is lower than if he were the only one to value maximize. Furthermore, value maximization is a dominant strategy for individual 2, and hence, individual 1 will also value maximize.

[INSERT TABLE 2]

These examples indicate that there are potential gains to the owners of a firm if they value maximize while other firms make the competitive investment in a mean-variance, homogeneous expectations model, but if all firms value maximize, those gains can be replaced by losses. Consequently, the potential gains to strategic behavior depend on the behavior of other individuals and firms. Certainly, more sophisticated strategies are possible, but they will not be pursued here.

Table 1

Symmetric Individuals

Endogenous Variables	Competitive Allocation for Both Firms	Value-Maximizing Allocation for Both Firms	Firm 1 Competitive; Firm 2 Value Maximizing	Firm 1 Value Maximizing; Firm 2 Competitive
r	1	0.355	0.550	0.747
x_{10}	2	1.323	2.450	1.127
x_{20}	4	2.323	2.225	4.253
V_1	2	3.541	2.450	3.237
V_2	4	8.541	8.399	4.253
V_3	3	4.427	3.868	3.434
$c_{10}=c_{20}$	1	2.178	1.663	1.310
$\hat{\alpha}_{13}=\hat{\alpha}_{23}$	.5	.5	.5	.5
$\mu_1=\mu_2$	19	12.615	14.575	18.013
$\sigma_1^2=\sigma_2^2$	20	7.147	10.953	19.358
$W_1=W_2$	9	8.366	8.498	8.876

Exogenous data: $I = 2$, $J = 3$, $\bar{\gamma}_1 = 4$, $\bar{\gamma}_2 = 6$, $\sigma_{11} = \sigma_{22} = 4$, $y_{10} = y_{20} = 6$,

$\beta_1 = \beta_2 = .5$, $x_{30} = 4$, $f_3(x_{30}) = 6$, $\bar{\alpha}_{11} = \bar{\alpha}_{12} = \bar{\alpha}_{13} = .5$

Table 2

Individual 1 Owns Firm 1; Individual 2 Owns Firms 2 and 3

Endogenous Variables*	Competitive Allocation For Both Firms	Value-Maximizing Allocation for Both Firms	Firm 1 Competitive; Firm 2 Value Maximizing	Firm 1 Value-Maximizing; Firm 2 Competitive
$\hat{\alpha}_{13}$	2/3	0	-.281	.890
$\hat{\alpha}_{23}$	1/3	1	1.281	.110
μ_1	20	9.615	9.889	20.353
μ_2	18	15.615	19.261	15.673
W_1	8.5	6.866	6.154	10.046
W_2	7.5	9.866	10.841	7.706

Exogenous data: Same as in Table 1 except that $\bar{\alpha}_{11} = 1$, $\bar{\alpha}_{22} = 1$, $\bar{\alpha}_{23} = 1$.

*The values of r, x_{10}, x_{20}, V_1, V_2, V_3, c_{10}, c_{20}, σ_1^2, and σ_2^2 are the same as in Table 1.

IV. Restoring Pareto Optimality in the Mean-Variance Model

In Section I perceived value maximization with the competitivity assumption was shown to be consistent with the best interests of the initial shareholders of the firm, and in Section II actual value maximization in a mean-variance model was shown to lead to investment plans that are not in the best interests of those shareholders. Since the competitivity condition is a necessary condition for Pareto optimality and unanimity among initial shareholders, it is necessary to restore competitivity to the mean-variance model if the inefficiency is to eliminated. The absence of optimality has, of course, not gone unnoticed, and this section is concerned with the two principal assumptions that have been utilzed to obtain the competitive investment allocation and Pareto optimality in a mean-variance model.

One justification of competitivity is the smallness argument that firms are small enough that individuals behave as if the investment allocation to any one firm does not alter their (implicit) prices. For instance, in the example of the previous section suppose that there are N firms that have the identical yield γ_{js}. The market value of one of these firms (j) is given by

$$\bar{\gamma}_j x_{j0} - \eta\sigma_{jj} x_{j0} \sum_n x_{n0} = (1+r)V_j,$$

where the summation is over the N firms. If all firms have identical investment levels $x_{n0} = x_0$, the actual value maximizing allocation x_0^o for firm j satisfies

$$x_0^o = \frac{\bar{\gamma}_j - (1+r)}{(N+1)\eta\sigma_{jj}}.$$

The total investment Nx_0^o is

$$Nx_0^o = \frac{\bar{\gamma}_j - (1+r)}{(1+1/N)\eta\sigma_{jj}},$$

which has as its limit the competitive investment allocation. Consequently, if there are many small firms with correlated yields, the competitive allocation would be approximately realized in a mean-variance model. The competitive allocation however cannot strictly obtain unless there are no firm-specific risks as indicated by Fama (1972).[33] The degree to which firm-specific risks are present is an empirical question, but if they are present, Pareto efficiency can result only as an approximation.

Rubinstein (1978), and from a slightly different perspective Svensson (1977), have recently argued that even though an individual firm may have a small impact on implicit prices (the interest rate in their certainty models) unanimity among initial shareholders will not result. This is evident from (5) because there is in general nothing to guarantee that all individuals evaluate the changes in their implicit prices in a similar manner. Rubinstein's conclusion is obtained because he assumes that the number of consumers and firms increase proportionately and hence the "per capita" effect on the interest rate does not go to zero as the number of firms (and consumers) increase. If however the number of firms increase and the number of consumers remain fixed, the effect of an individual firm on the interest rate becomes approximately zero,since production is then approximately characterized by constant returns to scale.[34] Unanimity among initial shareholders is approximately satisfied in that case. Casual empiricism however indicates that all firms are not small and do not receive a negligible share of the available investment funds.

Fama and Laffer have proposed a theory to deal with this issue by specifically assuming that while there may be only a few firms in an industry their behavior will in fact produce the perfectly competitive outcome. The objective of their theory is to give conditions that are sufficient to yield Pareto optimality and the coincidence of the objectives of perceived and actual value maximization. More specifically, they propose a concept of "perfect competition" that is intended to be logically consistent with the case in which implicit prices and hence η and $(1 + r)$, are actually invariant to any revision in the investment allocation. For the investment allocation to have no effect on the implicit prices, it is clear that it is necessary that the total supply of the commodity in any state must be fixed.[35]

In addition to fixed supplies in each state, Fama and Laffer (also see Fama(1978)) assume that a "reaction principle" governs the behavior of firms. Fama (1972, p.514) states that "This 'reaction principle' of offsetting output changes by other firms in response to output changes by an individual firm is the fundamental mechanism whereby the decisions of an individual firm have no effect on price or on industry output."[36] In the context of the model developed here, the reaction principle implies that $\sum_k \gamma_{ks} f_k(x_{k0})$ is independent, for each s, of the investment allocation $(x_{10},\ldots,x_{J0})$. Then, if firm j increases its input, the reaction must satisfy

$$\gamma_{js} f_j'(x_{j0})dx_{j0} + \sum_{k \neq j} \gamma_{ks} f_k'(x_{k0})dx_{k0} = 0, \; s=1,\ldots,S. \qquad (23)$$

If the yield vectors (γ_j) are linearly independent, this system of equations has a unique solution $f_j'(x_{j0})dx_{j0} = 0$ for all j, so the assumption that total output remains fixed implies that no firm can change its output. If the firms are in the same "industry" in the sense that for each s, $\gamma_{ns} = \gamma_s$, n=1,...,N, so that $\sigma_{nk} = \sigma^2$ and $\overline{\gamma}_n = \overline{\gamma}$ for all n and k, then differentiating V_n in (13) with respect to x_{n0} and using the reaction principle (23) within this industry yields the expression in (11) obtained with the competitivity assumption. Consequently, the reaction principle and perfectly correlated returns are sufficient assumptions to yield the competitive result.[37]

To determine if the reaction principle can serve as a basis for a theory of investment, three questions must be answered. 1) Can the aggregate outputs be assumed to be fixed in each state?; 2) Does the reaction principle form a plausible description how firms behave?; and 3) Does the reaction principle yield a theory of investment allocation among firms? The answer to each of these questions appears to be in the negative. First, since individuals can allocate their endowments between time 0 consumption and investment in firms, the total investment $\sum_j x_{j0}$ committed to firms is an endogenous rather than an exogenous variable. A theory of investment consequently should explain how the aggregate amount of investment, as well as its distribution among firms, is determined. As Brenner and Subrahmanyam indicate, the reaction principle is only relevant for intra-equilibrium analysis corresponding to an exogenous level of aggregate investment and cannot serve as a basis for a theory of aggregate investment.

Second, Fama and Laffer argue that their reaction principle is implied at a general equilibrium when there are two or more firms in an industry,[38,39] but their argument seems to pertain to the stability of an equilibrium in the sense that at an equilibrium a change in the output of a firm does not lead to a different equilibrium. If one firm proposes to increase its investment and output, other firms will have no reason to propose to reduce theirs when the implicit prices are understood not to change. Consequently, the firm that proposed to increase its output would, with free entry and exit, receive no investment funds, leave the industry and be replaced by another firm with the original investment level.[40]

Yawitz (1977) has interpreted the result that no firm can alter its output at an equilibrium as implying that the reaction principle represents "the decision of one firm to acquire another firm in the industry (p.1148)." That is, no firm can increase its output by simply increasing its level of investment, since no other firm would have any reason to reduce its output. The only way that a firm can increase its output is to purchase the output of another firm. He concludes that "the implication [of the reaction principle] for growth per se is unsatisfactory, since the general process of industry growth is precluded (pp.1148-9)."[41]

Third, Fama and Laffer base their reaction principle on the assumption of constant returns to scale for all firms, but in that case the theory of the firm and investment allocation is vacuous. To see this in the context of the model presented here, if $f_{js}(x_{j0})$ exhibits constant returns to scale, then $f_{js}(x_{j0}) = \gamma_{js}x_{j0}$, and with free entry, equal access to technologies, and perfectly mobile resources, no rents can be earned (i.e., $V_j = x_{j0}$). Hence, all individuals are indifferent to the investment allocation among firms. Consequently, there is no theory of investment allocation.[42] This is the same result as the Modigliani-Miller (1958) theorem that all individuals are indifferent to the investment level as well as to the financing of that investment (see Baron (1976)).

Neither the smallness nor the reaction principle, fixed supply assumptions have much descriptive power and when strictly valid have no predictive power. The next section briefly considers certain aspects of current research that may lead to progress in the search for a general theory of investment for shareholder-owned firms.

V. Aspects of the Search for an Investment Policy

The above analysis suggests that a general theory of investment policy for firms whose shares are traded in financial markets must recognize that firms are not negligible in size and that the expectations of individual investors regarding effects of investment reallocations on implicit prices must be dealt with explicitly. At one extreme is the mean-variance model with homogeneous expectations that permits an individual to conclude that his implicit prices are the same as those of all other individuals and to predict perfectly the effect of a change in investment on those implicit prices. At the other extreme is the case in which no restrictive assumptions are made regarding preferences or expectations. Thus, while individuals may realize that a revision in the investment allocation may affect their implicit prices, they have no way to determine how those prices will change and consequently, use their current implicit prices to evaluate proposed reallocations. In between these extremes is the case in which individuals form expectations regarding the implicit prices that would result if a proposed investment allocation were implemented. Unfortunately, the theory of expectations formation is not yet well developed. Furthermore, theoretical problems may well be present, since rational expectations equilibria do not exist in general unless individuals receive information in addition to the prices they are able to observe.

A further complication arises when the investment in one firm affects the return or yield on another firm. The study of these interactions is incomplete and involves serious problems. Equilibria in models with monopolistically competitive firms do not exist in general, and as Roberts and Sonnenschein (1977) indicate, serious difficulties arise in attempting to model monopolistically competitive behavior in an Arrow-Debreu economy.

Even if theories incorporating expectations formation and noncompetitive behavior are developed, there will remain the issue of determining an appropriate objective for a shareholder-owned firm. Unanimity and Pareto efficiency, even in a restricted sense, are unlikely to result in an incomplete securities market without competitivity, so there will in general be no unanimously-supported objective for guiding the allocation of investment. The threat of take-over bids is not a simple solution as Hart (1977) has indicated, because a take-over bid equilibrium need not be Pareto optimal when price discrimination is not permitted "since individual firms are too large relative to the aggregate economy. (p.82)" Furthermore, value maximization does not result unless there is multiplicative uncertainty as in Diamond's model. The search for an objective for firms has only recently begun, but the appropriate guide for this search should clearly center on the shareholders' right to choose among alternative levels and allocations of investment.

When unanimity is not present, the investment in a firm has the properties of a (local) public good. The impossibility theorems of social choice theory however do not leave much hope for the use of voting mechanisms to generate a non-dictatorial allocation mechanism.[43] Since social choice theory is not likely to yield an objective for the firm, shareholders' decisions regarding investment policy may perhaps best be viewed as a cooperative game. Hart (1977) has taken this approach in the study of take-over bid equilibria, and Drèze (1974) uses a similar approach and achieves a characterization of an optimal investment policy when side payments can be made. Hart however demonstrates that when the payment must be the same for all shareholders, a take-over bid equilibrium may not be a constrained Pareto optimum.

When advances are made in rational expectations, monopolistically competitive, and cooperative game models in a static context, the remaining task will be to extend the models to a dynamic framework. Until that time the theory of investment allocation through securities markets must remain based on assumptions regarding investment behavior. A researcher who wishes to use a mean-variance model because of its interpretational value or because of the ability to test empirically the predictions of that model must choose among the currently available alternatives. Since the mean-variance model itself is perhaps best thought of as an analytically tractable approximation to a more general model, basing predictions of investment behavior on the accurate assessment of changes in implicit prices seems unwarranted. From this viewpoint the use of the Pareto optimal investment allocation characterized in (10) and (11) is the more appealing. The predictions of the model then must be interpreted as arising from competitive price-taking behavior and perceived value maximization or from a steady-state (*ex post*) situation in which shareholders already hold the optimal portfolios given the unanimously-supported investment allocation.

Footnotes

*Morrison Professor of Decision Sciences, Graduate School of Management, Northwestern University. The author would like to thank Michael Adler, John Roberts, Robert A. Taggart, Jr., and Gustaaf Van Herck for their stimulating comments on the issues considered herein.

1. Diamond refers to the vector $(\gamma_{j1},\ldots,\gamma_{jS})$ as a "composite good" representing quantities of the commodity "contingent" on states.
2. The use of implicit prices facilitates the analysis of the difference between the value-maximizing investment allocation and the unanimously-preferred allocation as will be indicated in Section II.
3. An equilibrium in this economy involves both an equilibrium in the securities market corresponding to an investment allocation across firms and an equilibrium with respect to the allocation of consumers' endowments between present consumption and investment. An equilibrium allocation is a feasible consumption-portfolio plan $\hat{z}_i$ satisfying (1) and (2) for each individual, an investment plan x_{j0} for each j, and market values V_j, j=1,...,J, such that $\hat{z}_i$ maximizes the expected utility of the individual subject to his budget constraint, x_{j0} is the "preferred" investment level to be defined below, and such that the securities market clears, $\sum_i \hat{\alpha}_{ij} = 1$, j=1,...,J, and the market for the commodity at time 0 clears, $\sum_i c_{i0} + \sum_j x_{j0} = \sum_i y_{i0}$. Such an equilibrium is assumed to exist, although as Drèze (1974) indicates care must be taken in demonstrating existence, since the term $\alpha_{ij}x_{j0}$ is a bilinear function and does not yield a convex feasible set.

4. See Leland (1974), Baron (1976), Nielsen (1976), and Grossman and Stiglitz (1976)(1977), for derivations of this expression.

5. Grossman and Stiglitz (1976) define competitivity as follows:

> "Competitivity means that each consumer believes that if the output of any firm increases by 6% in each and every state of nature, then the value of the firm increases by 6%. (p.397)"

Diamond states

> "If, for example, the firm is considering doubling its input, this would be calculated as doubling the value of input payments (since the firm is a price taker). (p.768)"

Krouse refers to the competitivity assumption as "price taking in supply" and defines it as follows:

> "When shareholders as owners direct each firm to act as if changes in its supply decisions (by the choice of a project) do not affect the aggregate supply of claims in any state, and hence implicit claims prices, then there is said to be price-taking in supply. (p.767)"

Le Roy uses this approach and also interprets the competitivity assumption as one of price-taking. Merton and Subrahmanyam (1974) consider this assumption and argue that the "'price-taker' assumption is equivalent to taking the aggregate amount of investment in a project as fixed (p.146)." This interpretation is not employed in the model considered here because the aggregate investment provided to firms by shareholders is not fixed but instead is determined endogenously in the model. Instead, the price-taking assumption is treated as a behavioral postulate.

6. The competitivity assumption has a number of other important implications for financial policy. For example, a merger of two firms that will not affect their production functions is perceived to result in a value for the merged firm equal to the sum of the values of the individual firms as is evident from the valuation condition in (2).

7. This does not indicate unanimity regarding the change in investment, however, since all initial shareholders may not evaluate $\partial V_j^i/\partial x_{j0}$ similarly.

8. The spanning condition can also be stated in terms of the production sets of firms as in Radner. The production set is the set of all vectors $(x_{j0},x_{j1},\ldots,x_{jS})$ that are technologically feasible, and if that set is contained in the space $T \times L$, where L is the real line, the spanning condition is satisfied.

9. Whether or not spanning is present in actual markets and can be relied upon for the evaluation of investment allocations is an empirical question that remains open. If spanning is not present in actual markets, there exists no unambiguous objective for the firm, and even the possibility of take-over bids does not provide Pareto optimal investment allocations as indicated by Hart (1977).

10. Unanimity can also be established if preferences and expectations are restricted. For example, if expectations are identical and utility functions are restricted to the linear risk tolerance (HARA) class, unanimity results as demonstrated by Wilson (1968) and Rubinstein (1974).

11. The expression "perceived value maximization" is used because (6) and (7) are based on the assumption that individuals act as price takers with respect to their current implicit prices.

12. Milne (1974) presents a discussion of the complete market and the Diamond models, and Baron (1978) provides a comparison between complete and incomplete market models, as well as an interpretation of the unanimity results involving initial and final shareholders.

13. In an incomplete market shareholders will not be unanimous in supporting the creation of new securities by firms, since a security that changes the opportunity set of individuals will not be evaluated identically by different individuals. An individual, however, may have an incentive to create new securities, but the difficulty caused by the possibility of personal bankruptcy makes this issue quite complicated.

14. The difference between a complete securities market and an incomplete market with spanning is a delicate one, since, for example, Grossman and Stiglitz (1976) demonstrate that if a firm issues debt and this security does not alter the space of returns available in the securities market, that market is in fact complete. It is well known that the Modigliani-Miller theorem holds in this case, so complete markets is a necessary condition for all initial shareholders to be indifferent to the financing of a firm.

15. If trading is permitted after the investment decisions have been made, a stronger competitivity assumption is required as indicated by Grossman and Stiglitz (1976).

16. Hart (1975) has shown that this result cannot be extended to an economy with more than one commodity, and Grossman (1977) has provided a characterization of optimality for this case.

17. Mossin (1977, pp.33-6) discusses such a process in more detail.

18. With this approach it may be that $\partial V_k/\partial x_{j0} \neq 0$ for $k \neq j$.

19. The notion of a constrained Pareto optimum was introduced by Diamond to refer to the case in which consumption patterns are constrained to be those that can be generated through share purchases. Also, the Pareto efficiency considered here is defined in terms of expected utility. Starr (1973) has investigated conditions under which such an optimum in a complete market is also a Pareto optimum conditional on the state s that occurs.

20. Merton and Subrahmanyam present a non-tatonnement process and similarly conclude that "Because of the non-tatonnement nature of the approach to equilibrium, there are wealth distributional effects if trades actually take place at the 'false' interim prices. (p.155)"

21. The analysis pertains both to the case of complete and incomplete markets, although no special distinction will be made.

22. The random yield γ_j may have a normal distribution, or if $U_{is} = U_i$ is quadratic in c_{is}, any distribution with a finite mean and variance. Preference structures giving rise to a mean-variance utility function are considered by Chipman (1973), Baron (1977), and Fishburn (1978).

23. The portfolio optimality conditions corresponding to (1) and (2) for the individual are

$$\frac{\partial W_i}{\partial c_{i0}} - \lambda_i = 0$$

$$\left(\frac{\partial W_i}{\partial \mu_i} \bar{\gamma}_j + 2 \frac{\partial W_i}{\partial \sigma_i^2} \sum_k \sigma_{jk} f_k(x_{k0}) \hat{\alpha}_{ik}\right) f_j(x_{j0}) - \lambda_i V_j = 0, j=1,\ldots,J-1.$$

Dividing by $\partial W_i/\partial \mu_i$, letting $\eta_i \equiv -(\partial W_i/\partial \sigma_i^2)/(\partial W_i/\partial \mu_i)$, and letting $1 + r \equiv \lambda_i/(\partial W_i/\partial \mu_i) = f_J(x_{J0})/V_J$ yields

$$(\bar{\gamma}_j - 2\eta_i \sum_k \sigma_{jk} f_k(x_{k0}) \hat{\alpha}_{ik}) f_j(x_{j0}) - (1+r)V_j = 0, \ j=1,\ldots,J-1.$$

Then, dividing by η_i, summing over i, and defining $\eta \equiv 1/\sum_i (1/2\eta_i)$ and $\mathrm{cov}(\gamma_{js}, x_s) = \sum_k \sigma_{jk} f_k(x_{k0})$ yields (13).

24. The expression in (18) is derived as in (4)

$$\frac{\partial W_i}{\partial x_{j0}} = \frac{\partial W_i}{\partial \mu_i} \hat{\alpha}_{ij} \bar{\gamma}_j f_j'(x_{j0}) + 2\frac{\partial W_i}{\partial \sigma_j^2} (\sum_k \sigma_{jk} f_k(x_{k0}) \hat{\alpha}_{ik}) \hat{\alpha}_{ij} f_j'(x_{j0})$$

$$+ \lambda_i \left(\sum_k (\bar{\alpha}_{ik} - \hat{\alpha}_{ik}) \frac{\partial V_k^o}{\partial x_{j0}} - \bar{\alpha}_{ij}\right).$$

Multiplying and dividing the first-two terms by λ_i,

substituting $\eta_i^o = -(\partial W_i/\partial \sigma_i^2)/(\partial W_i/\partial \mu_i)$, $2\hat{\alpha}_{ik}\eta_i^o = \eta^o$, and $1/(1+r^o) = (\partial W_i/\partial \mu_i)/\lambda_i$, yields

$$\frac{\partial W_i}{\partial x_{j0}} = \lambda_i [(\bar{\gamma}_j - \eta^o \mathrm{cov}(\gamma_{js}, x_s)) \hat{\alpha}_{ij} f_j'(x_{j0})/(1+r^o)$$

$$+ \sum_k (\bar{\alpha}_{ik} - \hat{\alpha}_{ik}) \frac{\partial V_k^o}{\partial x_{j0}} - \bar{\alpha}_{ij})]$$

At x_{j0}^o, the condition in (17) implies that $\bar{\alpha}_{ij}(\frac{\partial V_j^o}{\partial x_{j0}} - 1) = 0$ and substituting $\frac{\partial V_j^o}{\partial x_{j0}}$ from (16) yields (18).

25. If the dependence of r^o on x_{j0} is taken into account, the terms $\frac{\partial(1+r^o)}{\partial x_{j0}} p_{is}(1+r^o)$ and $\frac{\partial V_J^o}{\partial X_{jo}} V_j^o/f_j(x_{j0})$ are added to the right sides of (21) and (22) respectively. Under this assumption the changes in the market values from (13) are

$$\frac{\partial(1+r^o)}{\partial x_{j0}} V_k + (1+r) \frac{\partial V_k}{\partial x_{j0}} = -\eta^o \operatorname{cov}(\gamma_{js}, x_{ks}) f_j'(x_{j0}), k \neq j$$

and $\frac{\partial(1+r)}{\partial x_{j0}} V_k + (1+r) \frac{\partial V_{kj}}{\partial x_{j0}}$ equals the right side of (16). For firm J, $\frac{\partial(1+r)}{\partial x_{j0}} = (1+r) \frac{\partial V_J}{\partial x_{J0}}$.

26. Greenberg, Marshall, and Yawitz (1978) have studied the relationship between risk and return in a variety of models, and because they value-maximize in a mean-variance model, they include terms analogous to (22).

27. Svensson (1977) indicates that the same issue arises in an intertemporal certainty model when the marginal rates of substitution are assumed to depend on the actions of a firm. Rubinstein (1978) provides an alternative view of the change in implicit prices. In the context of a certainty model he argues that even though an individual firm may have a slight effect on the interest rate (the inverse of the sum of the implicit prices), the consumption effect that the change in

interest rate generates "may dominate the firm's impact on the value of its own shares (which comprise a small portion of the consumer's portfolio). (p.282)" When individuals behave as if a firm can influence the (sum of the) implicit prices, the resulting investment allocation is Pareto inefficient. Rubinstein argues in addition that assuming that firms are "small relative to the rest of the economy" is not sufficient to justify the assumption that implicit prices will not be affected by the firm's investment level. This argument will be considered in Section IV.

28. In the context of ex post analysis Leland (1974) demonstrates that the unanimously preferred production level does not maximize the actual value of the firm. The proof of that result essentially involves implicit prices that depend on a firm's production.

29. An informative illustration of this issue is that provided by the paper by Mossin (1969) and the subsequent comment by Kumar (1972) and Mossin's response (1972). In his original paper Mossin implicitly made the competitivity assumption by assuming that if a firm increased its scale by a factor $(1+\delta)$, then its expected return would increase in the same proportion. Kumar pointed out, however, that when the covariance is actually computed, the relationship should include another term that cor-

responds to the term in (19) that represents the Pareto inefficiency. Mossin in his reply correctly observed that this last term must be assumed to be small to obtain a Pareto optimal allocation. He states, "Therefore, one way of 'saving' the M-M investment theory is to treat this change as a minor one and second-order effect by assuming that the number of firms is so large as to make the changes in marginal utilities negligible." The marginal utilities referred to are the implicit prices. In his later book Mossin (1973, p.129) utilizes the value-maximizing criterion rather than the competitivity assumption.

30. Stiglitz states "The reason for this misallocation is that the covariance of the firms with one another acts very much like an externality. A change in the level of investment of one firm affects the value of all other firms; although it may affect the value of any individual firm very little, when added up over all firms, the effect is nonnegligible. (p.46)"

31. Mayshar provides a good discussion of this view.

32. This utility function can be derived from a utility function

$$U_i(c_o,c_i) = -\exp(-2c_{i0}^{\frac{1}{2}})\exp(-\beta_i c_i)$$

with c_i (the γ_j) normally distributed.

33. As indicated by Merton and Subrahmanyam, Fama's result is inconsistent with his assumptions, but the case he analyzes does illustrate the inefficiency that results when there are firm-specific risks.

34. Specifically, he assumes that the production function is strictly increasing and strictly concave, so as each firm receives a smaller and smaller share of the available investment, production approximates that with constant returns to scale.

35. As Nielsen (p.589) states "The interpretation of this definition of a perfect capital market is most clear in the context of a state of the world model, since it simply requires that aggregate output in each state of the world is unchanged. If there exist separate claims on output for each state of the world, the prices of these claims have to be unchanged. And if there exist[s] a stock market, the price of a share with a given state dependent payoff has to remain unchanged."

36. Krouse (p.772) states that "the s fundamental state claim [implicit] prices depend only on the aggregate returns in the economy, and not at all on how they are made-up from the various securities."

37. Fama (1972, pp.521-528), however, does not reach the conclusion that the competitive result (that is, Pareto optimality) is obtained when his assumptions of free entry, equal access to technologies, and the reaction principle are made. This results because he assumes that the return of each firm in an industry is representable as a linear combination of a market return and a linearly independent, firm-specific risk. Fama, in his equations (24) and (25), rediscovers the term in

(19). Krouse (p.773) states "... he [Fama] applies his reaction principle to hold constant aggregate investment and not aggregate return. As the aggregate level of investment is distinct from the aggregate distribution of returns in economies where production technologies are not subject to constant returns to scale, and resulting returns are not identically distributed and perfectly correlated, it is not surprising that he finds the allocation of risk in his general competitive model to be inefficient for a mean-variance economy." Furthermore, as Merton and Subrahmanyam demonstrate, if there are constant stochastic returns, there are an infinite number of firms in any industry. Anderson (1977) has made an analogous point.

38. Their definition of perfect competition is inconsistent with the usual definition as given for example, by Knight (1965) who requires (p.190) "... negligible size in the marginal unit as a condition of effective competition,..." If production units are not negligible in size, Knight concludes that (p.192) "... they will combine and bargain as a unit; and the same incentive will urge them to keep on combining until a monopoly results." Free and costless entry would be required to eliminate this incentive and must in the limit result in firms with only a negligible size.

39. Fama and Laffer (1972)(1974) state that the reaction principle implies perfect competition, but Hori (1974) argues that their principle "does not exclude the classical Stackleberg duopoly solution...." Anderson has similarly concluded that the reaction principle is the basic assumption of the Bertrand oligopoly model. In response to Hori, Fama and Laffer (1974) reject this

type of argument on the basis that those oligopoly models do not permit entry, but this does not seem to be a sufficient defense. It is also interesting to note that the reaction principle is not inconsistent with the concept of a fixed price equilibrium as considered by Drèze (1975).

40. If firms were initially out of equilibrium, then two or more firms could simultaneously move to the minimum of their average total cost functions but this contradicts the assumption that the reaction principle holds at an equilibrium. The case of reactions by other firms is addressed by Ekern (1975) but without the assumption of fixed supplies.

41. Yawitz proposes an alternative to the reaction principle and the competitivity assumption for the case of an investment project that can be acquired by only one firm. If the project provides a return vector q_s, $s = 1,\ldots,S$, then with the competitivity assumption, the value V_q of that project will be $V_q = \sum_s p_{is}q_{is}$. With mean-variance preferences this differs from Yawitz's equation (11) by the term $-\eta\mathrm{cov}(q_s,q_s)$, which is analogous to the term in (19).

42. Fama and Laffer argue that the reaction principle is also valid when there is a finite optimal plant size (investment level) and all firms in an industry operate at multiples of that optimal size. Because this implies an indivisibility, their argument is strictly valid only if the optimal plant size is infinitesimally small in which case returns to scale are essentially constant.

43. Benninga and Muller (1978) have shown that if a firm has only one input, all individuals will have single-peaked preferences and hence the median of the investment levels preferred by individual shareholders will constitute an equilibrium under a majority rule voting procedure. Furthermore, individuals have no incentive to misrepresent their preferences. Unfortunately, single-peakedness will not in general result when there is more than one input.

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